A Renormaliazable Standard Model with a composite Higgs?

Jörg Jäckel

DESY Workshop September 2004

Holger Gies and Christof Wetterich

Phys.Rev.D69:105008,2004 (hep-ph/0312034)

Universität Heidelberg

1. Introduction

The Standard Model

+ Tested to very high precision

- Large number of parameters
- The hierarchy problem
- The triviality Problem
- Unknown origin of flavor physics
- Unknown origin of neutrino physics

The Hierarchy Problem

- We have a huge hierarchy in the standard model:

$$M_{GUT} \sim \Lambda_{UV} \gg \Lambda_{EW} \sim 10^{-13}\Lambda_{UV}$$

- The Higgs mass renormalizes quadratically

$$\underbrace{m_R^2}_{\sim 10^4\,\mathrm{GeV}^2} \sim \underbrace{m_\Lambda{}^2}_{\sim 10^{30}\left(1+\ldots 10^{-26}\right)\mathrm{GeV}^2} - \underbrace{\delta m^2}_{\sim 10^{30}(\mathrm{GeV})^2}$$

Incredible Finetuning: $1 : 10^{26}$

The Hierarchy Problem: caveat

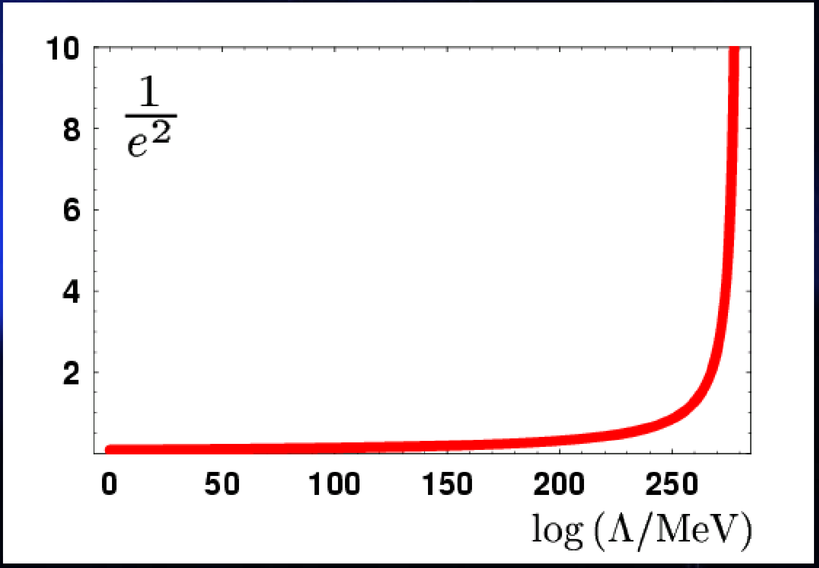
- However: It is a problem of naturalness or finetuning!
- It is not a „fundamental" problem.

The Triviality Problem

- The triviality problem appears in the U(1) and the Higgs sector of the SM
- E.g., in QED we have the well known Landau pole ➡ breakdown of perturbation theory:

$$\frac{1}{e_R^2} - \frac{1}{e_\Lambda^2} = \beta \log\left(\frac{\Lambda}{m_R}\right),$$

$$\beta = \frac{N_f}{6\pi^2}$$

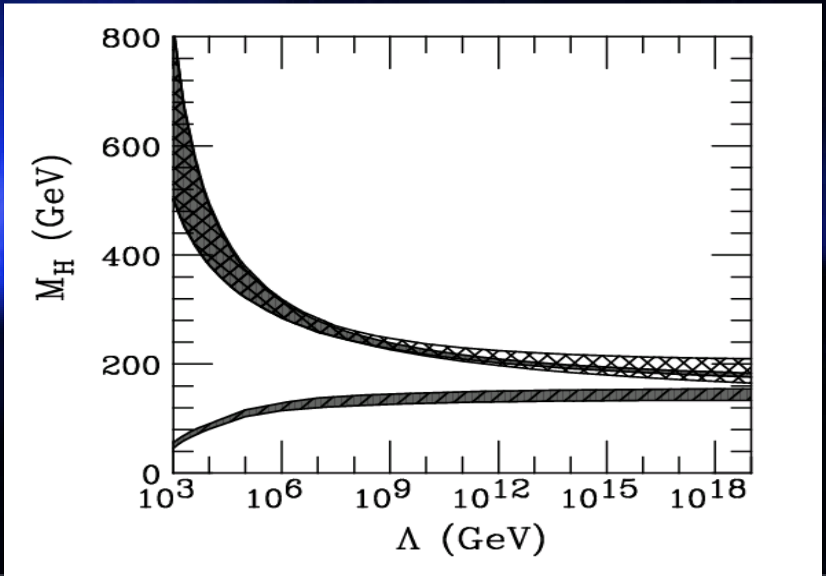


Landau 1955

The Triviality Problem

- It is a fundamental problem.
- A trivial theory must be an effective field theory with a finite UV cutoff!
- More important in the Higgs sector

→ bound on the Higgs mass

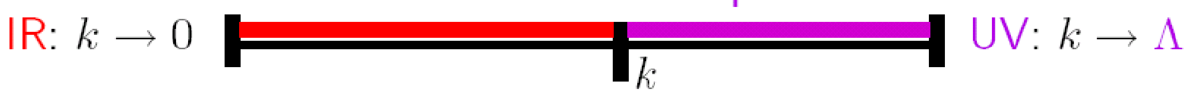


Hambye, Riesselmann, 1997

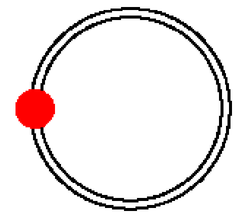
2. Renormalization Group Flow

A Flow Equation for the Effective Action

Exact RG Flow Equation

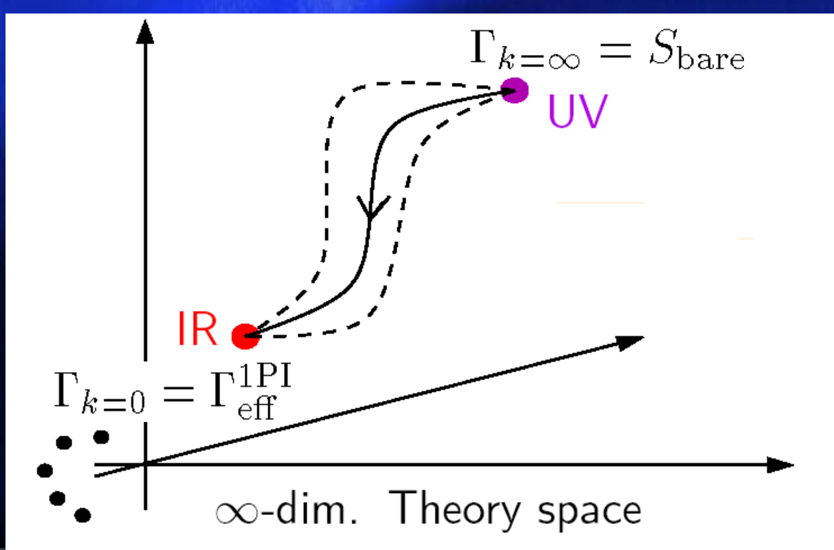


$$\partial_t \Gamma_k = \frac{1}{2}\,\mathrm{STr}\,\partial_t R_k \left(\Gamma_k^{(2)} + R_k\right)^{-1} =$$

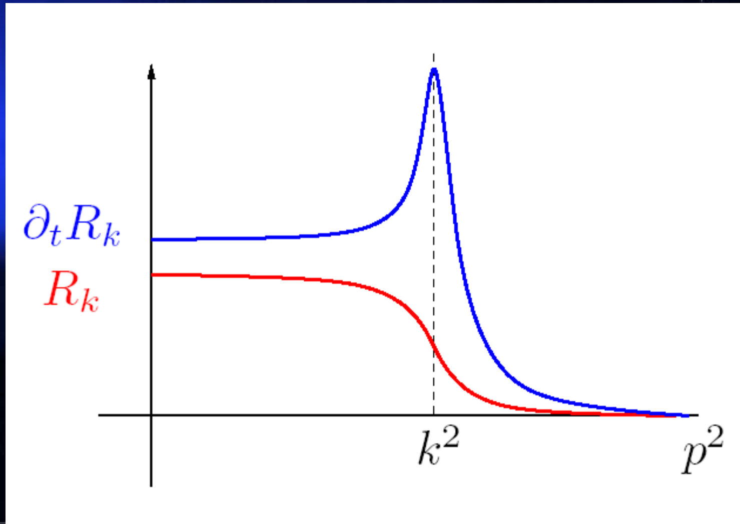


Wetterich, 1993

RG trajectory:



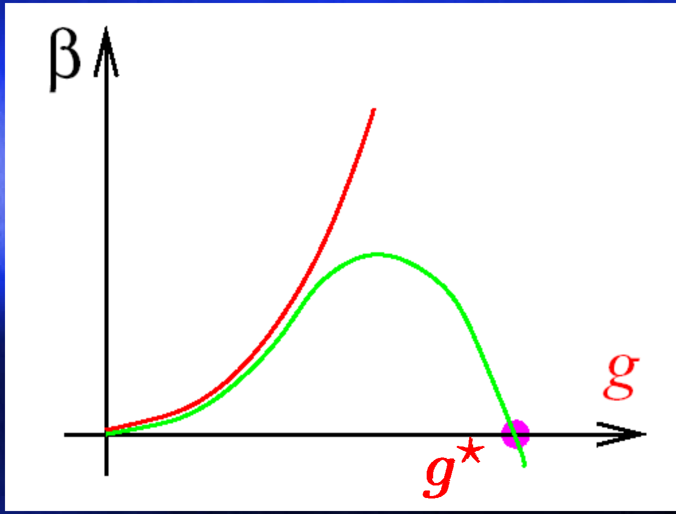
Cutoff function R_k



Non-perturbative Renormalizability

- Asymptotic safety: (non-gaussian) fixed point:

Weinberg, 1976



$$\beta_i(g_1^\star, g_2^\star, \ldots) = 0, \ \forall\, i$$

- Linearized flow near the FP

$$(g_i - g_i^\star) = \sum_I C^I V_i^I \left(\frac{k_0}{k}\right)^{\Theta_I}$$

Non-perturbative Renormalizability

- $\Theta_I > 0$ ➡ UV attractive ➡ relevant
- $\Theta_I < 0$ ➡ UV repulsive ➡ irrelevant
- $\Theta_I = 0$ ➡ it depends... ➡ marginal

- All relevant trajectories together form the critical surface
- Starting on the critical surface we end in g^* for $\Lambda \to \infty$

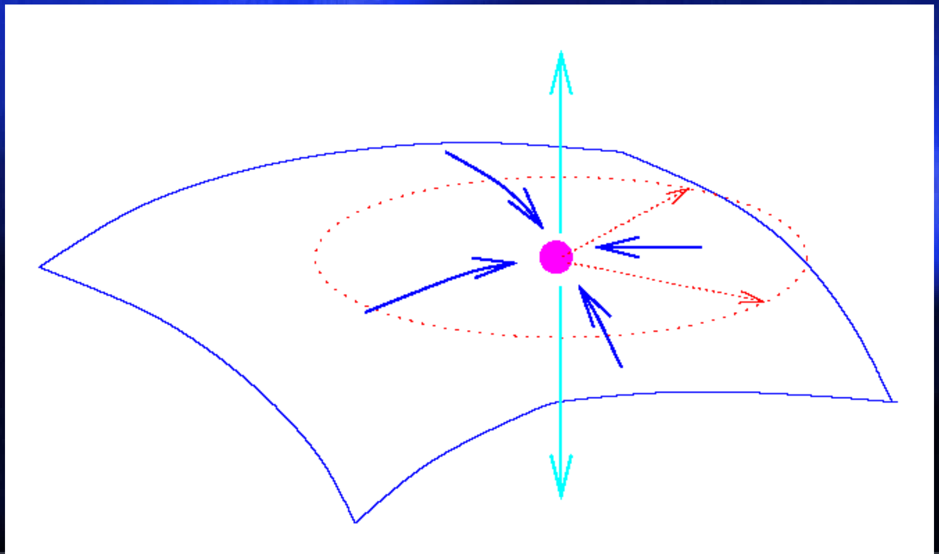
Non-perturbative Renormalizability

- The other way around: all other starting points have no finite UV-limit

  ➡ not allowed ($C^I_{\text{irrelevant}} = 0$)

- Finite dimensional critical surface

  ➡ finite number of parameters



➡ Predictive

The Hierarchy Problem Revisited

- In the RG-context: large relevant eigenvalues → „very relevant"

→ Hierarchy problem

- E.g., in the standard model (gaussian FP):

$$\partial_t \frac{m_H^2}{k^2 h^2} = -2\frac{m_H^2}{k^2 h^2} + \frac{1}{8\pi^2}$$

→ $$\Theta_I = 2 \gg 0$$

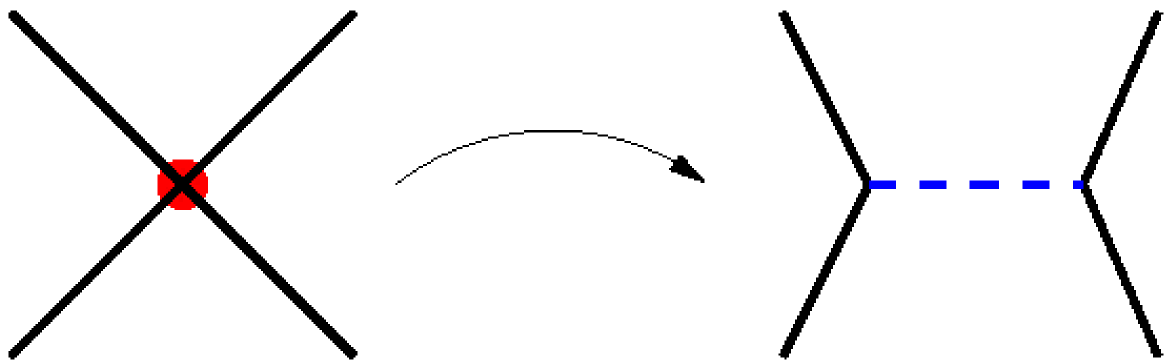
3. Our Model

A Composite Higgs ?

- Hierarchy and triviality caused by scalar particle

→ make it a composite $\bar{\psi}\psi \to \phi$

Nambu, Miransky, Tabanashi, Yamawaki, 1989



$$2\,\lambda_\sigma\,\bar{\psi}_L t_R\,\bar{t}_R \psi_L \quad \to \quad h\,(\bar{t}_R \psi_L\,\phi - \bar{\psi}_L t_R\,\phi^*) + m_\phi^2\,\phi^*\phi$$

However

- Not perturbatively renormalizable

$$[\lambda_\sigma] = -2$$

- Simplest models ruled out ?
- In bosonic formulation, still a hierarchy problem

Towards the Standard Model

- Toy model with $U(1)\times SU(N_c)$ gauge and $SU(N_f)\times SU(N_f)$ chiral symmetry

$$\Gamma_k = \int \bar\psi \left(\mathrm{i}Z_\psi \partial\!\!\!/ + Z_1 \bar g A\!\!\!/ + Z_1^{\mathrm{B}} \bar e B\!\!\!/\right)\psi + \frac{Z_{\mathrm{F}}}{4} F_z^{\mu\nu} F_{\mu\nu}^z + \frac{Z_{\mathrm{B}}}{4} B^{\mu\nu} B_{\mu\nu}$$
$$+\frac{1}{2}\left[\bar\lambda_- \,(\mathrm{V{-}A}) + \bar\lambda_+ \,(\mathrm{V{+}A}) + \bar\lambda_\sigma \,(\mathrm{S{-}P}) + \bar\lambda_{\mathrm{VA}} \left[2(\mathrm{V{-}A})^{\mathrm{adj}} + (1/N_{\mathrm{c}})(\mathrm{V{-}A})\right]\right]$$

where

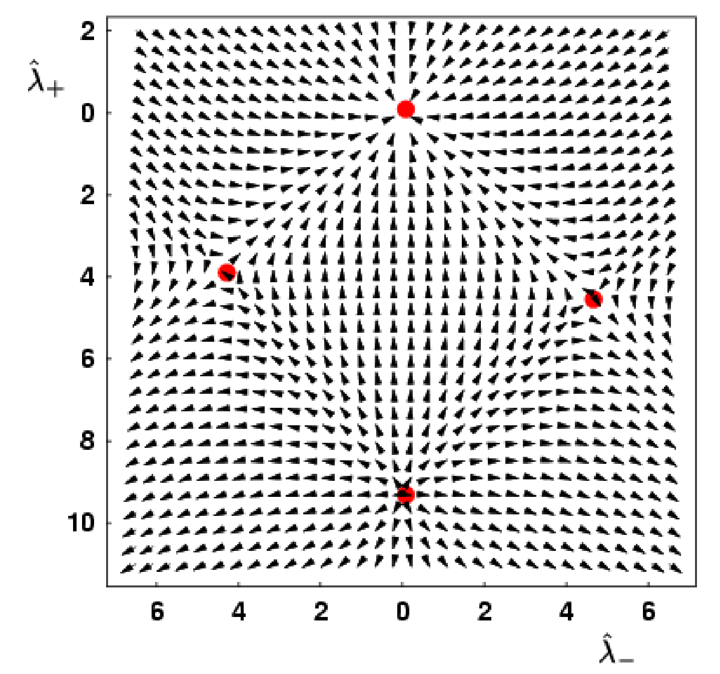
$$
\begin{aligned}
(\mathrm{V{-}A}) &= (\bar\psi\gamma_\mu\psi)^2 + (\bar\psi\gamma_\mu\gamma_5\psi)^2 \\
(\mathrm{V{+}A}) &= (\bar\psi\gamma_\mu\psi)^2 - (\bar\psi\gamma_\mu\gamma_5\psi)^2 \\
(\mathrm{S{-}P}) &= (\bar\psi^a\psi^b)^2 - (\bar\psi^a\gamma_5\psi^b)^2 \equiv (\bar\psi_i{}^a\psi_i{}^b)^2 - (\bar\psi_i{}^a\gamma_5\psi_i{}^b)^2 \\
(\mathrm{V{-}A})^{\mathrm{adj}} &= (\bar\psi\gamma_\mu T^z\psi)^2 + (\bar\psi\gamma_\mu\gamma_5 T^z\psi)^2
\end{aligned}
$$

Fermionic Sector

- Flow equations (for $g^2, e^2 = 0$)

$$\partial_t \lambda_i = 2\lambda_i + \lambda_j A_i^{jk} \lambda_k$$

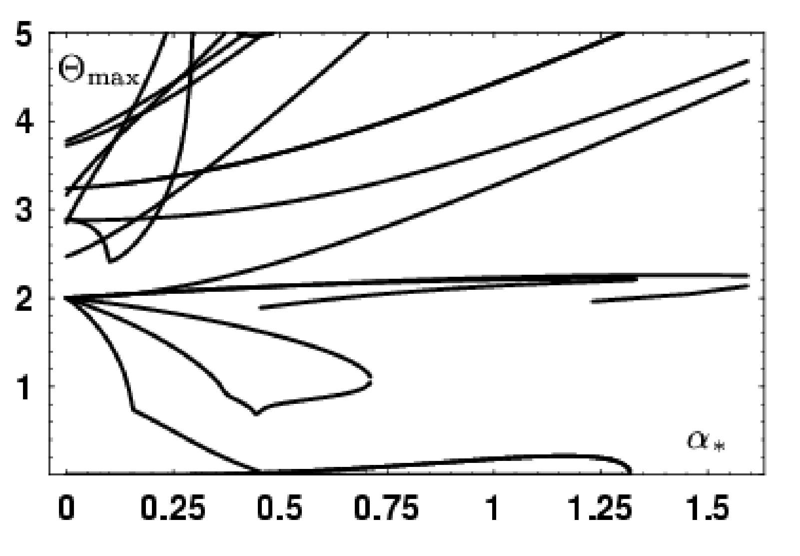
- 2 fixed points per λ ➡ 16 FP
- $\binom{4}{j}$ FP with j relevant directions
- All FP solve triviality problem



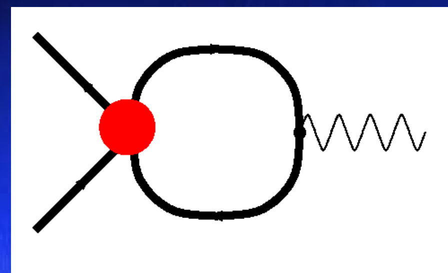
Hierarchy Problem?

- For vanishing gauge couplings...
- At least one eigenvalue $\Theta^{\max} \geq 2$
- BUT: non-vanishing gauge couplings may cure this problem

$$\partial_t \lambda_i = (2 + O(e^2, g^2))\lambda_i + \lambda_j A_i^{jk} \lambda_k + O(e^4, e^2 g^2, g^4)$$



Gauge Coupling Flow

- Naively:  $\Rightarrow$ $$\partial_t g^2 = \eta_{\mathrm{F}} g^2 + C_i \lambda_i g^2$$

- Ward identity $$\partial_t g^2 = \eta_{\mathrm{F}} g^2 + \frac{C_i \partial_t \lambda_i}{C_k \lambda_k} g^2$$

Gies, Wetterich, JJ, 2003

$\Rightarrow$ no contributrion at FP

does not induce FP

4. Conclusions

Conclusions

- Many fixed points → toy model may be asymptotically safe
- Hierarchy problem still severe without gauge couplings → need $g^2, e^2 \neq 0$
- Fermion interactions cannot induce fixed point in gauge couplings
- # of physical parameters may be less than in the SM

Outlook

- + $SU(2)_L$?
- Strongly coupled gauge sector
- Non-standard fermionic action

    no quadratic term?


The End