Isospin-3/2 and -1/2 resonances in one-pion electro- and neutrinoproduction

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Outline

- Why to study one pion production
- Which resonances contribute and how to study them
- What are the uncertainties: form factors, running width of the resonance
- Conclusion

Why to study one pion production

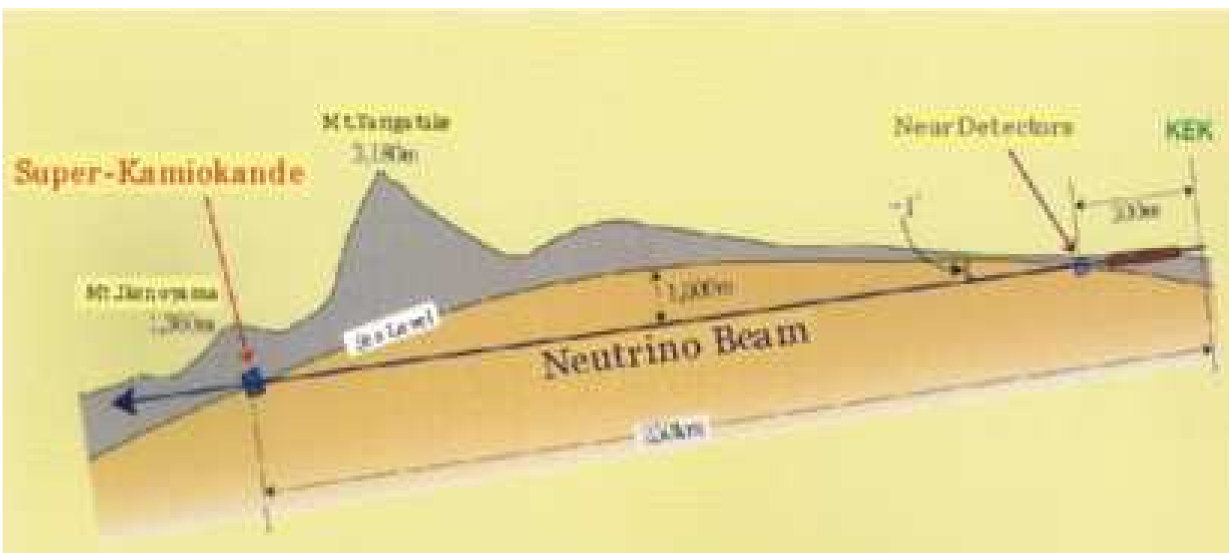
The first evidence about the neutrino oscillations came from SuperKamiokande, where the $\mu-$deficit was observed for atmospheric neutrinos. Then evidences came for solar neutrinos from SNO.

It became clear that for the subsequent measurements of the mixing matrix elements more precise measurements are needed. Thus one comes to the idea of ”artificial” neutrino sources and long baseline exepriments.

Among the artificial neutrinos there are ”reactor neutrinos”, which are $\bar{\nu}_e$ with the energy of few MeV. We are mainly interested in the $accelerator \ \ neutrinos$, 99% of which are ν_μ with the energy $few\ \mathrm{GeV}$.

Long baseline experiments

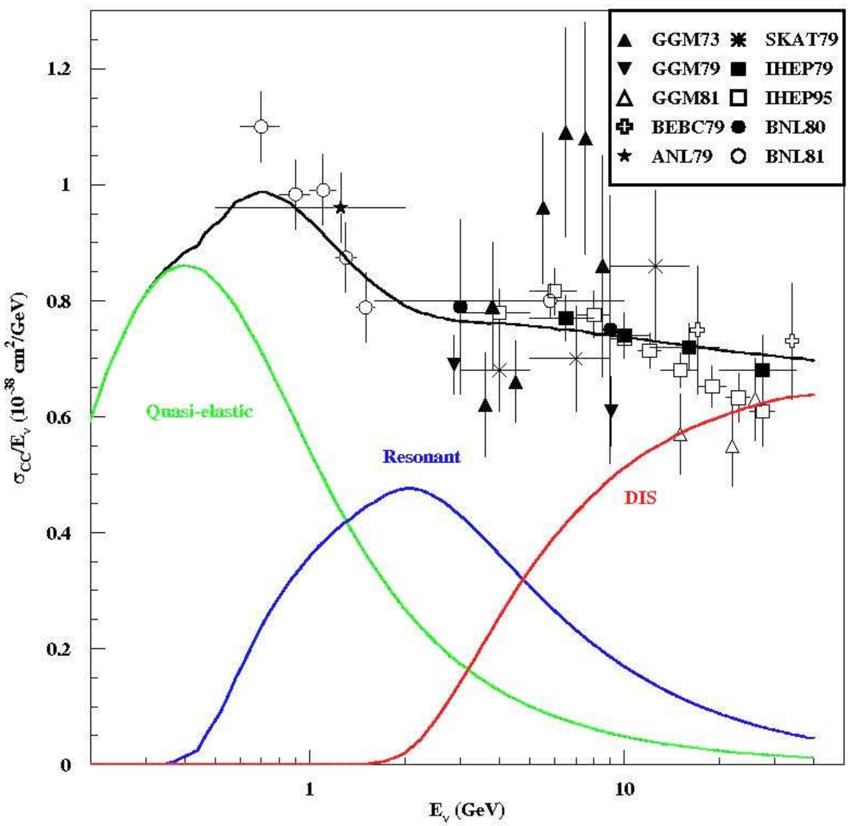
- T2K (Tokai to Kamioka) $\langle E_\nu \rangle \sim 0.7\ \mathrm{GeV}$ (planned)
- K2K (KEK to Kamioka) $\langle E_\nu \rangle \sim 1\ \mathrm{GeV}$ (operating)
- MINOS (Fermilab to Soudan) $\langle E_\nu \rangle \sim 3\ \mathrm{GeV}$
- CNGS (CERN to GranSasso) $\langle E_\nu \rangle \sim 17\ \mathrm{GeV}$ (under construction)



neutrino.kek.jp/news/

2004.06.10/index-e.html

The total cross section



$\sigma_{tot} = \sigma_{\rm QE} + \sigma_{\rm RES} + \sigma_{\rm DIS}$

1) quasi-elastic (QE)

$\nu_\mu n \to \mu^- p$

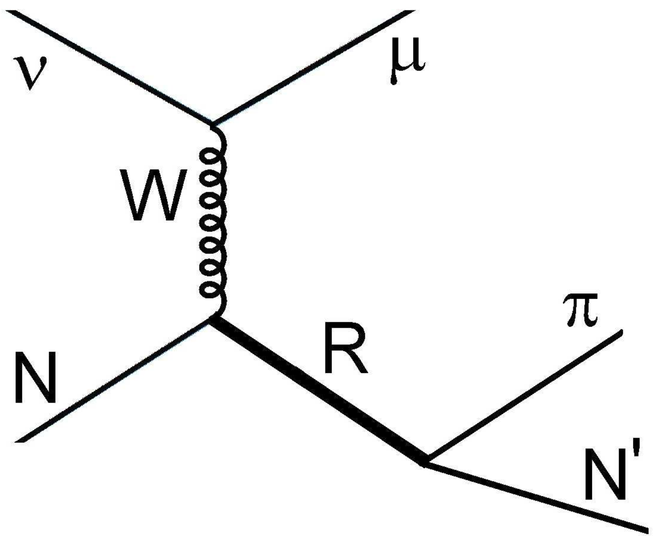
2) one-pion-production $\equiv$ resonance production (RES)

$\nu_\mu N \to \mu^- R \to \mu^- N' \pi$

3) deep inelastic scattering (DIS)

$\nu_\mu N \to \mu^- X$

Resonance production

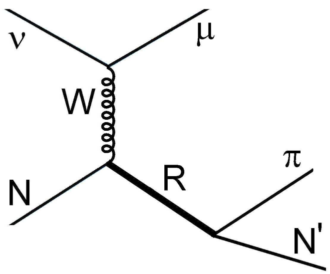


Experimentators - detection of the exclusive final state

Theorists - quantitative description of the processes

R	M_R, GeV	$\Gamma_{R(tot)}$, GeV	$\Gamma_R(R \to \pi N)/\Gamma_{R(tot)}$
$P_{33}(1232)(\Delta^{++}, \Delta^{+}, \Delta^{0}, \Delta^{-})$	1.232	0.120	0.995
$P_{11}(1440)(P_{11}^{+}, P_{11}^{0})$	1.440	0.350	0.65
$D_{13}(1520)(D_{13}^{+}, D_{13}^{0})$	1.520	0.125	0.56
$S_{11}(1535)(S_{11}^{+}, S_{11}^{0})$	1.535	0.150	0.45

Theoretical description



For $E_\nu \sim few\,\mathrm{GeV}$, $2m_N E_\nu << m_W^2$, so the weak vertex is described as Fermi 4-fermion intreraction (current-current interaction)

$$\frac{G_F}{\sqrt{2}} J^\nu_{(\mathrm{hadronic})} j_\nu^{(\mathrm{leptonic})}, \qquad J^\nu_{(\mathrm{hadronic})} = V^\nu - A^\nu$$

The hadronic current is parametrized in terms of the

nucleon-resonance form factors (vector and axial)

which depend on the transfered momentum

These form factors are independent of the flavor of the incoming nutrino (and correspondingly outgoing muon). So one can simulate them for ν_μ and then use for ν_e and ν_τ.

Question: how many form factors do we need for each resonance?

How many form factors do we need for N-R vertex

Form factors for $P_{33}(1232)$ ($J^P = \frac{3}{2}^+$) C.H. Llewellyn Smith, Phys. Rep. 3 (1972) 261

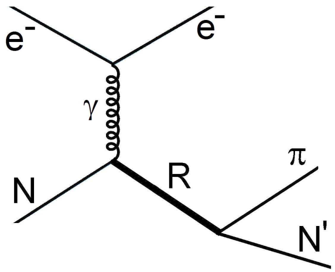
$$\langle\Delta|V^\nu|N\rangle = \bar\psi_\lambda^{(R)}\left[\frac{C_3^V}{m_N}(\not q g^{\lambda\nu} - q^\lambda\gamma^\nu) + \frac{C_4^V}{m_N^2}(q\cdot p g^{\lambda\nu} - q^\lambda p^\nu) + \frac{C_5^V}{m_N^2}(q\cdot p' g^{\lambda\nu} - q^\lambda p'^\nu) + C_6^V g^{\lambda\nu}\right]\gamma_5 u_{(N)}$$

$$\langle\Delta|A^\nu|N\rangle = \bar\psi_\lambda\left[\frac{C_3^A}{m_N}(\not q g^{\lambda\nu} - q^\lambda\gamma^\nu) + \frac{C_4^A}{m_N^2}(q\cdot p g^{\lambda\nu} - q^\lambda p^\nu) + C_5^A g^{\lambda\nu} + \frac{C_6^A}{m_N^2}q^\lambda q^\nu\right]u_{(N)}$$

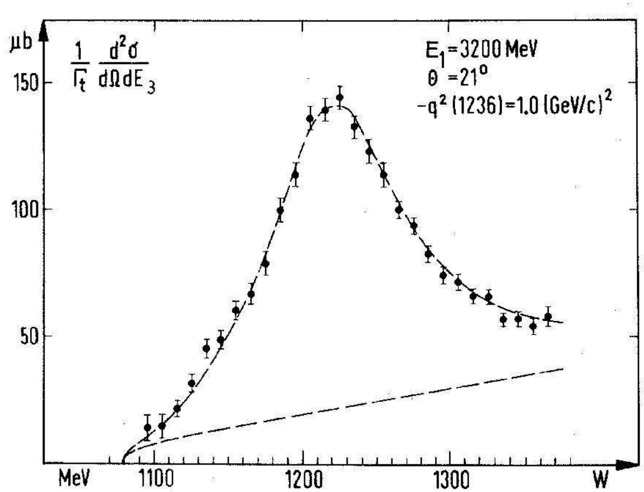
Can any of them be theoretically fi xed?

CVC $\Longrightarrow C_6^V = 0$, PCAC $\Longrightarrow C_5^A = 1.2$, $C_6^A = m_N^2 \frac{C_5^A}{m_\pi^2 - Q^2}$

Vector form factors



reveal theirselves in electro- (and neutrino-) production



Magnetic multipole dominance lead to

$$C_3^V(0) = 2.05, \quad C_4^V(0) = -\frac{m_N}{W} C_3^V, \quad C_5^V = 0$$

S. Galster et. al, Phys. Rev. D 5 (1972) 519, the curve is

the phenomenological fi t by the authors

As Q^2 increases, the form factors fall down steeper than dipole (that is the case for the nucleon) representing the larger size of the resonance states because of the mesonic cloud, surrounding them.

$$C^V(Q^2) = \frac{C^V(0)}{\left(1+\frac{Q^2}{{M_V}^2}\right)^2} \cdot \frac{1}{1+\frac{Q^2}{4{M_V}^2}}$$ vector mass $M_V = 0.84\ \mathrm{GeV}.$

New data form Jefferson Laboratory: the contribution of the electric multipole $E2 \sim -2.5\%$, of scalar multipole $Q2 \sim -5\%$

Axial form factors

reveal theirselves in neutrinoproduction

From PCAC $C_5^A = 1.2, \quad C_6^A = m_N^2 \frac{C_5^A}{m_\pi^2 - Q^2}$

C_5^A gives the main contribution to the cross section

C_6^A contribution is proportional to the mass of the outgoing lepton

Other form factors: in general free parameters, $C_4^A = -C_5^A/4, \quad C_3^A = 0$ is favoured

by Adler's model and gives good agreement with the experiment

As Q^2 increases, axial form factors also fall down steeper than dipole.

One of the choices $C^A(Q^2) = \dfrac{C^A(0)}{\left(1 + \dfrac{Q^2}{{M_A}^2}\right)^2} \cdot \dfrac{1}{1 + \dfrac{Q^2}{3{M_A}^2}}$

E.A. Paschos. M. Sakuda, J.-Y. Yu, Phys. Rev. D 69 (2004) 014013 (PYS)

The axial mass M_A is to be determined from both quasi-elastic scattering and resonance production, $M_A \sim 1$.

The running resonance width

The width of the $\Delta-$resonace (in the resonance rest frame averaged over spin states) on-mass-shell is

$$\Gamma = \frac{1}{4}\frac{g_{P33}^2}{6\pi M_R^2}\left[(M_R + m_N)^2 - m_\pi^2\right] \cdot (p_\pi(M_R))^3$$

Here $3 = 2l + 1$, where $l = 1$ is the angular momentum of the resonance

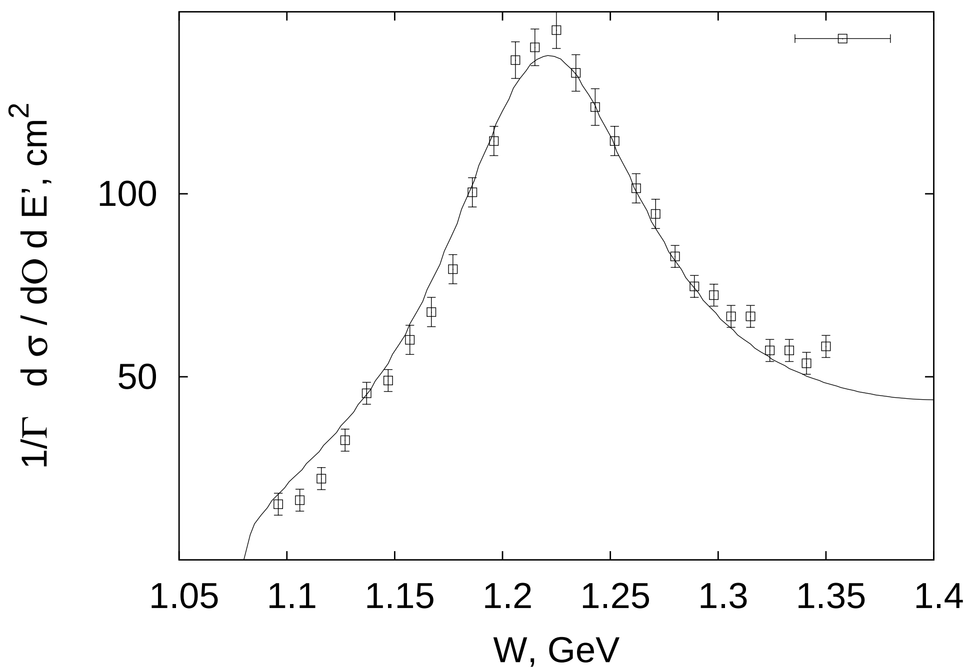
For one-pion production the resonance is off-mass-shell.

Thus it is reasonable to suppose that off-mass-shell

$$\Gamma(W) = \left(\frac{p_\pi(W)}{p_\pi(M_R)}\right)^{2l+1} = \left(\frac{p_\pi(W)}{p_\pi(M_R)}\right)^3$$

The running resonance width

However, the electroproduction experiment (performed in DESY in 1972) S. Galster et. al, Phys. Rev. D 5 (1972) 519 can be reasonably fi tted only with



$$\Gamma(W) = \left(\frac{p_\pi(W)}{p_\pi(M_R)}\right)^1$$

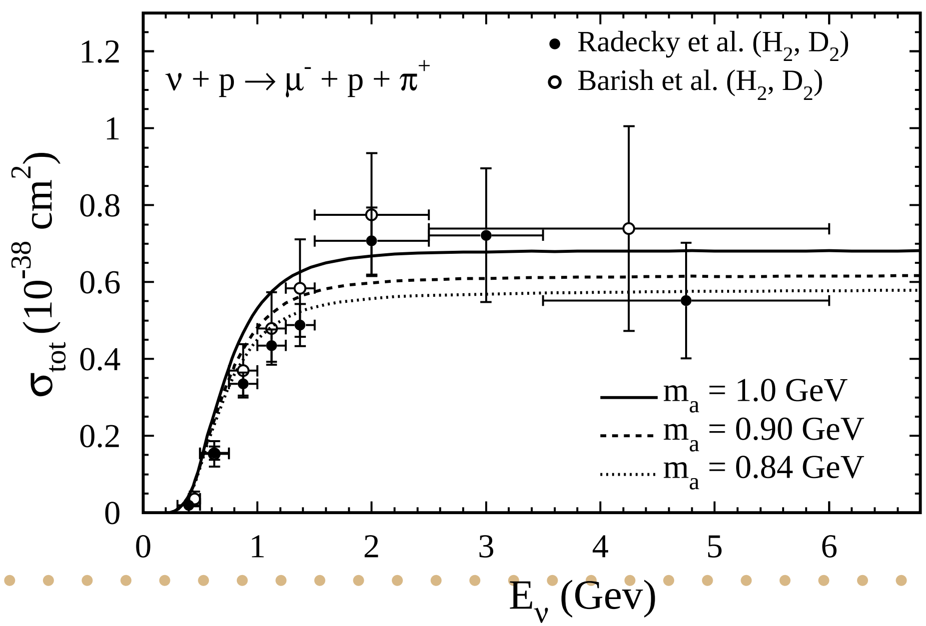
Nuclear corrections

The Long Baseline experiments measure reactions on nuclear targets, like $_8\mathrm{O}^{16}$, $_{18}\mathrm{Ar}^{40}$ and $_{26}\mathrm{Fe}^{56}$. The neutrino-nucleon cross section (which is calculated as described above) is affected

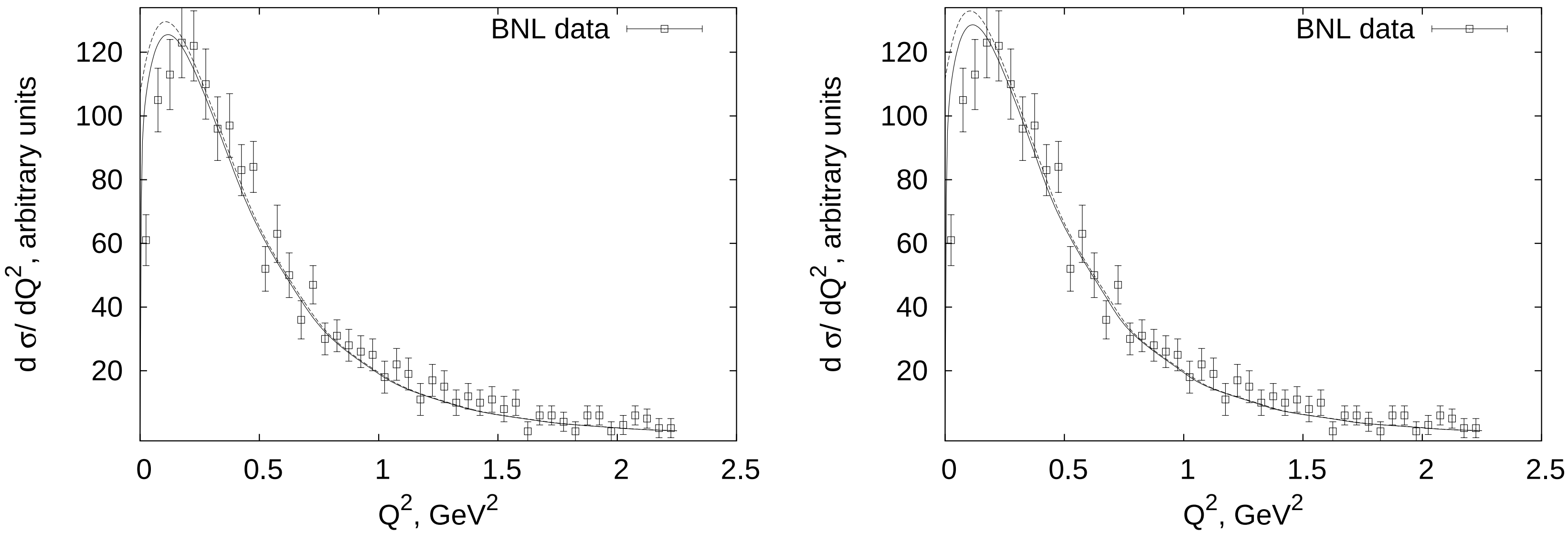
- by the Fermi–motion of the nuclei, which shifts the energy and
- by the Pauli–suppression (Pauli–blocking), i.e. when a nucleon receives a small momentum transfer, below the Fermi–sea, it cannot recoil because the state to which it must go is already occupied. For the simple geometrical model see E.A. Paschos. M. Sakuda, J.-Y. Yu, Phys. Rev. D 69 (2004) 014013 (PYS)
- final state interactions - as the pions wander through the nucleus, they scatter performing a random walk. This can leada to the charge exchange effect ($\pi^0 p \rightarrow \pi^+ n$). The first treatment of this effect is given in S. L. Adler, S. Nussinov. and E. A. Paschos, Phys. Rev. D 9, 2125, (1974).

The cross section

The cross section for the $\Delta-$resonance production has been calculated about 30 years ago and is usually expressed in terms of helicity amplitudes, introduced in 1971 by Zucker. The complete set of formulars can be found in the paper of Schreiner and Von Hippel Nuclear Physics B58 (1973) 333-362. The calcualtions have been done, neglecting muon mass, that is a valid approximation at $Q^2 >> m_\mu^2$.

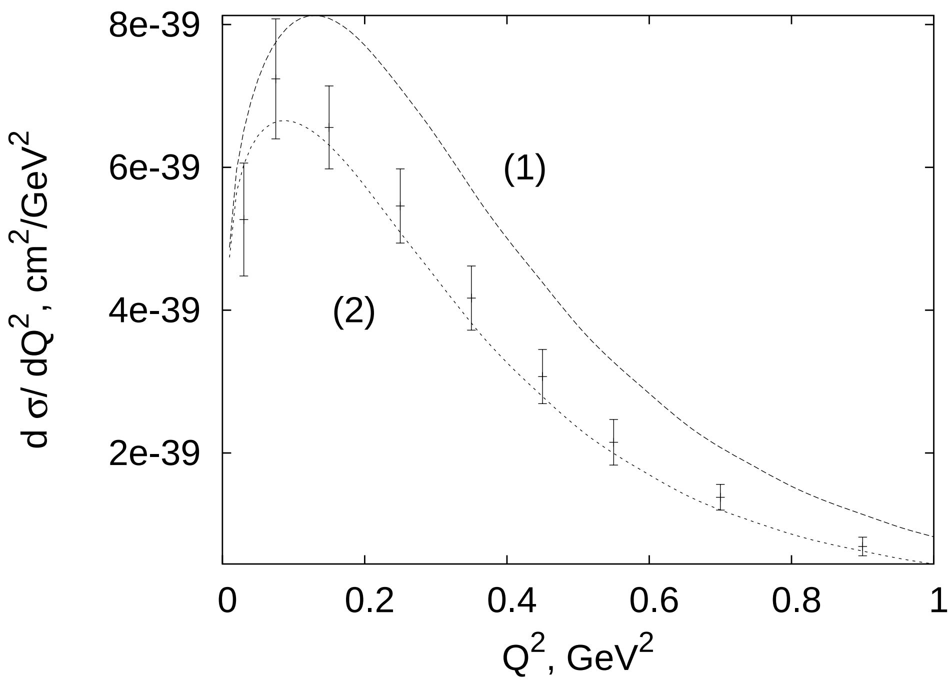


BNL data on the $d\sigma/dQ^2$.



The cross section for the running width $\Gamma \sim p^3_{(\pi)}$ (left figure) and $\Gamma \sim p^1_{(\pi)}$ (right figure). The full lines are for the case $m_\mu = 0.105$ GeV, the dashed lines are for the approximation $m_\mu = 0$.

ANL data on $d\sigma/dQ^2$



$$1)\quad C^A = \frac{C^A(0)}{\left(1+\frac{Q^2}{M_A{}^2}\right)^2}\cdot\frac{1}{1+\frac{Q^2}{3M_A{}^2}}$$

$$2)\quad C^A = \frac{C^A(0)}{\left(1+\frac{Q^2}{M_A{}^2}\right)^2}\cdot\frac{1}{1+\frac{2Q^2}{M_A{}^2}}$$

Isospin-1/2 resonances

$P_{11}(1440)$ $D_{13}(1535)$ $S_{11}(1520)$

- Old bubble chamber experiments - $W < 1.4\ \mathrm{GeV}$, the contribution is negligible
- The contribution to the integrated cross section does not exceed $10 - 15\%$
- SLAC experiment at JLAB investigates region $W > 1.4\ \mathrm{GeV}$
- To describe modern experimental data and to calculate the integrated cross section with the accuracy more than $10 - 15\%$ we need to understand the production of isospin-1/2 resonances theoretically

How many form factors do we need for each resonance?
It is reasonable to expect the same dependence of the formfactors on Q^2 for all resonances?

Isospin-1/2 resonances, spin-1/2

$P_{11}(1440)$ ($J^P = \frac{1}{2}^+$)

$$\langle P_{11}|V^\nu|N\rangle = \bar{u}^{(P_{11})}\left[g_1^V\gamma^\nu + \frac{i}{M_P+m_N}g_2^V\sigma^{\nu\lambda}q_\lambda + g_3^V q^\nu\right]\gamma_5 u_{(N)}$$
$$\langle P_{11}|A^\nu|N\rangle = \bar{u}^{(P_{11})}\left[g_1^A\gamma^\nu + \frac{i}{M_P+m_N}g_2^A\sigma^{\nu\lambda}q_\lambda + g_3^A q^\nu\right]u_{(N)}$$

CVC: $g_1^V = g_3^V = 0$

CP invariance: $g_2^A = 0$

PCAC: $g_3^A = g_1^A \frac{\not{q}}{Q^2}, \quad g_1^A = \frac{g_{\pi NR}f_\pi}{M_R+m_N}$

$S_{11}(1535)$ ($J^P = \frac{1}{2}^-$) (the similar formular)

Result: for spin-1/2 resonances one has 1 vector and 1 axial (theoretically fi xed from PCAC) form factors for each resonance

Isospin-1/2, spin-3/2

$D_{13}(1520)$ ($J^P = \frac{3}{2}^-$)

The formulas for this resonance are similar to that for P_{33},

the are 3 independent vector form factors

and 3 independent axial form factors, one of which is fi xed from PCAC

Summary

- Theoretically the contribution of the leading $P_{33}(1232)$ resonance is well understood, however the new data from JLAB should be taken into account
- Experimentally for $P_{33}(1232)$ production there is a general, but not the detailed agreement between the different experiments. In particular, ANL and BNL data on $d\sigma/dQ^2$ require different form factor behaviour.
- The muon mass effects are important for $Q^2 < 0.05 - 0.06\ \mathrm{GeV}^2$
- The contribution of the isospin-1/2 resonances in the kinematical region of $P_{33}(1232)$ production $W < 1.4\ \mathrm{GeV}$ is experimentally known to be small, does not exceed 1%.
- Isospin-1/2 resonances at $W > 1.4\ \mathrm{GeV}$ are investigated in modern experiments (CLAS) and should be understood theoretically.