DESY Theory Workshop
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Non-Abelian gravitating solitons with negative cosmological constant

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The EYM theory

Is defined by the action

$$S_{EYM} = \frac{1}{4\pi}\int\Big(-\frac{c^3}{4G}R - \frac{1}{4g^2c}F^a_{\mu\nu}F^{a\mu\nu}\Big)\sqrt{-g}d^4x \ ,$$

The spherically symmetric metric can be parameterized as

$$ds^2 = A^2Bdt^2 - \frac{dr^2}{B} - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \ .$$

It is common to express $B(r)$ through the "mass function" $m(r)$ defined by

$$B(r) = 1 - 2m(r)/r.$$

For the $SU(2)$ gauge field we take 'monopole' ansatz

$$W_0^a = 0, \quad W_i^a = \epsilon_{aij}\frac{n^j}{r}(1 - W(r)) \ .$$

$W_\infty = \pm 1$ - neutral
$W_\infty = 0$ - monopole

From two coupling constants

$$[G] = L^3T^{-2}M^{-1}, \quad [g] = L^{-1}T^{1/2}M^{-1/2} ,$$

one can form

$$l_{EYM} = \sqrt{G/cg^2}, \quad m_{EYM} = \sqrt{c/Gg^2} .$$

In dimensionless variables the reduced action is

$$S^{\mathrm{red}}_{\mathrm{EYM}} = \sqrt{\frac{c^5}{Gg^2}} \int A \left[\frac{1}{2}(B + rB' - 1) + \left(BW'^2 + \frac{(1-W^2)^2}{2r^2} \right) \right] dr .$$

Resulting field equations for $W, U \equiv W', B$ and A are

$$(BW')' = \frac{W(W^2-1)}{r^2} - \frac{2BW'^3}{r} ,$$

$$B' = \frac{1}{r}\left(1 - B - BW'^2 - \frac{(1-W^2)^2}{r^2}\right) ,$$

$$\frac{A'}{A} = \frac{2W'^2}{r} .$$

These Eqs have regular singular points at $r = 0, \infty$ and at some r_h, where $B(r_h) = 0$.
At $r \to 0$ one-parameter family b:

$$W(r) = 1 - br^2 + O(r^4) \,,$$
$$B(r) = 1 - 4b^2r^2 + O(r^4) \,.$$

At $r \to \infty$ two-parameter family c, m:

$$W(r) = \pm(1 - \frac{c}{r} + O(\frac{1}{r^2})) \,,$$
$$B(r) = 1 - \frac{2m}{r} + O(\frac{1}{r^4}) \,.$$

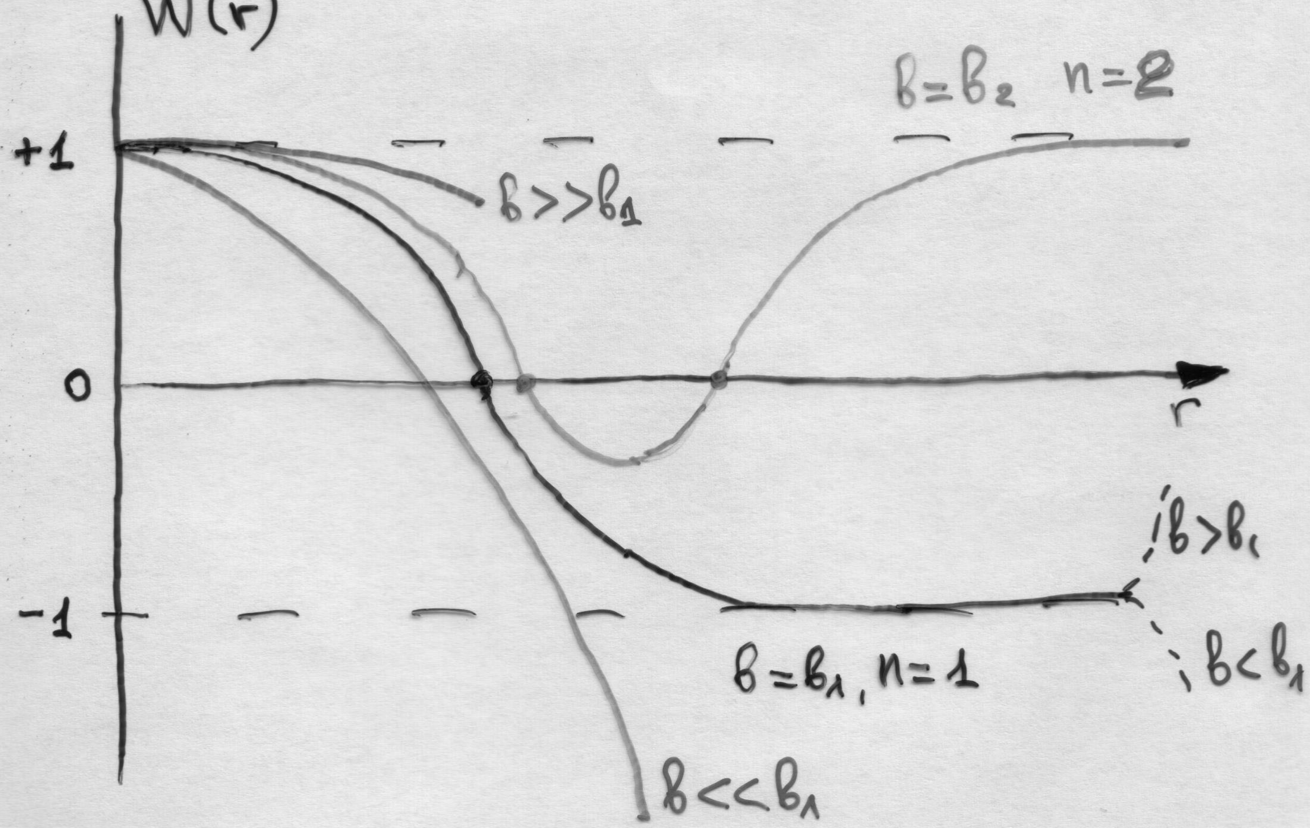
At $r \to r_h$ two-parameter family r_h, W_h:

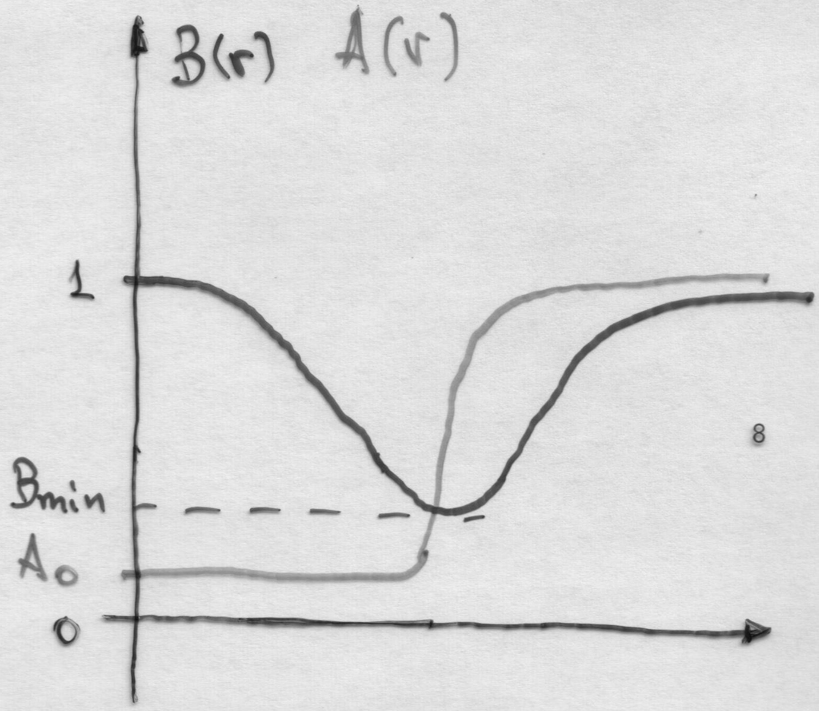
$$W(r_h + \rho) = W_h + W_h'\rho + O(\rho^2) \,,$$
$$B(r_h + \rho) = B_h'\rho + O(\rho^2) \,,$$

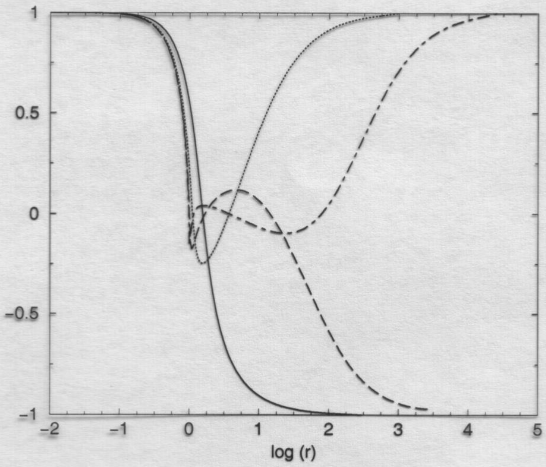
with $\rho = r - r_h$ and

$$B_h' = \frac{1}{r_h}(1 - \frac{(W_h^2 - 1)^2}{r_h^2}) \,, \quad W_h' = \frac{W_h(W_h^2 - 1)}{B_h' r_h^2} \,.$$

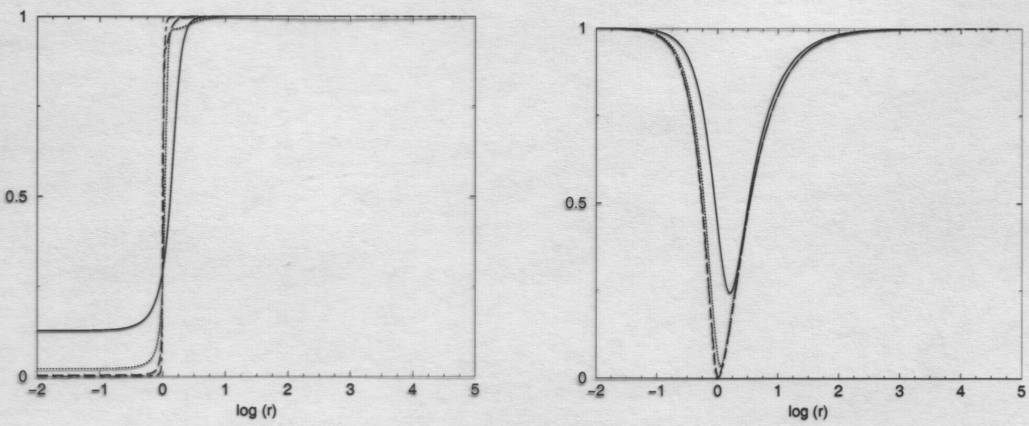
Qualitative picture







The gauge field amplitude $W(r)$ for the first four globally regular solution of EYM theory.



The metric functions $A(r)$ (left) and $B(r)$ (right) for the first four globally regular solution of EYM theory.

EYM_Λ theory with $\Lambda > 0$

The action of EYM theory with the cosmological constant Λ has the form:

$$S_{EYM_\Lambda} = \frac{1}{4\pi}\int\Big(-\frac{1}{4G}(R+2\Lambda)-\frac{1}{4g^2}F^a_{\mu\nu}F^{a\mu\nu}\Big)\sqrt{-g}d^4x \ .$$

It turns out that the situation is rather different in case of positive and negative values of cosmological constant.

$$(BW')' = \frac{W(W^2-1)}{r^2} - \frac{2BW'^3}{r} \ ,$$

$$m' = (BW'^2 + \frac{(W^2-1)^2}{2r^2}) \ ,$$

$$\frac{A'}{A} = \frac{2W'^2}{r}$$

with $B(r) = 1 - \frac{2m(r)}{r} - \frac{\Lambda}{3}r^2$.
Note that equation for A decouples as in pure EYM case.

At $r \to 0$ one-parameter family b (for given Λ):

$$W(r) = 1 - br^2 + O(r^4) ,$$
$$B(r) = 1 - (4b^2 + \frac{\Lambda}{3})r^2 + O(r^4) ,$$

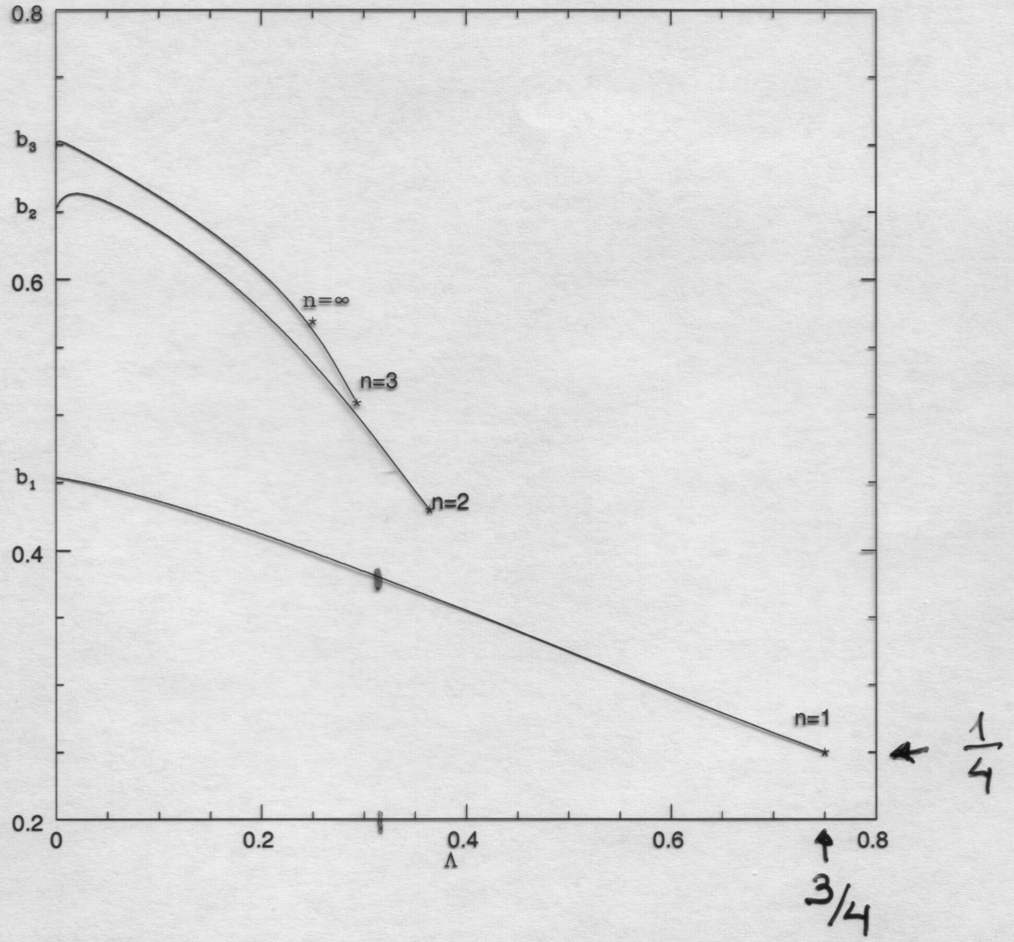
In $\Lambda > 0$ case the solutions have a cosmological horizon, where $B(r)$ vanishes. Close to it one finds

$$W(r) = W_h + W_h' x + O(x^2), \qquad B(r) = B_h' x + O(x^2) ,$$

where $x = r - r_h$ and W_h' and $B_h' < 0$ are some functions.
At $r = \infty$ for asymptotically de Sitter solutions

$$W(r) = W_\infty + \frac{p}{r} - \frac{3W_\infty({W_\infty}^2 - 1)}{4\Lambda r^2} + O\left(\frac{1}{r^3}\right) ,$$
$$B(r) = 1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2 + \frac{Q_{eff}^2}{r^2} + O\left(\frac{1}{r^3}\right) ,$$

with $Q_{eff}^2 = ({W_\infty}^2 - 1)^2 - \frac{2}{3}\Lambda p^2$. Here b, r_h, W_h, W_∞, p and M are six "shooting" parameters.



Shooting parameter b versus cosmological constant (for $\Lambda > 0$) for different solutions of the EYM$_\Lambda$ theory.

n	$\Lambda_{crit}(n)$	$\Lambda_\diamond(n)$	$\Lambda_\star(n)$
1	0.330	0.334	0.75
2	0.239	0.250	0.364
3	0.237	0.247	0.293

The special values of Λ for the lowest n's.

EYM_Λ theory with $\Lambda < 0$

In this case no cosmological horizon anymore. Many conditions are relaxed.

* Solutions can be nodeless in W ($\rightarrow$ stability!)

* Solutions with non-zero electric part are allowed

* W_∞ is not necessary ± 1

As a result it was found that there are
E.Winstanley 1999; J. Bjoraker, Y.Hosotani 2000

* Neutral solutions

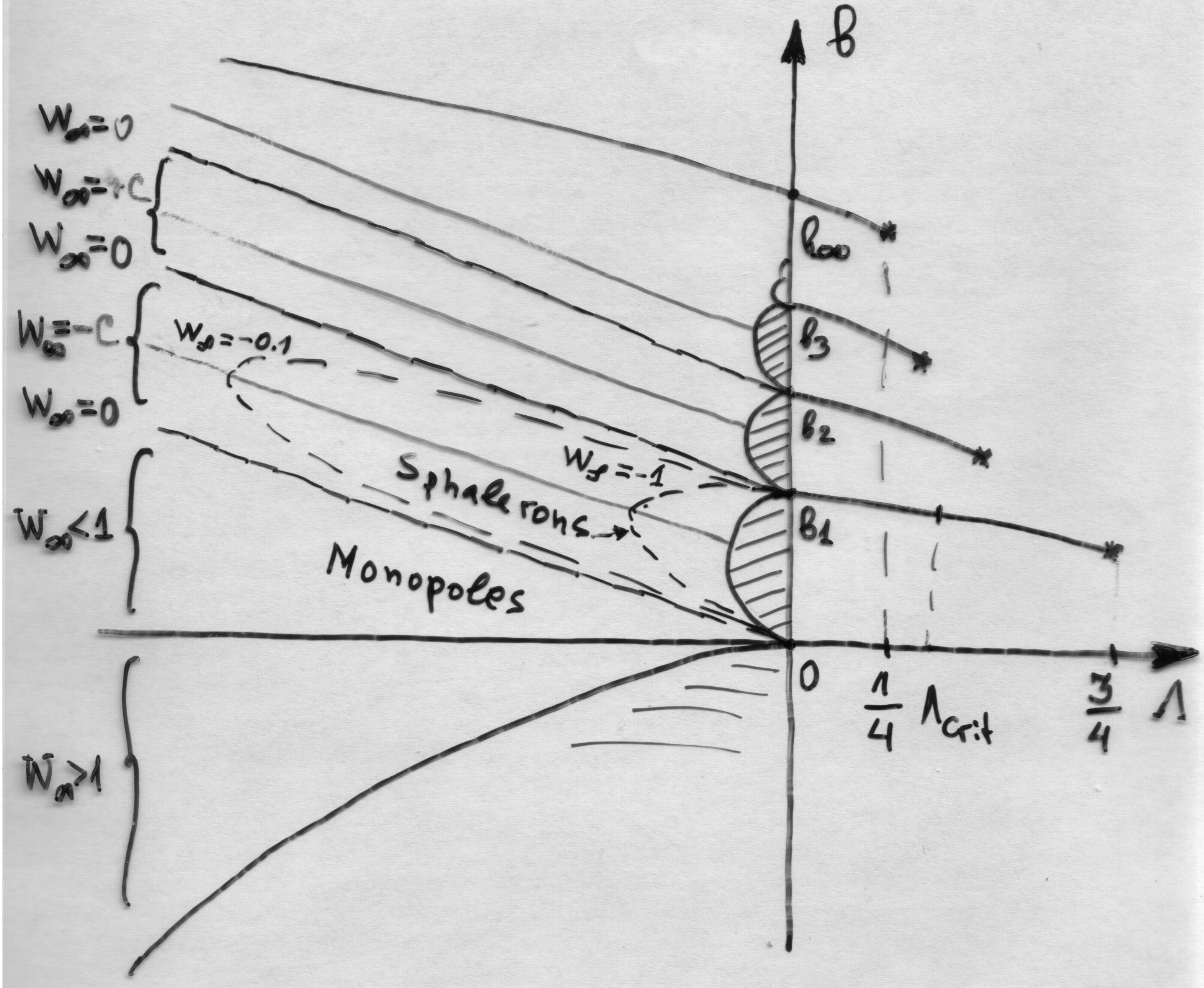
* Magnetically charged

* Electrically charged

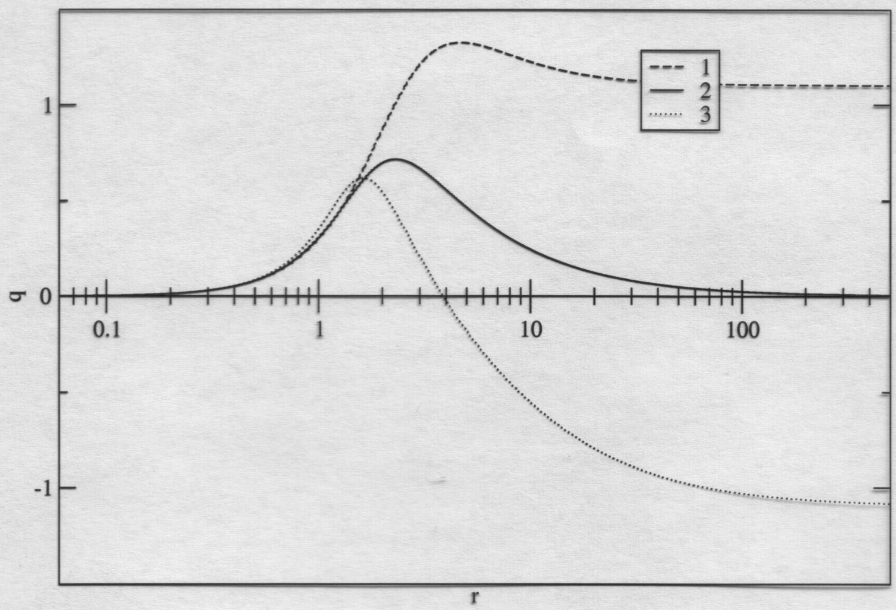
* Dyons

with any number of nodes in W, including nodeless and corresponding black holes.

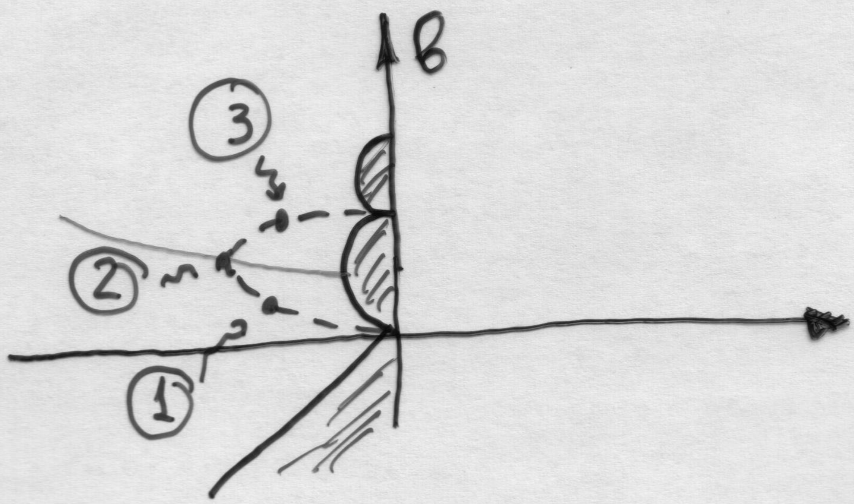
P. Breitenlohner, G. L., D. Maison '04



Initial data for different solutions in EYM_Λ theory



Zero energy wave functions of Schrödinger equation of linear perturbations (gravitational sector) about background soliton for three different values of parameters Λ, b: 1-the wave function for under critical values (dashed line) has no nodes, 2-the wave function for critical values (solid line) is a normalizable zero mode, 3-the wave function for above critical values (dotted line) has one node.



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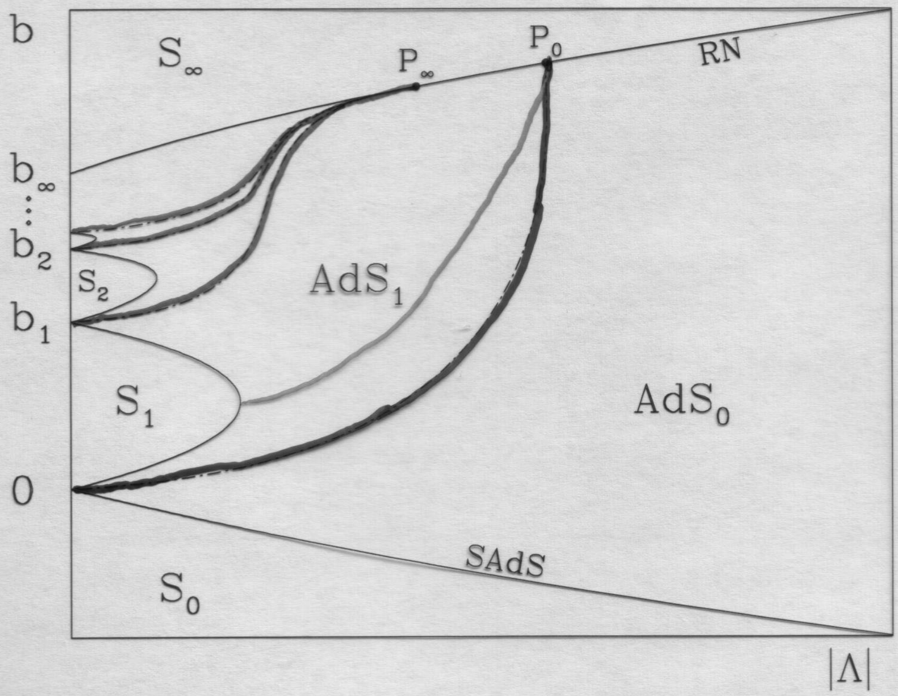
Moduli space of the EYM_Λ solutions for $\Lambda < 0$.



"—" $W_\infty = 0$

"—" zero mode curves

A - stable

B - 1 gr. inst

C - 1 sph. inst

D - 2 unst. modes