Spontaneous compactification as self-tuning the cosmological constant

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Outline

- A review of self-tuning with codimension-one branes
- Codimension-two brane case: flux compactification
- Compactification by $\sigma-$model scalars
- Comparison to hyperbolic $\sigma-$model
- Conclusion

Self-tuning for codimension-one branes

[N.Arkani-Hamed,S.Dimopoulos,N.Kaloper,R.Sundrum, hep-th/0001197]

[S.Kachru,M.B.Schulz,E.Silverstein, hep-th/0001206]

- Cosmological constant problem: the most serious hierarchy problem

$$\rho_\Lambda^{(obs)} \sim 10^{-120} M_{Pl}^4 <<< \rho_\Lambda^{(natural)} \sim M_{Pl}^4$$

- Self-tuning: To obtain a flat solution independently of brane tension by choosing parameters of the bulk solution, but no fine-tuning of Lagrangian parameters.

- Consider a 3-brane with tension Λ and a massless scalar field ϕ coupled to gravity in 5d flat bulk:

$$S = \int d^4x dy \sqrt{-G}(R - \frac{4}{3}(\partial\phi)^2) \tag{1}$$

$$- \int d^4x \sqrt{-g} \Lambda e^{\frac{4}{3}\phi} \Big|_{y=0} . \tag{2}$$

- The obtained solutions are

$$ds^2 = \left|c + \frac{4}{3}|y|\right|^{1/2} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \tag{3}$$

$$\phi(y) = -\frac{4}{3} \ln \left|c + \frac{4}{3}|y|\right| + \phi_0 \tag{4}$$

with the condition

$$\Lambda = 4e^{\frac{4}{3}\phi_0}. \tag{5}$$

- Self-tuning via the bulk shift symmetry with ϕ_0
- No nearby curved solution
- $c < 0$ for a finite Planck mass, but there exist naked singularities at $|y| = -\frac{3}{4}c$!

- The naked singularities must be cured, e.g. by introducing additional branes at the singularities:

$$\Delta S = -\int d^4x\sqrt{-g}\Lambda_+ e^{\frac{4}{3}\phi}\Big|_{y=-\frac{3}{4}c} \tag{6}$$

$$- \int d^4x\sqrt{-g}\Lambda_- e^{\frac{4}{3}\phi}\Big|_{y=\frac{3}{4}c} \tag{7}$$

with fine-tuning conditions

$$\Lambda_+ = \Lambda_- = -\frac{1}{2}\Lambda. \tag{8}$$

[S.Förste,Z.Lalak,S.Lavignac,H.P.Nilles, hep-th/0002164]

- A nonzero bulk cosmological constant leads to nearby curved solutions.

[C.Csaki,J.Erlich,C.Grojean,T.Hollowood, hep-th/0004133]

[S.Förste,Z.Lalak,S.Lavignac,H.P.Nilles, hep-th/0006139]

Flux compactification with codim-two branes

[Z.Horvath,L.Palla,E.Cremmer,J.Scherk, NPB127 (1977) 57]

[S.M.Carroll, M.M.Guica, hep-th/0302067]

[I.Navarro, hep-th/0302129]

- Codimension-two branes do not curve the space, but only introduce a deficit angle: promising for self-tuning.

[J.W.Chen,M.A.Luty,E.Ponton,hep-th/0003067]

- Consider a $U(1)$ gauge field and a nonzero bulk cosmological constant in 6d Einstein gravity:

$$S = \int d^4x d^2y \sqrt{-g}\left(\frac{1}{2}R - \Lambda_b - \frac{1}{4}F_{MN}F^{MN}\right). \quad (9)$$

- Taking a nonzero gauge flux $F_{mn} = \epsilon_{mn}E$, the obtained solution is

$$ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu + R_0^2(d\theta^2 + \beta^2 \sin^2\theta d\phi^2). \quad (10)$$

  – One needs a bulk tuning: $E^2 = 2\Lambda_b$.
  – Radius stabilized with $R_0^{-2} = 2\Lambda_b$
  – In supergravity case, the dilaton equation of motion guarantees the bulk tuning.

[A.Salam,E.Sezgin, PLB147(1984) 47]

[Y.Aghababaie,C.P.Burgess,S.L.Parameswaran,F.Quevedo, hep-th/0304147]

- Two 3-branes with nonzero tensions(Λ_1, Λ_2) can be accommodated at the poles of a sphere with deficit angle:

$$\Lambda_1 = \Lambda_2 = 2\pi(1-\beta) \tag{11}$$

  – The bulk tuning remains the same, i.e. is independent of the brane tensions.

  – The fine-tuning between brane tensions can be avoided by Z_2 symmetry around the equator.

- However, there appears a quantization(or conservation) of gauge flux:

$$E = \frac{n}{2g\beta R_0^2}, \quad n = \text{integer}. \tag{12}$$

  – Thus, the brane tensions appear in the bulk tuning condition via the deficit angle.

  – Even in the supergravity case, one cannot avoid this problem.

- Is it possible to compactify extra dimensions without gauge flux?

Compactification by $\sigma-$model scalars

[S.Radjbar-Daemi,V.Rubakov, hep-th/0407176]

[H.M.L.,A.Papazoglou, hep-th/0407208]

- Consider a two-dimensional non-linear sigma model $\{\phi^1, \phi^2\}$ coupled to gravity in 6d:

$$S = \int d^4x d^2y \sqrt{-g}\left(\frac{1}{2}R - \frac{1}{2}k f_{ij}(\phi)\partial_M \phi^i \partial^M \phi^j\right). \quad (13)$$

- Take a spherical manifold of $S^2 = SU(2)/U(1)$:

$$d\sigma_f^2 = (d\phi^1)^2 + \sin^2\phi^1 (d\phi^2)^2 \quad (14)$$

or for a complex scalar $\Phi = (\tan\frac{\phi^1}{2})e^{i\phi^2}$,

$$d\sigma_f^2 = \frac{4d\Phi d\bar{\Phi}}{(1+|\Phi|^2)^2}. \quad (15)$$

- For a flat solution, assume the factorized extra dimensions.

- Take the ansatze for the internal metric and the scalar field in complex coordinates

$$ds_2^2 = r_0^2 e^{2A(z,\bar{z})} dz d\bar{z}, \quad (16)$$

$$\Phi = \Phi(z, \bar{z}). \quad (17)$$

  - The (zz) Einstein equation leads to a (anti-)holomorphic Φ.

– The remaining Einstein equations and the scalar equation are also satisfied for any (anti-)holomorphic Φ.

- Taking $\Phi = e^{ic} z^b$ with c a constant phase and $b > 0$($b < 0$ case is its dual by $z \to 1/z$), the obtained solution for the metric is

$$ds_2^2 = r_0^2 |z|^{-2a} \frac{dz d\bar{z}}{(1 + |z|^{2b})^{2k}} \tag{18}$$

with 3-brane tensions located at $z = 0$ and $z = \infty$

$$\Lambda_1 = 2\pi a, \quad \Lambda_2 = 2\pi(2 - a - 2kb). \tag{19}$$

- The radius is not fixed: a massless modulus in four dimensions.

- Potential curvature singularities at $r = 0, r = \infty (r^2 \equiv z\bar{z})$ from

$$R = \frac{8b^2 k}{r_0^2} r^{2(a+b-1)} (1 + r^{2b})^{2(k-1)} \tag{20}$$

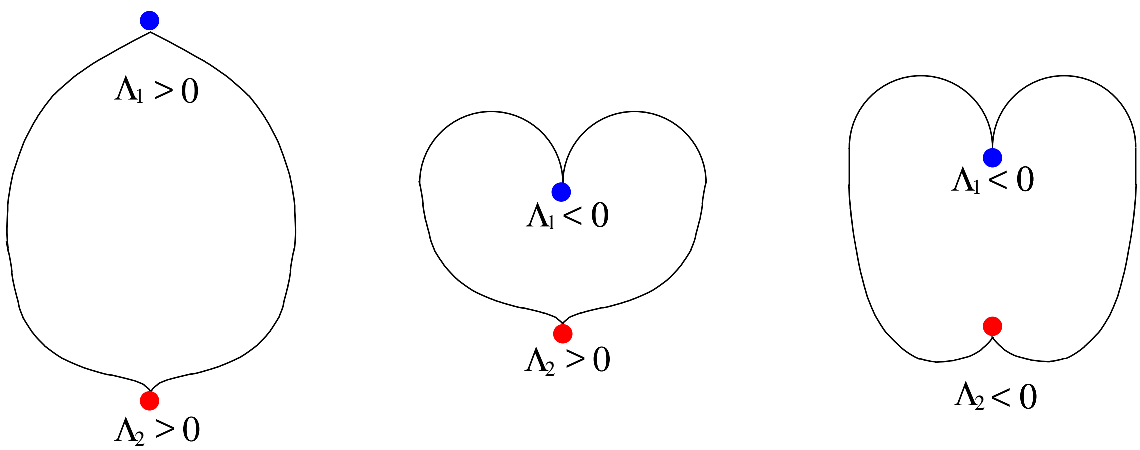
are avoided for $a + b \geq 1$ and $a + b(2k - 1) \leq 1$.

- Volume of the internal space

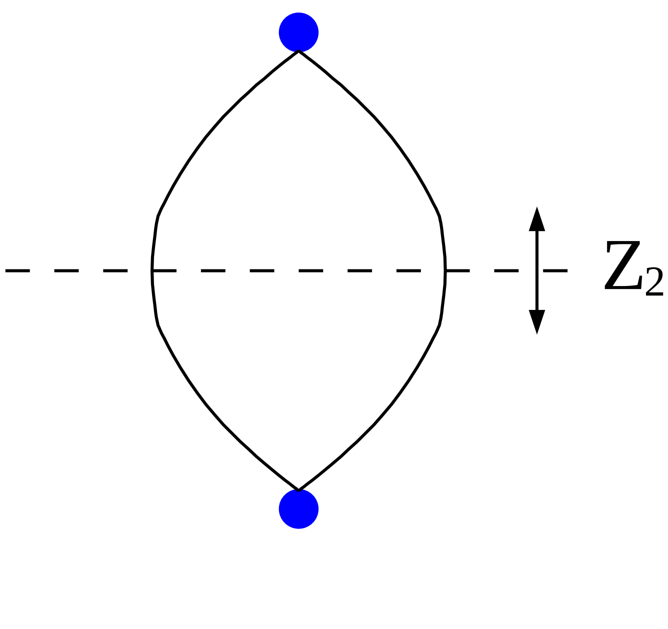
$$V = \int_0^\infty dr \frac{r^{1-2a}}{(1 + r^{2b})^{2k}} \tag{21}$$

is finite for $a < 1$ and $a + 2kb > 1$.

- The general internal spaces look like



- "Football-shaped" extra dimensions for $k = 1$ and $a + b = 1$



Single-valuedness of the scalar field

[H.M.L.,A.Papazoglou, hep-th/0407208]

- In complex coordinate $z = re^{i\psi}$, the scalar field Φ should be single-valued for a 2π rotation along ψ

$$\Phi(r, 0) = \Phi(r, 2\pi) \Rightarrow e^{2\pi i b} = 1 \Rightarrow b = \text{integer}.$$

- An integer b leads to a quantized brane tension.
- When the solution is written as

$$\Phi = (\tan\frac{\phi^1}{2})e^{i\phi^2} \qquad (22)$$

with $\phi^1 = 2\arctan(r^b)$ and $\phi^2 = b\psi + c$, move to a field

$$X = (\tan\frac{\phi^1}{2})e^{iK(\phi^2)}. \qquad (23)$$

- The periodic condition for X becomes

$$K[\phi^2(2\pi)] = K[\phi^2(0)] + 2\pi n. \qquad (24)$$

  – e.g. $K(\phi^2) = \phi^2 + \epsilon(\phi^2)^2$ gives a single-valued X if

$$c = -\pi b + \frac{1}{2\epsilon}\left(\frac{n}{b} - 1\right). \qquad (25)$$

- One can choose a nontrivial mapping from a holomorphic multi-valued Φ to a non-holomorphic single-valued X for the SAME spacetime metric.

Nearby curved solutions

[H.M.L.,A.Papazoglou, hep-th/0407208]

- Add a nonzero bulk cosmological constant Λ_b.
- For a maximally symmetric 4d solution, assume the factorizable extra dimensions:

$$ds^2 = g^{(4)}_{\mu\nu}(x)dx^\mu dx^\nu + ds_2^2 \tag{26}$$

with $R^{(4)}_{\mu\nu} = 3\lambda g^{(4)}_{\mu\nu}$.

- One finds a solution with "football-shaped" extra dimensions

$$ds_2^2 = r_0^2|z|^{-2a}\frac{dzd\bar{z}}{(1+|z|^{2(1-a)})^2}, \quad \Phi = e^{ic}z^{1-a} \tag{27}$$

with

$$\lambda = \frac{\Lambda_b}{6}, \quad \Lambda_1 = \Lambda_2 = 2\pi a. \tag{28}$$

- The radius r_0 is fixed

$$r_0^2 = \frac{8(1-k)(1-a)^2}{\Lambda_b}. \tag{29}$$

- The flat solution with $k=1, \Lambda_b = 0$ is continuously connected to curved solutions: dS_4 for $0<k<1, \Lambda_b>0$, and AdS_4 for $k>1, \Lambda_b<0$.

Comparison to hyperbolic σ−model

[M.Gell-Mann,B.Zwiebach, PLB147 (1984) 111]

[M.Gell-Mann,B.Zwiebach, NPB260 (1985) 569]

[A.Kehagias, hep-th/0406025]

- Take a hyperbolic manifold of $H_2 = SU(1,1)/U(1)$ for a complex scalar field:

$$d\sigma_f^2 = \frac{4d\Phi d\bar{\Phi}}{(1-|\Phi|^2)^2}. \tag{30}$$

- $k = \frac{1}{2}$ and $\Lambda_b = 0$ by supersymmetry.
- For $\Phi = e^{ic}z^b$ and factorized extra dimensions, a flat solution("Tear-drop") was found

$$ds_2^2 = r_0^2|z|^{-2a}(1-|z|^{2b})dzd\bar{z} \tag{31}$$

with

$$\Lambda_1 = 2\pi a. \tag{32}$$

- Naked singularity at $|z| = 1$

$$R = \frac{4b^2}{r_0^2}\frac{|z|^{2(a+b-1)}}{(1-|z|^{2b})^3}. \tag{33}$$

- One restricts to $|z| < 1$(open disc) and imposes b.c.'s to field fluctuations for no flow of energy, angular momentum and $U(1)$ charge to $|z| = 1$.
- Finite volume for $a < 1, b > 0$.

Conclusion

- The deficit angle of two extra dimensions gives a new possibility of self-tuning.
- Flux compactifications are not working for self-tuning because of flux quantization.
- Compactification by $\sigma-$model scalars leads to a self-tuning without the same problem.
- In spherical $\sigma-$model,
  - No naked singularies, finite volume of internal space
  - Positive brane tensions possible
  - $\Lambda_b = 0$ not guaranteed in spherical sigma model: Warped solutions for $\Lambda_b \neq 0 \Rightarrow$ a non-holomorphic Φ?
- In hyperbolic $\sigma-$model,
  - Noncompact extra dimensions
  - Boundary conditions at the naked singularity needed
  - $\Lambda_b = 0$ guaranteed by supersymmetry
  - Broken supersymmetry at low energy spoils the self-tuning?
- Radius stabilization with the self-tuning?