A Brane model with two asymptotic regions.

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Plan of the talk

- *The Kip Thorne Interpretation Of The Melvin Solution.*
  - $\rho = \infty$*: asymptotic region near the symmetry axis.*
- *Inclusion Of A Complex Higgs Field.*
  - *divergence of the energy.*

- Solution Interpolating Between The Cosmic String And The Melvin Solution.

  - potential.
  - causal structure.

- No Trapping of Gravity.

References.

- Kip.S.Thorne, Phys.Rev. 138(1965) B251.
- Gary W. Gibbons, Daisuke Ida, Tetsuya Shiromizu, Phys.Rev.D66(2002)044010.
- J. Louko, D.L. Wiltshire, JHEP 0202:007,2002.
- B. de Carlos, J.M. Moreno, JHEP 0311:040,2003.
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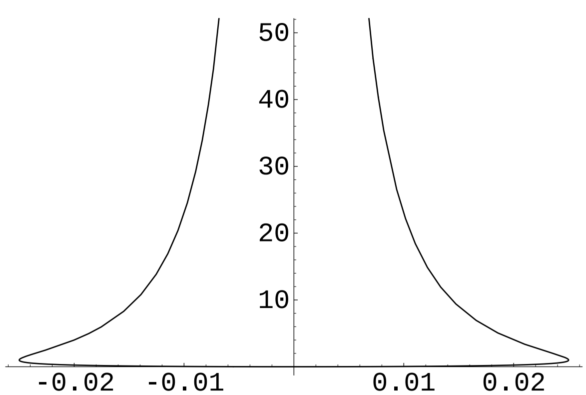
The Kip Thorne Interpretation Of The Melvin Solution.

$$S = \int d^d x \sqrt{|g|} \left[-\frac{1}{4} F_{ab} F^{ab} + \frac{R}{16\pi G} \right].$$

$$\begin{aligned} ds^2 &= \left(1 + \frac{\rho^2}{a^2}\right)^{2/(d-3)} \left(\eta_{\mu\nu} dx^\mu dx^\nu - d\rho^2\right) \\ &- \rho^2 \left(1 + \frac{\rho^2}{a^2}\right)^{-2} d\phi^2 \quad , \\ F &= B_0 \left(1 + \frac{\rho^2}{a^2}\right)^{-2} \rho \, d\rho \wedge d\phi \quad , \end{aligned}$$



$\rho = \infty$: asymptotic region near the symmetry axis.

Inclusion Of A Complex Higgs Field.

Gibbons et al: real scalar field.

$$S = \int d^6x \sqrt{-g} \left[\frac{1}{2} D_\mu \Phi D^\mu \Phi^* - \frac{\lambda}{4} (\Phi\Phi^* - v^2)^2 \right].$$

$$\begin{aligned} ds^2 &= \beta^2(\rho)\eta_{\mu\nu}dx^\mu dx^\nu - \gamma^2(\rho)d\rho^2 - \alpha^2(\rho)d\phi^2 \\ &\quad \text{where} \quad \mu = 0, \cdots 3 \quad , \\ \Phi &= v f(\rho) e^{i\phi} \quad \text{and} \quad A_\phi = \frac{1}{e}(1 - p(\rho)) \quad . \end{aligned}$$

Energy density

$$\begin{aligned} \epsilon(\rho) &= \sqrt{|g|} \left[\frac{1}{2} g^{\rho\rho} |D_\rho \Phi|^2 + \frac{1}{2} g^{\phi\phi} |D_\phi \Phi|^2 + \frac{1}{4} F_{\rho\phi} F^{\rho\phi} \right. \\ &+ \left. \frac{\lambda}{4} (\Phi^* \Phi - v^2)^2 \right]. \end{aligned}$$

Higgs potential

$$\begin{aligned} \rho &= \infty \quad \text{"symmetry axis"} \Rightarrow \quad f(\infty) = 0 \\ &\quad V(\Phi) \sim \rho^{7/3} \lambda v^4 \end{aligned}$$

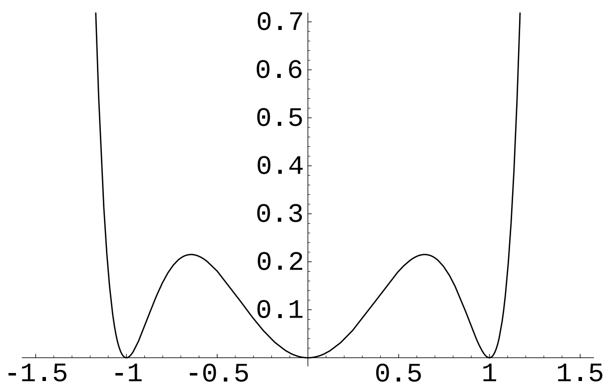
divergence.

Solution Interpolating Between The Cosmic String And The Melvin Solution.

$$V(0) = 0 \quad .$$

Susy like potential

$$V(\Phi) = \lambda e^{w^2 \Phi\Phi^*} \Phi\Phi^* (\Phi\Phi^* - v^2)^2 \quad .$$



$$\mu = G v^2 \quad , \quad \nu = \lambda^2 G^{-3} \quad , \quad \tau = e^2 v \quad \text{and} \quad \sigma = w^2 v^2 \quad .$$

$$\rho = L\, x \qquad \text{where} \quad L = \frac{1}{2\sqrt{\pi}\mu^{5/4}\nu^{1/4}} \frac{1}{\sqrt{v}}$$

Boundary conditions

ρ: a length(cartesian) coordinate along the meridian.

$\rho \longrightarrow -\infty$:far region of the cosmic string solution:

$$\begin{aligned} A(x) &\sim x \quad , \quad B(x) \sim 1 \quad , \quad \bar{\gamma} \sim 1 \quad , \\ f(x) &\sim 1 \quad , \quad p(x) \sim 0 \end{aligned}$$

$\rho \longrightarrow \infty$: asymptotic region of the Melvin solution:

$$\begin{aligned} A(x) &\sim \frac{a^2}{x} \quad , \quad B(x) \sim \bar{\gamma}(x) \sim \left(\frac{x^2}{a^2}\right)^{1/3} \quad , \\ f(x) &\sim 0 \quad , \quad p(x) \sim 1 \quad . \end{aligned}$$

True radial coordinate.

$$r = \int_0^\rho \gamma(y) dy$$

Field Equations

$$
\begin{aligned}
& e^{\sigma F^2(x)} B(x) F^2(x) \bar{\gamma}^2(x)(1-F^2(x))^2 + \frac{A'(x)B'(x)}{A(x)} \\
+ \quad & 3\frac{B'^2(x)}{B(x)} - \frac{B'(x)\bar{\gamma}'(x)}{\bar{\gamma}(x)} - 2\frac{B(x)P'(x)^2}{A^2(x)} + B''(x) = 0 \quad ,
\end{aligned}
$$

$$
\begin{aligned}
& \frac{1}{4} e^{\sigma F^2(x)} B(x) F^2(x) \bar{\gamma}^2(x)(1-F^2(x))^2 + 2\pi\mu B(x) F'^2(x) \\
- \quad & \frac{B(x)A'(x)\bar{\gamma}'(x)}{4A(x)\bar{\gamma}(x)} - \frac{B'(x)\bar{\gamma}'(x)}{\bar{\gamma}(x)} + 3\frac{B(x)P'^2(x)}{2A^2(x)} \\
+ \quad & \frac{B(x)A''(x)}{4A(x)} + B''(x) = 0 \quad ,
\end{aligned}
$$

$$
\begin{aligned}
& e^{\sigma F^2(x)} B(x) F^2(x) \bar{\gamma}^2(x)(1-F^2(x))^2 \\
+ \quad & \frac{2\tau}{\pi\mu^{5/2}\sqrt{\nu}} \frac{F^2(x)\bar{\gamma}^2(x)P^2(x)}{A(x)} + 4\frac{A'(x)B'(x)}{B(x)} - \frac{A'(x)\bar{\gamma}'(x)}{\bar{\gamma}(x)} \\
& +6\frac{P'^2(x)}{A(x)} + A''(x) = 0 \quad ,
\end{aligned}
$$

$$
\begin{aligned}
& \frac{\tau}{2\pi\mu^{5/2}\sqrt{\nu}} F^2(x)\bar{\gamma}^2(x)P(x) - \frac{A'(x)P'(x)}{A(x)} + 4\frac{B'(x)P'(x)}{B(x)} \\
- \quad & \frac{\bar{\gamma}'(x)P'(x)}{\bar{\gamma}(x)} + P''(x) = 0
\end{aligned}
$$

$$
\frac{1}{2\pi\mu} e^{\sigma F^2(x)} F(x) \bar{\gamma}^2(x)(1 + (-4+\sigma)F^2(x) \quad +
$$

$$(3-2\sigma)F^4(x)+\sigma F^6(x)) - \frac{\tau}{4\pi^2\mu^{7/2}\sqrt{\nu}}\frac{F(x)\bar{\gamma}^2(x)P(x)}{A^2(x)}$$

$$\frac{A'(x)F'(x)}{A(x)} + 4\frac{B'(x)F'(x)}{B(x)} - \frac{F'(x)\bar{\gamma}'(x)}{\bar{\gamma}(x)} + F''(x) = 0 \quad .$$

The causal structure.

$$\left.\begin{array}{c}\bar{u}\\ \bar{v}\end{array}\right\} = \arctan[t \mp \sigma(\rho)] \quad ,$$

$$\sigma(\rho) = \int_0^\rho d\xi \frac{\gamma(\xi)}{\beta(\xi)}$$

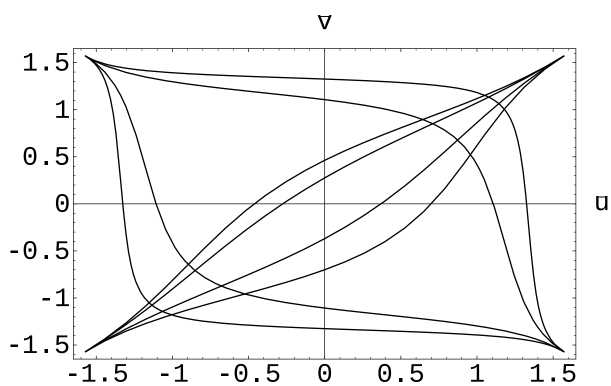


Figure 1: The causal structure of the solution is given. The time-like infinities are $I^+(\bar{u} = \bar{v} = \pi/2)$ and $I^-(\bar{u} = \bar{v} = -\pi/2)$. As ρ can change sign, there are two space infinities : $I_o^>(-\pi/2, \pi/2)$ and $I_o^<(\pi/2, -\pi/2)$. The curves which begin at I^- and end at I^+ correspond to fixed values of ρ while the others correspond to fixed t.

Trapping Of Gravity.

$$\int dx^4 dx^5 g^{00} \sqrt{|g|} < \infty$$

not satisfied.

Conclusions.

- Melvin solution, $\rho \longrightarrow \infty$, vanishing complex scalar field. Higgs potential excluded.
- Potential with one maximum and two minima.
- Solution interpolating between The Melvin and the cosmic string solutions.
- No trapping of gravity.