Gauge Couplings

at

High Temperature

and the

Relic Gravitino Abundance

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DESY workshop '04

Based on W. Buchmüller, K. Hamaguchi & M.R., Phys. Lett. **B** 574, 156 (2003)
W. Buchmüller, K. Hamaguchi, O. Lebedev & M.R., hep-th/0404168

Outline

1 Main idea: Temperature effects for field-dependent couplings

2 Example: Gauge couplings at high temperature

3 Application: Solution of the gravitino problem

Temperature effects for field-dependent couplings

■ (Gauge) couplings at finite T

☞ **Generic situation**:
couplings are determined by expectation values of (singlet) fields

$$g = g(\langle S \rangle)$$

☞ At finite temperature: potential of S changes

$$\mathscr{V}_T(S) = \mathscr{V}_0(S) + \mathcal{F}(S,T)$$

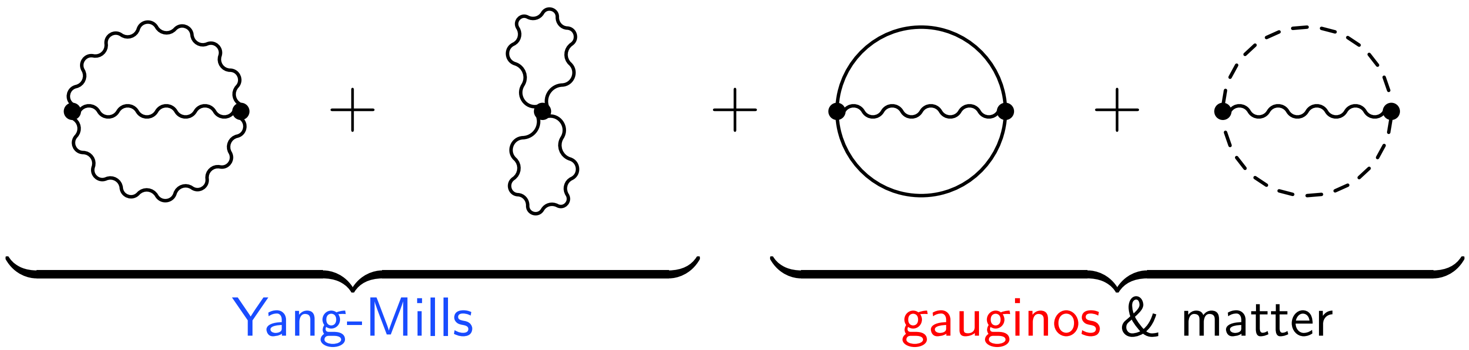
➥ Expectation value of S is shifted (= minimum of $\mathscr{V}_T$)

➥ **Main message**: Couplings change at finite temperature:

$$g(\langle S \rangle_T) \neq g(\langle S \rangle_0) \quad \curvearrowright \quad g(T>0) \neq g(T=0)$$

Free energy

☞ Free energy of super Yang-Mills theory (at two-loop)



☞ e.g. $\mathrm{SU}(N_c)$ theory with N_f matter multiplets in fundamental representation

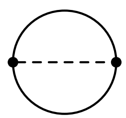
$$\mathcal{F}(g,T) = -\frac{\pi^2 T^4}{24} \times \left\{ a_0 + a_2\, g^2 + \mathcal{O}(g^3) \right\}$$

$$a_0 = N_c^2 + 2N_cN_f - 1$$

$$a_2 = -\frac{3}{8\pi^2}(N_c^2 - 1)(N_c + 3N_f)$$

☞ **Crucial**: $a_2 < 0 \Rightarrow \mathcal{F}$ increases with increasing g

. . . also true at higher loop or lattice level

☞ Analogous analysis applies for Yukawa couplings as well ()

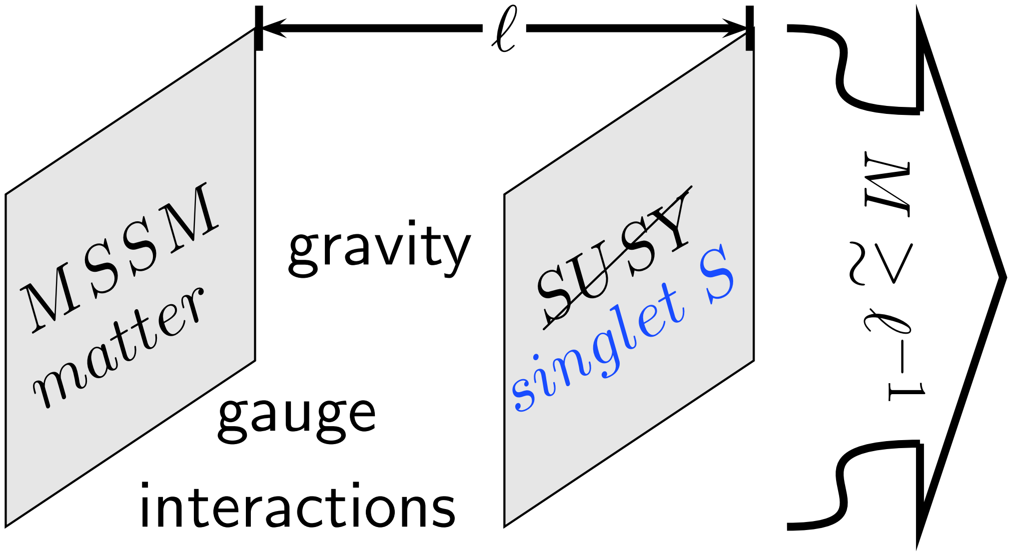
Gauge couplings
at
high temperature

. . . using gaugino mediation as an example

Field-dependent gauge couplings

☞ E.g. gaugino mediation

Kaplan, Kribs & Schmaltz '99
Chacko, Luty, Nelson & Ponton '99



$$\langle S|_{\theta=0}\rangle_{T=0} = 0$$

$$\mathscr{L}_{\text{eff}}^{4\text{D}} \supset \int \mathrm{d}^2\theta \left(\frac{1}{g_0^2} + \frac{S}{M}\right) W^\alpha W_\alpha + \text{h.c.}$$

➥ $g^2 = g_0^2 \dfrac{1}{1 + g_0^2\left(\dfrac{\langle\phi\rangle}{M} + \dots\right)}$ and $m_{\widetilde{g}} = \dfrac{g^2}{2}\dfrac{\langle F_S\rangle}{M} \overset{T=0}{=} \dfrac{g_0^2}{2}\dfrac{\langle F_S\rangle}{M}$

$$\phi = \mathrm{Re}\, S|_{\theta=0}$$

➥ Gauge couplings field-dependent in effective theory

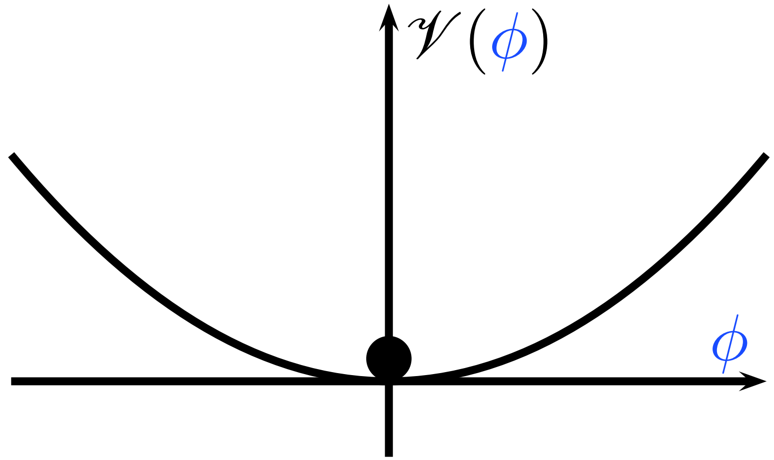
. . . true under different/more general assumptions

Relations at $T = 0$

☞ At $T = 0$

$$\mathscr{V}(\phi) = \frac{1}{2} m_\phi^2 \, \phi^2 + \ldots$$

$$m_\phi^2 = \xi \, m_{3/2}^2$$



☞ Usual assumption: $\xi = \mathscr{O}(1)$ (. . . not crucial)

☞ Mass relations:

$\mathscr{O}(1)$

$$m_{3/2} = \eta \frac{\langle F_S \rangle}{M_\mathrm{P}} < m_{\widetilde{g}} = g_0^2 \frac{\langle F_S \rangle}{M} \qquad \text{for } M < M_\mathrm{P}$$

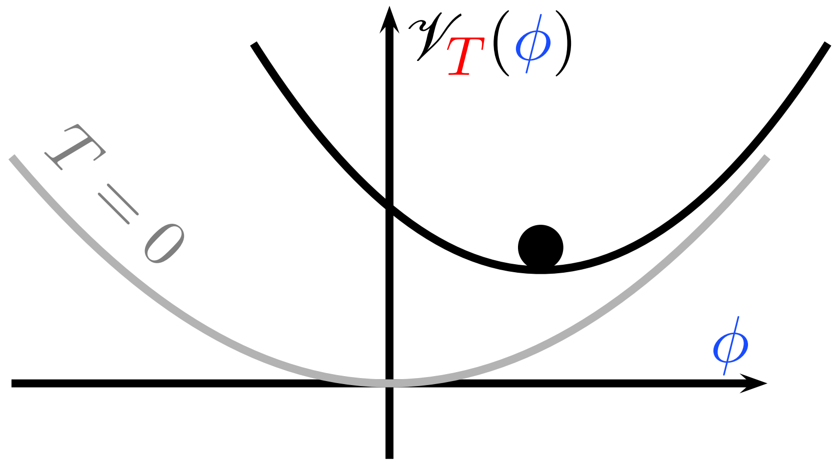
➥ Gravitino is naturally LSP !

■ Gauge couplings at $T > 0$

☞ Free energy: $\mathcal{F} = -a_0 T^4 + a_2\, g^2(\phi)\, T^4 + \ldots$

➥ Effective potential for ϕ

$$\mathscr{V}_T = \frac{m_\phi^2}{2}\phi^2 + \frac{a_2\, g_0^2}{1 + g_0^2\left(\frac{\phi}{M} + \ldots\right)} T^4 + \ldots$$



☞ $T, \phi \ll M$: $\langle\phi\rangle_T = \frac{a_2\, g_0^4}{\xi}\frac{T^4}{m_{3/2}^2\, M}$

☞ Include higher order terms ↷

$$\boxed{\langle\phi\rangle_T \sim M\ (T/T_*)^{\alpha}}$$

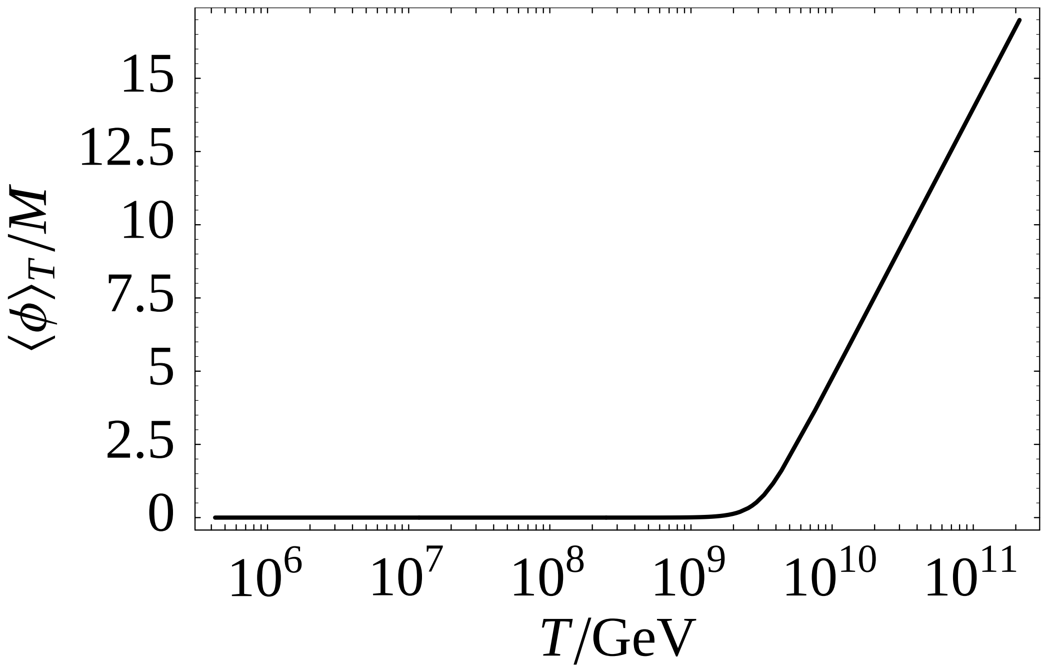
$\alpha > 1$

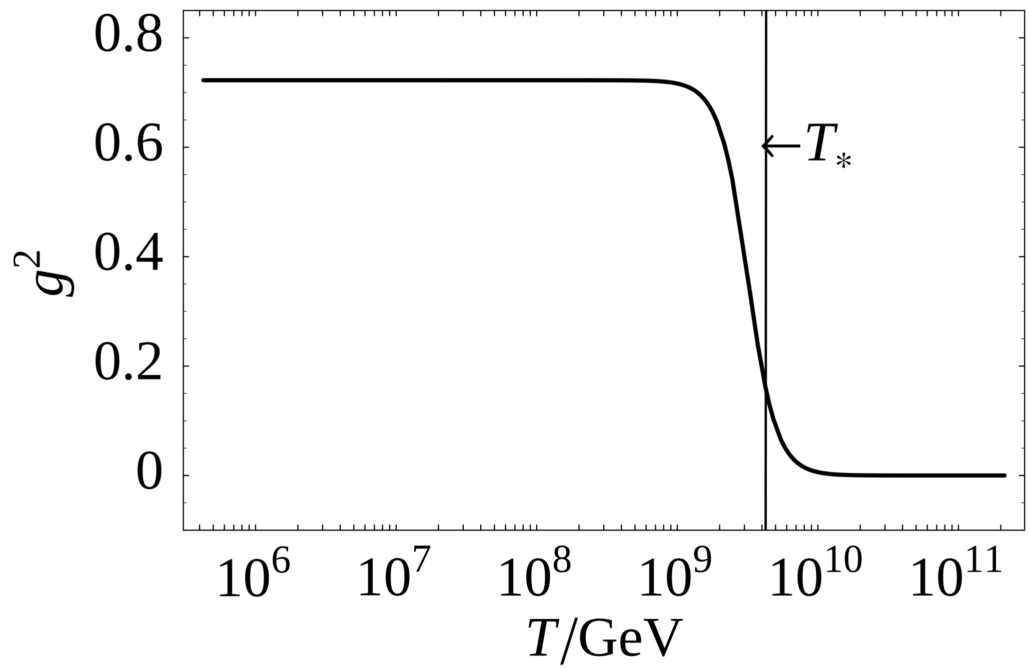
■ Gauge couplings at $T > 0$

☞ Insert $\langle\phi\rangle_T$ into $g(\phi)$ ↷ Temperature-dependent gauge couplings

$$g^2(T) = g_0^2 \frac{1}{1 + \left(\frac{T}{T_*}\right)^\alpha}$$

$$T_* \simeq \xi^{1/4} \sqrt{m_{3/2} M}$$





Solution
of the
gravitino problem

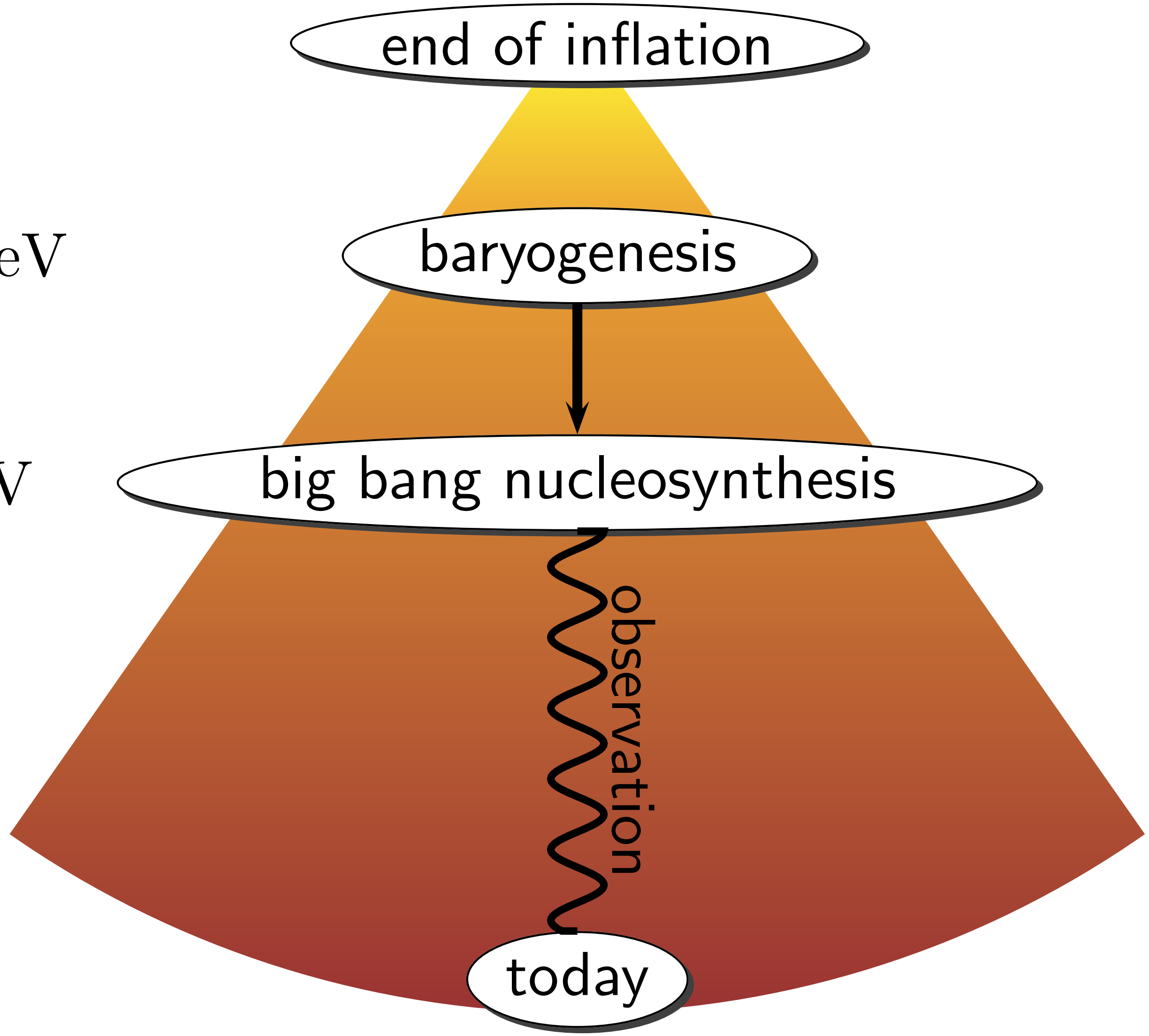
. . . using gauge couplings at high temperature

Gravitino problem

Thermal history of the universe

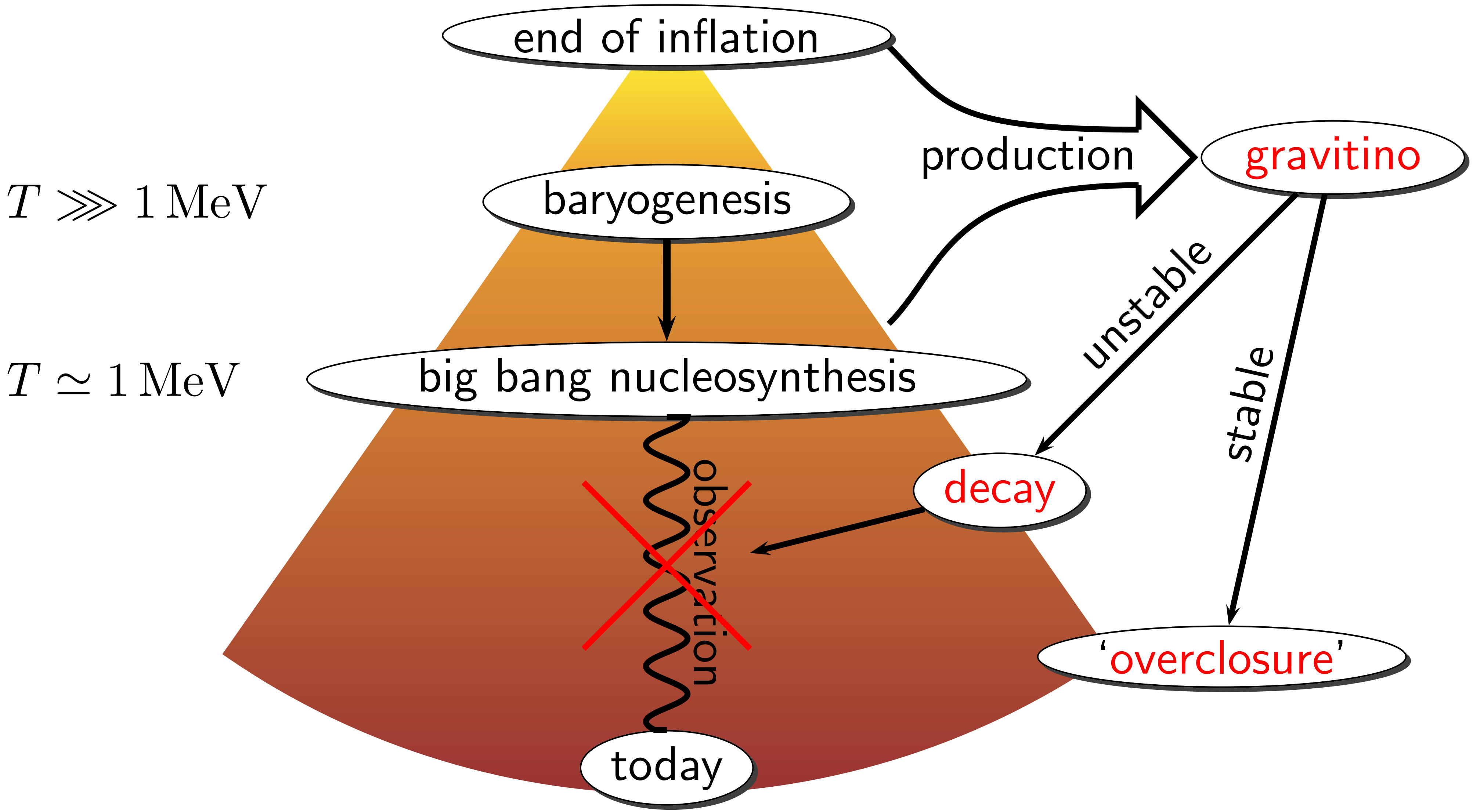
$T \ggg 1\,\mathrm{MeV}$

$T \simeq 1\,\mathrm{MeV}$



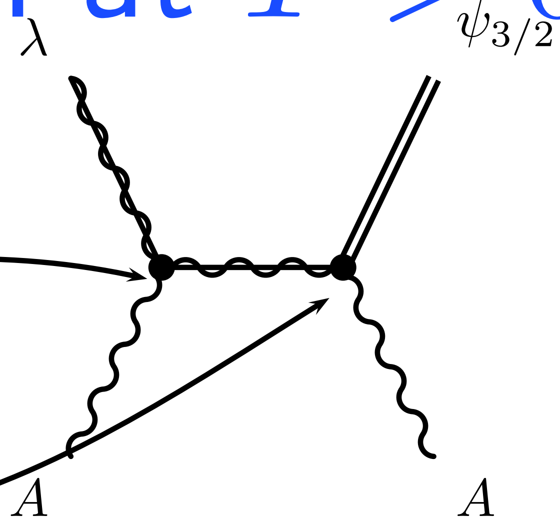
Gravitino problem

Thermal history of the universe with gravitino



■ Gravitino production at $T > 0$

☞ Gravitino production collision term



$$C_{3/2} \simeq 0.03 \times g^2 \frac{T^6}{M_{\mathrm{P}}^2} \left(1 + \frac{m_{\widetilde{g}}^2}{3\, m_{3/2}^2} \right) \times \text{(thermal correction factor)}$$

Bolz, Brandenburg & Buchmüller '00

☞ Include behavior of gauge coupling

$$g^2 \to g^2(T) \quad \text{and} \quad m_{\widetilde{g}}^2 = \frac{g_0^4}{4} \left(\frac{\langle F_S \rangle}{M} \right)^2 \to \frac{g^4(T)}{4} \left(\frac{\langle F_S \rangle}{M} \right)^2$$

➥ Gravitino production strongly suppressed for $T \gtrsim T_*$

The relic gravitino abundance

☞ For $m_{\tilde{g}} \gg m_{3/2}$ — Bolz, Brandenburg & Buchmüller '00

$$\Omega_{3/2} h^2 = 0.21 \times \left(\frac{T_R}{10^{10}\,\mathrm{GeV}}\right) \times \left(\frac{100\,\mathrm{GeV}}{m_{3/2}}\right) \times \left(\frac{m_{\tilde{g}}}{1\,\mathrm{TeV}}\right)^2$$

☞ Switch off production above T_* $\curvearrowright$ replace T_R by T_*

$$\left(\frac{\rho_{\mathrm{crit}}}{s} M_{\mathrm{P}}\right) \times \Omega_{3/2} h^2 \propto m_{\tilde{g}}^2 \frac{T_*}{m_{3/2}} \sim \xi^{1/4} m_{\tilde{g}}^2 \sqrt{\frac{M}{m_{3/2}}} \sim \xi^{1/4} m_{\tilde{g}}^{3/2} \sqrt{M_{\mathrm{P}}}$$

$$T_* \simeq \xi^{1/4} \sqrt{m_{3/2} M}$$

$$M / M_{\mathrm{P}} \sim m_{3/2} / m_{\tilde{g}}$$

The relic gravitino abundance

W. Buchmüller, K. Hamaguchi & M.R. '03

☞ Final result

$$\Omega_{3/2}h^2 \simeq 0.1 \times \left(\frac{m_{\widetilde{g}}}{1\,\mathrm{TeV}}\right)^{3/2} \times \left(\frac{\xi}{\eta^2}\right)^{1/4}$$

☞ Invert (by using $\Omega_{3/2} \stackrel{!}{=} \Omega_{\mathrm{CDM}}$)

$$m_{\widetilde{g}} = \left(\frac{\eta^2}{\xi}\right)^{1/6} \times (0.5 - 2)\,\mathrm{TeV}$$

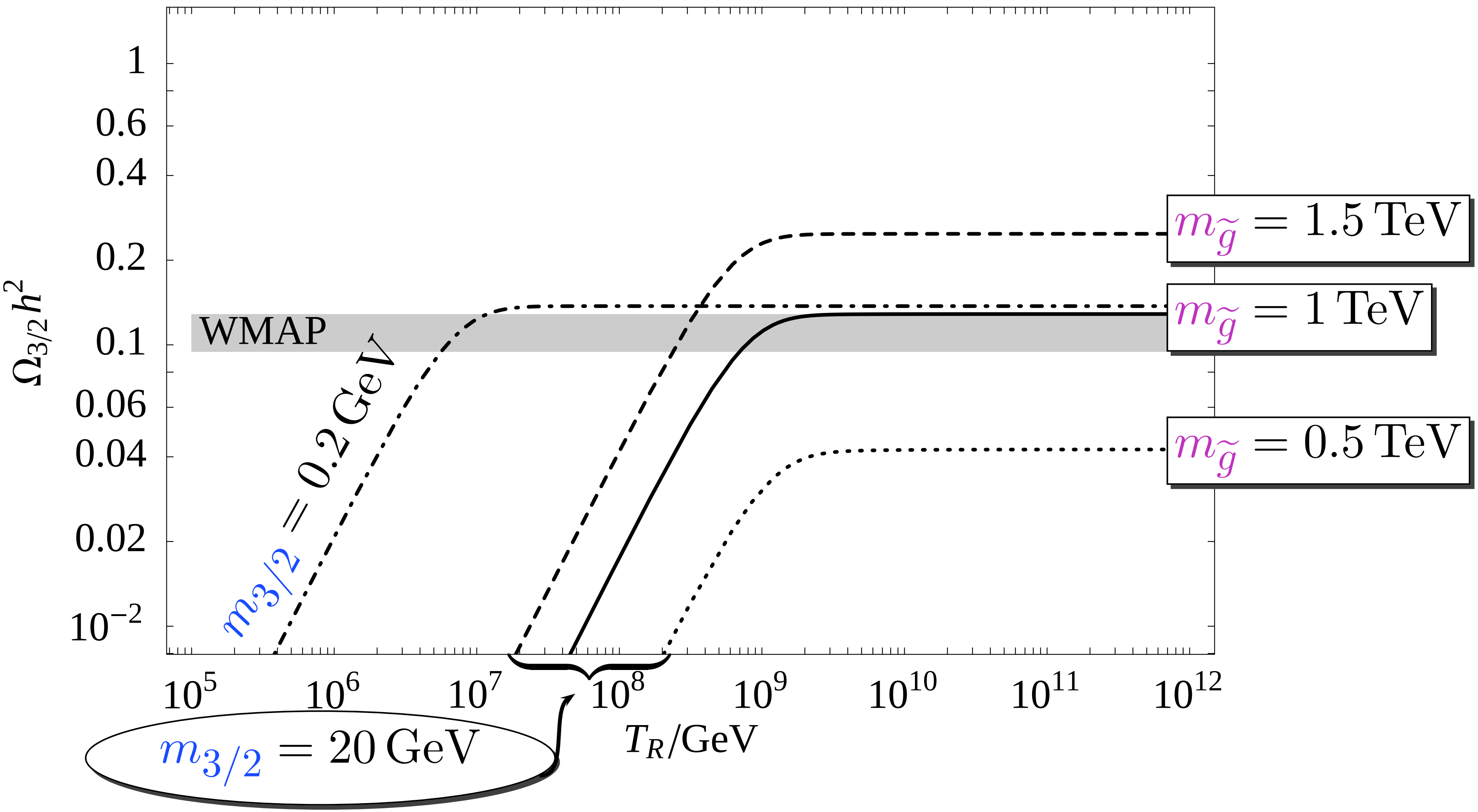
➥ Gluino mass in terms of only present dark matter density and M_{P} !

➥ Confirm or rule out at LHC !

➥ Test the gravitino LSP and prove the existence of supergravity in nature

The relic gravitino abundance

W. Buchmüller, K. Hamaguchi & M.R. '03



Summary

☞ Buy

$$\mathscr{L}_{\text{eff}} \supset \left(\frac{\phi}{M} + \ldots\right)\left(-\frac{1}{4}F^{a}_{\mu\nu}F^{\mu\nu}_{a} + \ldots\right)$$

➥ Gauge couplings switch off at $T_* \sim \sqrt{M\, m_\phi}$ (independently of SUSY)

☞ Buy (in addition) $m_\phi \sim m_{3/2}$ within SUSY

➥ Gravitino (overclosure) problem solved in an elegant way provided that:

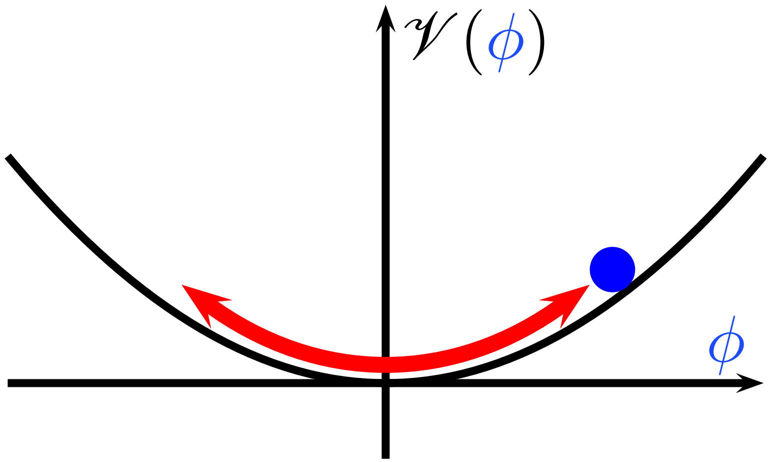
- $m_{\widetilde{g}} \simeq 1\,\mathrm{TeV}$ and
- gravitino is LSP

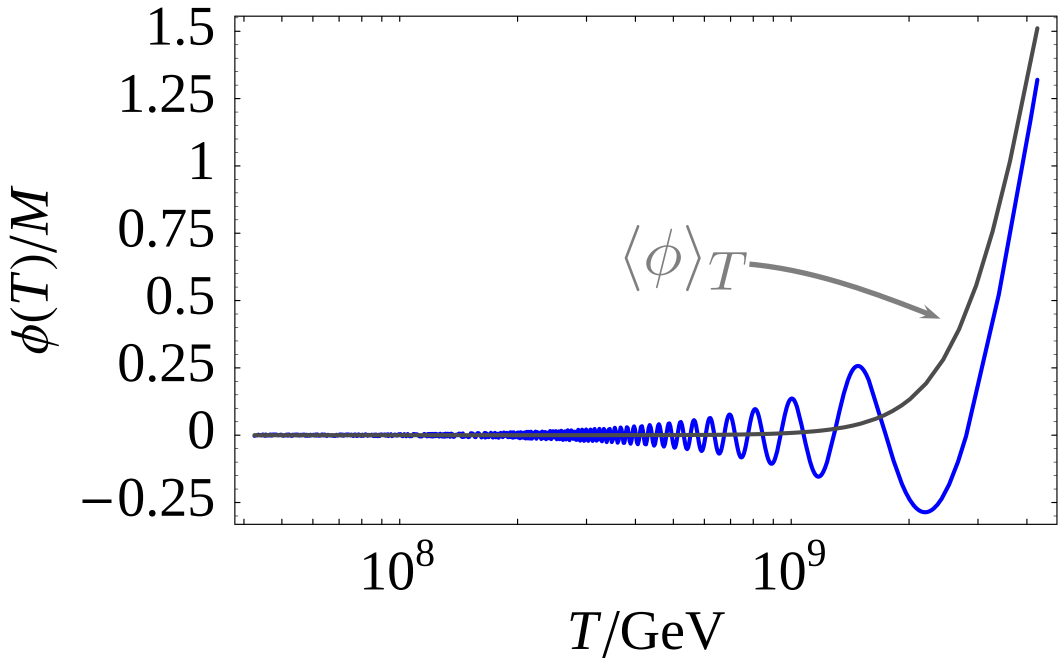
☞ Mechanism works more generally: gravity mediation, gauge mediation, etc.

■ However: 'moduli problem'

☞ ϕ does not follow the thermal expectation value

➥ Oscillation around minimum



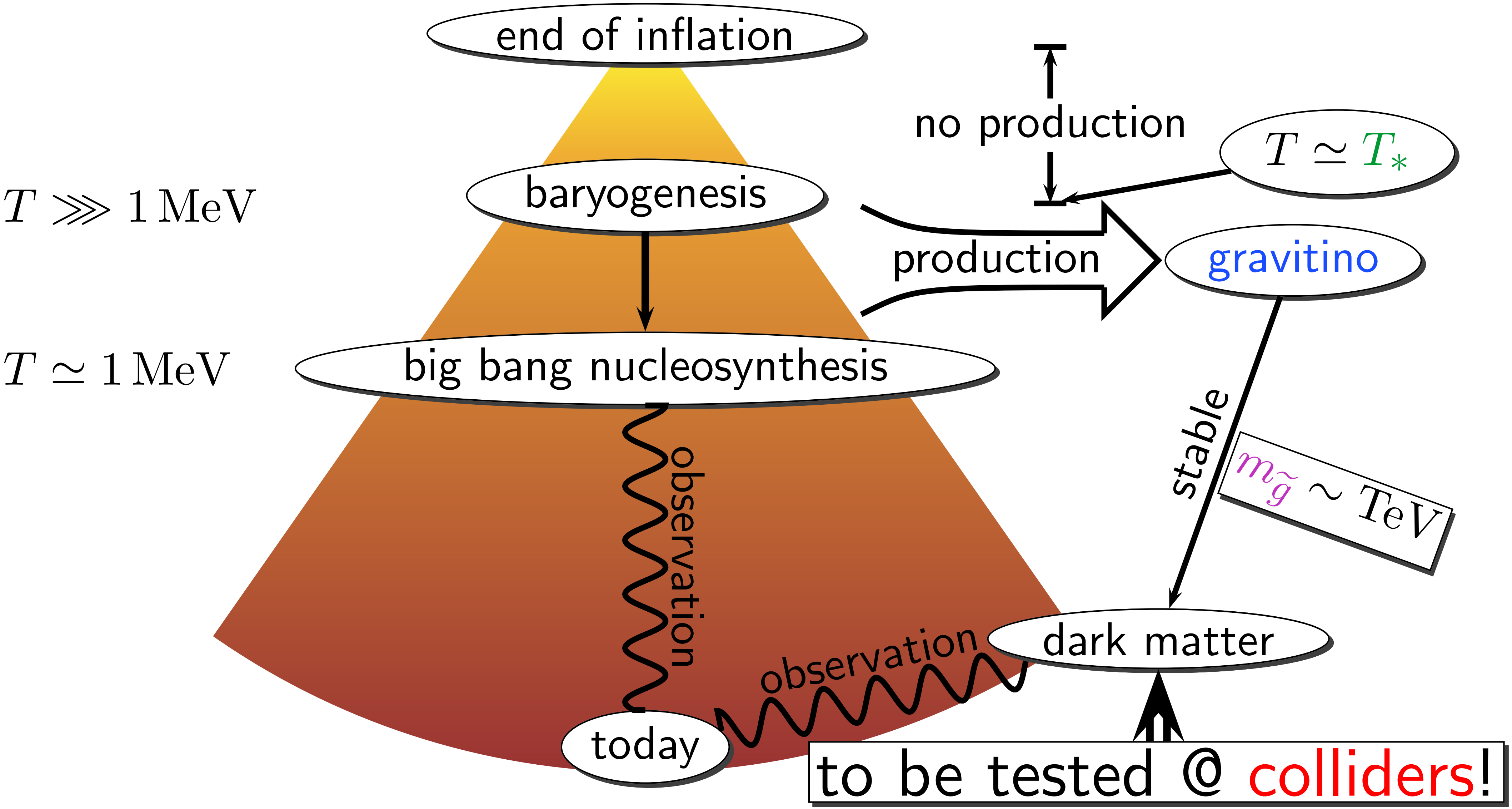


➥ ρ_ϕ tends to dominate universe (and to spoil BBN)

. . . currently under discussion

Summary: Gravitino ~~problem~~

Thermal history of the universe with gravitino



Additional slides

■ Unstable gravitinos

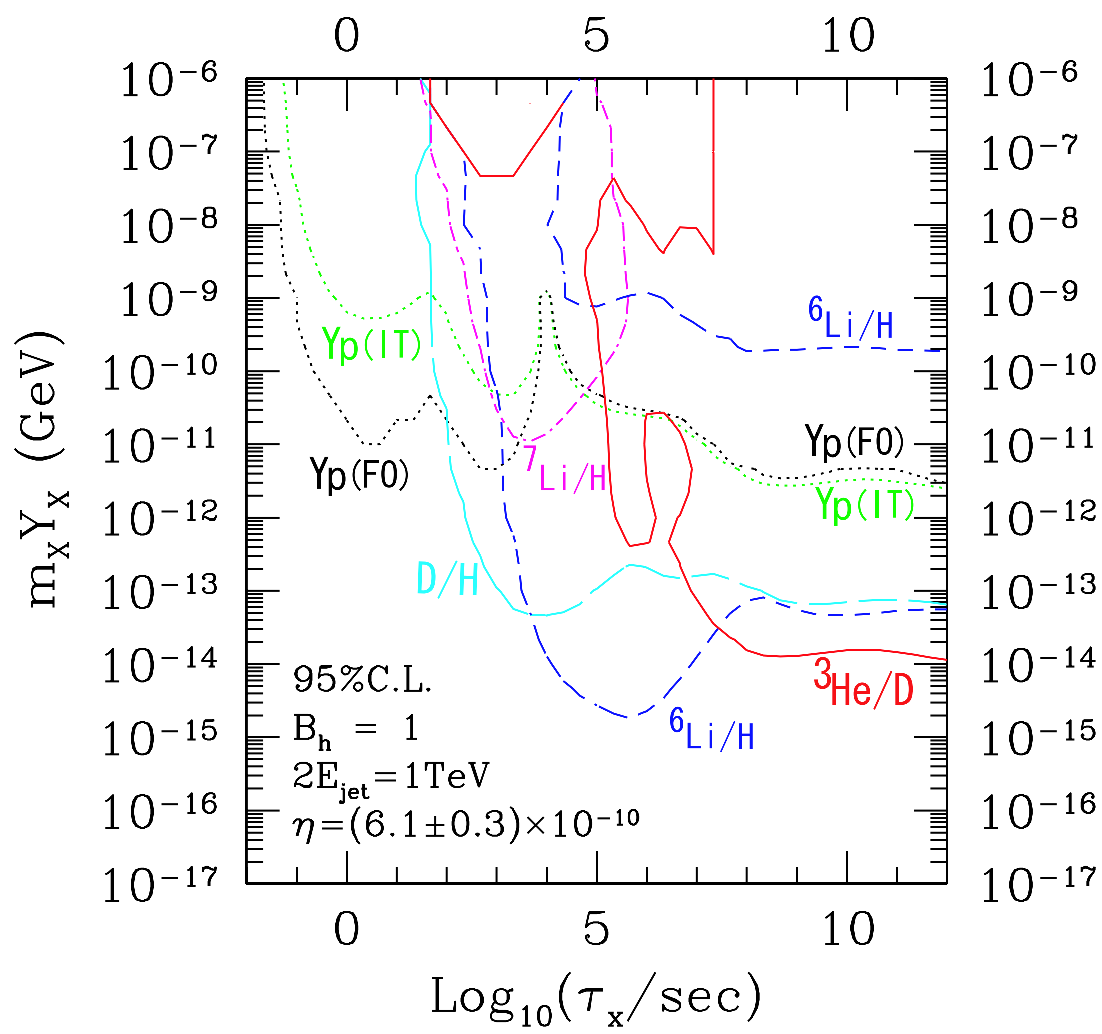
. . . ; Kawasaki, Kohri & Moroi '04; . . .

☞ Late decay spoils BBN

➥ Lower bound on $m_{3/2}$
and/or
upper bound on T_R (in GeV)

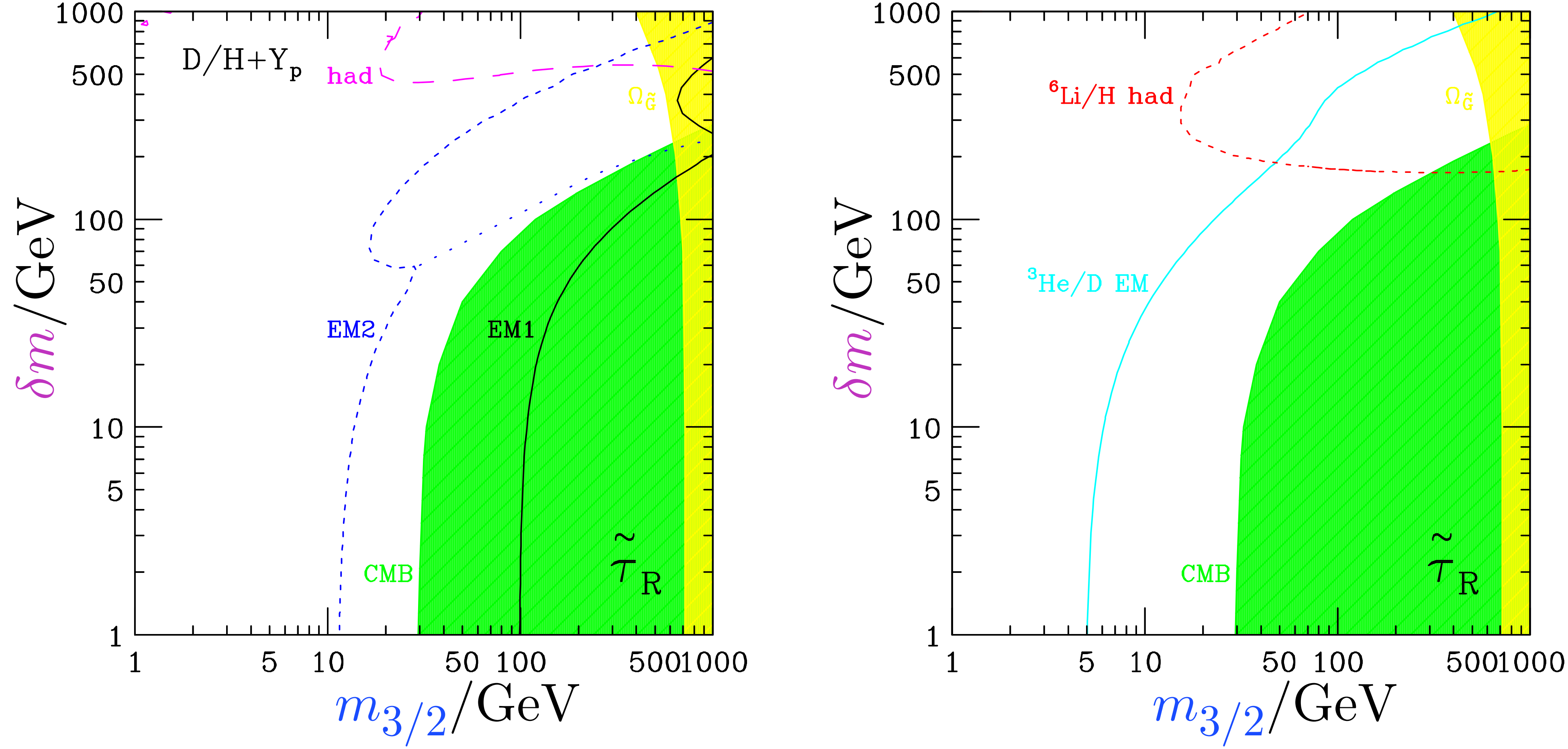
$m_{3/2}$	$B_h = 10^{-3}$	$B_h = 1$
100 GeV	2×10^6	1×10^6
300 GeV	3×10^6	5×10^4
1 TeV	2×10^7	2×10^4
3 TeV	4×10^7	3×10^5

(B_h: hadronic branching ratio)



$\tilde{\tau}$ NSP vs. BBN

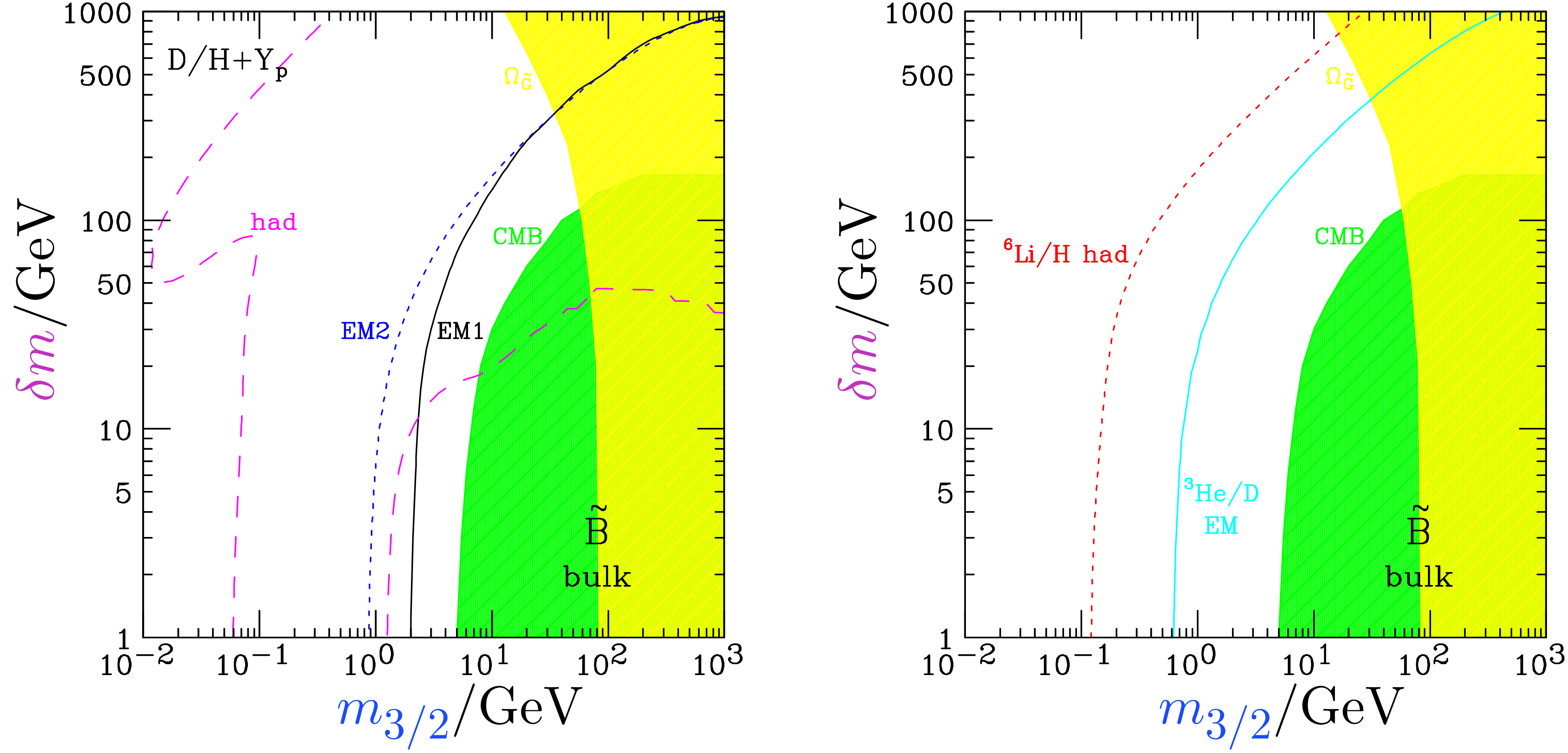
Feng, Su & Takayama '04



$$\delta m = m_{\tilde{\tau}} - m_{3/2} - m_Z$$

Bino NSP vs. BBN

Feng, Su & Takayama '04



$$\delta m = m_{\tilde{B}} - m_{3/2} - m_Z$$