Radially Inhomogeneous Cosmological Models with Cosmological Constant

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- Introduction and motivation
- CMB isotropy and cosmological principle
- Smooth fusion of local spactimes with the cosmological background
- Raidally inhomogeneous spacetimes with cosmological constant (conventional and brane cosmology)
- Concluding remarks

- Does the observed isotropy of CMB really imply that the universe is homogeneous? Inhomogeneous spacetimes have been found which allow every [comoving] observer to see an isotropic CMB (Clarkson et al. gr-qc/0302068). (I will not follow this line of work).
- How does a locally curved spacetime (e.g. that around a galaxy cluster) evolve with the expansion of the universe?
- As early as 1930s, cosmological models were investigated which described the evolution of spherical clumps in a cosmological background (the LTB models). See astro-phy/0006083

Cosmological Principle and Cosmological Spacetimes

- Provided that the spacetime (on large enough scales) is homogeneous and isotropic, the cosmological metric should be RW:
- The spatial part of RW is maximally symmetric.
- The 4-dimensional maximally symmetric spacetime is the deSitter spacetime: This spacetime is supported by the cosmological constant. It has a horizon at

$$r = \int_0^\infty \frac{c\,dt}{a(t)} = \int_0^\infty \exp(-Ht)c\,dt = c/H.$$

- This horizon has a Hawking temperature $kT_{deS} = hH/2\pi$
- The RW metric requires a perfect fluid as its matter source.
- The high isotropy of CMB when combined with the Copernican principle is NOT enough to draw conclusions about the spatial homogeneity of the universe.

Fusion of locally inhomogeneous spacetimes with a cosmological background

Examples include:

- Schwarzschild-deSitter,
- Reissner-Nordstrom-deSitter,
- LTB models,
- More exotic models like the ones I will talk about

Reissner-Nordstrom-deSitter

We have naked singularities if M<|Q|. For M>|Q| we have Hawking radiation until the temperature of the two horizons become equal and the black hole becomes extreme RN.

$$ds^2 = -V(R)dT^2 + V^{-1}(R)dR^2 + R^2 d\Omega^2, \qquad A_T = -\frac{Q}{R},$$

$$V(R) = 1 - \frac{2M}{R} + \frac{Q^2}{R^2} - \frac{1}{3}\Lambda R^2,$$

In cosmological coordiantes, the extreme RNdS spacetime takes the following form:

$$ds^2 = -\Omega^{-2}dt^2 + a^2(t)\Omega^2\left(dr^2 + r^2 d\Omega^2\right), \qquad A_t = \Omega^{-1},$$

$$\Omega = \left(1 + \frac{m}{ar}\right), \qquad a(t) = e^{Ht}, \qquad H = \pm\sqrt{\frac{\Lambda}{3}}$$

For m=0, this reduces to the usual deSitter spacetime.
The deSitter horizon and the black hole horizon are located at

$$r_\pm = \frac{1}{2a(t)|H|}\left(1 - 2M|H| \pm \sqrt{1 - 4M|H|}\right).$$

In fact, the H>0 metric describes a white hole inside a deSitter background. A q=m test particle stays at constant radial comoving coordinate, but moves out of the white hole and deSitter horizons, as the universe expands (Kastor and Traschen, 1992).

Raidally inhomogeneous spacetimes with cosmological constant

- Ansatz metric:

$$ds^2 = -dt^2 + R(t)^2[(1+a(r))dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2]$$

- Generalized anisotropic equation of state:

$$\rho + \alpha P_r + 2\beta P_t = 0,$$

- Exact solutions (class one):

$$R(t) = R_o \left[\left(Be^{a\kappa t} - 1 \right)^{-2/\kappa} + e^{at/3} \right],$$

$$a(r) = \frac{r^A}{r_0^A - r^A} \qquad A = \frac{\alpha - 1}{1 - \beta}$$

Various cases:

- A=2: The metric reduces to RW
- A>0, and A nonzero: radially deformed RW
- A<0: central wormhole (by gluing two copies of the spacetime at r=r_0), asymptotically RW.

Exoticity parameter:

$$\xi = -\frac{\rho + P_r}{\mid \rho \mid} = \frac{1-\alpha}{\alpha - \beta}.$$

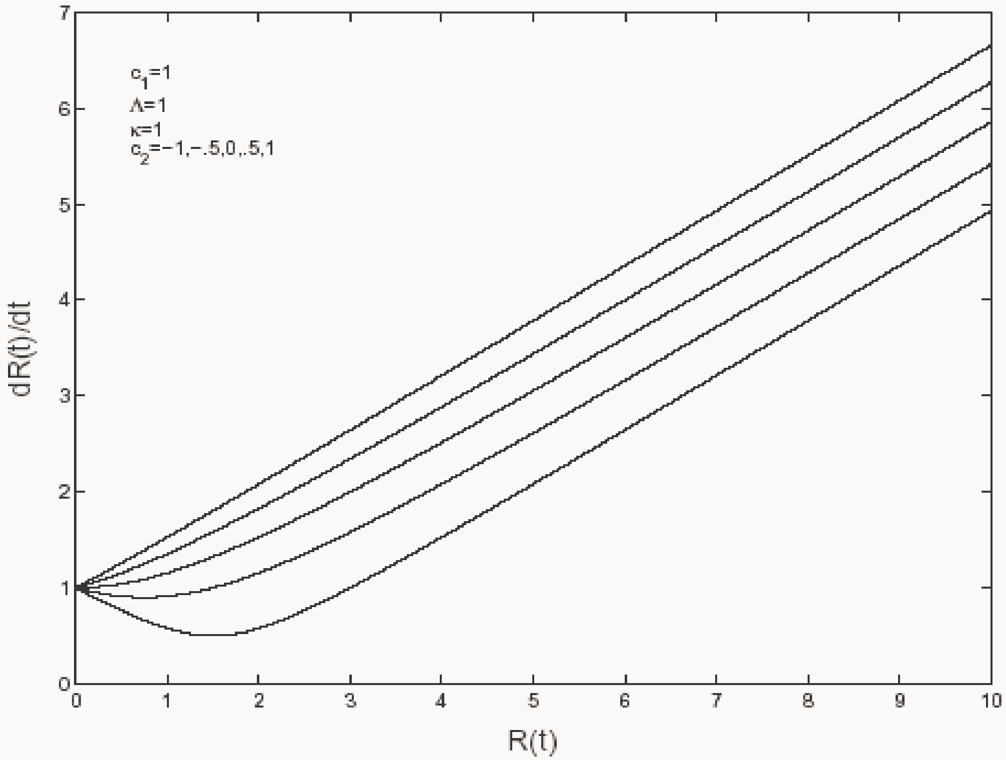
Exotic (xi>0): $\begin{cases} 1 > \alpha > \beta \\ \text{or} \\ 1 < \alpha < \beta \end{cases}$ Non-exotic (xi<0): $\begin{cases} 1 > \alpha, \ \alpha < \beta \\ \text{or} \\ 1 < \alpha, \ \alpha > \beta. \end{cases}$

Other exact solutions (class two):

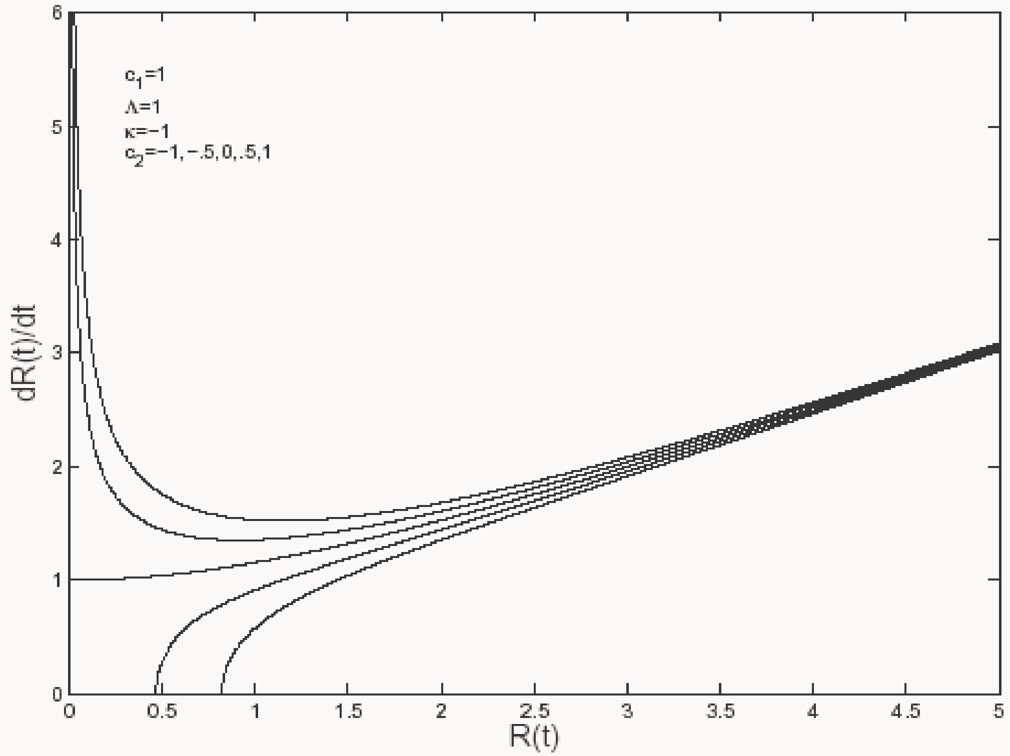
$$\dot{R}^2 = c_1 + \frac{\Lambda}{3} R^2 + c_2 R^\kappa ,$$

$$1 + a(r) = [1 + c_0 r^{-B} + \frac{C}{3 - \alpha - 2\beta} r^2]^{-1} ,$$

There are many different cases according to the values of the integration constants. Once again, the solutions include deformations of RW, naked singularities at r=0, and expanding wormholes.



c_1=1
Λ=1
κ=1
c_2=−1,−.5,0,.5,1
dR(t)/dt
R(t)


c_1=1
Λ=1
κ=−1
c_2=−1,−.5,0,.5,1
dR(t)/dt
R(t)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
6

Same problem in brane cosmology

$$\tilde{G}_{AB} = \tilde{R}_{AB} - \frac{1}{2}\tilde{g}_{AB}\tilde{R} = k^2\tilde{S}_{AB},$$

$$\tilde{S}_{AB} = \tilde{T}_{AB} - \frac{1}{\mu^2}\sqrt{\frac{g}{\tilde{g}}}\delta(y)\left[R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right]\delta^{\mu}_{A}\delta^{\nu}_{B}.$$

$$T^{A}_{B}|_{brane} = \delta(y)\mathrm{diag}(-\rho, P_r, P_t, P_t, 0),$$

$$ds^2 = -n^2(t,y)dt^2 + a^2(t,y)\left[(1+b(r))dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2\right] + dy^2,$$

Einstein Equations in Bulk

$$\tilde{G}_{ty} = 3\left(\frac{n'}{n}\frac{\dot{a}}{a} - \frac{\dot{a}'}{a}\right) = k^2 \tilde{T}_{05},$$

$$\tilde{G}_{rr} = -\frac{b}{r^2} - (1+b)\left[2\frac{a\ddot{a}}{n^2} + \frac{\dot{a}^2}{n^2} - a'^2 - 2aa'' - 2aa'\frac{n'}{n} - 2a\dot{a}\frac{\dot{n}}{n^3} - a^2\frac{n''}{n}\right]$$

$$= k^2 a^2 (1+b)\left(-\Lambda_5 + P_r\delta(y)\right) - \frac{k^2}{\mu^2}\delta(y)\left[-\frac{b}{r^2} - (1+b)(2a\ddot{a} + \dot{a}^2)\right],$$

$$\tilde{G}_{\theta\theta} = -\frac{r\frac{\partial b}{\partial r}}{2(1+b)^2} + r^2\left[a'^2 + 2aa'' + 2a\dot{a}\frac{\dot{n}}{n^3} - 2a\frac{\ddot{a}}{n^2} + 2aa'\frac{n'}{n} - \frac{\dot{a}^2}{n^2} + \frac{a^2 n''}{n}\right]$$

$$= k^2 r^2 a^2 \left(-\Lambda_5 + P_t\delta(y)\right) - \frac{k^2}{\mu^2}\delta(y)\left[-\frac{r\frac{\partial b}{\partial r}}{2(1+b)^2} - r^2(2a\ddot{a} + \dot{a}^2)\right],$$

$$\tilde{G}_{\phi\phi} = \sin^2\theta\, \tilde{G}_{\theta\theta},$$

$$\tilde{G}_{yy} = -3\frac{A(r)}{a^2} + 3\left[\frac{a'^2}{a^2} + \frac{\dot{a}\dot{n}}{an^3} + \frac{a'n'}{an} - \frac{\ddot{a}}{an^2} - \frac{\dot{a}^2}{a^2n^2}\right] = -k^2\Lambda_5.$$

Friedmann Equation

$$\epsilon\sqrt{H^2 - \frac{k^2}{6}\Lambda_5 - \frac{c}{a^4} + \frac{A(r)}{a_o^2}} = \frac{k^2}{2\mu^2}(H^2 + \frac{A}{a_o^2}) - \frac{k^2}{6}\rho,$$

$$\epsilon = \mathrm{sign}(\frac{a'}{a}|_{y=0}) = \pm 1.$$

$$ds^2 = -dt^2 + a_o^2(t)\left[\frac{dr^2}{1 + \beta r^2 - \frac{b_o}{r}} + r^2 d\Omega^2\right],$$

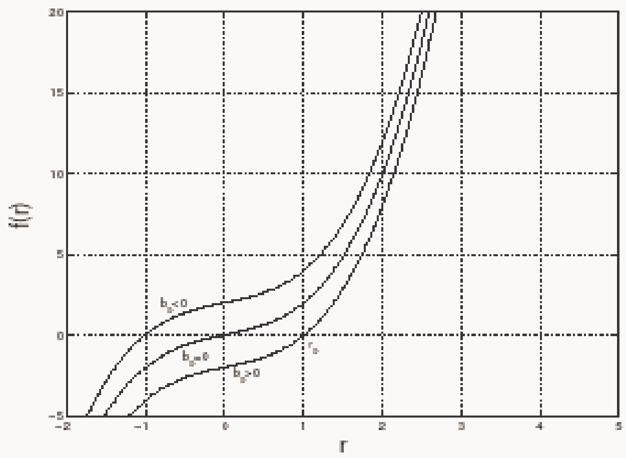


FIG. 1: The behavior of $f(r)$ for $\beta = 1$, $b_o = 2, 0, -2$.

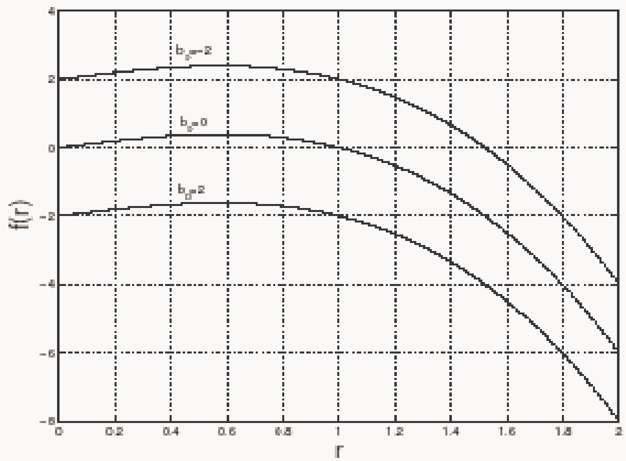


FIG. 2: The behavior of $f(r)$ for $\beta = -1$, $b_o = 2, 0, -2$.

Concluding remarks

- Our ansatz metric, together with the equation of state $\rho + \alpha P_r + 2\beta P_t = 0,$

  admits solutions which fall into three categories: radial deformations of FRW, naked singularities, and wormholes in a FRW background.
- The scale factor asymptotically conforms with the deSitter behavior with and without initial singularity.
- The redshift relation is similar to the one in standard FRW cosmology and does not depend on the radial function a(r). Our models differ from LTB in this respect.
- Similar models are worked out in the context of brane cosmology.

Thank you for your attention!