Astrophysical Probes of Quantum Gravity

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DESY THEORY WORKSHOP 2004

OUTLINE

- Manifestation of quantum gravity in “low” energy phenomena
- Spectral time lags in emissions from GRBs
- Synchrotron radiation constraint and the principle of equivalence
- Conclusions

in collaboration with J. Ellis, N. Mavromatos and D. Nanopoulos

Quantum gravity approaches

The existance of the lower bound at which space-time responses actively to the presence of energy, may lead to violation of Lorentz symmetry. In fact, different approaches to this theory lead to this result.

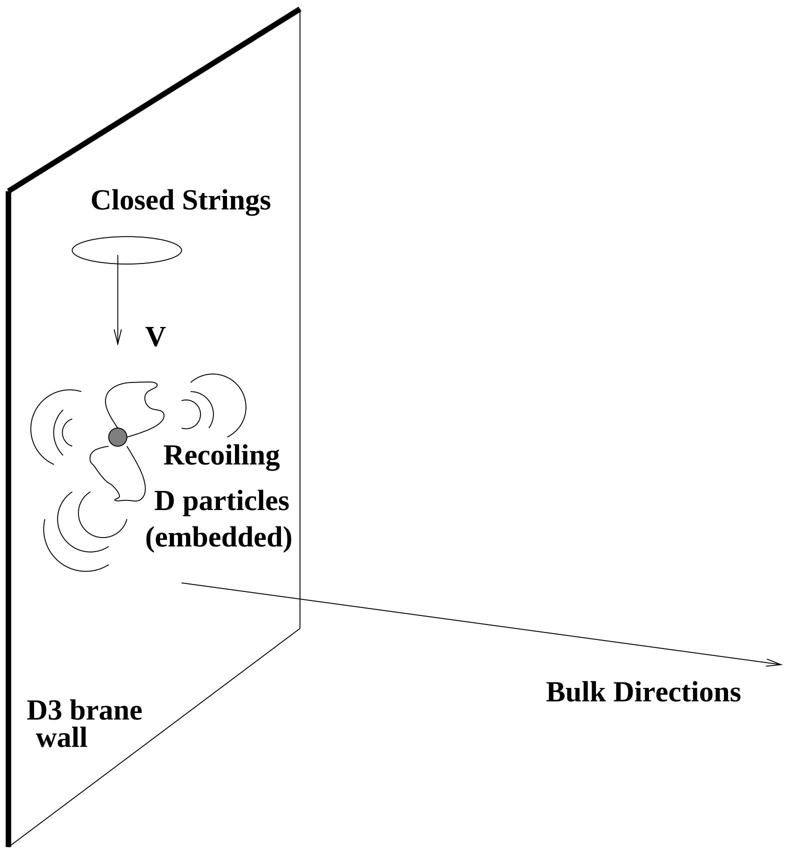
- Liouville strings (J. Ellis, N. Mavromatos, D. Nanopoulos 1997, 1998, 1999)
- Effective field theory approach (R.C. Mayers, M. Pospelov 2003)
- Space-time foam (L.J. Garay 1998)
- Loop quantum gravity (R. Gambini, J.Pullin 1999)
- String theory (A. Matusis, L Suskind, N. Tombas 2000)
- Noncomutative geometry (G. Amelino-Camelia 2001)

In the approximation $E \ll M$, the distortion of the standard dispersion relations may be represented as an expansion in E/M:

$$E^2 = m^2 + p^2(1 + \xi_1(p/M) + \xi_2(p/M)^2 + \ldots)$$

$$\xi_1 < 0; \quad v = c(1 - \frac{E}{M}); \quad n(E) = 1 + \frac{E}{M}$$

D-brain recoil



$$G_{ij} = \delta_{ij}, \quad G_{00} = -1, \quad G_{0i} \sim \bar{U}_i$$

Metric perturbations $h_{\mu\nu}$ about the flat spacetime

$$h_{0i} \simeq \bar{U}_i \quad \bar{U}/c = O(E/M_D c^2)$$

Dispersion analysis

The background metric

$$G_{00} \equiv -h, \qquad \mathcal{G}_i = -\frac{G_{0i}}{G_{00}}, \quad i = 1, 2, 3$$

Maxwell's equations in a medium with $1/\sqrt{h}$ playing the rôle of the **electric and magnetic permeability**

$$\nabla \cdot B = 0, \qquad \nabla \times H = \frac{1}{c}\frac{\partial}{\partial t} D = 0$$

$$\nabla \cdot D = 0, \qquad \nabla \times E = -\frac{1}{c}\frac{\partial}{\partial t} B = 0$$

$$D = \frac{E}{\sqrt{h}} + H \times \mathcal{G}; \quad B = \frac{H}{\sqrt{h}} + \mathcal{G} \times E$$

$$\frac{1}{c^2}\frac{\partial^2}{\partial^2 t} B - \nabla^2 B - 2\left(\overline{U}.\nabla\right)\frac{1}{c}\frac{\partial}{\partial t} B = 0$$

$$\frac{1}{c^2}\frac{\partial^2}{\partial^2 t} E - \nabla^2 E - 2\left(\overline{U}.\nabla\right)\frac{1}{c}\frac{\partial}{\partial t} E = 0$$

The one dimensional wave solution

$$E_y(x,t) = E_0 e^{ikx-\omega t}; \quad B_z(x,t) = B_0 e^{ikx-\omega t}$$

with dispersion relation

$$k^2 - \omega^2 - 2\overline{U}k\omega = 0$$

Modification by dimention 5 operators

Lorentz symmetry is broken by introducing a background four vector n^a (with $n \cdot n = 1$)

1. One more derivative than the usual kinetic term
2. Not reducible to lower dimension operators by the equations of motion

Scalar:

$$\mathcal{L}_0 = |\partial\Phi|^2 - m^2|\Phi|^2$$

$$\Phi \sim \exp(-ik \cdot x), \quad (-k^2 + m^2)\Phi(k) = 0$$

$$\mathcal{L}_s = i\frac{\eta}{M}\bar{\Phi}(n \cdot \partial)^3\Phi$$

odd under CPT and charge conjugation

$$(\nabla^2 + m^2)\Phi = i\frac{\eta}{M}(n \cdot \partial)^3\Phi$$

In momentum space (with $n \cdot \partial \sim -iE$), and in the Lorentz frame where $n^a = (1, 0, 0, 0)$, we have the dispersion relation

$$E^2 \simeq |\vec{p}|^2 + m^2 + \frac{\kappa}{M}|\vec{p}|^3$$

References

- **Ultra high energy cosmic rays, anomalous particle production thresholds**
  J. Alfaro and G. Palma (2003); T. Jakobson, S. Liberati, D. Mattingly (2001, 2002, 2003); T.J. Konopka, S.A. Major (2002); G. Amelino-Camelia, Y.J. Ng, H. Van Dam (2002); R. Alosio, P. Blasi, A. Galante, A.F. Grillo (2000, 2003); J. Ellis, E. Gravanis, N.E. Mavromatos, D.V. Nanopoulos (2002), F.W. Stecker, S.L. Glashow (2001)
- **Multi-TeV AGN and infrared background**
  R.J. Protheroe, H. Meyer (2000); G. Amelino-Camelia (2002); F.W. Stecker (2003)
- **GRBs, AGN (time of fligh)**
  G. Amelino-Camelia, J. Ellis, N.E. Mavromatos, D.V. Nanopoulos, S.Sarkar (1998); S.D. Biller et all (1999), J. Ellis, N.E. Mavromatos, D.V. Nanopoulos, A.S. Sakharov (2003), B.E. Schaefer (1999)
- **Synchrotron, polarisation ... T. Jakobson, S. Liberati, D. Mattingly (2003); J. Ellis, N.E. Mavromatos, A.S. Sakharov (2003); T. Jakobson, S. Liberati, D. Mattingly, F.W. Stecker (2003), R. Ragazzoni, M. Turano, W. Gaessler (2003); Y.J. Ng, H. Van Dam, W.A. Christiansen (2003); J. Ellis, E. Gravanis, N.E. Mavromatos, D.V. Nanopoulos (2003)**

Light propagation

Light propagation from remote objects is affected by the expansion of the Universe and depends upon the cosmological model: $\Omega_{total} = \Omega_\Lambda + \Omega_M = 1$; $\Omega_\Lambda \simeq 0.7$

$$dt = -H_0^{-1} \frac{dz}{(1+z)h(z)}; \quad h(z) = \sqrt{\Omega_\Lambda + \Omega_M (1+z)^3}$$

$$udt = -H_0^{-1} \frac{udz}{(1+z)h(z)}$$

$$\Delta L = H_0^{-1} \int_0^z \frac{\Delta u dz}{(1+z)h(z)}$$

We consider two photons traveling with velocities very close to c**, whose present day energies are** E_1 **and** E_2**. At earlier epochs, their energies would have been blueshifted by a factor** $1+z$**.**

$$\Delta u = \frac{\Delta E(1+z)}{M}; \quad \Delta u = \frac{\Delta E^2 (1+z)^2}{M^2}$$

$$\Delta t = H_0^{-1} \frac{\Delta E}{M} \int_0^z \frac{dz}{h(z)}$$

$$\Delta t = H_0^{-1} \left(\frac{\Delta E}{M}\right)^2 \int_0^z \frac{(1+z)dz}{h(z)}$$

Time-of-flight studies

The modification of the group velocity would affect the simultaneity of the arrival times of photons with different energies from distant transient sources.

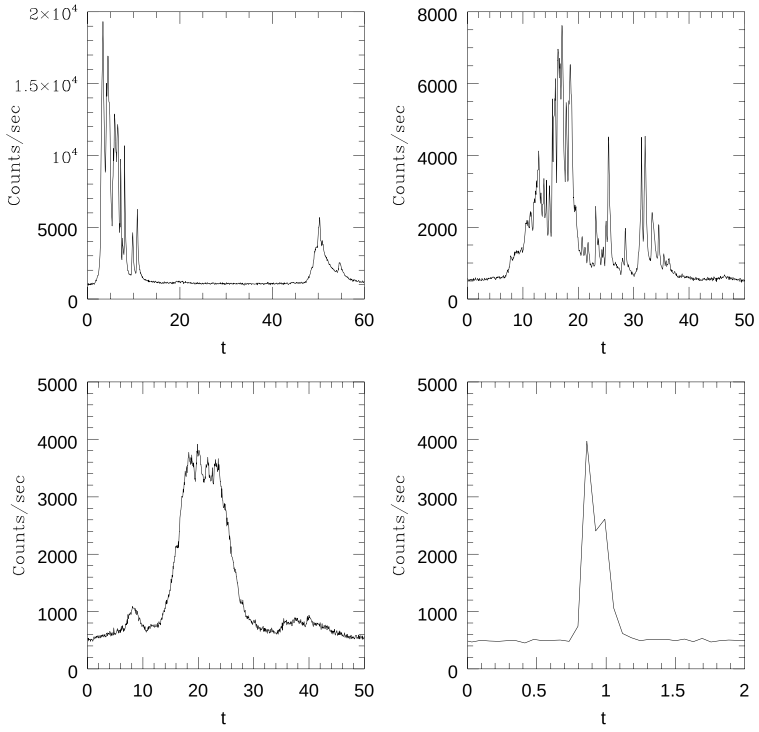
- **Pulsars**
- **Blazars (Markarian 421, 501)**
- **Gamma Ray Bursts**

A key issue in such probes is to distinguish the effects of the quantum-gravity medium from any intrinsic delay in the emission of particles of different energies by the source.

Any quantum-gravity effect should increase with the redshift of the source, whereas source effects would be independent of the redshift in the absence of any cosmological evolution effects. It is preferable to use transient sources with a known spread in distances (redshifts)

GRB's

GRBs are short, non-thermal bursts of low energy γ-rays.



The angular distribution of GRBs' positions on the sky is perfectly isotropic. The distance is cosmological $z \simeq 1$.

We look for the spectral time lags of the light curves recorded by **BATSE** in the $115 - 320$ **KeV** energy band relative to those in the lowest $25 - 55$ **KeV** energy band. We also compare the rather more energetic light curves accumulated by **OSSE** $3 - 6$ **MeV** with the $115 - 320$ **KeV BATSE** light curves.

GRBs and Wavelet Shrinkage

A discrete GRB signal $f[n]$ of size N is contaminated by addition of a noise $W[n]$, whose probability distribution is unknown:

$$X[n] = f[n] + W[n], \quad \tilde{F} = DX$$

A thresholding estimator of a $n = 2^J$-component discrete signal $X[n]$ decomposed at the resolution level L

$$\tilde{F} = \sum_{j=L}^{J-1} \sum_{m=0}^{2^{j-1}} \rho_T(d^X_{j,m})\psi_{j,m} + \sum_{m=0}^{2^{L-1}} \rho_T(c^X_{J,m})\phi_{J,m}$$

The function ρ_T provides a soft threshold

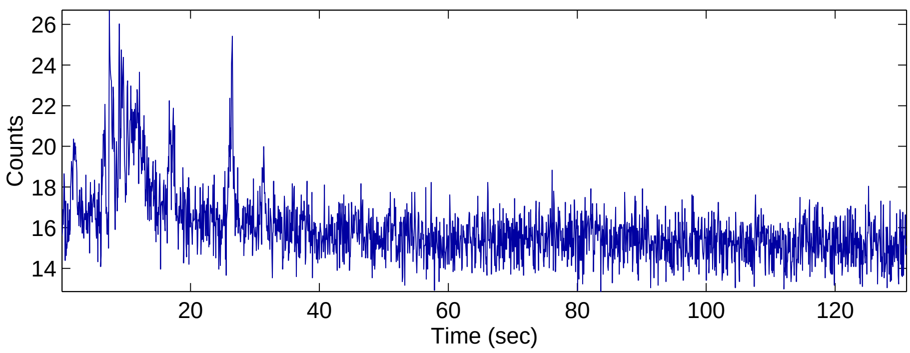
$$\rho_T(x) = \begin{cases} x - T & \text{if} \quad x \geq T \\ x + T & \text{if} \quad x \leq -T \\ 0 & \text{if} \quad |x| \leq T \end{cases} \qquad T = \sigma\sqrt{2\log n}$$

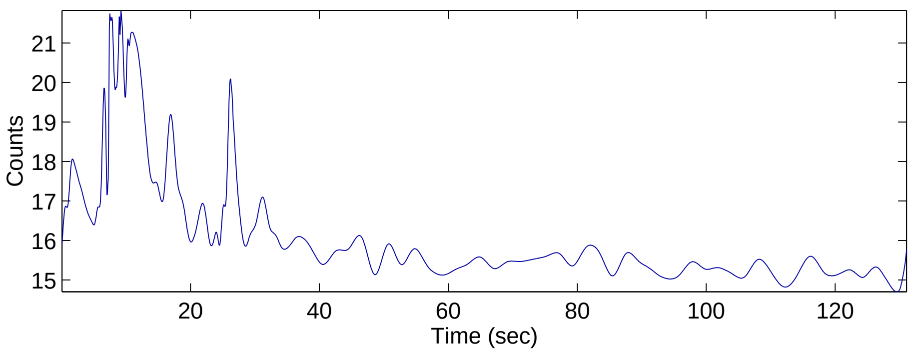
The wavelet shrinkage procedure guarantees with high probability that $|d^{\tilde{F}}_{j,m}| \leq |d^f_{j,m}|$, implying that the estimator $\tilde{F}$ is at least as regular as the 'original' intensity profile f, because its wavelet coefficients have smaller amplitudes.

Nonparametric estimation of GRBs

After applying a DWT to GRB light curves, the **structure** therein is represented by relatively **few** **"large"** wavelet coefficients, while the **noise** is spread out among the remaining **"small"** coefficients.

With an appropriate **thresholding procedure**, it is possible to remove the **noise** from the signal by setting all coefficients beneath the threshold to zero and applying the **inverse of the original wavelet transform** to the result.





The intensity profile of GRB990308 estimated by the wavelet shrinkage procedure at the level $L = 6$.

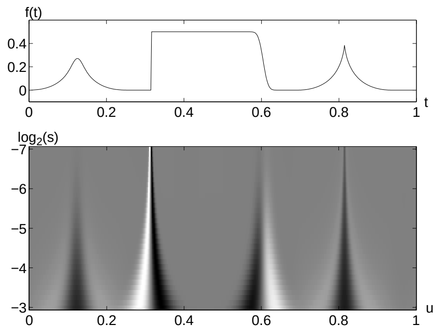
Lipschitz regularity and CWT

$$|f(t) - p_\nu(t)| \le \mathcal{K}|t-\nu|^\alpha$$

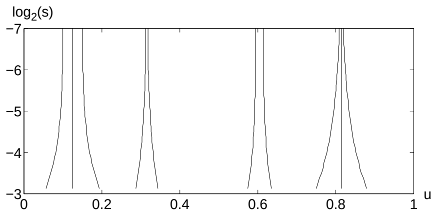
If f is $m \le \alpha$ times continuously differentiable in a negborhood of ν, then p_ν is the Taylor expansion of f at ν. If $\nu \le \alpha < 1$, then $p_\nu(t) = f(\nu)$, and the Lipschitz condition becomes

$$|f(t) - f(\nu)| \le \mathcal{K}|t-\nu|^\alpha$$

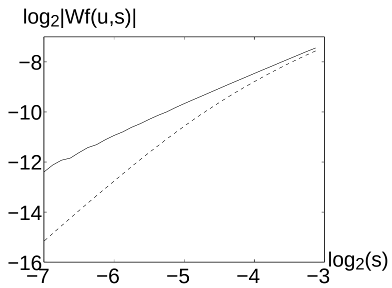
The CWT amplitude decays with the scale $|Wf(u,s)| \propto s^{\alpha+1/2}$.



(a)



(b)

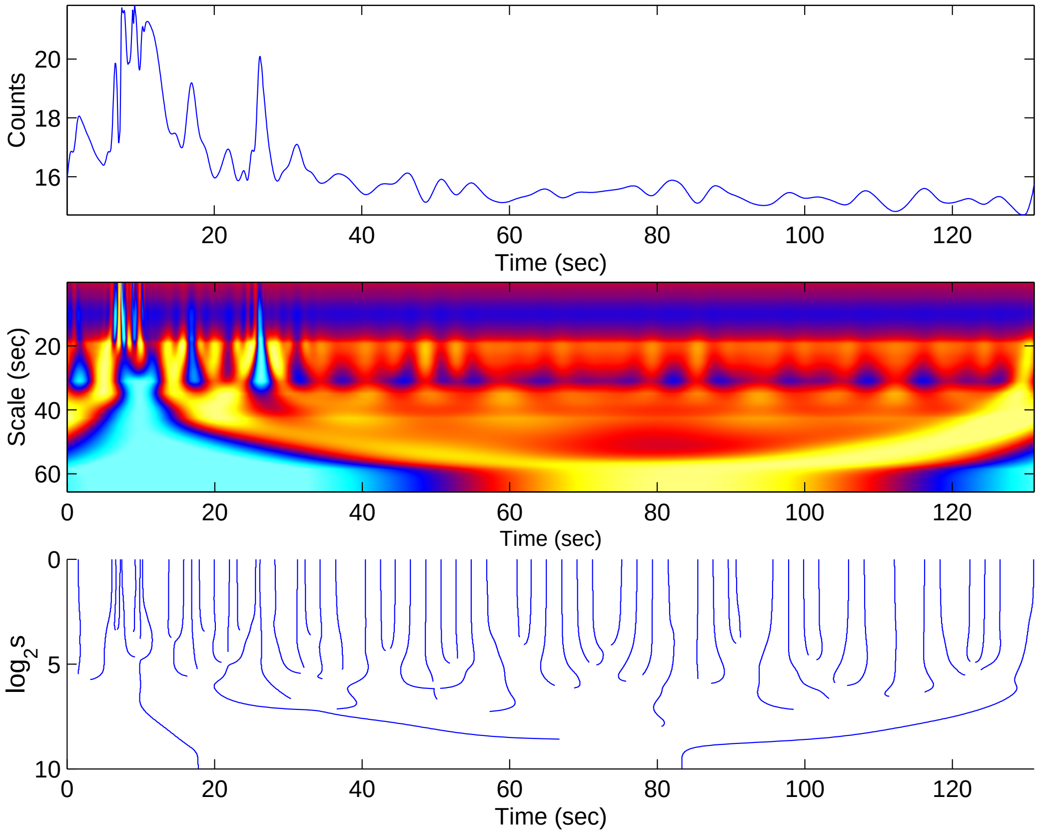


(c)

Detection of singularities with CWT

Singularities of a function f are detected by finding the abscissa where the wavelet modulus maxima converge on fine scales in the scale-time plane (u, s).

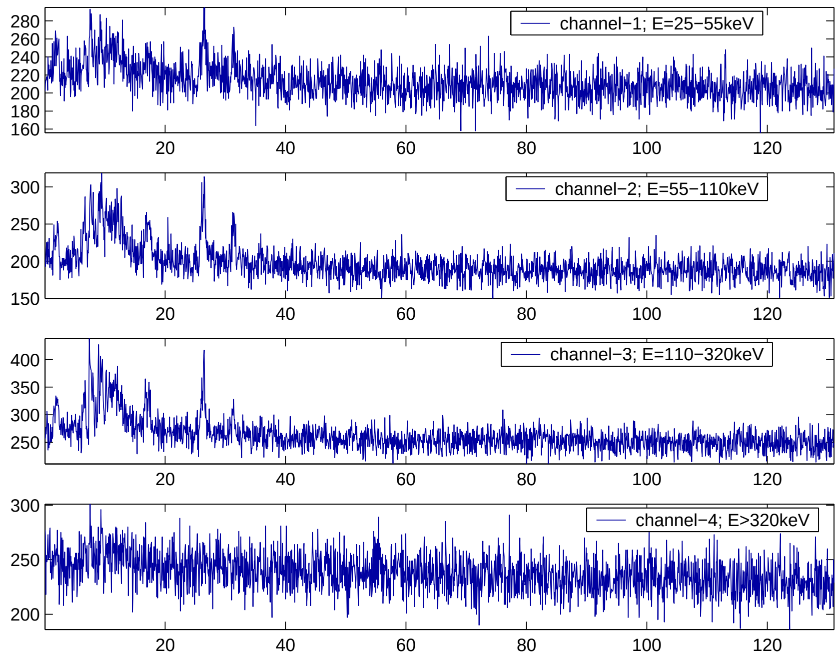
$$\Psi(t) = \frac{2}{\sqrt{3}}\pi^{1/4}(t^2 - 1)\exp(-\frac{t^2}{2})$$

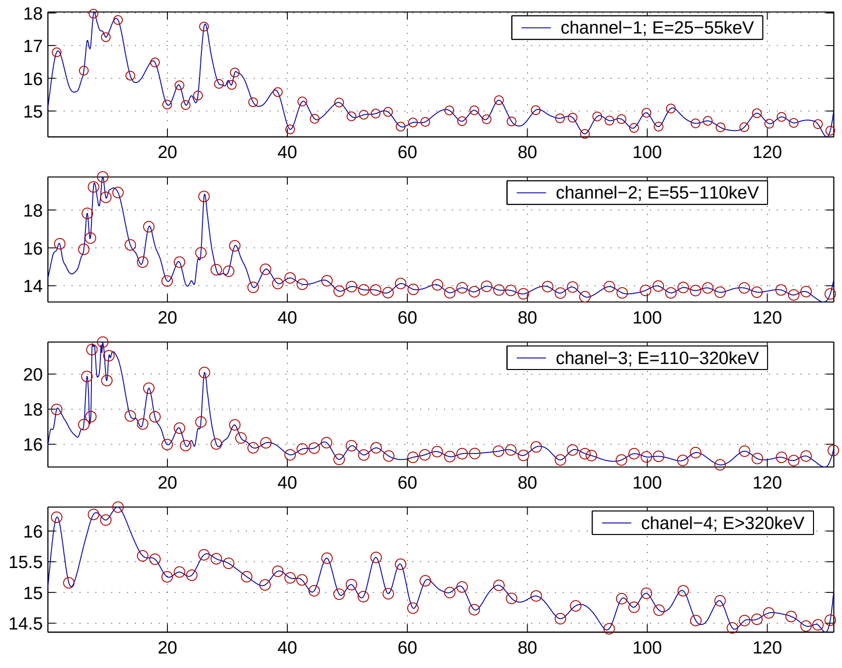


$$\log_2 |Wf(u,s)| \approx \left(\alpha + \frac{1}{2}\right)\log_2 s + const$$

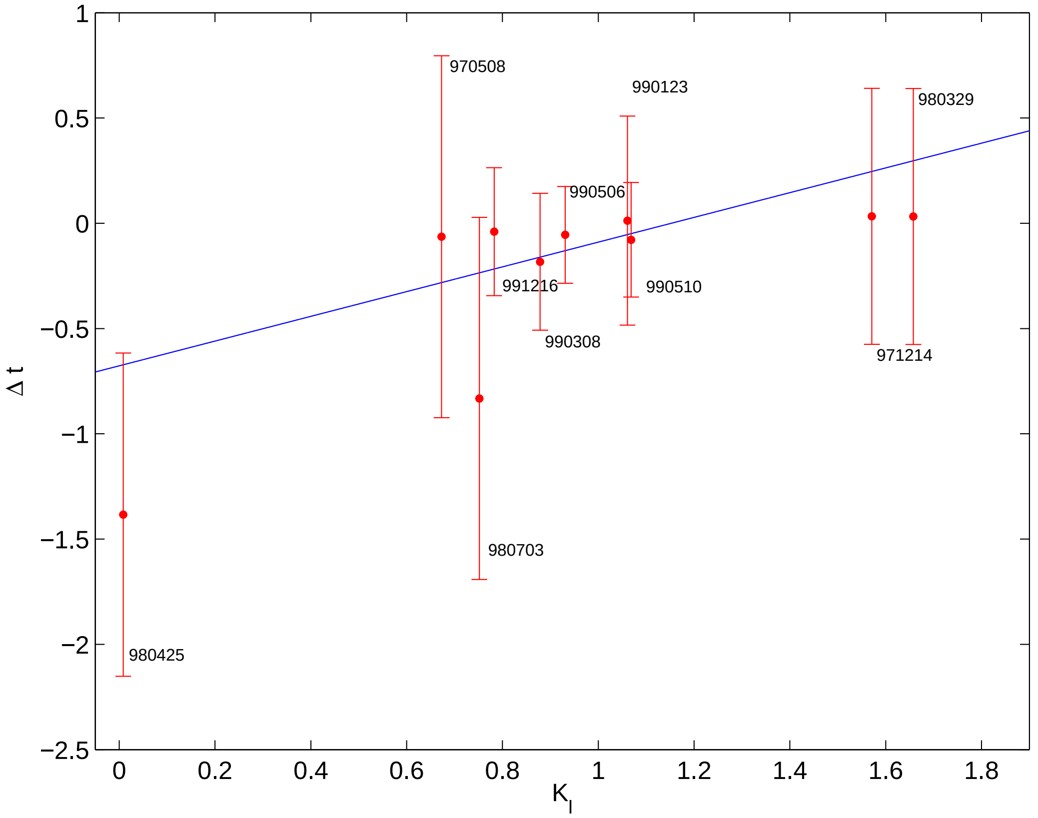
$$\alpha + 1/2 \leq 1.5$$

Genuine variation points





Linear Regression using 64ms BATSE

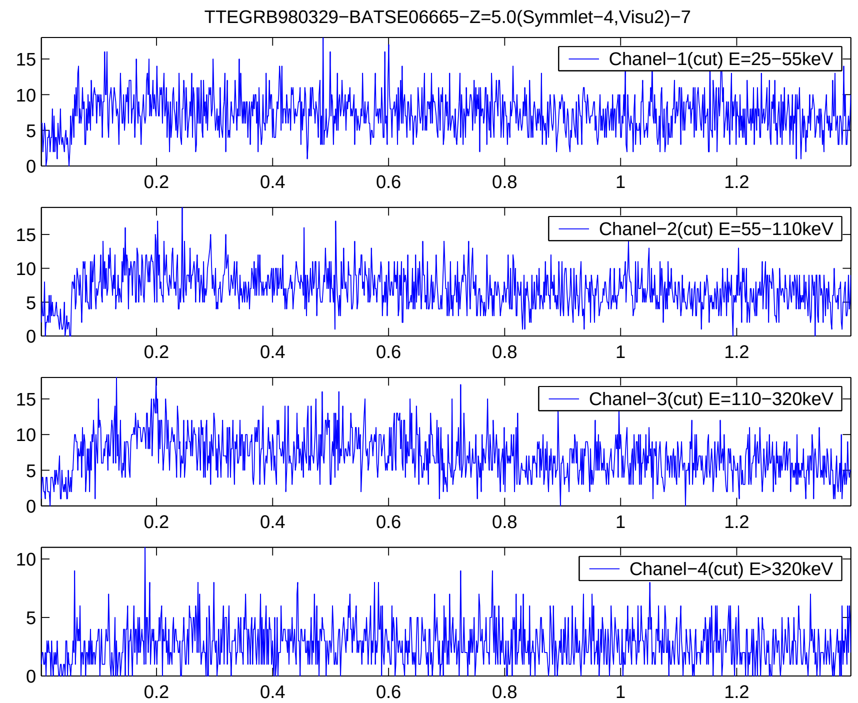


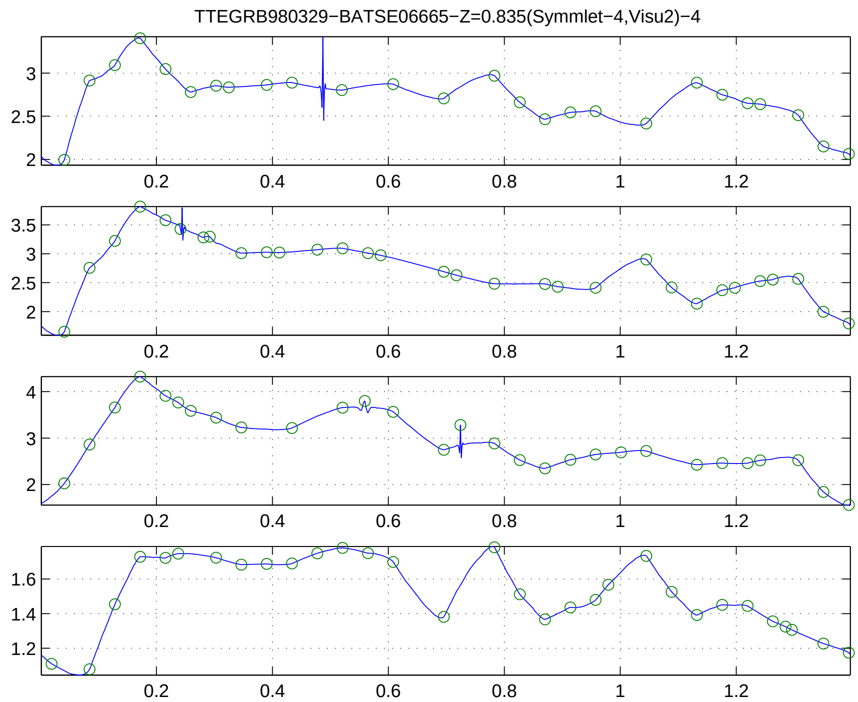
$$\Delta t = aK + b; \quad K_l = \int_0^z \frac{dz}{h(z)}; \quad K_q = \int_0^z \frac{(1+z)dz}{h(z)}$$

$$\Delta t = 0.59(\pm 0.46)K_l - 0.68(\pm 0.46)$$

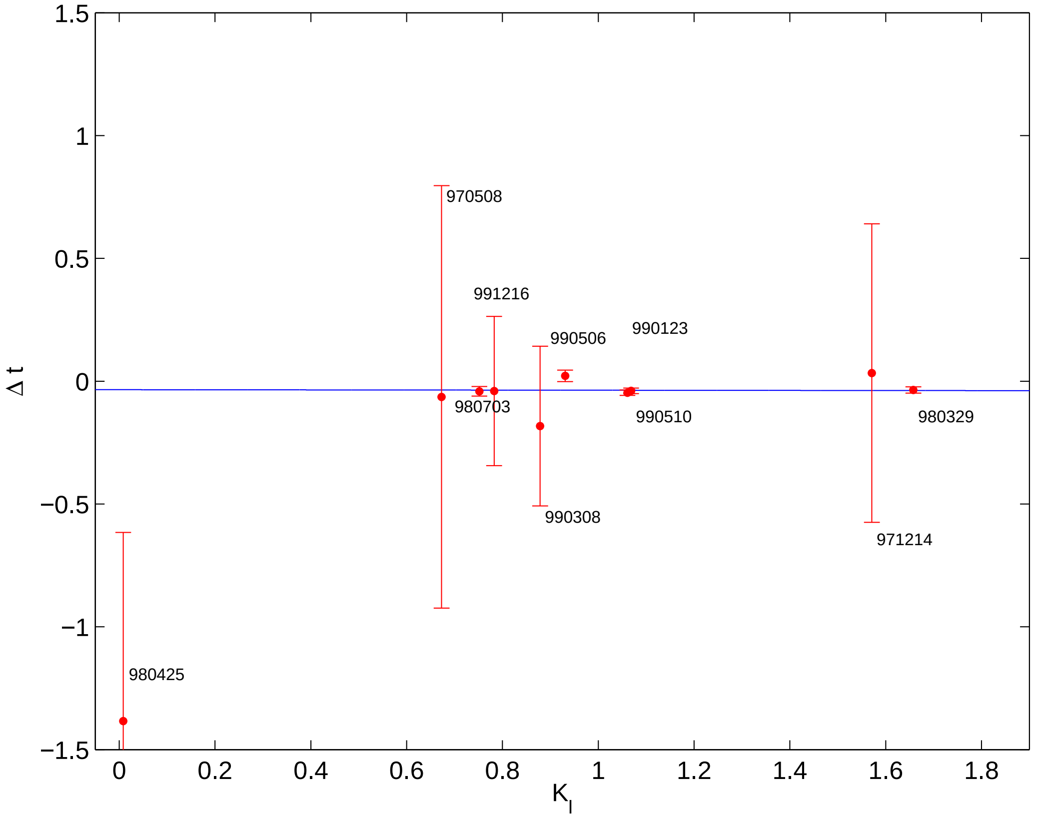
$$\Delta t = 0.16(\pm 0.17)K_q - 0.37(\pm 0.32)$$

TTE bin ≈ 1 *ms*


TTEGRB980329−BATSE06665−Z=5.0(Symmlet−4,Visu2)−7
Chanel−1(cut) E=25−55keV
Chanel−2(cut) E=55−110keV
Chanel−3(cut) E=110−320keV
Chanel−4(cut) E>320keV


TTEGRB980329−BATSE06665−Z=0.835(Symmlet−4,Visu2)−4

Lenear regression: all data compiled



$$\Delta t = aK + b; \quad K_l = \int_0^z \frac{dz}{h(z)}; \quad K_q = \int_0^z \frac{(1+z)dz}{h(z)}$$

$$\Delta t = -0.002(\pm 0.021)K_l - 0.03(\pm 0.020)$$

$$\Delta t = -0.0003(\pm 0.006)K_q - 0.040(\pm 0.015)$$

Compilation of quantum gravity limits

$$\frac{\int_{\mathcal{M}}^{\infty} L(\xi)d\xi}{\int_{0}^{\infty} L(\xi)d\xi} = 0.95; \quad \mathcal{M} = 10^{19}\text{GeV}$$

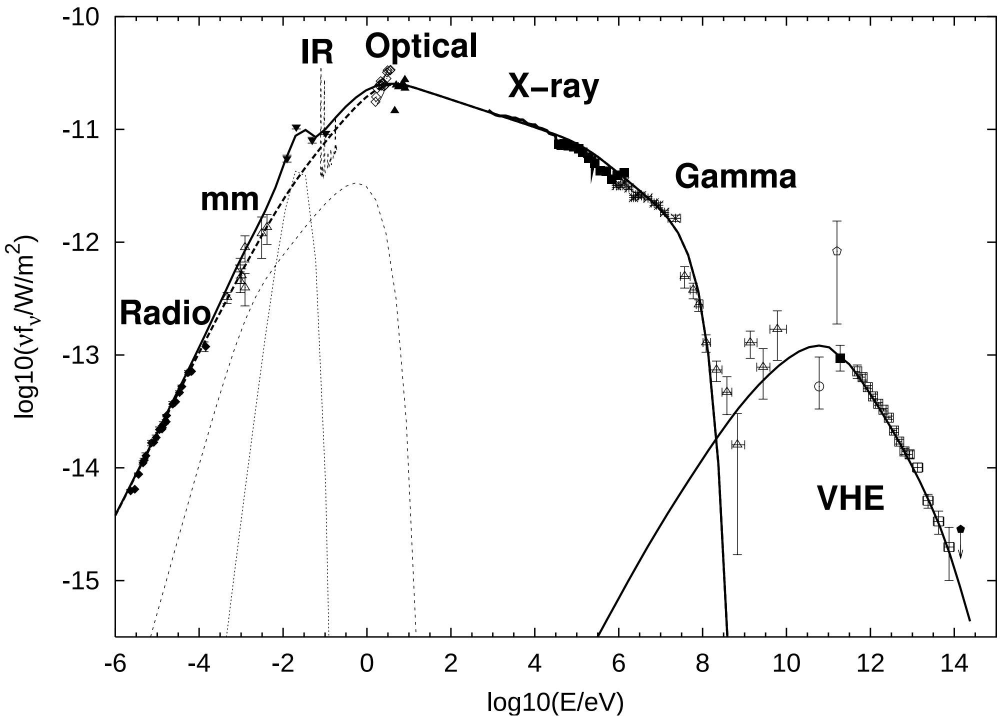
$$L \propto \exp(-\chi^2(M)/2)$$

We use the fact that only the coefficient a is related to the quantum-gravity scale, whereas b includes a possible unknown spectral time lag inherited from the sources, which we assume to be universal for our data set.

$$\chi^2(M) = \sum_{\mathcal{D}} \left[\frac{\Delta t_i - b_{shift} - a(M)K_i}{\sigma_i}\right]^2$$

$$M \geq 1.3 \cdot 10^{16} \ \text{GeV}$$

Synchrotron hump



Synchrotron emission is observed up to energies of about 0.5 GeV, where the inverse Compton hump begins to dominate the spectrum.

In the standard Lorentz-invariant theory, the 0.5 GeV synchrotron emission in the magnetic field of the Crab nebula ($\sim$ mG) is generated by electrons of energy 5×10^4 TeV.

Synchrotron from the Crab Nebula

$$\omega_c^{LI} = \frac{3}{2}\frac{eH}{m_0}\frac{1}{1-\beta^2}$$

$$\omega_0^* = \frac{\omega_0}{sin^2\theta}; \quad \omega_0 = \frac{\beta_\perp}{R}$$

$$E^2(p) = m_0^2 + p^2 - \frac{p^{2+\alpha}}{\mathcal{M}^\alpha}$$

The maximal synchrotron frequency gets modified, because of the modification of the orbital frequency as well as the velocity factor β of the electron:

$$\omega_1 = \frac{\omega_0^{QG}}{1-\beta_{QG}} = \frac{3}{2}\frac{eH}{m_0}\frac{1}{1-\beta_{QG}^2}\frac{R_0}{R_{QG}}$$

QG modified synchrotron radiation

The modification of the orbital frequency is connected with QG effects on the radius of the orbit, due to the back-reaction of the foamy QG medium on the propagation of the electron.

$$\frac{R_{QG}}{R_0} \approx \frac{1}{\sqrt{2}} \left(1 + \sqrt{2 - \frac{1}{\eta^2}}\right)^{1/2} ; \quad \eta = 1 - \left(\frac{E}{\mathcal{M}}\right)^{\alpha}$$

$$(E/\mathcal{M})^{\alpha} < 1 - \frac{1}{\sqrt{2}} \simeq 0.293$$

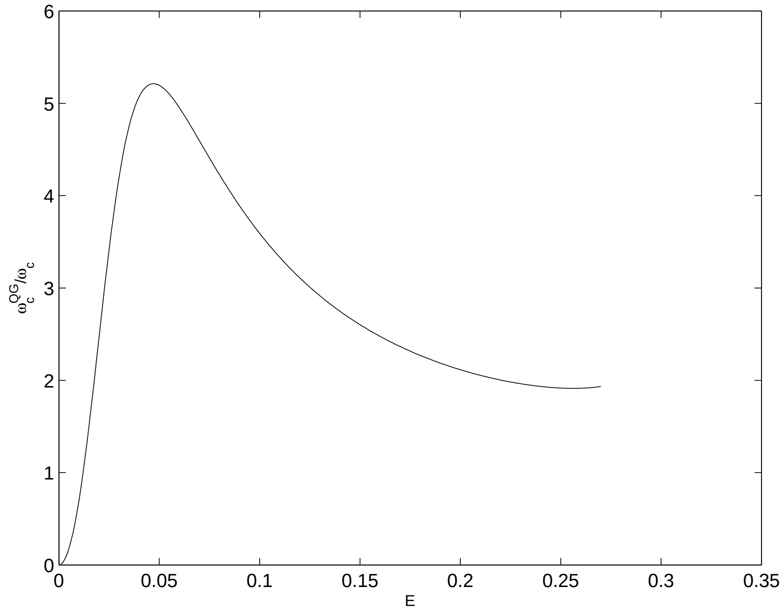
$$\beta_{QG} \equiv \frac{\partial E}{\partial p} = \frac{p}{E} \left(1 - \left(\frac{\alpha}{2} + 1\right) \left(\frac{E}{\mathcal{M}}\right)^{\alpha}\right)$$

$$1 - \beta_{QG}^2 \simeq \frac{m_0^2}{E^2} + (\alpha + 1) \left(\frac{E}{\mathcal{M}}\right)^{\alpha}$$

$$\omega_c^{QG} = \frac{3}{\sqrt{2}} \frac{eH}{m_0} \frac{1}{(1+\sqrt{2-1/\eta^2})^{1/2} \left(\frac{m_0^2}{E^2} + (\alpha+1)\left(\frac{E}{\mathcal{M}}\right)^{\alpha}\right)}$$

$$E_{max} = (2m_0^2 \mathcal{M}^{\alpha} / \alpha(\alpha + 1))^{1/(2+\alpha)}$$

Synchrotron constraint on QG



$$E^2(p) = m_0^2 + p^2 - \xi_e^\alpha \frac{p^{2+\alpha}}{M_P^\alpha}$$

$$|\xi_e| < \left(\frac{3eH}{m_0}\right)^{\frac{\alpha+2}{2\alpha}} \left(\frac{M_P}{m_0}\right) \left(\frac{2}{\alpha(\alpha+1)}\right)^{1/\alpha} \left(\frac{\alpha}{\alpha+2}\right)^{(\alpha+2)/2\alpha}$$

For the average magnetic field of Crab Nebula
$H_{cons} = 260\mu\text{G}$

$$|\xi_e| < 6.8 \cdot 10^{21} \left(8.8 \cdot 10^{-21}\right)^{\frac{\alpha+2}{2\alpha}} \; ; \quad \alpha \geq 1.7$$

For Vela pulsar's magnetic field $H_{Vela} \sim 3\mu\text{G}$

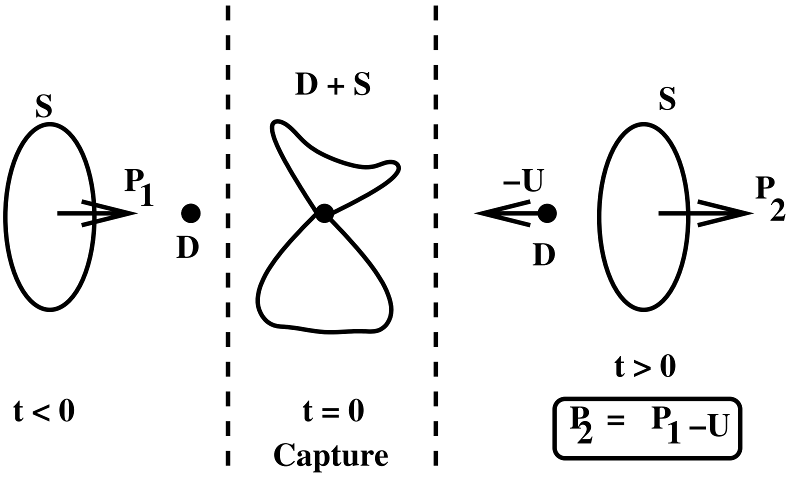
$$\alpha \geq 2.04$$

Violation of the equivalence

$$G_{00} = -1 \ ; \ G_{ij} = \delta_{ij}$$

$$G_{0i} = g_s \Delta p_i / M_s \sim \frac{1}{2} g_s p_i / M_s$$

$$p_\mu p_\nu G^{\mu\nu} = -m_0^2$$



$$\vec{p} \cdot u^i \sim \frac{\vec{p}\cdot\vec{p}}{2\mathcal{M}} < 0$$

The violation of the equivalence is predicted by Liouville string/D-particle model for space-time foam, only gauge bosons might have QG-modified dispersion relations, and not charged matter particles

Conclusions

We find that the combination of all the available data on GRBs prefers marginally a linear violation of Lorentz invariance between 10^{15} GeV and 10^{16} GeV, although the effect is not significant.

We prefer to interpret the data as giving a limit on the linear quantum-gravity scale $M \geq 1.3 \cdot 10^{16}$ GeV, which we consider to be the most robust and model-independent currently available.

The interactions of different particle species with the foamy space-time fluctuations expected in quantum gravity theories may not be universal, in which case different types of energetic particles may violate Lorentz invariance by varying amounts, violating the equivalence principle.