From heaviness to lightness during inflation

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Introduction

- Quantum fluctuations of scalar fields are one of the cornerstone of modern cosmology.

- In standard inflation the quantum fluctuations of the inflaton are responsible for the origin of cosmological perturbations

- **Lightness** ($m << H$) is the crucial ingredient: any light field inherits a scale invariant spectrum during inflation.

Purpose: study the quantum fluctuations of a "light" scalar field χ whose mass varies with time during inflation due to the coupling to a heavier field ϕ.

Preceding related works: ***Preheating***. Quantum fluctuations of a field χ are amplified when inflaton oscillates after the end of inflation.

Scalar field fluctuations during inflation

Evolution equation: $\ddot{\chi} + 3H\dot{\chi} + m_\chi^2 \chi = 0$

Fourier modes: $\ddot{\delta\chi}_k + 3\frac{\dot{a}}{a}\dot{\delta\chi}_k + \left(\frac{k^2}{a^2} + m_\chi^2\right)\delta\chi_k = 0$

As an example of (non-realistic but simple) inflation we consider **de Sitter spacetime**:

Scale factor: $a(t) = e^{Ht}$ Scale factor in **conformal time** (*dτ=dt/*a): $a(\tau) = -\frac{1}{H\tau}$

Canonical variable: $u_k \equiv a\delta\chi_k$

$$u_k'' + \left[k^2 - \frac{2 - (m_\chi^2/H^2)}{\tau^2}\right] u_k = 0$$

After *vacuum normalization* on small scales (*k>>aH*) we can take two limits for the **power spectrum** at late time:

$$\mathcal{P}_{\delta\chi}(k) \equiv \frac{k^3}{2\pi^2}\frac{|u_k|^2}{a^2}$$

$$\simeq \left(\frac{H}{2\pi}\right)^2 \qquad (k \ll aH)$$ **massless** scalar field $m_\chi << H$

$$\simeq 0 \qquad (k \ll aH)$$ **massive** scalar field $m_\chi >> H$

Two scalar fields tale

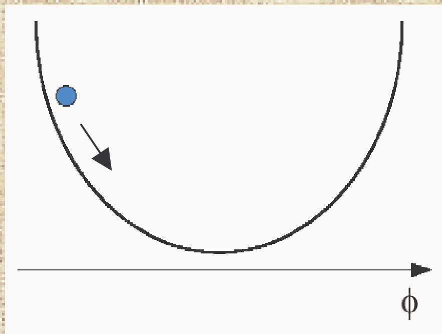
Consider two fields during inflation: one is heavy and the other light: $m >> m_\chi$

$$\mathcal{L} = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\chi}^2 - V(\phi,\chi), \quad V(\phi,\chi) = \frac{1}{2}m^2\phi^2 + \frac{1}{2}m_\chi^2\chi^2$$

Heavy field:

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0$$

- Initially $H>>m$: ϕ is frozen



- Eventually $H<<m$: ϕ **starts oscillating**

$$\phi = \Phi e^{-\frac{3}{2}Ht}\cos(mt)$$

Damped oscillations: $\phi \sim a^{-3/2} \sim \exp(-3/2Ht)$

Light field:

$$\ddot{\chi} + 3H\dot{\chi} + m_\chi^2\chi = 0$$

$H>>m_\chi$: Frozen field.

Practically $m_\chi = 0$

The model

We add a coupling between the two fields:

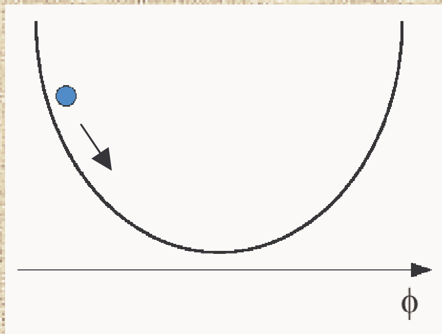
$$V(\phi,\chi) = \frac{1}{2}m^2\phi^2 + \frac{1}{2}m_\chi^2\chi^2 + \frac{1}{2}g^2\phi^2\chi^2$$

coupling

Heavy field:

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0$$

- Initially *H>>m*: ϕ is frozen



- Eventually *H<<m*: ϕ **starts oscillating**

$$\phi = \Phi e^{-\frac{3}{2}Ht}\cos(mt)$$

Damped oscillations: $\phi \sim a^{-3/2} \sim \exp(-3/2Ht)$

Light field:

$$m_{\mathrm{eff}}^2(t) = g^2\phi^2(t)$$

H << $m_{\mathrm{eff}}^2 = g^2\phi^2$ = contant:

χ **is initially heavy!**

$\chi(t)$ quickly rolls to zero; $\chi(t) \to 0$

Field with **oscillating mass during inflation:**

$$\ddot{\delta\chi}_k + 3\frac{\dot{a}}{a}\dot{\delta\chi}_k + \left(\frac{k^2}{a^2} + g^2\phi^2\right)\delta\chi_k = 0$$

$$m_\chi^2 = g^2\Phi^2 e^{-3Ht}\cos^2(mt)$$

χ later **it becomes light**: $m_{\mathrm{eff}}^2 \to 0$

Preheating

This situation is similar to the preheating scenario. After inflation an inflaton ϕ is coupled to a field χ into which it decays. If we **neglect the expansion** of the universe:

$$\frac{d^2 u_k}{d\tau^2} + \left(k^2 + g^2\Phi^2\cos^2(m\tau)\right) u_k = 0.$$

canonical variable

$$u_k \equiv a\delta\chi_k$$

Oscillator with a periodically changing frequency

We can rewrite this into the well known **Mathieu equation**:

$$\frac{d^2 u_k}{dz^2} + (A_k + 2q_0\cos 2z)\, u_k = 0 \ ,$$

$$z = m\tau, \quad A_k = \frac{k^2}{m^2} + 2q_0,$$

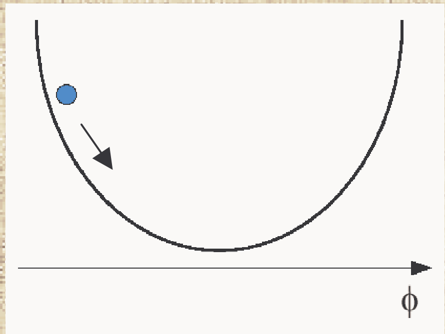
Here we are interested in the regime where the parameter

$$q_0 \equiv \frac{g^2\Phi^2}{4m^2}$$

is large: **broad resonance regime**. In this regime we have an abundant particle production.

Particles are produced when **adiabaticity** is violated:

$$\left|\frac{d\omega_k}{d\tau}\right| \gtrsim \omega_k^2$$



Using $\omega_k^2 = k^2 + 4m^2 q_0\cos^2(m\tau)$ we obtain $k \lesssim m q_0^{1/4}$

χ fluctuations

In analogy to the preheating scenario we study the fluctuations of χ. Our situation is however *very different from preheating* because **we are during inflation.**

$$\frac{d^2 u_k}{d\tau^2} + \left(k^2 - U(\tau)\right) u_k = 0$$

$$U(\tau) = \frac{a''}{a} - g^2\phi^2 a^2$$

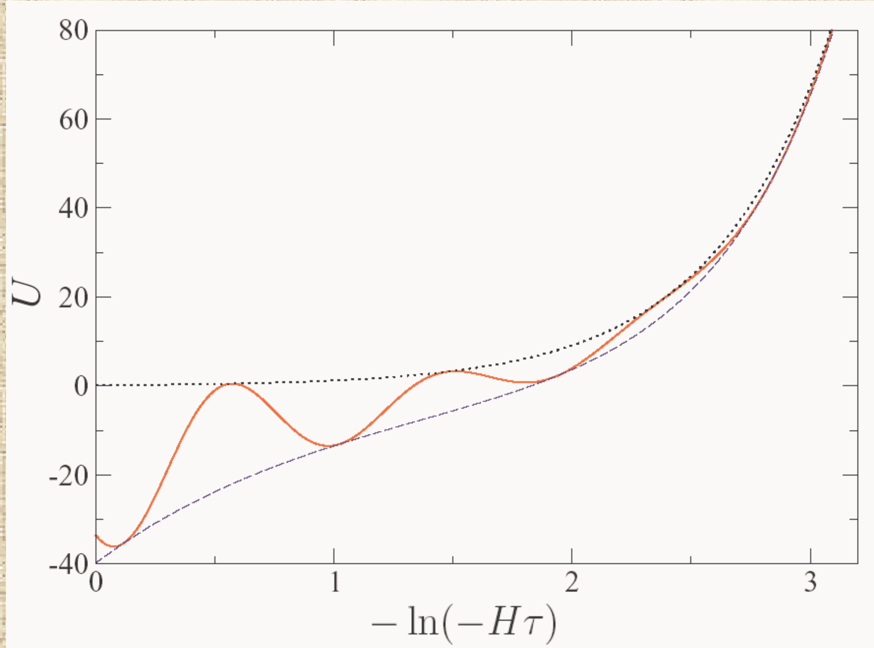
$$= \frac{2}{\tau^2} + g^2\Phi^2 H\tau \cos^2\left[\frac{m}{H}\ln(-H\tau)\right]$$

In de Sitter

$$a(\tau) = -\frac{1}{H\tau}$$

ϕ oscillations

Schrödinger-like equation with potential $U(\tau)$



In this example $q_0 = g^2\Phi^2/(4m^2) = 10$ and $m/H = 2\sqrt{3}$.

We can write the equation in this form:

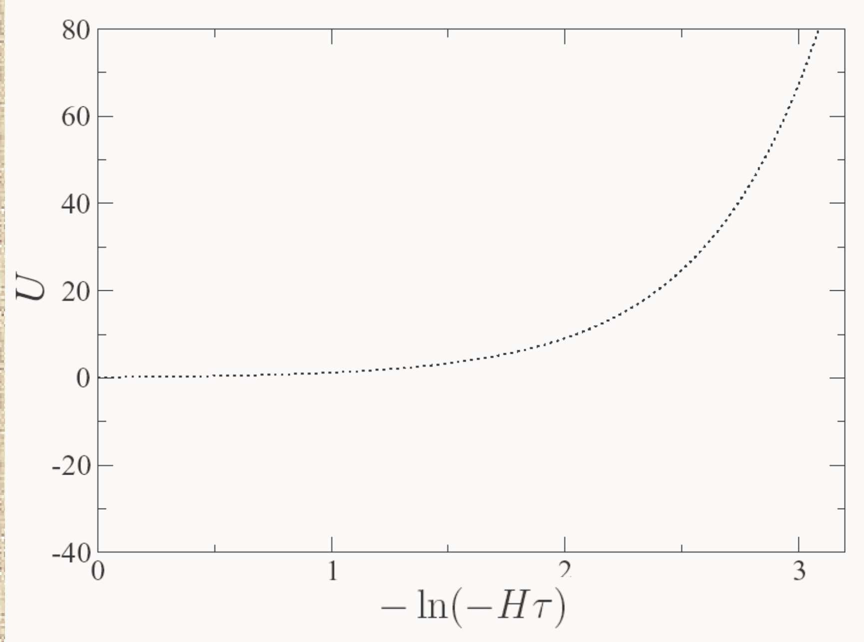
$$\frac{d^2 u_k}{dz^2} + \left\{\frac{k^2}{m^2} - \frac{2}{z^2} + 4q_0\frac{H}{m} z\cos^2\left[\frac{m}{H}\ln\left(-\frac{H}{m}z\right)\right]\right\} u_k = 0$$

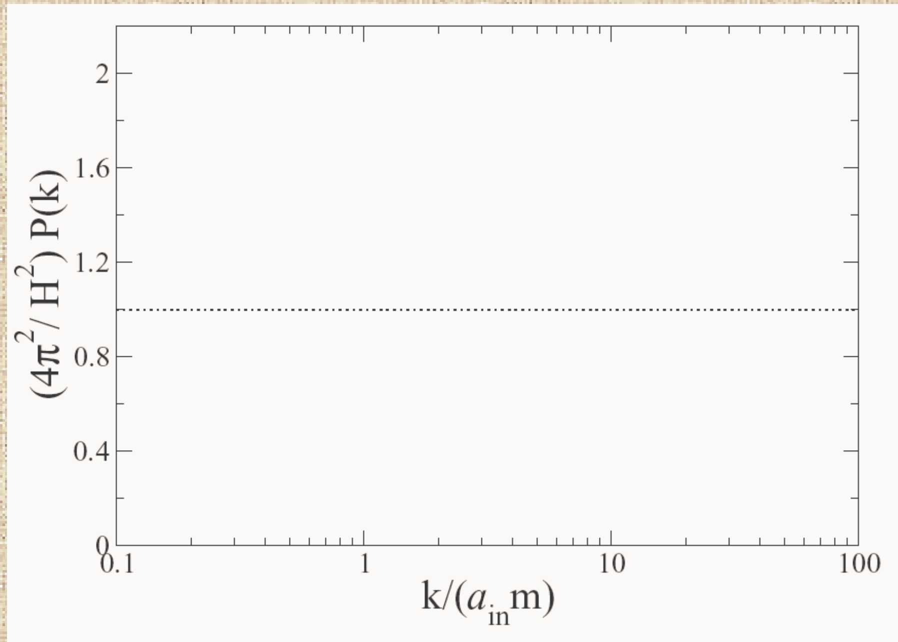
$$z = m\tau$$

Two parameters:

$$q_0 = \frac{g^2\Phi^2}{4m^2}, \quad \text{and} \quad \frac{m}{H}$$

Spectrum from inflation



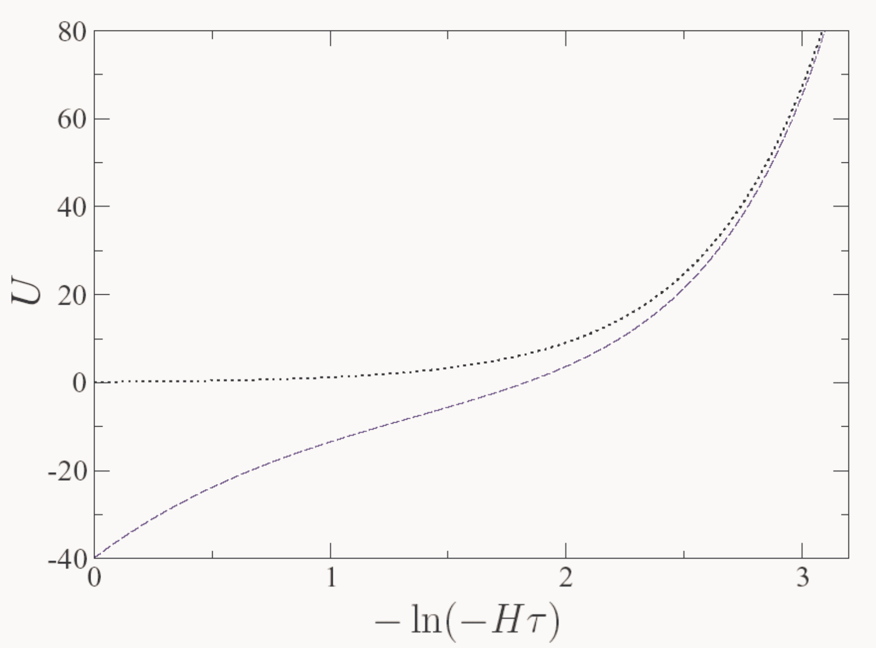


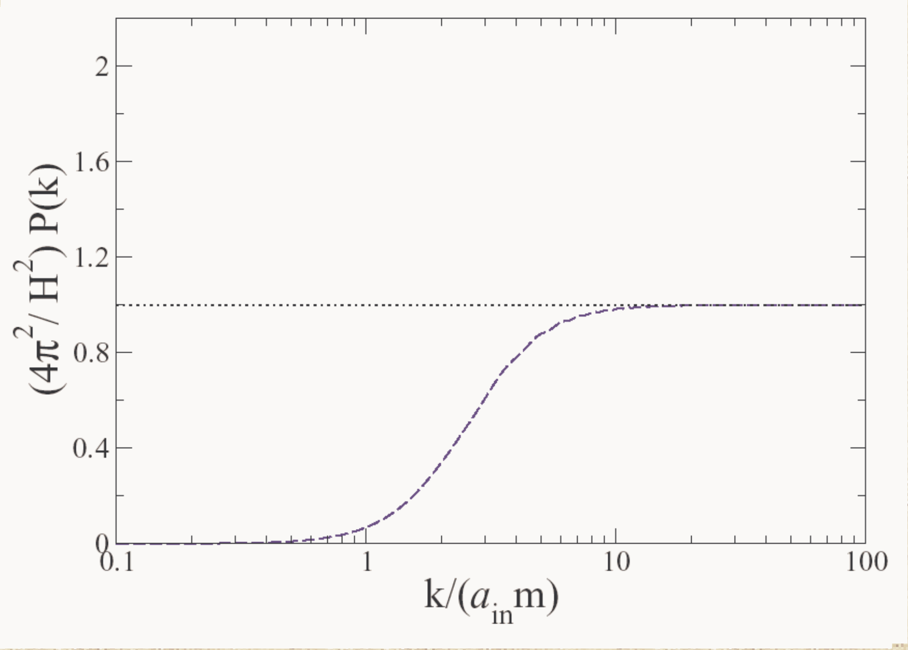
$$\frac{d^2 u_k}{d\tau^2} + \left(k^2 - U(\tau)\right) u_k = 0,$$

$$U(\tau) = \frac{2}{\tau^2}$$

$$\mathcal{P}_{\delta\chi}(k) \simeq \left(\frac{H}{2\pi}\right)^2 \qquad (k \ll aH)$$

Spectrum with varying mass



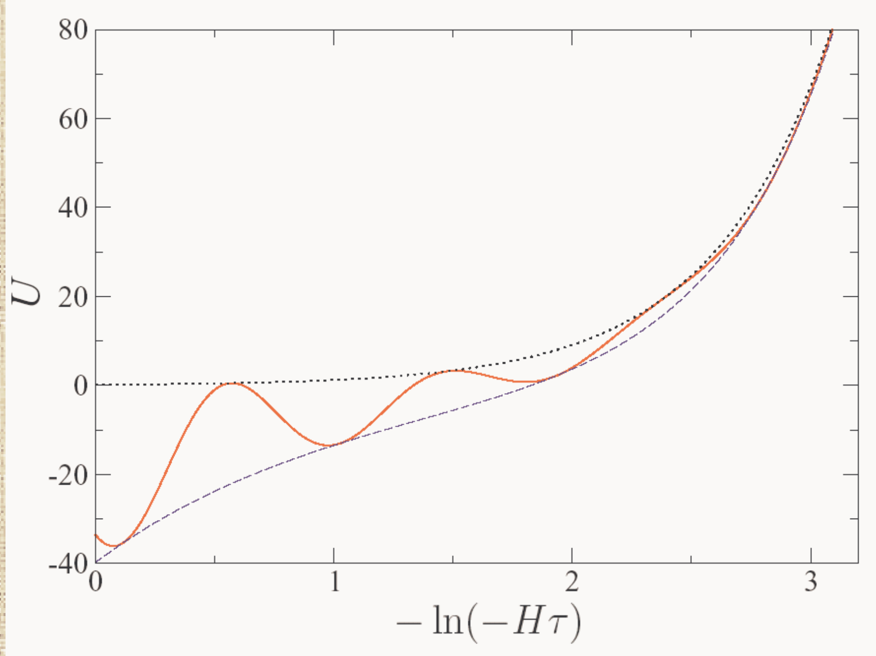


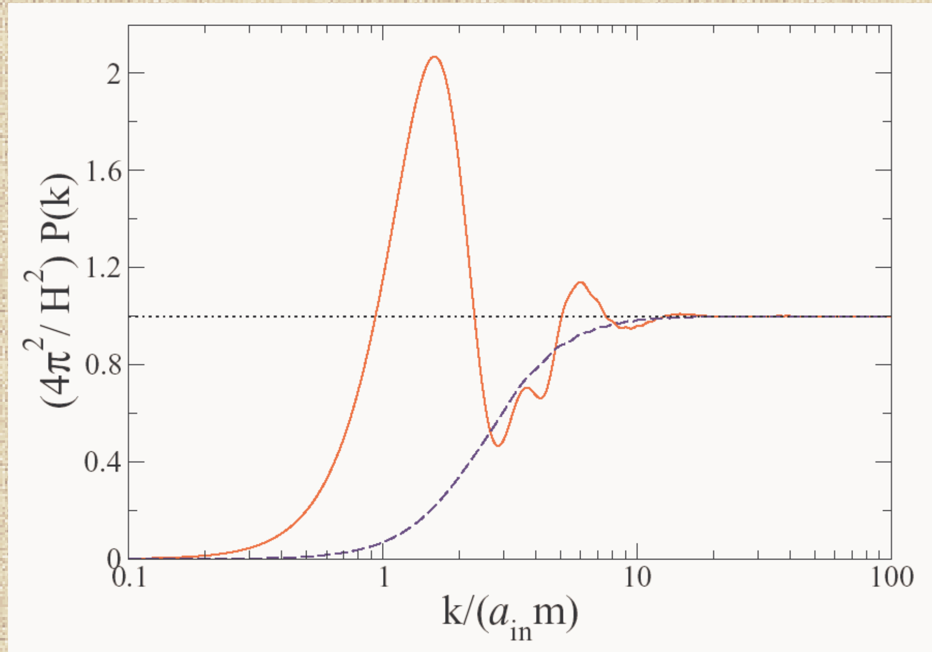
$$\frac{d^2u_k}{d\tau^2} + \left(k^2 - U(\tau)\right) u_k = 0,$$

$$U(\tau) = \frac{2}{\tau^2} + g^2\Phi^2 H\tau$$

The mass of χ is varying (decreasing) without oscillations

Spectrum with oscillating mass





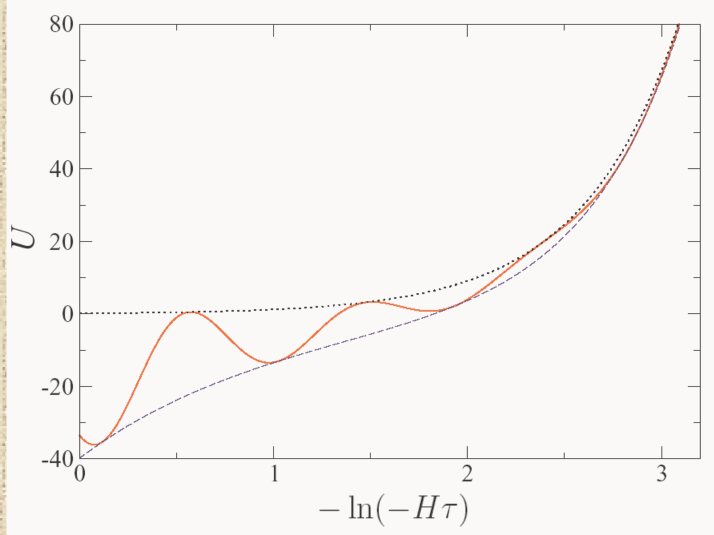
$$\frac{d^2u_k}{d\tau^2} + \left(k^2 - U(\tau)\right) u_k = 0,$$

$$U(\tau) = \frac{2}{\tau^2} + g^2\Phi^2 H\tau \cos^2\left[\frac{m}{H}\ln(-H\tau) + \varphi\right].$$

Discussion

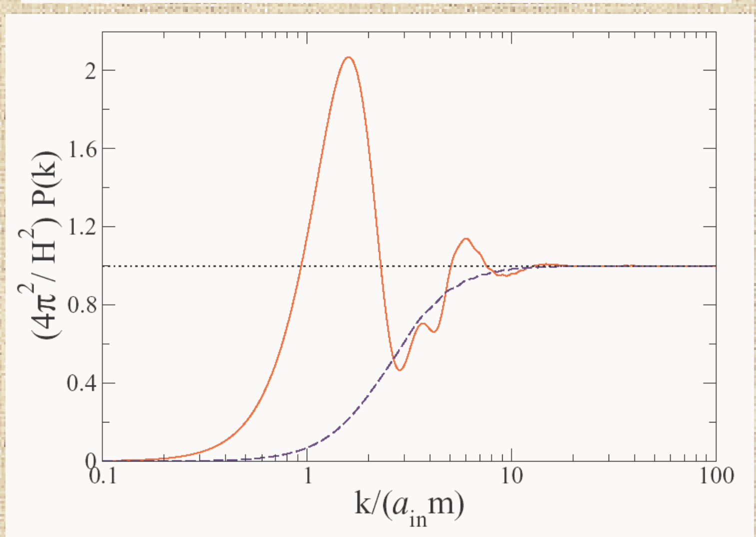
It is instructive to attempt an analysis similar to the preheating scenario:

$$\frac{d^2 u_k}{d\tau^2} + \underbrace{\left\{ k^2 - \frac{2}{\tau^2} + 4 q_0 m^2 H \tau \cos^2 \left[\frac{m}{H} \ln(-H\tau) \right] \right\}}_{\omega_k^2(\tau)} u_k = 0$$



Subtle effects: **two sources of particle production**, sometimes acting in opposite way

Adiabaticity violation: $\left| \frac{d\omega_k}{d\tau} \right| \gtrsim \omega_k^2$



- Predict **when parametric resonance stops** being important for particle production: after the j_{max} oscillation, j_{max} being a function of q_0 and m/H.

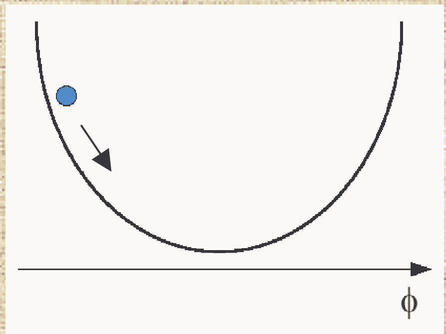
Maximal mode amplified: $k_{max} \equiv \sqrt{2} m \left(a(\tau_{j_{max}}) q_0 \right)^{1/4}$

Observational consequences

If χ is not the inflaton, **is it relevant for cosmological perturbations**?

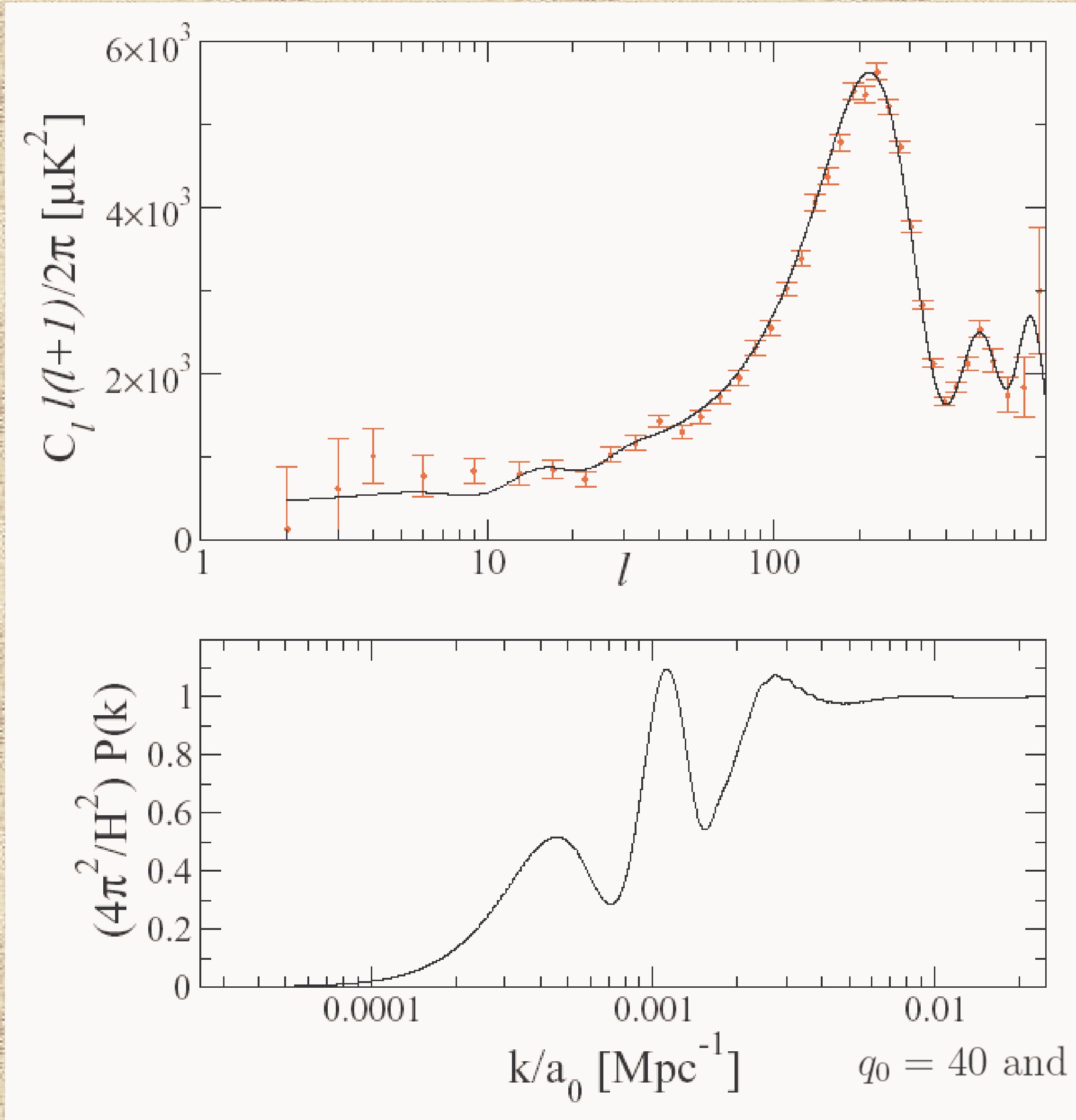
Cosmological perturbations may have been generated by a light field other than the inflaton:
curvaton or modulated fluctuations

The **curvaton** is light field which oscillates late after inflation and decays before nucleosynthesis. If during oscillations it dominates the universe, **it imprints its perturbations**.



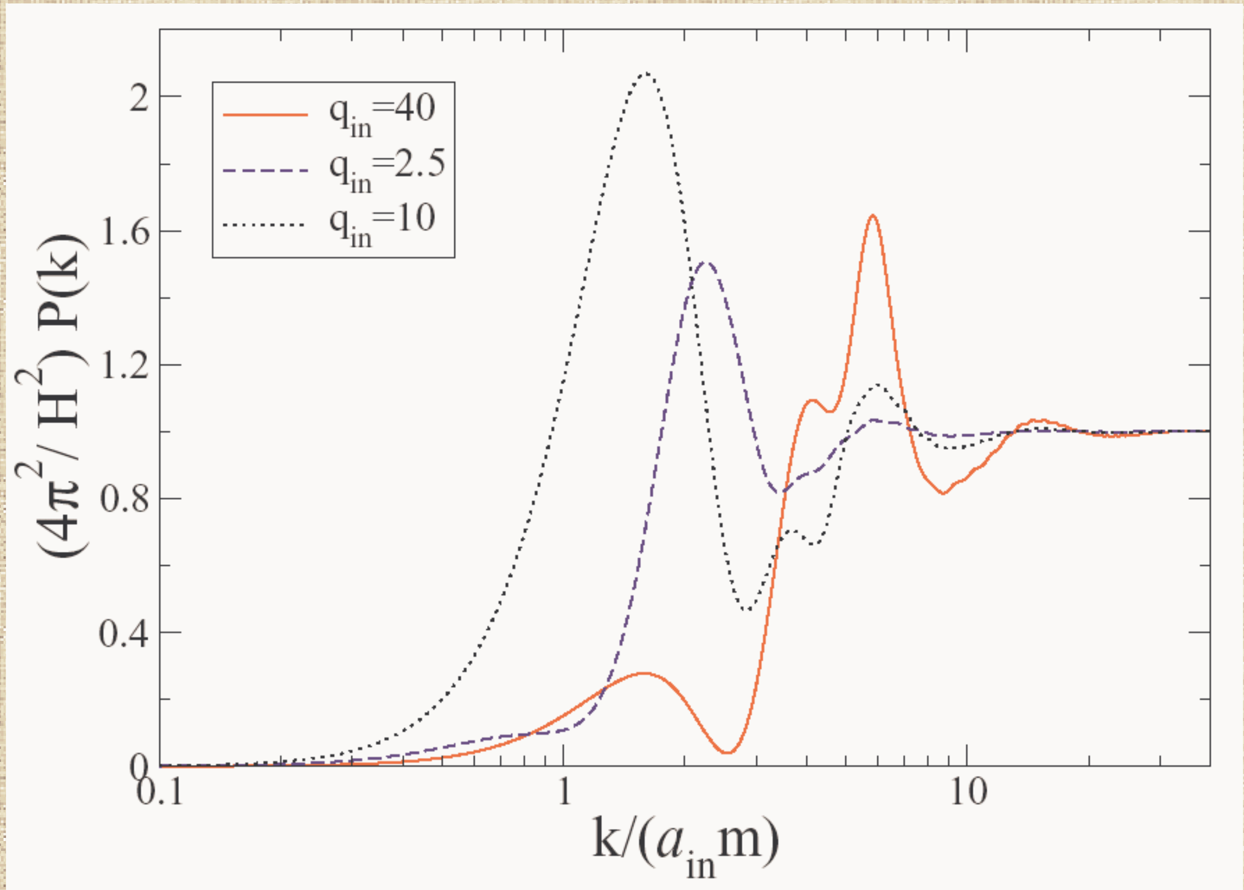
$$V(\phi,\chi) = \frac{1}{2}{m_\phi}^2\phi^2 + \frac{1}{2}g^2\phi^2\left(\chi-\chi_0\right)^2 + \frac{1}{2}m_\chi^2\chi^2$$

CMB spectrum

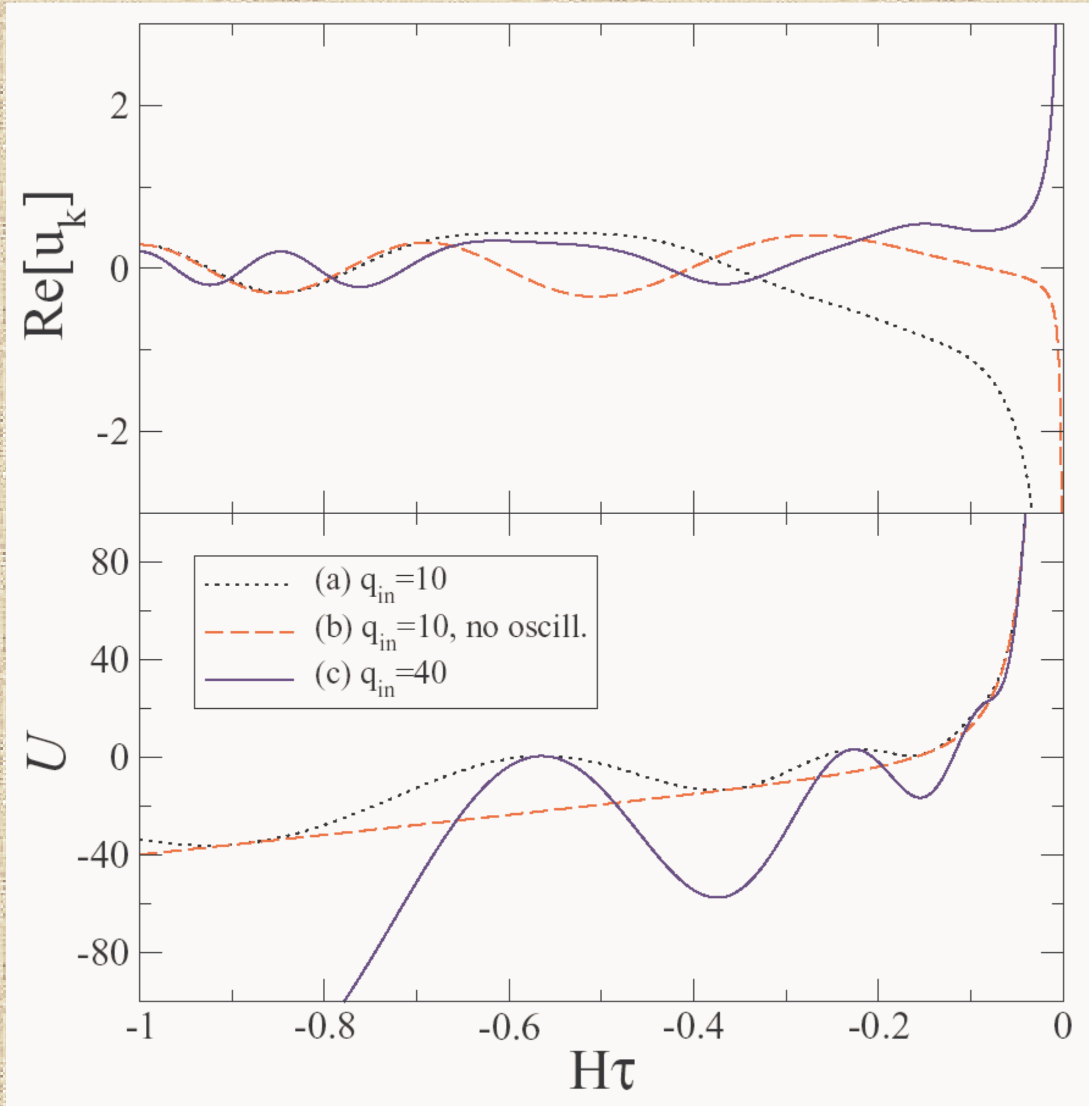


$q_0 = 40$ and $m/H = 2\sqrt{2}$

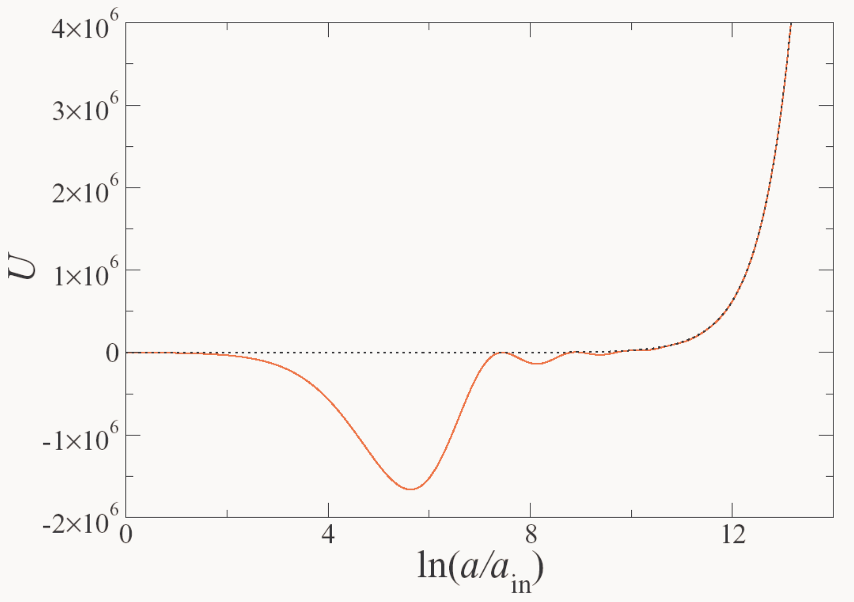
Varying q_0

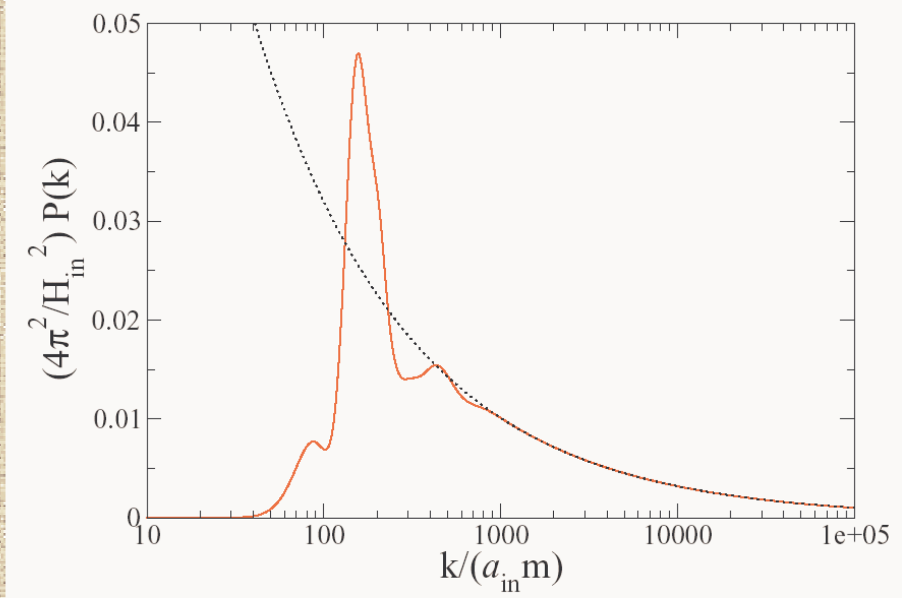


Time evolution of modes



Power-law inflation





$$a(t) \propto t^p, \qquad p \gg 1$$

$$\ddot{\phi} + \frac{3p}{t}\dot{\phi} + {m_\phi}^2 \phi = 0$$

$$\phi = \phi_0 \Gamma(\nu + 1) \left(\frac{m_\phi t}{2}\right)^{-\nu} J_\nu(m_\phi t), \quad \nu = (3p - 1)/2$$

$$\Phi \sim \phi_0 \left(\frac{3H}{4 m_\phi e}\right)^{3p/2}$$

The potential