Can dark energy evolve to the Phantom?

Alexander Vikman
LMU Munich

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Motivation

A) Late acceleration of the Universe. (DE)

- 1) Observations of the transition from $W>-1$ to $W<-1$?
  Alam, Sahni, Starobinsky 2003/4
- 2) Dynamical **DE** + $W\approx-1$ →
  Can **DE** evolve through $W=-1$?
- 3) if $W_{NOW}<-1$ observed, coincidence problem
- 4) „big rip" singularity for $W<-1$
- 5) Stability

3) Early Universe

- 1) „big crunch" singularity in pre-big bang scenarios
- 2) exit from superexponential inflation

C) General ?

Dynamical violation of DEC
- or WEC

Model

GR + 1 scalar field φ
(dominance of DE)

$$S = \int d^4x \sqrt{-g} \left[-\frac{R}{2} + p(\varphi, X) \right]$$

$$X \equiv \frac{1}{2} \nabla_\mu \varphi \nabla^\mu \varphi$$

$$T_{\mu\nu} = p_{,X} \nabla_\mu \varphi \nabla_\nu \varphi - p\, g_{\mu\nu} \rightarrow$$

„perfect fluid"

pressure $= p(\varphi, X)$

energy density $\varepsilon = 2X p_{,X} - p$

Armendáriz-Picón, Damour, Mukhanov 1999

Armendáriz-Picón, Mukhanov, Steinhardt 2000

EE for spatially flat FRW Univers

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\varepsilon + 3p)$$

$$H^2 = \frac{\varepsilon}{3} \longrightarrow \varepsilon \geqslant 0 \text{ +dominan}$$

$$\rightarrow \varepsilon > 0$$

continuity equation

$$\dot{\varepsilon} = -3H(\varepsilon + p)$$

Whole dynamics in Eq. of Motion

$$\varepsilon_{,X}\ddot{\varphi} + \dot{\varphi}\sqrt{3\varepsilon}\, p_{,X} + \varepsilon_{,\varphi} = 0$$

$(\varphi, \dot{\varphi})$ - Phase space

Phase diagram needed

$$w \equiv \frac{p}{\varepsilon} = -1 + \frac{2X}{\varepsilon} p_{,X}$$

$$X \geqslant 0 \quad \varepsilon > 0 \rightarrow p_{,X}$$

should change the sign for the dynamical evolution from $w \geqslant -1$ to $w < -1$

We do not live in an ideal FRW Universe! → Perturbations

sound speed

$$C_s^2 \equiv \frac{p_{,X}}{\varepsilon_{,X}} = \frac{(w+1)\varepsilon}{2X\varepsilon_{,X}}$$

Garriga, Mukhanov 1999

$$C_s^2 = \frac{(w+1)\,\mathcal{E}}{2X\,\mathcal{E}_{,X}} \longrightarrow \text{ if } X \neq 0,\ \mathcal{E}_{,X} \neq 0$$

$(w+1)$ changes the sign $\Leftrightarrow$

C_s^2 changes the sign!

For Stability $C_s^2 > 0$! $\longrightarrow$

Transition destroys $\approx$FRW Universe!

Consider the transition via $X = 0$. $\underline{C_s^2\ ?}$

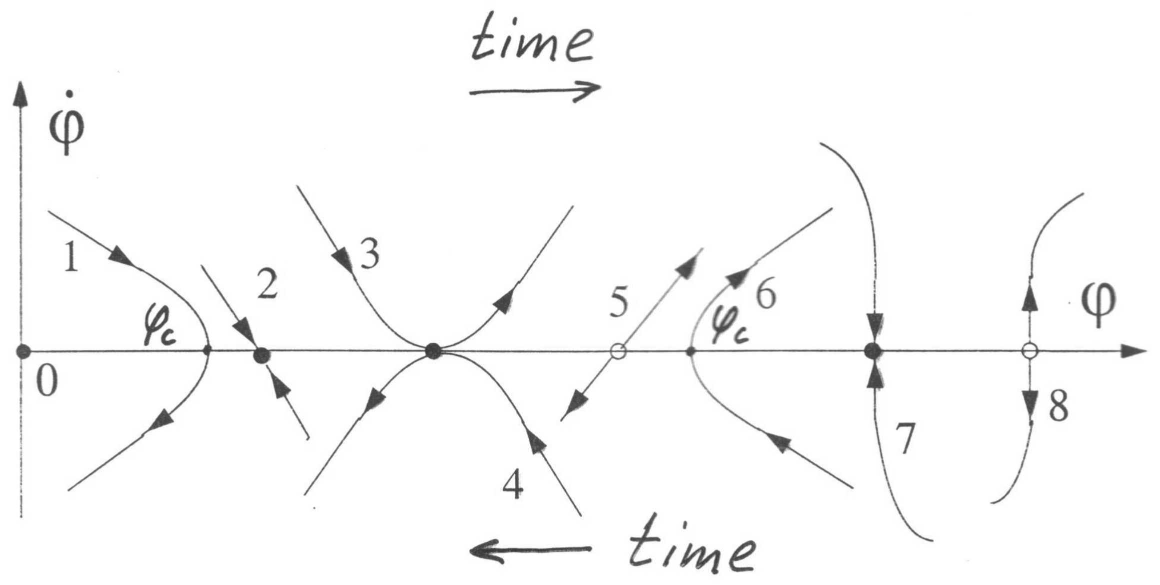

Figure 1: Possible phase curves in the neighborhood of the φ−axis. Only on the curves 1 and 6 the system pass the φ−axis. The curves 2,3,4,7 have an attractor as a shared point with the φ−axis, whereas the curves 5,8 have an repulsor. These attractors and repulsors can be a fixed point solutions or singularities as well.

$$\frac{d\dot{\varphi}}{d\varphi} = \frac{\ddot{\varphi}}{\dot{\varphi}}$$

- For phase curves 1 and 6

$$\varphi - \varphi_c \sim \dot{\varphi}^{2n} = X^n \qquad n \geq 1$$

thus $X p_{,X}(X^n, X)$ cannot change the sign.

- For phase curves 3 and 4

$$\dot{\varphi} \sim (\varphi - \varphi_c)^{2n} \quad \text{and} \quad n \geq 1$$

$$t = \int_{\varphi_{in}}^{\varphi_c} \frac{d\varphi}{\dot{\varphi}(\varphi)} \sim \int_{\varphi_{in}}^{\varphi_c} \frac{d\varphi}{(\varphi - \varphi_c)^{2n}} = \infty$$

The last possibility for transition the points $\Psi_c \equiv (\varphi_c, \dot{\varphi}_c)$ where

$$p_{,x}(\Psi_c) = 0, \quad \mathcal{E}_{,x}(\Psi_c) = 0$$

$$X_c \neq 0$$

<u>Eq. of Motion</u> Leads to the Eq. for phase curves

$$\frac{d\dot{\varphi}}{d\varphi} = -\frac{\dot{\varphi}\, p_{,x}\sqrt{3\mathcal{E}} + \mathcal{E}_{,\varphi}}{\dot{\varphi}\, \mathcal{E}_{,x}}$$

$$\exists\, \ddot{\varphi}\big|_{\Psi_c} \longrightarrow \mathcal{E}_{,\varphi}(\Psi_c) = 0$$

if $\mathcal{E}_{,\varphi}(\Psi_c) \neq 0$ then $\nexists\, \ddot{\varphi}$ and

$$\frac{d^2\varphi}{d^2\dot{\varphi}} = -\frac{\dot{\varphi}^2}{\mathcal{E}_{,\varphi}}\, \mathcal{E}_{,xx} \neq 0 \rightarrow$$

Transition impossible

Ψ_c is a singularity of Eq. Motion

Let us study the type of the singularity.

Use QTDE. Linearization.

- Why it is important to investigate this type?

- The form of the Eq. Motion excludes nodal point!

- The only mathematically allowed transition is absolutely unstable. The measure of the solutions which realize the transition is zero in $(\varphi, \dot{\varphi})$

Conclusions

- For dominating DE with $p(\varphi, \nabla_\mu \varphi)$

1) the transitions from $w \geqslant -1$ to $w < -1$ (or vice versa) are physically implausible.
2) no exit from superexp. infl.
3) smooth bounce cannot be realized

- DE with $p(\varphi, \nabla_\mu \varphi) = \frac{1}{2}(\nabla_\mu \varphi)^2 f(\varphi) - V($ cannot evolve from $w \geqslant -1$ to $w < -1$ (or vice versa).

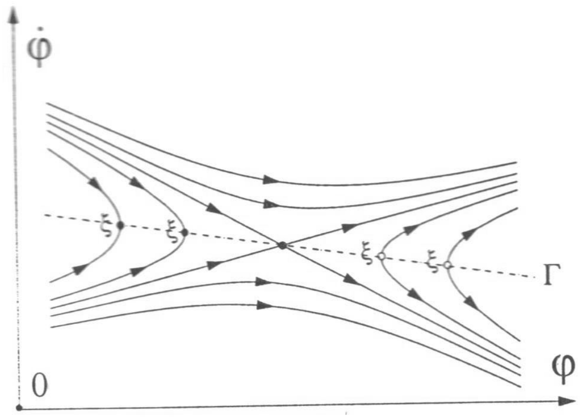

Figure 2: Phase curves in the neighborhood of the singular point Ψ_c^+ are plotted for the case of the real λ. At the points ξ the solutions $\varphi(t)$ do not exist. These points together with Ψ_c^+ build the curve Γ on which $\varepsilon_{,x}(\Gamma) = 0$.

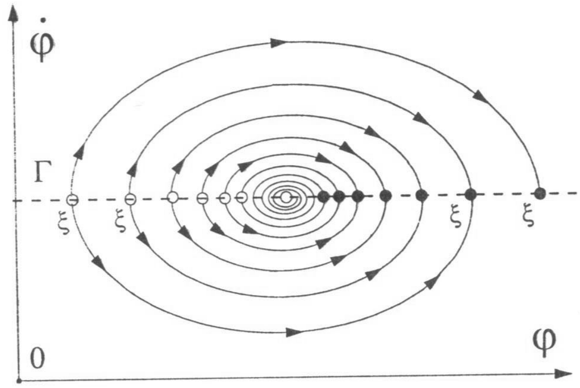

Figure 3: Phase curves in the neighborhood of the singular point Ψ_c^+ are plotted for the case of the pure imaginary λ. Her we assume that the singular point is a focus.

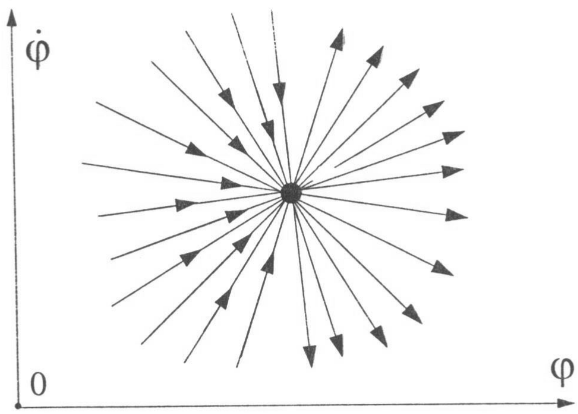

Figure 4: If **A** had the eigenvalues $\lambda_1 = \lambda_2$ then the singular point Ψ_c^+ would be a nodal point and there would be a continuous set of trajectories passing through it. To illustrate this we plot here the phase curves in the particular case of an degenerate nodal point. The form of the equation of motion excludes such type of the singular points and therefore prevents the possibility of such transitions.

$p = K(\varphi)X + U$

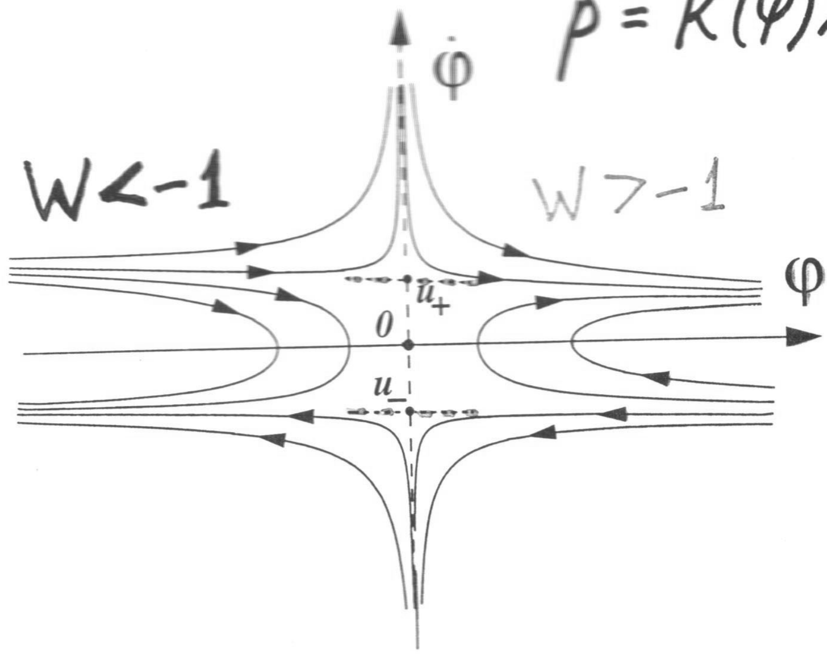

Figure 5: The typical behavior of the phase curves in the neighborhood of the "critical" line where $K(\varphi) = 0$ (here $\dot{\varphi}$–axis), is plotted for the case when $K_c' > 0$, $V_c' < 0$. Horizontal dashed lines are the analytically obtained separatrices $\dot{\varphi}_{\pm}$, $(0, u_{\pm})$ are the points of transition.

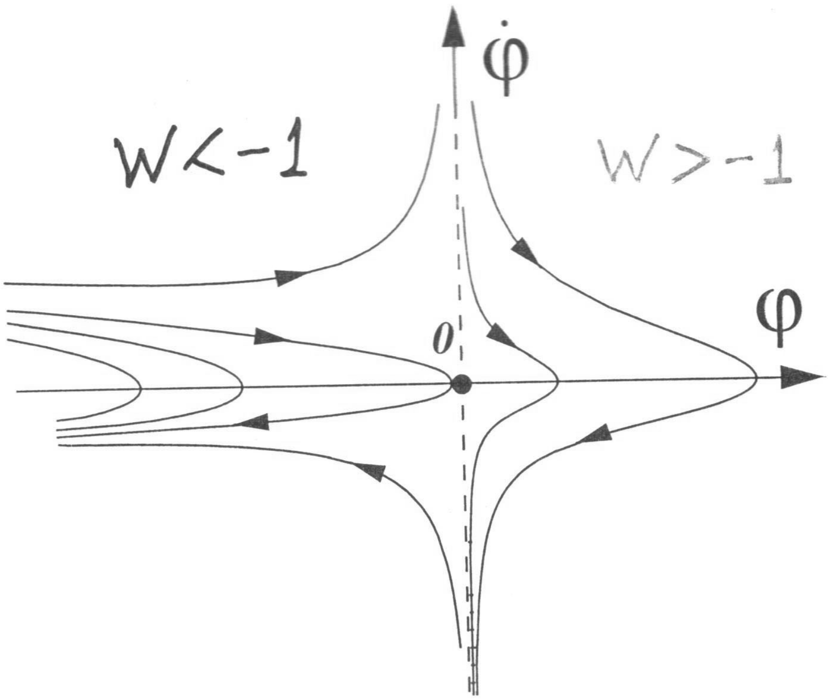

Figure 6: The typical behavior of the phase curves in the neighborhood of the "critical" line where $K(\varphi) = 0$ (here $\dot{\varphi}$–axis), is plotted for the case when $K_c' > 0$, $V_c' = 0$, $V_c'' > 0$.

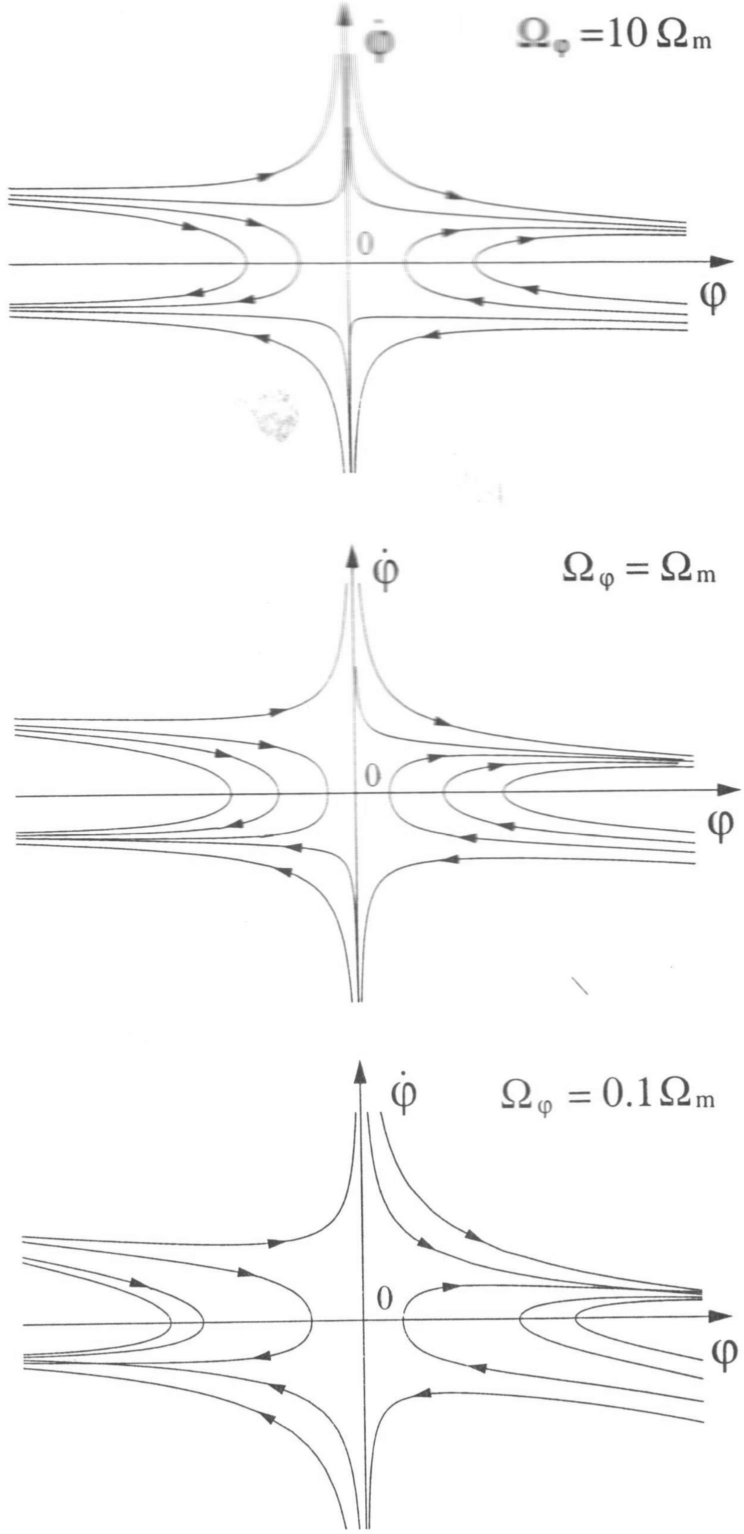

Figure 7: Numerically obtained trajectories of the dark energy described by a Lagrangian linear in X are plotted for the cases $\Omega_\varphi = 10\Omega_m$, $\Omega_\varphi = \Omega_m$ and $\Omega_\varphi = 0.1\Omega_m$.