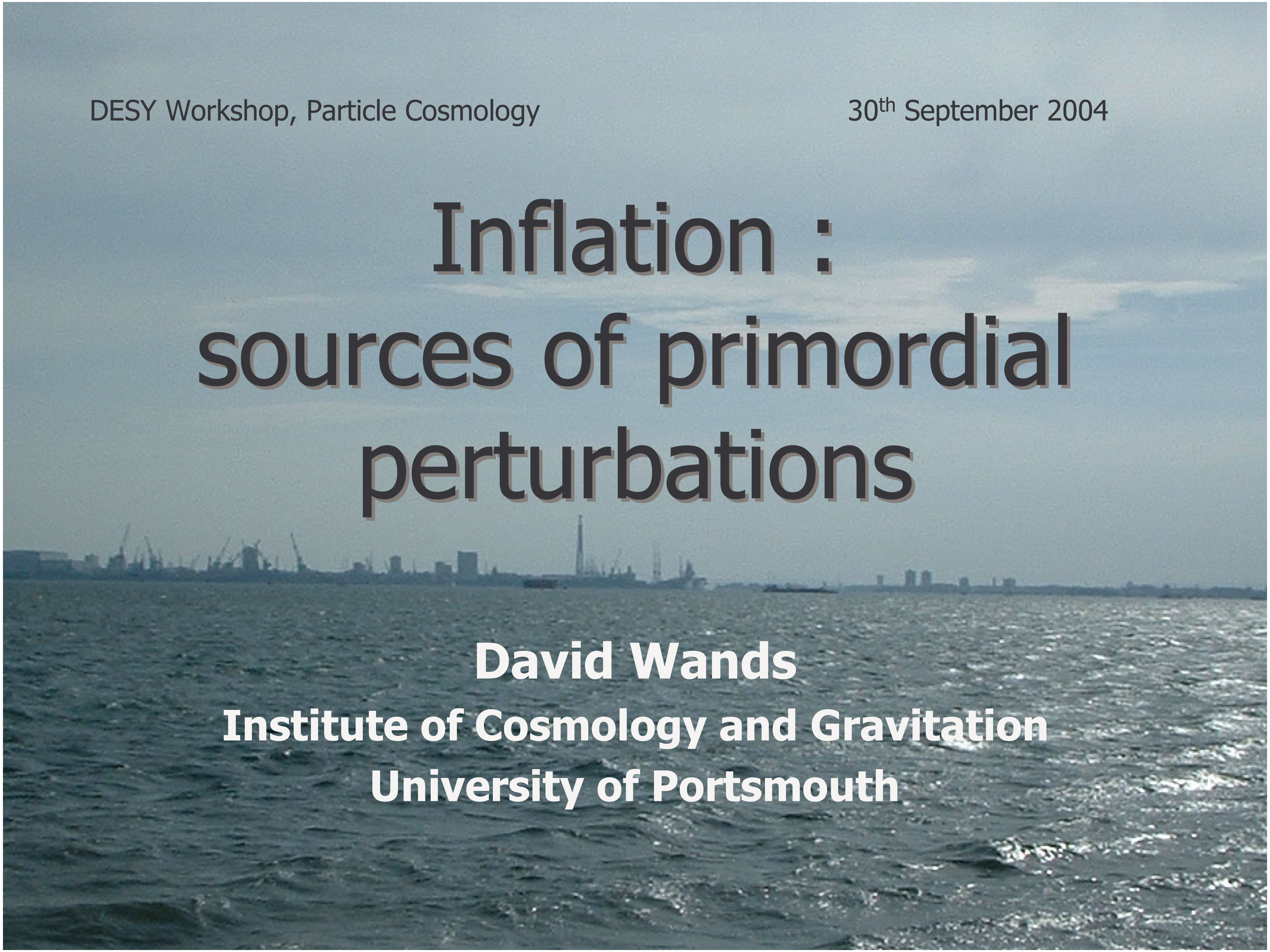

DESY Workshop, Particle Cosmology

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Inflation : sources of primordial perturbations

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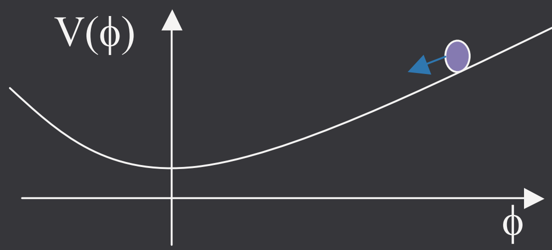
Cosmological inflation:

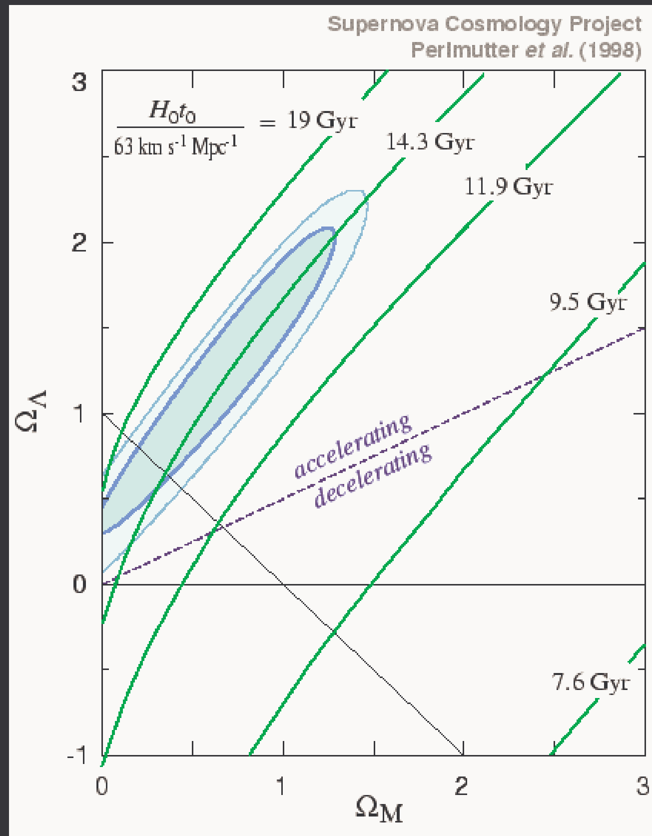
Starobinsky (1980)
Guth (1981)

- period of accelerated expansion
  in the very early universe
- requires negative pressure

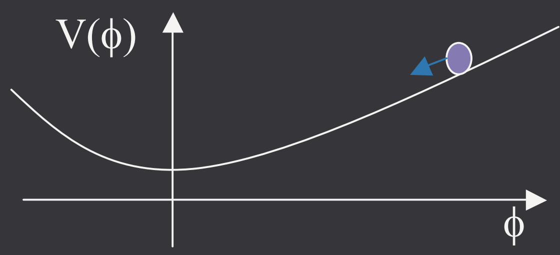
  e.g. self-interacting scalar field



- speculative and uncertain physics
- just the kind of peculiar cosmological
  behaviour we observe today
  dark energy!



Inflationary dynamics:



- N self-interacting scalar fields:

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{dV}{d\varphi} = 0$$

- Homogeneous Hubble expansion:

$$3H^2 = 8\pi G\left(V + \frac{1}{2}\dot{\varphi}^2\right)$$

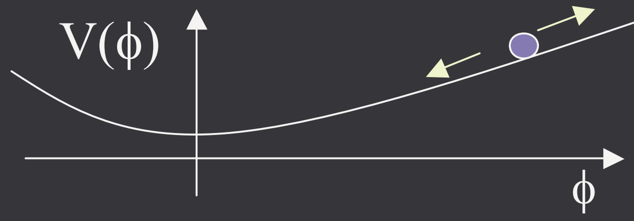
inflation ⇔ *acceleration* ⇔ $V > \dot{\varphi}^2$

- inhomogeneous field perturbations, wavenumber k, coupled to metric perturbations ψ

$$\delta\ddot{\varphi} + 3H\delta\dot{\varphi} + \left(\frac{k^2}{a^2} + m^2\right)\delta\varphi = 3\varphi\dot{\psi} + \ldots$$

Vacuum fluctuations

Hawking '82, Starobinsky '82, Guth & Pi '82



- *small-scale/underdamped zero-point fluctuations* $\delta\phi_k \approx \frac{e^{-ik\eta}}{a\sqrt{2k}}$
- *large-scale/overdamped perturbations in growing mode*
  linear evolution $\Rightarrow$ *Gaussian random field*

$$\left\langle \delta\phi^2 \right\rangle_{k=aH} \approx \frac{4\pi k^3 \left|\delta\phi_k{}^2\right|}{(2\pi)^3} = \left(\frac{H}{2\pi}\right)^2$$

fluctuations of any scalar light fields (m<3H/2) `frozen-in' on large scales

weakly scale-dependent

$$\frac{d \ln\langle \delta\phi^2 \rangle}{d \ln k} \equiv n_\phi - 1$$

- *slowly changing Hubble rate* -2ε
- *slow evolution outside horizon*
  - *finite mass* $+2\eta$
  - *metric back-reaction (self-gravity)* -4ε

Slow-roll parameters: $\varepsilon \equiv -\frac{\dot{H}}{H^2}$, $\eta \equiv \frac{1}{3}\frac{m^2}{H^2}$

$$n_\phi - 1 = -6\varepsilon + 2\eta$$

Self-gravity of inflaton field in GR

Consistently include linear metric perturbations, ψ, using Mukhanov/Sasaki variable:

$$Q \equiv \delta\phi - \frac{\dot{\phi}}{H}\psi$$

obeys 4D wave equation:

$$\ddot{Q} + 3H\dot{Q} + \left(\frac{k^2}{a^2} + m^2 + \frac{8\pi G}{a^3}\left(\frac{a^3\dot{\phi}^2}{H}\right)^{\cdot}\right)Q = 0$$

used to calculate corrections,
e.g., in slow-roll expansion

Sources of structure:
(1) density perturbations during inflation

Inflaton field perturbations lead to adiabatic density perturbations on super-Hubble scales during inflation:

$$\delta\rho \approx V'\delta\phi$$

Conserved perturbation: *Bardeen, Steinhardt & Turner (1983)*

$$\zeta = -\frac{H}{\dot{\rho}}\delta\rho - \psi \approx -\frac{H}{\dot{Q}}Q$$

Local energy conservation => conserved perturbation always exists for adiabatic perturbations on large scales

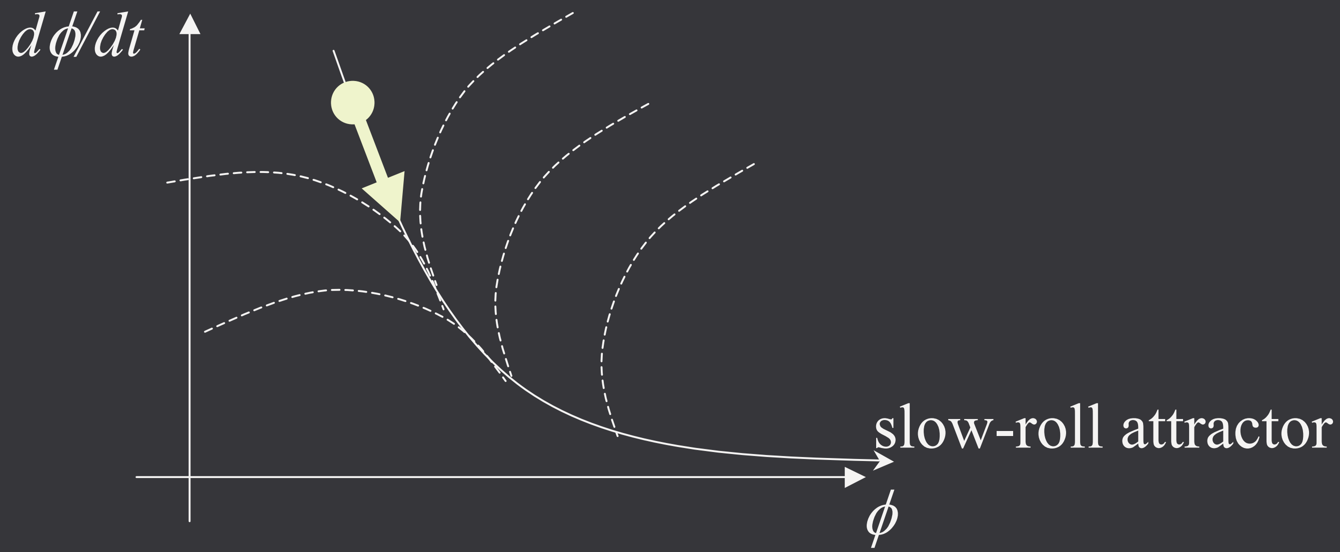
Wands, Malik, Lyth & Liddle (2001)

adiabatic perturbations on large scales

- **adiabatic perturbations** *e.g.,* $\delta\left(\frac{n_\gamma}{n_B}\right) \propto \frac{\delta n_\gamma}{n_\gamma} - \frac{\delta n_B}{n_B} = 0$
  - *perturb along the background trajectory*

$$\frac{\delta x}{\dot{x}} = \frac{\delta y}{\dot{y}} = \delta t$$

  - ***adiabatic perturbations stay adiabatic on large scales***



observables in inflaton scenario:

- **primordial density perturbation -> window onto inflation**
  - *amplitude*

$$\langle \zeta^2 \rangle \approx \left(\frac{H}{\dot\phi}\right)^2 \left(\frac{H}{2\pi}\right)^2_{k=aH} \approx \left(\frac{1}{\varepsilon}\frac{V}{M_{Pl}^4}\right)_{k=aH}$$

  - *scale-dependence*

$$n_\zeta - 1 \approx (-6\varepsilon + 2\eta)_{k=aH}$$

  - *tensor-scalar ratio*

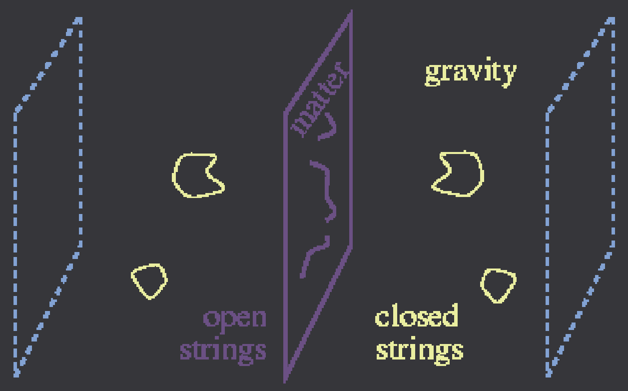
$$\frac{\langle T^2 \rangle}{\langle \zeta^2 \rangle} \approx (16\varepsilon)_{k=aH} \approx -2n_T$$

observational consistency test *Liddle & Lyth '93*

brane-world inflaton scenario:

Maartens, Wands, Bassett & Heard (2000)

- 4D inflaton on our brane-world



- full 5D metric perturbations at high energies too complicated to solve in general
- Calculate ζ during quasi-de Sitter inflation on the brane (negligible metric back-reaction for V=constant)
- ζ conserved on large scales until low energies where we can use 4D gravity Langlois, Maartens, Sasaki & Wands (2001)
- but can't yet include metric perturbations for vacuum fluctuation on small scales, $l_{5D,Planck} < \lambda < l_{AdS}$
  Koyama, Langlois, Maartens & Wands (2004)
- hence can't yet calculate as slow-roll corrections for linear perturbations

brane-world inflaton scenario:

- **primordial density perturbation**
  - *amplitude*

$$\langle \zeta^2 \rangle \approx \left(\frac{H}{\dot{\phi}}\right)^2 \left(\frac{H}{2\pi}\right)^2_{k=aH} \approx \left(\frac{1}{\varepsilon}\frac{V}{M_{Pl}^4}\right)_{k=aH} \times G^2\left(Hl_{AdS}\right)$$

  - *tensor-scalar ratio*

$$\frac{\langle T^2 \rangle}{\langle \zeta^2 \rangle} \approx (16\varepsilon)_{k=aH} \times \frac{F^2\left(Hl_{AdS}\right)}{G^2\left(Hl_{AdS}\right)} \approx -2n_T$$

same observational consistency test *Huey & Lidsey 2000*

multi-field perturbations

orthogonal fields ϕ, χ => uncorrelated vacuum fluctuations

"adiabatic" $Q \equiv \delta\phi + \frac{\dot{\chi}}{\dot{\phi}}\delta\chi$ "entropy" $\delta s \equiv \delta\chi - \frac{\dot{\chi}}{\dot{\phi}}\delta\phi$

wave equations:

$$\ddot{Q} + 3H\dot{Q} + \left(\frac{k^2}{a^2} + \frac{d^2V}{d\phi^2} + \frac{8\pi G}{a^3}\left(\frac{a^3\dot{\phi}^2}{H}\right)^{\cdot}\right)Q \approx O\left(\frac{d\theta}{d\sigma}\delta s\right)$$

$$\delta\ddot{s} + 3H\delta\dot{s} + \left(\frac{k^2}{a^2} + \frac{d^2V}{ds^2}\right)\delta s \approx 0$$

additional source for density perturbations

Sasaki & Stewart '96; Gordon et al '01

Scale dependence of isocurvature fluctuations

$$\frac{d\ln\langle \delta s^2 \rangle}{d\ln k} \equiv n_{\delta s} - 1$$

- *slowly changing Hubble rate* -2ε
- *slow evolution outside horizon*
  - *finite mass* $+2\eta_{ss}$
  - ~~*metric back-reaction (self-gravity)* -4ε~~

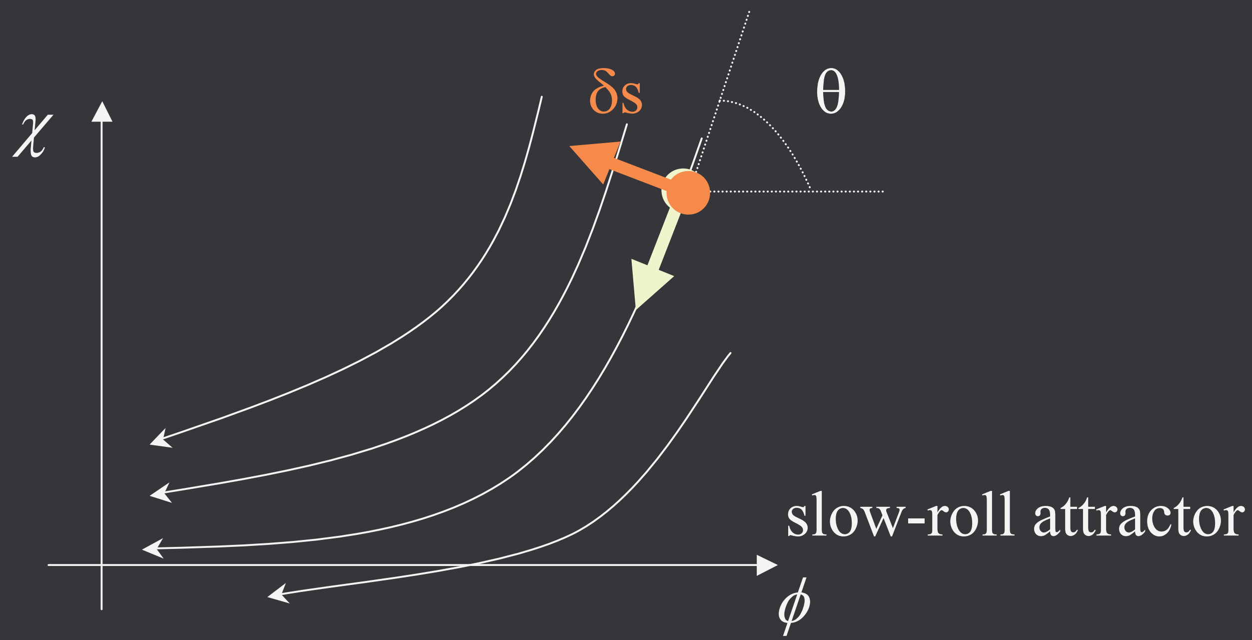
Slow-roll parameters: $\varepsilon \equiv -\frac{\dot{H}}{H^2}$, $\eta_{ss} \equiv \frac{1}{3}\frac{m_s^2}{H^2}$

$$\boxed{n_{\delta s} - 1 = -2\varepsilon + 2\eta_{ss}}$$

entropy perturbations

-> density perturbations on large scales

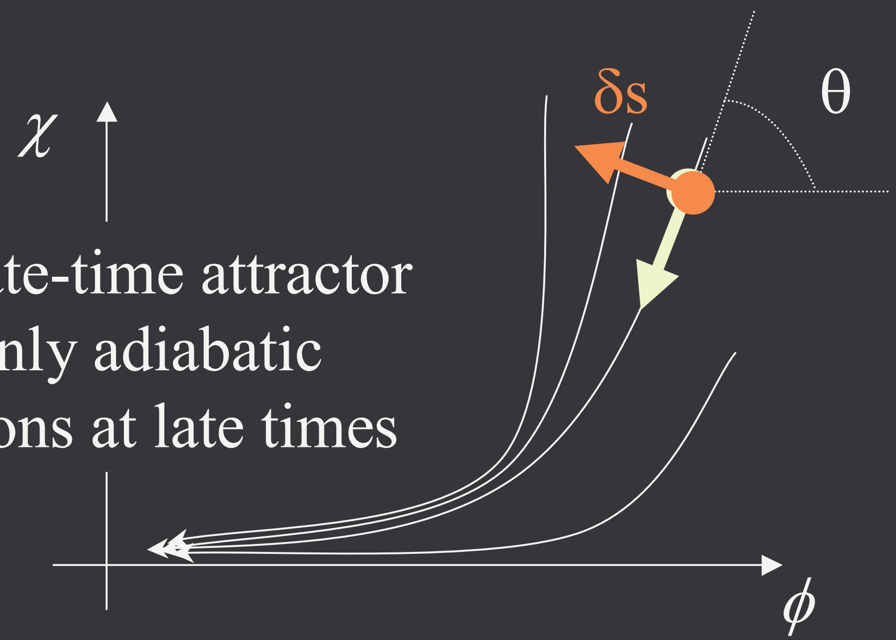
$$\dot{\zeta} \approx 2H \frac{d\theta}{d\sigma} \delta s$$



entropy perturbations

-> *adiabatic* density perturbations on large scales

$$\dot{\zeta} \approx 2H \frac{d\theta}{d\sigma} \delta s$$



a unique late-time attractor
gives only adiabatic
perturbations at late times

two-field scenario: Wands, Bartolo, Matarrese & Riotto 2002

- **primordial density perturbation enhanced after Hubble exit**
  - *amplitude*

$$\langle \zeta^2 \rangle \approx \frac{1}{\sin^2 \Delta}\left(\frac{H}{\dot\phi}\right)^2 \left(\frac{H}{2\pi}\right)^2_{k=aH} \approx \frac{1}{\sin^2\Delta}\left(\frac{1}{\varepsilon}\frac{V}{M_{Pl}^4}\right)_{k=aH}$$

  - *scale-dependence*

$$n_\zeta - 1 \approx -\left(6 - 4\cos^2\Delta\right)\varepsilon + 2\left(\eta_{\phi\phi}\sin^2\Delta + 2\eta_{\phi s}\sin\Delta\cos\Delta + \eta_{ss}\cos^2\Delta\right)$$

  - *gravitational waves*

$$\frac{\langle T^2 \rangle}{\langle \zeta^2 \rangle} \approx (16\varepsilon)_{k=aH}\sin^2\Delta \approx -2n_T\sin^2\Delta$$

  - *plus*
    - *residual isocurvature modes? correlation angle* $\cos\Delta$
    - *non-Gaussianity?*

Sources of structure:
(2) inhomogeneous reheating at end of inflation

Inflaton decay rate, Γ, sensitive to moduli fields

$$\delta\Gamma \approx \Gamma' \delta\chi$$

earlier decay changes local radiation density after

Kofman (2003); Dvali, Gruzinov & Zaldarriaga (2003)

$$\zeta = -\frac{H}{\dot{\rho}}\delta\rho - \psi = \left(\frac{\delta\rho_\gamma}{4\rho_\gamma}\right)_{\psi=0} = -\frac{1}{6}\frac{\delta\Gamma}{\Gamma}$$

Sources of structure:
(3) late-decaying scalars after the end of inflation

Enqvist & Sloth; Lyth & Wands; Moroi & Takahashi (2001)

Polonyi/moduli problem

- weakly-coupled massive scalar fields
- displaced from true vacuum
- begin to oscillate when H<m, with $\rho_\chi \propto a^{-3}$
- come to dominate over radiation, $\rho_\gamma \propto a^{-4}$
- must decay into radiation before nucleosynthesis

$$\zeta = \left(\frac{\delta \rho_\gamma}{4 \rho_\gamma} \right)_{\psi=0} \approx \Omega_{\chi,\mathrm{decay}} \zeta_\chi$$

curvaton mechanism

observational signature?
non-Gaussianity

simplest kind of non-Gaussianity:

$$\zeta \approx \zeta_1 + f_{NL} \zeta_1^2$$

Komatsu & Spergel (2001)

Wang & Kamiokowski (2000)

recall that for curvaton

$$\zeta \approx \Omega_{\chi,\text{decay}} \zeta_\chi \approx \Omega_{\chi,\text{decay}} \left(\frac{\delta\chi}{\chi} + \left(\frac{\delta\chi}{\chi} \right)^2 \right)$$

corresponds to

$$\zeta_1 \approx \Omega_{\chi,\text{decay}} \left(\frac{\delta\chi}{\chi} \right) \quad , \quad f_{NL} \approx \frac{1}{\Omega_{\chi,\text{decay}}}$$

Lyth, Ungarelli & Wands '02

constraints on f_{NL} from WMAP $f_{NL} < 134$

hence $\Omega_{\chi,\text{decay}} > 0.01$ and $10^{-5} < \delta\chi/\chi < 10^{-3}$

c.f. non-Gaussianity from inflaton scenario

single-field consistency relation:

$$f_{NL} = \frac{n_s - 1}{4} \approx \frac{-3\varepsilon + \eta}{2} << 1$$

Maldacena (2002)

Gruzinov; Creminelli & Zaldarriaga (2004)

"easy" to disprove the (simplest) inflaton scenario

Conclusions:

1. ***Precise data*** allows/requires us to study more detailed models of inflation and cosmological perturbations
2. ***Several mechanisms*** can produce ***primordial density perturbations on large-scales:***
   - inflaton perturbations during inflation
   - inhomogeneous rehating at end of inflation
   - late-decaying scalars (“curvaton”) after end of inflation
3. ***Distinctive observational tests:***
   - gravitational waves
   - (non-)Gaussianity
   - residual (correlated) entropy/isocurvature perturbations