SERGEI WINITZKI
(LMU MUNICH)

Creation of particles in a tunneling universe

J. Hong, A. Vilenkin, S.W.

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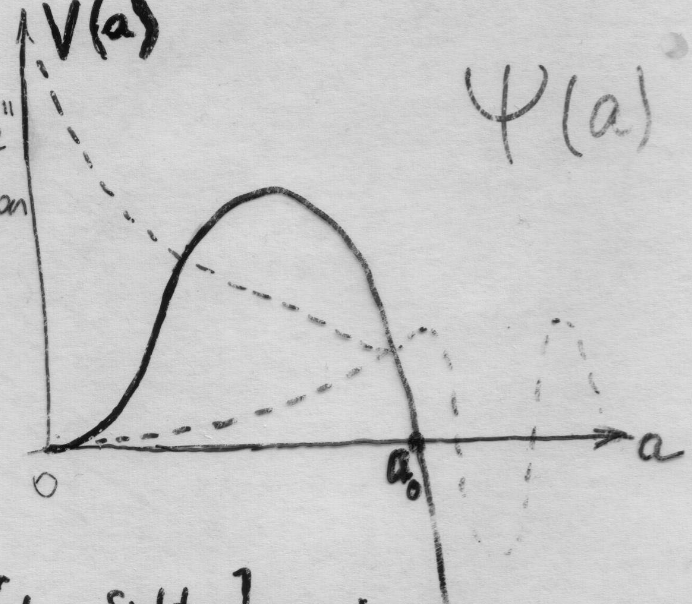
Outline

- Elementary quantum cosmology
  - Quantum mechanics of FRW universes
  - Wheeler - De Witt equation in minisuperspace
    $$\hat{\mathcal{H}}\Psi = 0$$
  - Creation of the universe by tunneling
  - Tunneling "from something" vs "from nothing"
- Inhomogeneous matter ("perturbative minisuperspace")
  - Tunneling in quantum mechanics: analogy
  - Interpretation of Ψ in QFT + FRW spacetime
- Claim: "catastrophic particle production" in tunneling
  [Rubakov et al. 1984 - 2002]
  - Contradicts other results (no particles)
- Resolution, part 1: massless case (explicit)
  - "Regularity conditions" and "branching" of Ψ
  - Dependence on the initial state "before" tunneling
- Resolution, part 2: massive case (approximate)
  - Construction of a "good" initial state
  - Interpretation and particle creation
- Conclusions

Minisuperspace of pure gravity

Wheeler-DeWitt equation

$$\left[\frac{\partial^2}{\partial a^2} - \underbrace{a^2 + H^2 a^4}_{V(a)}\right]\Psi(a) = 0.$$



- "Quantum-mechanical particle"
- Stationary Schrödinger equation
- Boundary conditions?

- Interpretation:
  - a semiclassical [de Sitter] universe
  - nucleated at $a = a_0$
  - "tunneling from nothing"

Elementary quantum cosmology

or, **Quantum mechanics of the universe**

- FRW metric: (spatially closed universe)

$$ds^2 = \underbrace{N^2(t)}_{\text{Lapse function}}\, dt^2 - \underbrace{a^2(t)}_{\text{scale factor}}\, \underbrace{d\Omega^2}_{\text{metric on } S^3}$$

- Einstein-Hilbert action: (pure gravity)

$$S_G = \int d^4x \sqrt{-g}\left(\frac{R}{l_p^2} - \Lambda\right) = \int \mathcal{L}_G\, dt$$

$$\mathcal{L}_G = -\frac{a\dot{a}^2}{2N} + \frac{Na}{2} - \frac{Na^3}{2}H^2 \qquad H^2 \equiv \left(\frac{4G}{3}\right)^2 \Lambda$$

- Canonical quantization:

$$P_a = -\frac{a\dot{a}}{N} \longrightarrow i\frac{\partial}{\partial a}$$

$$\mathcal{H}_G = \frac{N}{2}\left(-\frac{P_a^2}{a} - a + a^3H^2\right) \longrightarrow \hat{\mathcal{H}}_G$$

$$\Rightarrow \mathcal{H}_G \equiv 0 \longrightarrow \hat{\mathcal{H}}_G \Psi = 0$$

  - Factor ordering issue

$$\frac{1}{a}\frac{\partial^2}{\partial a^2},\ \frac{\partial}{\partial a}\frac{1}{a}\frac{\partial}{\partial a}, \ldots$$

- Hamiltonian constraint, a.k.a. WDW equation

$$\hat{\mathcal{H}}_G \Psi = 0$$

$$\Psi = \Psi(a)$$

"minisuperspace"

Quantum Cosmology with inhomogeneous matter fields

- Non-minimally coupled scalar field ϕ:

$$S_M = \int d^4x \sqrt{-g} \left[\frac{1}{2} (\partial_\mu \phi)^2 - m^2 \frac{\phi^2}{2} - \frac{\xi}{2} R \phi^2 \right]$$

$\xi = \frac{1}{6}$ - conformal field

- Expand in spherical harmonics on $S^3 \ni x$

$$\phi(x,t) = \frac{1}{a(t)} \sum_{nlp} \underbrace{f_{nlp}}_{\to f_n}(t)\, Q^{nlp}(x)$$

– obtain

- WDW equation for $\Psi(a, \{f_n\})$

$$\left[\frac{\partial^2}{\partial a^2} - V(a) + E + \sum_n -\frac{\partial^2}{\partial f_n^2} + (n^2 + m^2 a^2) f_n^2 \right] \Psi = 0$$

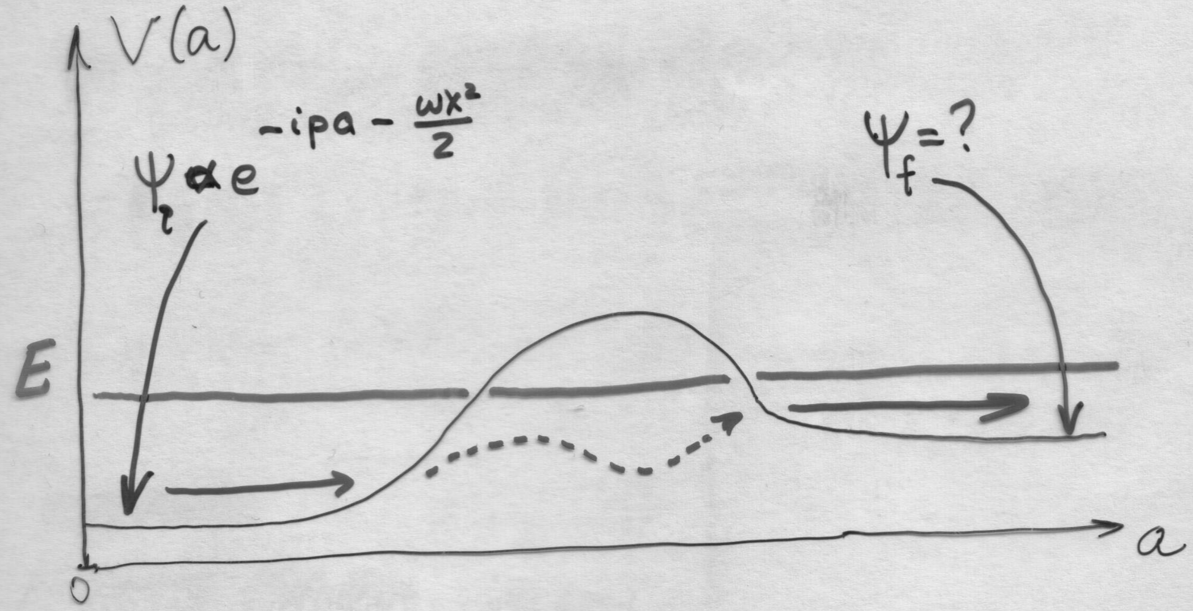
- Disregard $f_n^4 \ll f_n^2$

▶ Question:

Describe the quantum state of the nucleated universe.

Analogy with quantum mechanics

- **A tunneling oscillator:** (a, x)



$$\left[-\frac{\partial^2}{\partial a^2} + V(a) - E - \frac{\partial^2}{\partial x^2} + \omega^2 x^2\right]\psi(a,x) = 0$$

$\omega = \omega(a)$

- **Question:** determine the asymptotic state ψ_f at $a \to +\infty$ given the initial state ψ_i

- **Answer:** ψ_f is nearly equal to the ground state of the oscillator

Tunneling from something

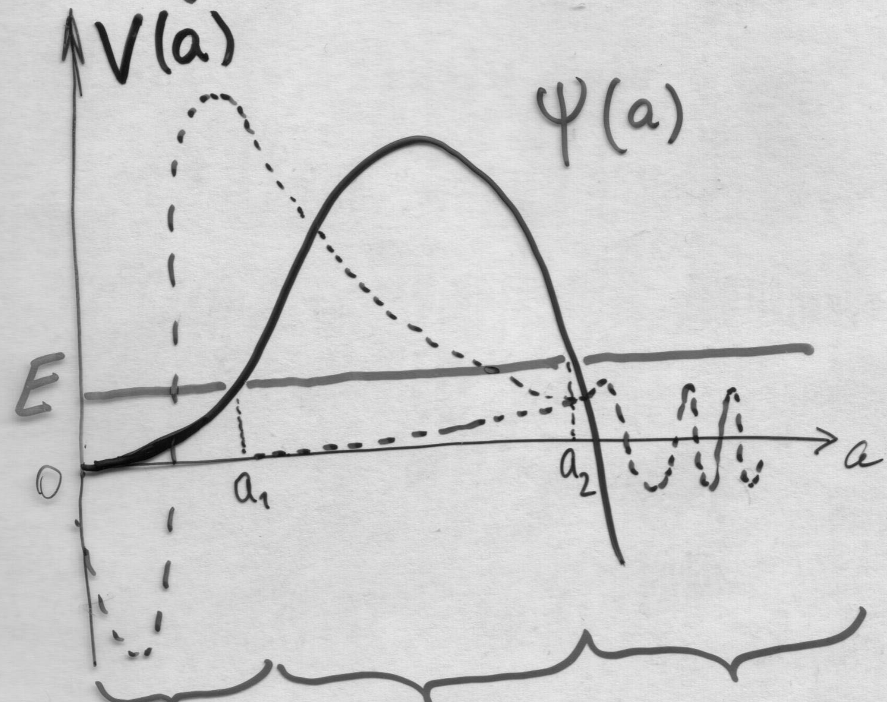
- Add homogeneous radiation:

$$\rho \propto \underbrace{H^2}_{\text{dark energy}} + \underbrace{\frac{E}{a^4}}_{\text{bright energy}}$$

- WDW equation:

$$\left[\frac{\partial^2}{\partial a^2} - V(a) + E\right]\Psi(a) = 0$$

$$\underline{V(a) = a^2 - H^2 a^4}$$



- recollapsing universe $0 < a < a_1$
- Euclidean regime $a_1 < a < a_2$
- expanding universe $a_2 < a < +\infty$

Interpretation of $\Psi(a, f_n)$

- **A vacuum state of QFT in the Schrödinger picture**: $|0\rangle \to \Psi(f_n)$

$$\hat{f}_n(t) = \hat{a}_n v_n^*(t) + \hat{a}_n^\dagger v_n(t) \longrightarrow f_n$$

$$\frac{d\hat{f}_n}{dt} = \hat{a}_n \dot{v}_n^* + \hat{a}_n^\dagger \dot{v}_n = \hat{\pi}_n \longrightarrow i\frac{\partial}{\partial f_n}$$

- Normalization of the mode functions:

$$\dot{v}_n^* v_n - v_n^* \dot{v}_n = i$$

$$\boxed{v_n \sim e^{-i\int \omega\, dt}}$$

- Vacuum wave function:

$$\hat{a}_n |0\rangle = 0 \to \left[\frac{\partial}{\partial f_n} + i\frac{\dot{v}_n}{v_n} f_n\right]\Psi = 0$$

$$\to \Psi(f_n) = \exp\left[-\frac{i}{2}\frac{\dot{v}_n}{v_n} f_n^2\right]$$

- **Wave function from Gaussian ansatz:**

$$\Psi \sim \exp\left[-S(a) - \frac{1}{2} S_n f_n^2\right]$$

- Conformal time t: $\dfrac{da}{dt} = \sqrt{E - V(a)}$

$$\to \quad i\frac{dS_n}{dt} = S_n^2 - \omega_n^2$$

- Identify v_n from $\quad S_n(t) = i\dfrac{\dot{v}_n}{v_n}$

$$\to \ddot{v}_n + \omega_n^2 v_n = 0$$

Conclusions

- "Catastrophic particle production" is really "branching" of the wave func.
- Real, observable particle production is small
- "Tunneling from something" works if we choose a good quantum state of "something"
- "Good state" $\Leftrightarrow$ one semiclassical geometry
- A "Gaussian vacuum" prescription:
  - always well-defined, unique vacuum
    - all approximations are valid
    - coincides with adiabatic vacuum up to its accuracy
- Tunneling from nothing <u>can</u> be viewed as a well-behaved limit of "tunneling from something"