DESY THEORY WORKSHOP 2004, 28 Sept - 1 Oct, PARTICLE COSMOLOGY

Thermal Leptogenesis

Leptogenesis

Pasquale Di Bari (Max Planck, Munich)

in collaboration with W. Buchmüller and M. Plümacher

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Baryon asymmetry of the Universe

• CMB acoustic peaks (WMAP) + large scale structure (SLOAN):

$$\eta_B^{CMB} = \left. \frac{n_B - n_{\bar{B}}}{n_\gamma} \right|_{t_{\rm rec}} = (6.3 \pm 0.3) \times 10^{-10}$$

(Tegmark et al. 2003)

• very good agreement with SBBN + primordial Deuterium determination :

$$\eta_B^{SBBN} = \left. \frac{n_B - n_{\bar{B}}}{n_{\gamma}} \right|_{t_{\rm BBN}} = (6.1 \pm 0.5) \times 10^{-10}$$

(Cyburt et al. 2001, Kirkman et al. 2003)

1. A simple GUT Baryog. model (Kolb,Turner)



 $(\Delta_{B-L} = +1)$

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• sphaleron conversion:

$$N_B^{\rm f} \simeq \frac{1}{3} N_{B-L}^{\rm f} \simeq -\frac{1}{2} N_L^{\rm f}$$

(Kuzmin, Rubakov, Shaposhnikov'85; Khlebnikov, Shaposhnikov'88; Harvey, Turner'90):

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(Kuzmin, Rubakov, Shaposhnikov'85; Khlebnikov, Shaposhnikov'88; Harvey, Turner'90):

• CP asymmetry parameter:

• Total decay parameter:

$$\varepsilon = \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}} \ \Delta_{B-L} > 0$$

$$\Gamma_D = \Gamma + \bar{\Gamma} = \Gamma_D^{\text{rest}} \left\langle \frac{1}{\gamma} \right\rangle$$

Thermal Leptogenesis

Out of equilibrium decays

Out of equilibrium decays



Out of equilibrium decays



Decays and Inverse Decays

$$\frac{dN_X}{dz} = -D N_X + D N_X^{\text{eq}}$$
$$\frac{dN_{B-L}}{dz} = -\varepsilon \frac{dN_X}{dz} - W_{ID} N_{B-L}$$

$$W_{ID} = m \, {N_X^{
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 ('RIS' $\Delta L = 2$ processes included)

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$$N_{B-L}(z; K, z_i) = N_{B-L}^{\text{in}} e^{-\int_{z_i}^z dz' W_{ID}(z')} + \varepsilon \kappa(z)$$

$$\kappa(z; K, z_{\rm i}) = -\int_{z_{\rm i}}^{z} dz' \left[\frac{dN_X}{dz'}\right] e^{-\int_{z'}^{z} dz'' W_{ID}(z'')}$$

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- Weak wash-out regime for $K \lesssim 1$ (out-of-equilibrium picture recovered for $K \to 0$)
- Strong wash-out regime for $K\gtrsim 1$

Strong wash-out regime

(Kolb, Turner'90; Buchmüller, PDB, Plümacher'04)

 $\Delta(z) \equiv N_X(z) - N_X^{\rm eq}(z) \ll 1$

Close-to-equilibrium approximation

$$\frac{dN_X}{dz} \simeq \frac{dN_X^{\rm eq}}{dz}$$

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Thermal Leptogenesis

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 $\kappa_{\rm f} = \int_0^\infty dz' \, e^{-\psi(z',\infty)}$

Baryogenesis temperature $T_B = M_X/z_B$



• only non relativistic stage matters

•
$$\kappa_{\rm f}(z_i=0)\simeq\kappa_{\rm f}(z_i=z_B-\Delta z_B)$$

•
$$\kappa_{\rm f} \simeq \frac{2}{K z_B} \left(1 - e^{-\frac{K z_B}{2}} \right)$$



Neutrino production

(Fry, Turner '81; Buchmüller, PDB, Plümacher '04)





Thermal Leptogenesis

Neutrino production

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Final efficiency factor

DECAYS+INVERSE DECAYS



10²

Neutrino production

10¹

(Fry, Turner '81; Buchmüller, PDB, Plümacher '04)

10^c

10

Final efficiency factor

DECAYS+INVERSE DECAYS



Negligible

dependence on the initial conditions in the strong

wash-out regime

 10^{2}

Neutrino production

10¹

(Fry, Turner '81; Buchmüller, PDB, Plümacher '04)

10^c

10

Final efficiency factor

DECAYS+INVERSE DECAYS



Negligible (strong) dependence on the initial conditions in the strong (weak) wash-out regime

Thermal Leptogenesis

Seesaw \Rightarrow Leptogenesis (Fukugita, Yanagida '86)

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• seesaw formula

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 m_D , m_{ν} and M are complex matrices \Rightarrow natural source of CP violation

• 3 new heavy RH neutrinos N_1, N_2, N_3 with masses $M_{
m ew} \ll M_1 \leq M_2 \leq M_3$

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- 3 new heavy RH neutrinos N_1, N_2, N_3 with masses $M_{
 m ew} \ll M_1 \leq M_2 \leq M_3$
- lightest RH neutrinos play the role of the decaying particles $\,X o N_1\,,\;\;arepsilon oarepsilon_1$
- total decay rate

$$\Gamma_D^{\text{rest}} = \frac{\widetilde{m}_1 \, M_1^2}{8 \, \pi \, v^2}$$

• effective neutrino mass

$$\widetilde{m}_1 \equiv \frac{(m_D^{\dagger} m_D)_{11}}{M_1}$$

decay parameter and equilibrium neutrino mass

$$K = \frac{\Gamma_D^{\text{rest}}}{H|_{z=1}} = \frac{\widetilde{m}_1}{m_\star} \qquad m_\star = \text{const} \, \frac{v^2 \sqrt{g_\star}}{M_{Pl}} \simeq 10^{-3} \,\text{eV}$$

Thermal Leptogenesis

Range of \widetilde{m}_1

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(Fujii,Hamaguchi,Yanagida '02):

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(Fujii,Hamaguchi,Yanagida '02):

- for fully hierarchical neutrinos ($m_1 \ll m_{\rm sol}$):
- $\mathcal{O}(m_{
 m sol} \simeq 0.008 \, {\rm eV}) < \widetilde{m}_1 < \mathcal{O}(m_{
 m atm} \simeq 0.05 \, {\rm eV})$

 $m_{
m atm,sol} \equiv \sqrt{\Delta m_{
m atm,sol}^2}$

Leptogenesis K range

Translating \widetilde{m}_1 in terms of $K = \widetilde{m}_1/m_\star$:

Range of \widetilde{m}_1

• Seesaw orthogonal matrix (Casas, Ibarra '01):

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$$m_{\rm atm,sol} \equiv \sqrt{\Delta m_{\rm atm,sol}^2}$$



Neutrino mixing data favor leptogenesis

to lie in the strong wash-out regime

Thermal Leptogenesis

 $\Delta L = 2$ processes (Fukugita, Yanagida'86; Luty'92; Plumacher'97; Buchmüller, PDB, Plümacher'02; Pilaftsis, Underwood'03; Giudice et al.'03)



• ΔW dominates at low temperatures: $\Delta W(z \ll 1) \propto M_1 \,\bar{m}^2 / z^2$

 $\bar{m}^2 \equiv \sum_i m_{\nu_i}^2$

contributes to determine an upper bound

on the absolute neutrino mass scale

in the case of hierarchical neutrinos is negligible for $M_1 \ll 10^{14} \, {\rm GeV}$

 $\Delta L = 2$ processes Scatterings (Fukugita, Yanagida'86; Luty'92; Plumacher'97; Buchmüller, PDB, Plümacher'02; Pilaftsis, Underwood'03; Giudice et al.'03)



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- contributes to determine an upper bound on the absolute neutrino mass scale
- in the case of hierarchical neutrinos is negligible for $M_1 \ll 10^{14} \, {\rm GeV}$

1. involving the top quark (Higgs mediated):

 $N_1 l \leftrightarrow t q + \dots$

2. involving the g. bosons (Higgs mediated): $N_1 A \leftrightarrow H + \overline{l} + \dots$

$$\frac{dN_X}{dz} = -(D+S)(N_X - N_X^{eq})$$
$$\frac{dN_{B-L}}{dz} = \varepsilon D(N_X - N_X^{eq}) - (W_{ID} + W_S) N_{B-L}$$

- 1. in the weak wash-out regime: enhance the neutrino production source of large theoretical uncertainties
- 2. in the strong wash-out regime: contribute (sub-dominantly) to the wash-out

Theoretical uncertainties on $\kappa_{\rm f}$

unstable results in the weak wash-out regime:

- I.R. cut-off on the Higgs mass $\Rightarrow \kappa_{\rm f} \propto K$ (Luty'92,Plumacher'96,Barbieri et. al.'00, Buchmuller,PDB,Plumacher '02,'03)
- thermal corrections to the Higgs mass+ running Yukawa coupling $\Rightarrow \kappa_{\rm f} \propto K^2$

(Barbieri et. al.'03;Giudice et.al.'03)

• addition of scatterings involving gauge bosons $\Rightarrow \kappa_{\rm f} \propto K$

(Pilaftsis, Underwood'03; Giudice et.al.'03)

• 'spectator processes' $\Rightarrow \mathcal{O}(1)$ factor sup-

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pression (Buchmuller, Plumacher'01)
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 $\sim 50\%$ uncertainties in the strong wash-out regime



CP asymmetry

• Interference between tree level and (vertex + self energy) one-loop diagrams

$$\Rightarrow \varepsilon_1 \simeq -\frac{1}{8\pi v^2 (m_D m_D^{\dagger})_{11}} \sum_{i=2,3} \operatorname{Im} \left[(m_D m_D^{\dagger})_{i1}^2 \right] \times \left[f_V \left(\frac{M_i^2}{M_1^2} \right) + f_S \left(\frac{M_i^2}{M_1^2} \right) \right]$$

(Flanz, Paschos, Sarkar'95; Covi, Roulet, Vissani'96; Buchmüller, Plümacher'98)

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• barring RH neutrino mass degeneracies and strong phase cancellations: (Hamaguchi,Murayama,Yanagida '01; Davidson,Ibarra '02; Hambye,Lin,Notari,Papucci,Strumia '04)

 $\varepsilon_1 \simeq \varepsilon_{\max}(M_1, m_1, \widetilde{m}_1) \sin \delta_L(m_1, \widetilde{m}_1, \Omega_{j1}^2)$
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 $\varepsilon_1^{\max}(M_1, m_1, \widetilde{m}_1) = \varepsilon_1^{\max}(M_1) \,\beta(m_1, \widetilde{m}_1), \quad \beta(m_1, \widetilde{m}_1) \leq 1$

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 $\beta(m_1 = 0, \widetilde{m}_1) = 1 \implies \varepsilon_1$ maximum for fully hierarchical neutrinos

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$$\varepsilon_1^{\max}(M_1) \equiv \frac{3}{16\pi} \frac{M_1 m_{\text{atm}}}{v^2} \simeq 10^{-6} \left(\frac{M_1}{10^{10} \,\text{GeV}}\right)$$

CMB constraints in the full hierarchical case

(Buchmuller, PDB, Plumacher '02)

 $\eta_B^{\max}(M_1, \widetilde{m}_1)|_{m_1=0} \simeq d \varepsilon_1^{\max}(M_1) \kappa_f(M_1, \widetilde{m}_1)$

$$\eta_B^{
m max}(M_1, \widetilde{m}_1)|_{m_1=0} \propto M_1 \, e^{-rac{M_1}{10^{14}\,{
m GeV}}}$$

$$d \simeq \frac{1}{3 N_{\gamma}^{\rm rec}} \simeq 10^{-2}$$

CMB bound:

 $\eta_B^{\max}(M_1, \widetilde{m}_1)|_{m_1=0} \ge \eta_B^{CMB}$

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Lower bound on M_1 and on $T_{ m i}$



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Lower bound on M_1 and on $T_{ m i}$



 \widetilde{m}_1 (eV)



What if we do not use $m_3^{\min} = m_{\min}$?

The allowed region is characterized by:

- $\widetilde{m}_1^{\text{max}} \propto 1/m_3^{\text{min}}$
- $M_1^{\rm min} \propto 1/m_3^{\rm min}$
- $M_1^{\rm max} \propto 1/(m_3^{\rm min})^2$

to be compared with $\widetilde{m}_1 \sim m_3^{
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to be compared with $\widetilde{m}_1 \sim m_3^{
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- if $m_3^{\min} \ll 10^{-3} \,\mathrm{eV}$:
 - weak wash-out regime favored

(problem with initial conditions)

• $M_1^{\min} \gg 10^{11} \,\mathrm{GeV}$



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- $M_1^{\min} \gg 10^{11} \, \text{GeV}$
- if $m_3^{\min} \gg 1 \,\mathrm{eV} \Rightarrow \widetilde{m}_1^{\max} \ll m_3^{\min}$:
- \Rightarrow the experimental result:

 $\mathcal{O}(10^{-3} \,\mathrm{eV}) < m_{\mathrm{atm}} < \mathcal{O}(1 \,\mathrm{eV})$

is a successful test for thermal leptogenesis !



CP asymmetry bound for arbitrary m_1

 $\varepsilon_1 = \varepsilon_1^{\max}(M_1) \,\beta(m_1, \widetilde{m}_1) \sin \delta_L(m_1, \widetilde{m}_1, \Omega_{j1}^2)$

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general: $\beta(m_1, \widetilde{m}_1) = rac{m_{\mathrm{atm}}}{m_3 + m_1} f(m_1, \widetilde{m}_1)$

 $1 = m_1$) $0 \le f(m_1, \tilde{m}_1) \le 1$ $(\tilde{m}_1/m_1 \to \infty)$

(Davidson, Ibarra '02; Buchmüller, PDB, Plümacher '03)

CP asymmetry bound for arbitrary m_1

 $\varepsilon_{1} = \varepsilon_{1}^{\max}(M_{1}) \beta(m_{1}, \widetilde{m}_{1}) \sin \delta_{L}(m_{1}, \widetilde{m}_{1}, \Omega_{j1}^{2})$ or general: $\beta(m_{1}, \widetilde{m}_{1}) = \frac{m_{\text{atm}}}{m_{3} + m_{1}} f(m_{1}, \widetilde{m}_{1})$ $1 = m_{1}) \quad 0 \leq f(m_{1}, \widetilde{m}_{1}) \leq 1 \quad (\widetilde{m}_{1}/m_{1} \to \infty)$ (Davidson,Ibarra '02; Buchmüller,PDB,Plümacher '03)

$$(m_1, \widetilde{m}_1) = \frac{m_3 + m_1}{\widetilde{m}_1} \, \mathbf{Y}(m_1, \widetilde{m}_1) \, ; \, X, Y \equiv \operatorname{Re}, \operatorname{Im}(\Omega_{31}^2)$$

$$m_1 \sqrt{Y^2 + (1 - X)^2} + m_3 \sqrt{Y^2 + X^2} = \tilde{m}_1$$

$$\frac{m_3 X}{\sqrt{X^2 + Y^2}} = \frac{m_1 (1 - X)}{\sqrt{(1 - X)^2 + Y^2}}$$
(PDB '04)

$C\!P$ asymmetry bound for arbitrary m_1

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• quasi-degenerate neutrinos ($m_1/m_{\rm atm} \gg 1$): $f(m_1, \tilde{m}_1) = \sqrt{1 - \frac{\tilde{m}_1^2}{m_1^2}}$

(Hambye, Lin, Notari, Papucci, Strumia'04; PDB '04)

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$$\eta_B^{0^0} \frac{10^4 - 10^3 - 10^2 - 10^4 - 10^6}{10^4 - 10^2 - 10^4 - 10^6} \frac{10^6}{10^4} \frac{10^6}{10^4} \frac{10^6}{10^6} \frac$$

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(iii) running of neutrino masses (Antusch et al'03) $\Rightarrow m_i \leq 0.12$ eV

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The lower bound on the RH neutrino mass is much more sensitive to some variation than the upper bound on the light neutrino masses

The supersymmetric (MSSM) case

(Davidson et al. '92; Covi,Roulet,Vissani '96; Plumacher '97; Giudice et al. '03; PDB '04)

- 1. $N_1 \longrightarrow N_1, \widetilde{N}_1^c$
- 2. $N_{\gamma}^{\mathrm{rec}} \longrightarrow \sim 2 N_{\gamma}^{\mathrm{rec}}$
- 3. $\varepsilon_1^{\max} \longrightarrow 2 \varepsilon_1^{\max}$
- 4. $g_{\star} \longrightarrow 2 g_{\star} \Rightarrow H(1) \longrightarrow \sqrt{2} H(1)$

- 5. $\Gamma_D^{\text{rest}} \longrightarrow 2 \, \Gamma_D^{\text{rest}}$
- 6. $\Gamma_{\Delta L=2} \longrightarrow 5 (\Gamma_{\Delta L=2})/3$
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$$1+2+3+4+5+6+7 \Rightarrow \frac{m_i^{MSSM}}{m_i} < 0.15 \,\mathrm{eV}$$

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$$g_{\rm rec}^{\rm dec} \lesssim 1 \Rightarrow \xi_{g_{\star}}^{1/8} \lesssim 2$$

Degenerate leptogenesis

What if one relaxes the approximation of a hierarchical RH neutrino spectrum ?

- $\varepsilon_1^{\max} \to \xi_{\varepsilon} \, \varepsilon_1 \Rightarrow \eta_B^{\max} \to \xi_{\varepsilon} \, \eta_B^{\max}$
- dependence on all other seesaw parameters \Rightarrow great model dependence
- extreme situation: resonant leptogenesis (Pilaftsis, Underwood '04): no bounds at all !

Assuming equal degeneracies of light and heavy neutrinos:

- 'normal' heavy neutrino spectrum $\Rightarrow m_i^{\rm bound} \lesssim 0.2\,{\rm eV}$ (PDB '04)
- 'inverted' heavy neutrino spectrum $\Rightarrow m_i^{\rm bound} \lesssim 0.6\,{\rm eV}$

(Hambye,Lin,Notari,Papucci,Strumia '04)

A 'too-short-blanket' problem

(Buchmüller, PDB, Plümacher'03, PDB'04)



The assumption of hierarchical heavy neutrino spectrum seems to be reasonable for the most interesting region of the allowed parameter space ! More investigation is needed.

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- Smoking gun ? Very difficult, but the problem becomes somehow easier if leptogenesis is studied in connection with the other seesaw phenomenologies and it is important to have in mind that the seesaw mechanism predicts neutrinoless double beta decay.