DARK ENERGY & NON-LINEAR STRUCTURE FORMATION

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Here: DE is an uncoupled scalar field.

OUTLINE:

1.) LINEAR Perturbations in models WITH DE

2.) Non-linear perturbations: the spherical collapse model

3.) Observational consequences (?)

4.) Outlook

1) LINEAR PERTURBATIONS

Linear perturbations in the DE component are described by

$$\boxed{(\delta\phi)_k^{\cdot\cdot} + 3H(\delta\phi)_k^{\cdot} + \left[\frac{k^2}{a^2} + V''\right](\delta\phi)_k = \dot{\phi}\,\delta_{CDM}}$$

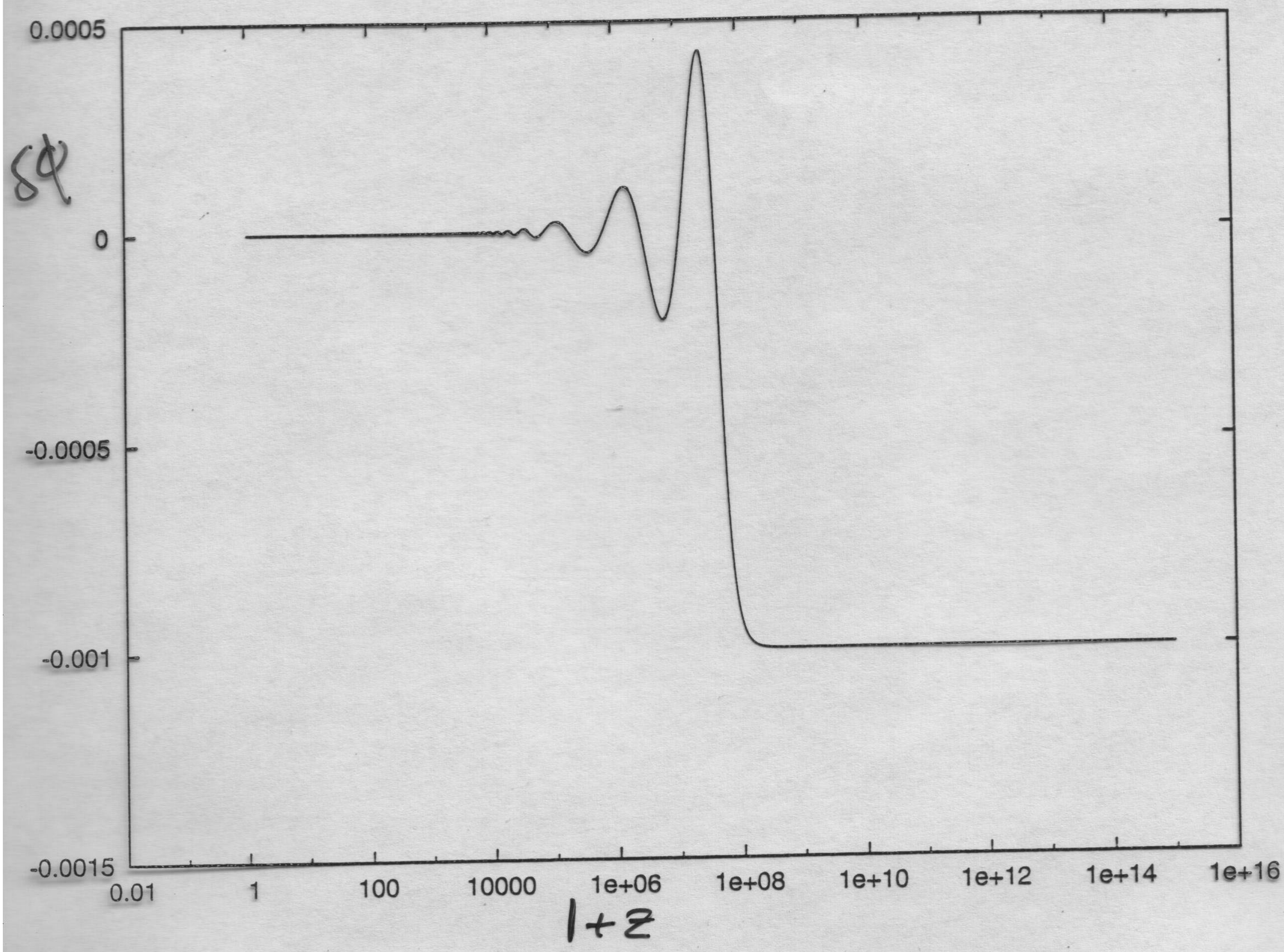
$$m_\phi^2 = \frac{\partial^2 V}{\partial \phi^2}$$

$$\lambda_J \approx \frac{2\pi}{\sqrt{V''}} = \frac{2\pi}{m_\phi}$$

In scalar field models of DE one finds (MA et. al. (1999))

$$\lambda_J \approx \lambda_H \qquad \text{VERY LARGE!}$$

LINEAR PERTURBATIONS:



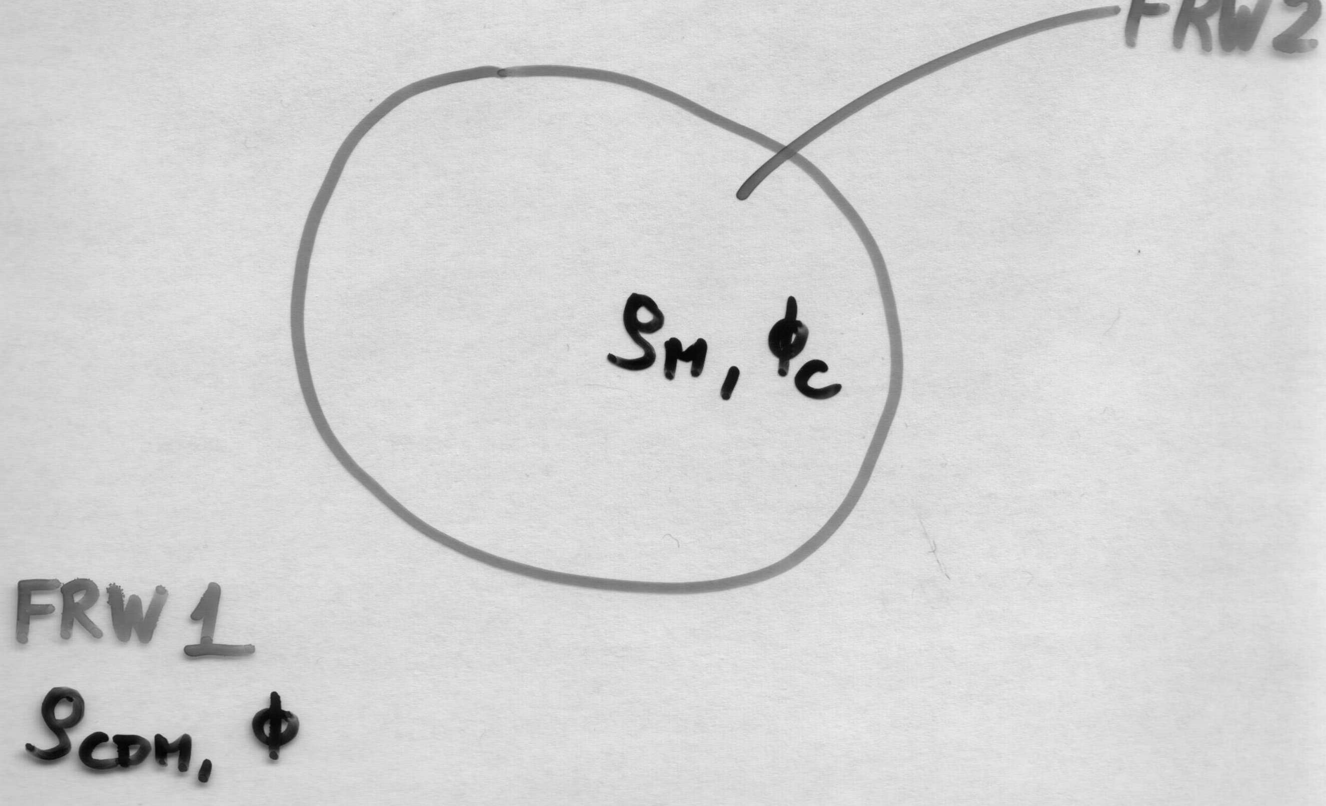
$k \approx 0.1\ Mpc^{-1} \rightarrow \lambda_p \ll \lambda_J$

However:

- WHAT ABOUT BACKREACTION EFFECTS FROM DM?
- WHAT ABOUT INTERACTION WITH DM? (Coupled quintessence)
- WHAT IS THE EFFECT OF A CHANGING EQUATION OF STATE?

2) THE SPHERICAL COLLAPSE MODEL

HERE: SIMPLIFIED MODEL TO STUDY NON-LINEARITIES (NON-PERTURBATIVE!)



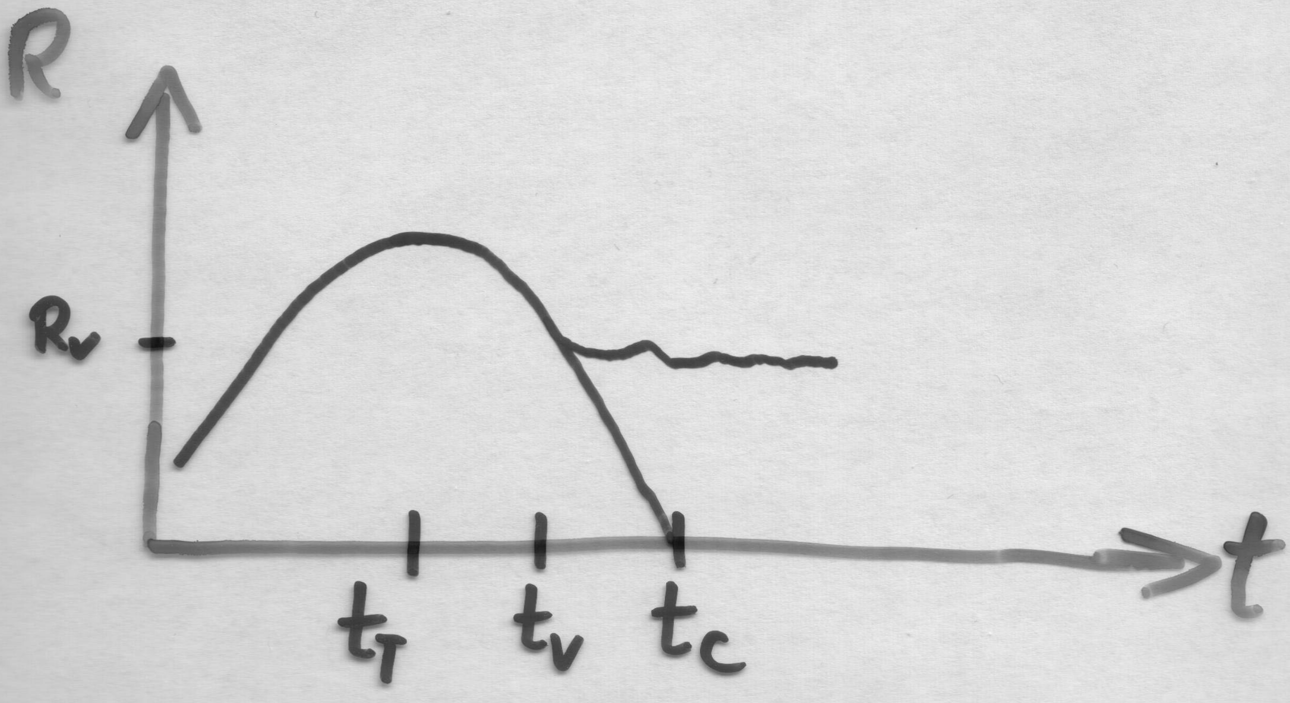
INITIALLY:

$$\rho_H = \rho_{CDM}\left[1 + \delta(R_i, t_i)\right]$$

$$\phi(t_i) = \phi_c(t_i) \text{ (see later)}$$

3 phases during evolution:

- TURNAROUND
- COLLAPSE $(R \rightarrow 0)$
- VIRIALISATION



EQUATIONS:

a) BACKGROUND

$$3H^2 = 8\pi G(\rho_{CDM} + \rho_\phi)$$

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0$$

HERE:

- $V(\phi) = M\{e^{\beta\phi} + e^{\gamma\phi}\}$
- $V(\phi) = M\{e^{\gamma/\phi} - 1\}$
- $V(\phi) = M\{A + (\phi - B)^2\}e^{-\gamma\phi}$
- $V(\phi) = M \cdot \frac{\exp \phi^2}{\phi^\gamma}$

b) Overdensity.

$$3\ddot{R} = -4\pi G\left[\rho_c + \rho_{\phi_c}(1+3w)\right]R$$

$$\ddot{\phi}_c + 3\frac{\dot{R}}{R}\dot{\phi}_c + \frac{\partial V}{\partial \phi}(\phi_c) = \frac{\Gamma}{\dot{\phi}}$$

OUTFLOW OF DE

VIRIALISATION CONDITION:

$$\frac{1}{2}R\frac{\partial}{\partial R}(U_G + U_{\phi_c})$$

$$+ U_G + U_{\phi_c}\Big|_{z_v} = U_G + U_{\phi_c}\Big|_{z_T}$$

N.B. $\Delta\Phi = 4\pi G(\rho + 3p)$

THE CHALLANGE

IS TO COMPUTE Γ.

THIS QUANTITY DESCRIBES
THE ENERGY OUTFLOW
OF THE DE COMPONENT.

Γ IS NOT COMPUTABLE
WITHIN THE SPHERICAL
COLLAPSE MODEL.

ANSATZ FOR Γ:

a) ASSUME DE is SMOOTH THROUGHOUT SPACE. PUT

$$\Gamma = -3\left[\frac{\dot{a}}{a} - \frac{\dot{R}}{R}\right] \dot{\phi}_c^2$$

with

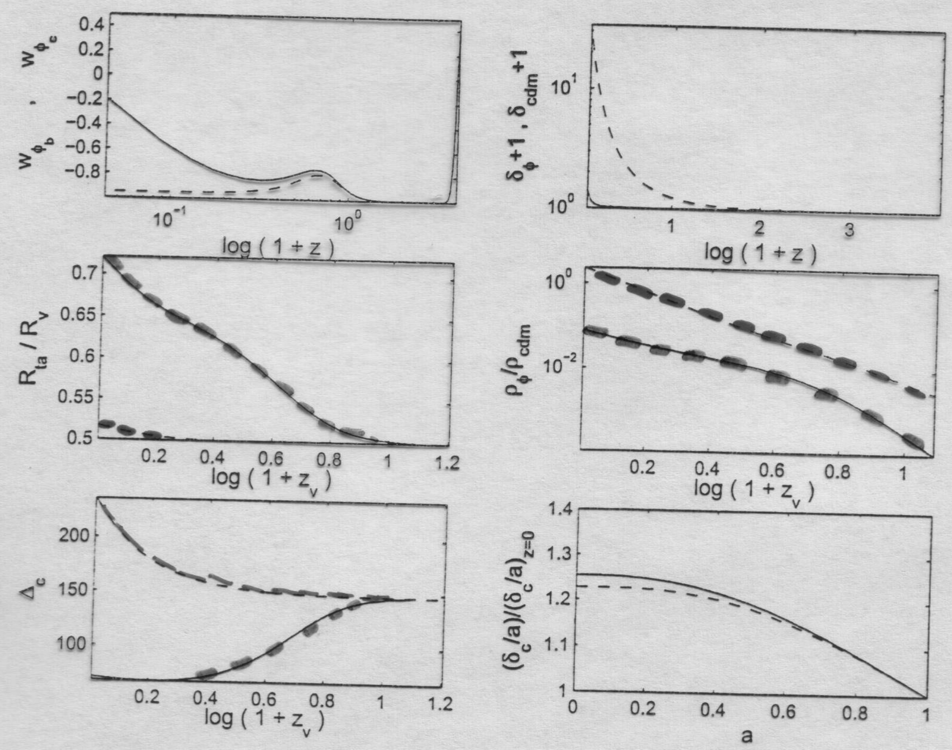
$$\phi_c(t_i) = \phi(t_i)$$

$$\dot{\phi}_c(t_i) = \dot{\phi}(t_i)$$

b) ASSUME DE COLLAPSES TOGETHER WITH DM. PUT

$$\Gamma = 0$$

$$V = M\left[\exp\left(\frac{\gamma}{\phi}\right) - 1\right]$$



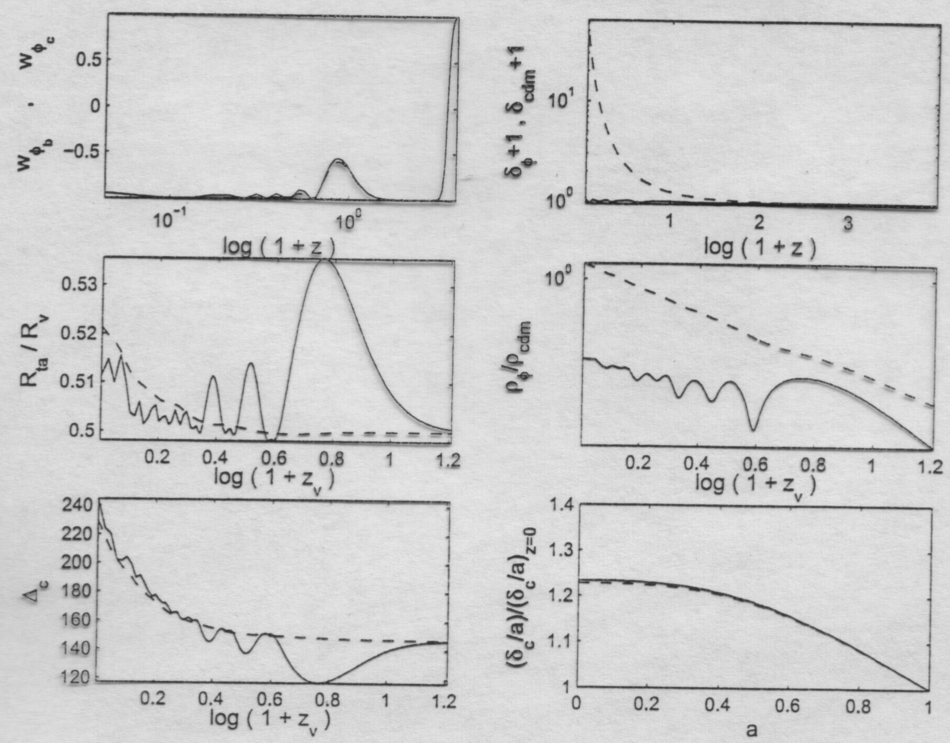
$$\delta_i + 1 \equiv \frac{\rho_i\,(\text{inside})}{\rho_i\,(\text{outside})}$$

— $\Gamma \neq 0$

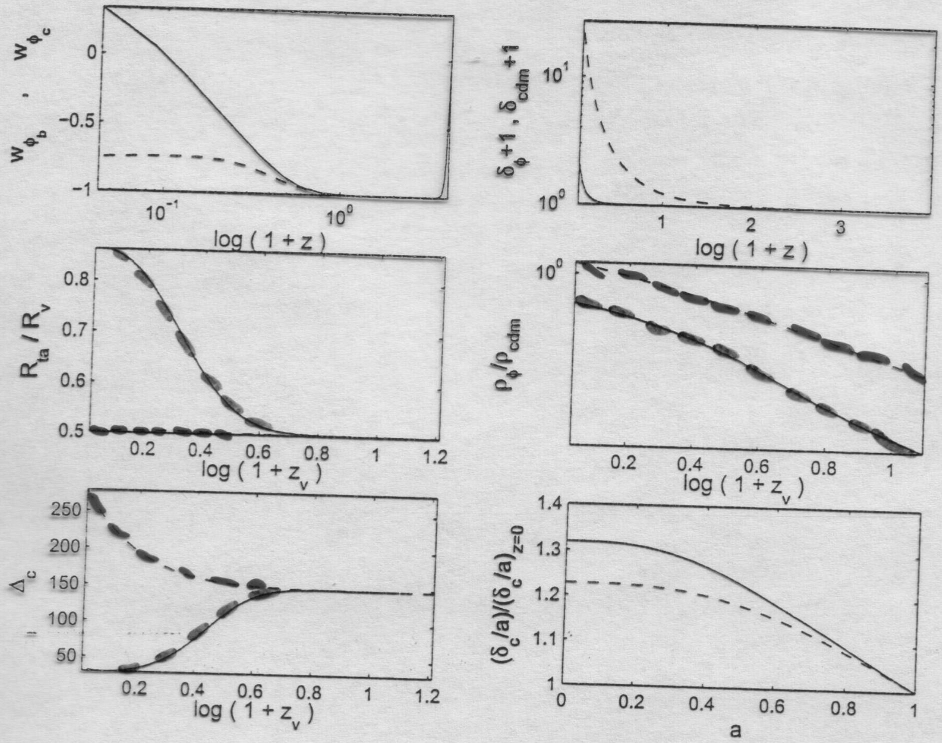
— $\Gamma = 0$

$$\Delta_c = \frac{\rho_M(t_v)}{\rho_{CDM}(t_v)}$$

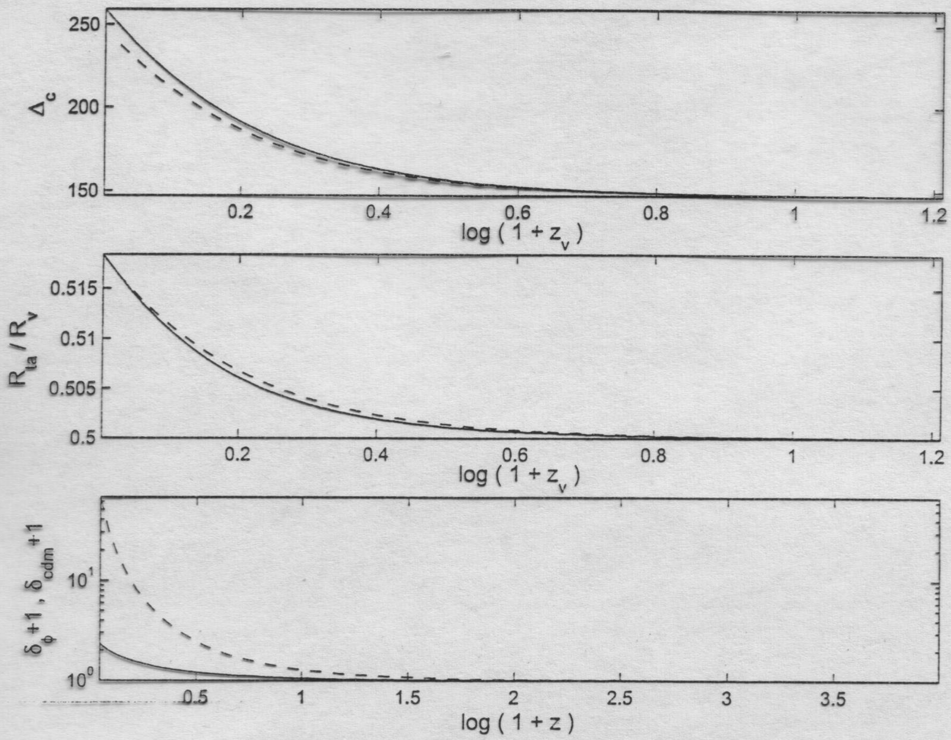
$$V = M\left[A + (\phi - B)^2\right] e^{-\gamma\phi}$$



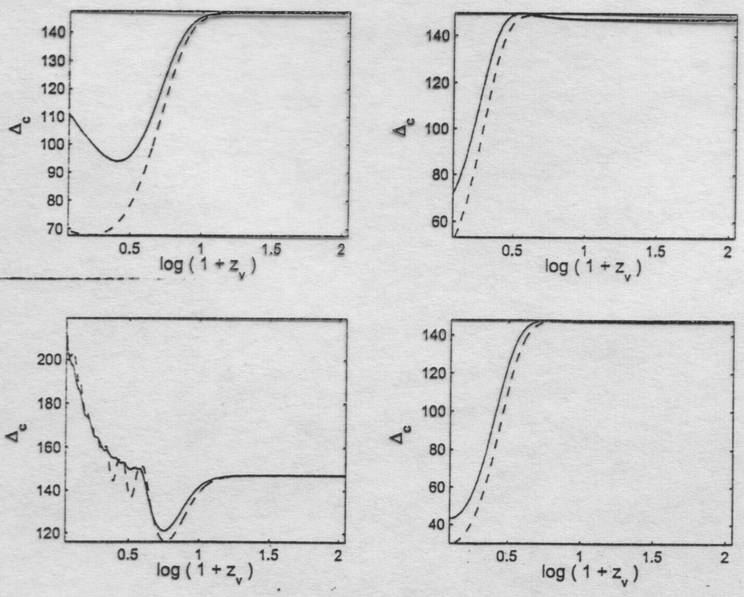
$$V = M\exp(\phi^2)/\phi^\gamma$$



$$W = \frac{P_\phi}{\rho_\phi} = \text{const} = -0.8$$



Delayed collapse
of DE:



3. Observational consequences (?)

(Lopes, Mota, Miller A&A 2004)

DM halo modelled with NFW profile:

$$\rho(r) = \frac{\rho_s}{\left(\frac{r}{r_s}\right)\left(1+\left(\frac{r}{r_s}\right)\right)^2}$$

$$\rho_s = \delta_c \, \rho_{crit}$$

$$\delta_c = \frac{\Delta_{vir}}{3} \frac{c_{vir}^3}{\ln(1+c_{vir}) - \frac{c_{vir}}{1+c_{vir}}}$$

$$c_{vir} \equiv \frac{r_{vir}}{r_s} \quad (\text{"concentration"})$$

$$M_{vir} = \frac{4\pi}{3} r_{vir}^3 \, \Delta_{vir} \, \rho_{crit}$$

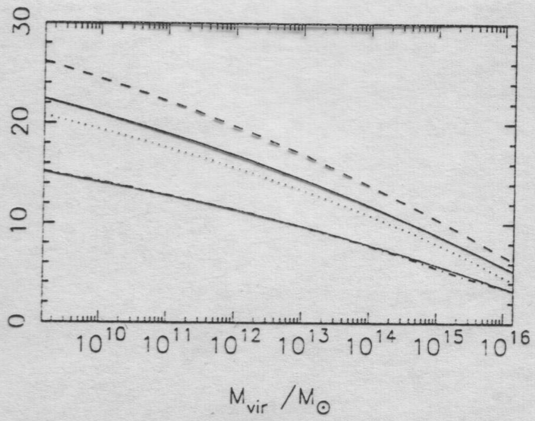
Prescription by Eke et al (2001):

$$c_{vir}^3 = \frac{\Delta_{vir}(z_c)}{\Delta_{vir}(z_0)} \frac{\Omega_m(z_0)}{\Omega_m(z_c)} \left(\frac{1+z_c}{1+z_0}\right)^3$$

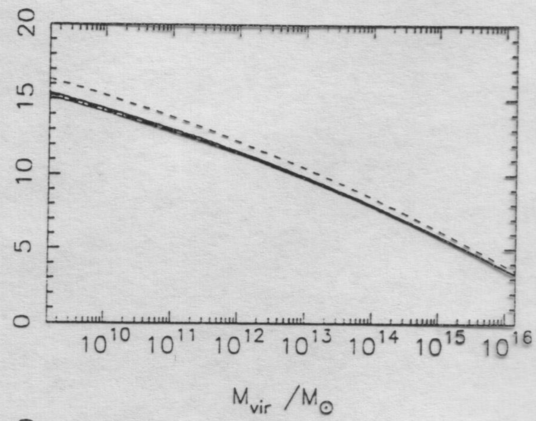
Eke et al. provide an algorithm to calculate z_c.

concentration:

Γ=0:



Γ≠0:



4. Outlook

- Predictions of spherical collapse model depend on clustering properties of DE, even if DE is only in the mildly non-linear regime.

- Applications:

  - Coupled quintessence
  - varying constants
  - backreaction effects

IMPROVEMENTS:

NEED A FULLY RELATIVISTIC TREATMENT:

- Matching interior metric with exterior metric
- Follow $R(t)$ from junction condition

Not easy in the presence of a scalar field ...