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Single Pion Production by Linearly Polarized Photons at High Energies

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Abstract: There exists a one-to-one correspondence between the dependence of the single-pion-photoproduction amplitude on the linear polarization of the photons at high energies and small momentum transfer in the direct channel $\gamma + N \rightarrow \pi + N$ on one hand and the parity of the exchanged particle-system in the crossed channel $\gamma + \pi \rightarrow N + \bar{\bar{N}}$ on the other.

1. Introduction

It is a well known fact that in the resonance region ($E_\gamma < 1\,\mathrm{BeV}$) linearly polarized photons are a useful tool for the determination of the parity of the multipole absorption[1] in the process $\gamma + N \to \pi + N$. In this energy region the excitation of the nucleon isobars is the dominating mechanism for photoproduction[2].

At higher energies more and more multipoles contribute. Therefore, the parametrization of the photoproduction amplitude in terms of multipole amplitudes has no longer any advantage. Peripheral collisions now become the dominating mechanism which may be described by the exchange of particles and resonances (either as 'elementary' particles or as Regge poles) in the crossed channel[3] $\gamma + \pi \to N + \bar{N}$. It can be shown that there is again a simple connection between the parity of these exchanged particles or systems and the polarization dependence of the photoproduction amplitude. This connection is stated in form of a theorem in section 3 of the present paper.

The kinematical notations and an appropriate form of the photoproduction amplitude are introduced in section 2.

The importance of the results of section 3 for experimental tests of the peripheral model is discussed in section 4.

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1) Verganelakis, A.: Nuovo cimento 31, 1121 (1964)
Hoff, G.T.: Phys. Rev. 122, 665 (1961)

2) Gourdin, M. and Ph. Salin: Nuovo cimento 27, 193 (1963)

3) Kramer, G. and P. Stichel: Zeitschr. f. Physik (in press) and the literature quoted there.

2. Kinematics

Let us denote the momenta of the incoming photon resp. outgoing pion in the CMS of the process $\gamma + N \rightarrow \pi + N$ as $\underset{\sim}{k}$ resp. $\underset{\sim}{q}$. The polarization state of the photon may be described by a vector $\underset{\sim}{\varepsilon}$ with the transversality condition $\underset{\sim}{\varepsilon} \cdot \underset{\sim}{k} = 0$, if we use the coulomb gauge in the CMS.

Then the differential cross section may be written as[4)]

$$\frac{d\sigma}{d\Omega} = \frac{q}{k} \left| \chi_f \mathcal{F} \chi_i \right|^2 \tag{1}$$

where the photoproduction amplitude $\mathcal{F}$ is a 2 x 2 matrix in spin space and is taken between Pauli spinors for the initial resp. final nucleon spin states.

The general form of $\mathcal{F}$ is written as follows

$$\begin{aligned} \mathcal{F} = {} & i \sin\varphi \left[C_1 (\underset{\sim}{\sigma} \cdot \hat{\underset{\sim}{k}}) + C_2 (\underset{\sim}{\sigma} \cdot \hat{\underset{\sim}{n}} \times \hat{\underset{\sim}{k}}) \right] \\ & + \cos\varphi \left[C_3 + i (\underset{\sim}{\sigma} \cdot \hat{\underset{\sim}{n}}) C_4 \right] \end{aligned} \tag{2}$$

where we have introduced an explicit representation for $\underset{\sim}{\varepsilon}$ by means of an angle φ which describes the orientation of $\underset{\sim}{\varepsilon}$ in a plane orthogonal to $\underset{\sim}{k}$

$$\underset{\sim}{\varepsilon} = - \cos\varphi \, \hat{\underset{\sim}{n}} + \sin\varphi \, (\hat{\underset{\sim}{n}} \times \hat{\underset{\sim}{k}}) \tag{3}$$

with $\underset{\sim}{n} \equiv \hat{\underset{\sim}{k}} \times \hat{\underset{\sim}{q}}$

and the symbol $\hat{\underset{\sim}{a}}$ for a unit vector in the direction of $\underset{\sim}{a}$.

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4) Chew, G.F., M.L. Goldberger, F.E.Low and Y.Nambu: Phys.Rev. 106, 1345 (1957)

The $\underset{\sim}{\sigma}$ are the Pauli-spin-matrices and the C_i are functions of the total CMS-energy squared s and the invariant momentum transfer t.

The form of $\mathcal{F}$ given in equ. (2) is equivalent to the representation given in CGLN[4] equ. (7.2). But equ. (2) is more appropriate for our purposes, because the components of $\underset{\sim}{\sigma}$ resp. $\underset{\sim}{\varepsilon}$ with respect to the orthonormal set of vectors $\hat{\underset{\sim}{n}} \times \hat{\underset{\sim}{k}}$, $\hat{\underset{\sim}{n}}$, $\hat{\underset{\sim}{k}}$ are fixed in the limit of high energies and fixed momentum transfer.

We further note, that the functions C_i add inkoherently in the cross section equ. (1) if summed over the nucleon spin states

$$\overline{\frac{d\sigma}{d\Omega}} \equiv \frac{1}{2}\,\frac{q}{k}\;\mathrm{Spur}\,(\mathcal{F}\mathcal{F}^{\dagger}) = \frac{q}{k}\left[\sin^2\varphi\left(|C_1|^2+|C_2|^2\right)+\cos^2\varphi\left(|C_3|^2+|C_4|^2\right)\right] \tag{3}$$

3. Polarization Theorem

With the notations of section 2 we are now able to state the desired polarization theorem.

Theorem:

Provided that to the amplitude $\mathcal{F}(\varphi)$ for $\gamma + N \rightarrow \pi + N$ only exchange of particles or particle-systems with total angular momenta j and either parity $\pi_j = (-1)^{j+1}$ or $\pi_j = (-1)^j$ in the crossed channel $\gamma + \pi \rightarrow N + \overline{N}$ contributes, then the φ-dependence of $\mathcal{F}(\varphi)$ at high energies s and small momentum transfer t becomes:

a) $\mathcal{F}(\varphi) \sim \sin\varphi$ for $\pi_j = (-1)^{j+1}$ resp.

b) $\mathcal{F}(\varphi) \sim \cos\varphi$ for $\pi_j = (-1)^j$

For the understanding of this theorem two remarks are in place:

1) Because of crossing symmetry (which we assume), the whole amplitude $\mathcal{F}$ can be analysed in terms of exchange of particles and particle-systems with definite j and parity π_j in the crossed channel[3]. Therefore, the starting point of our theorem is much more general than a simple peripheral model with exchange of just one particle.

2) If we believe in the Regge pole hypothesis[3], the theorem can be extended to complex angular momenta. Let us consider a Regge trajectory for bosons whose physical points (i.e. physical j) belong either to particles with parity $\pi_j = (-1)^{j+1}$ or $\pi_j = (-1)^j$, then the whole trajectory belongs to case a) resp. case b) of the theorem.

For the proof of this theorem we need the decomposition of the amplitudes C_i, which we have introduced in section 2, with respect to a set of amplitudes for fixed angular momentum j and parity π_j in the crossed channel.

Most of the formulae we need in the following have been derived already in ref.[3] (hereafter called I). Therefore, we give only a rough scetch for the proof of the theorem:

In I the decomposition of the invariant amplitudes A_i (which are defined in CGLN[4]) into helicity amplitudes in the crossed channel has been given (equ. (I-33) and (I-37)). We get the connection between the amplitudes C_i and A_i in the high energy limit by means of the amplitudes $\mathcal{F}_i$ (defined by CGLN[4] in equ. (7.2)) and equ. (I-65), (I-67) as follows:

$$C_1 = |\sin\theta| \left[(\mathcal{F}_2 + \mathcal{F}_3) + \cos\theta \; \mathcal{F}_4 \right] \xrightarrow[s\to\infty]{} \frac{\sqrt{-ts}}{8\pi} A_3$$

$$C_2 = (\mathfrak{F}_1 - \cos\theta\, \mathfrak{F}_2) + \sin^2\theta\, \mathfrak{F}_4 \xrightarrow[s\to\infty]{} \frac{\sqrt{s}}{8\pi}(A_1 + t\, A_2)$$

$$C_3 = |\sin\theta|\, \mathfrak{F}_2 \xrightarrow[s\to\infty]{} \frac{\sqrt{-ts}}{8\pi} A_4 \qquad (4)$$

$$C_4 = -(\mathfrak{F}_1 - \cos\theta\, \mathfrak{F}_2) \xrightarrow[s\to\infty]{} -\frac{\sqrt{s}}{8\pi} A_1$$

Now we consider the case $\pi_j = (-1)^{j+1}$. In this case only the helicity amplitudes d_j^2 and d_j^4 are nonvanishing (for the definition of the $d_j^{\varkappa}$ see I) and therefore, according to equ. (I-33) and (I-37)

$$A_{2,3} \neq 0 \qquad \text{and} \qquad A_{1,4} \sim \frac{1}{s} A_3$$

Then the statement a) of the theorem is obvious according to equ. (2) and (4).

The arguments for $\pi_j = (-1)^j$ are very similar. In this case only the helicity amplitudes d_j^1 and d_j^3 are nonvanishing and therefore we get according to equ. (I-33) and (I-37)

$$A_{1,4} \neq 0\,, \qquad A_2 = -\frac{1}{t} A_1\,, \quad A_3 \sim \frac{1}{s} A_1$$

Statement b) follows then directly from equ. (2) and (4).

Because the selection of the contributing $d_j^{\varkappa}$ is an intrinsic property of a whole Regge trajectory (compare I), the remark 2) is established.

4. Conclusions

From our theorem we have learned, that the differential cross section $\frac{d\sigma}{d\Omega}$ for unpolarized nucleons in the initial state and without measuring the polarization in the final state is proportional to $\sin^2\varphi$ resp. $\cos^2\varphi$ if only exchange of particles or systems with angular momenta j and parity $\pi_j = (-1)^{j+1}$ resp. $\pi_j = (-1)^j$ in the crossed channel $\gamma + \pi = N + \bar{N}$ contributes.

Further we note that the components of the polarization of the recoil nucleon in the directions $\hat{k}$ resp. $(\hat{n} \times \hat{k})$ vanish in both cases, because these components are proportional[5] to $\sin\varphi \cdot \cos\varphi$.

Thus, the stated connection between the φ-dependence of the photoproduction amplitude and the parity of the exchanged system in the crossed channel may be very useful for testing the peripheral model by experiments with linearly polarized photons [+]. It should be possible to determine in such experiments the nature of the exchanged particle or Regge pole. This question is difficult to answer from measurements on $\frac{d\sigma}{d|t|}$ alone, because there are several possibilities for the description of the amplitude with the exchange of one particle: 'Elementary' particle exchange with or without formfactors at the outer vertices, or Regge pole exchange.

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5) Hoff, G.T.: Phys.Rev. 122, 665 (1961)

+) The necessary beams of linearly polarized photons of some BeV should soon be available at the bigger electron accelerators.