DEUTSCHES ELEKTRONEN-SYNCHROTRON DESY

DESY 66/24
Oktober 1966
Theorie

Equal Time Commutators of Renormalized Current Operators in Perturbation Theory

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Abstract

The conjectures derived by applying canonical commutation relations to formally defined currents are tested by forming the actual commutators of the renormalized current operators obtained from perturbation theory (up to second order) an taking an "equal time limit". It is shown that consideration of renormalization phenomena leads to conclusions differing from those obtained in previous perturbational calculations; in certain models the commutator of two zeroth components of vector or axial vector currents diverge at equal times.

§1. The calculation by Johnson and Low [1] of the vacuum to one-particle matrix elements of the current commutators obtained from perturbation theory seems to have anticipated all the difficulties occurring in first order that come from setting the times equal. They do not work with the operators themselves, but use Feynman diagrams which contain all the essential information about the divergences in first order.

In this paper, the operators are constructed in the first two nontrivial orders of perturbation theory, the commutator (still an operator) is formed, and the limit of equal times is studied. In §2, the equal time limit is discussed and illustrated in zeroth order where the currents are constructed from free fields. The treatment of the first order commutator of "zeroth components" in §3 shows a discrimination between broken and exact symmetries. If the masses are split or the coupling constant is a matrix not proportional to the identity, then it seems rather to be accidental features of traces of Dirac matrices than any dependable fundamental reasons which are responsible for the cancellation of the divergences in the equal time limit.

After a study of the unwanted terms in the second order commutator in §4 a summary follows in which a connection between renormalization subtractions and divergences in the equal time commutators is suggested.

Except at the end of §3, the computational details are kept to a minimum in the hope of retaining clarity. After the general discussion in first order, however, a fairly rigorous calculation of one of the troublesome terms is given to prove that it will not disappear in the equal time limit.

§2. Preliminaries

The Yukawa-type coupling is given by the interaction Lagrangian $\mathcal{L}_I = G_{\alpha\beta\gamma}\bar{\psi}^\alpha A^\beta \psi^\gamma = \bar{\psi}\varphi\psi$ where we define $\varphi_{\alpha\gamma} = G_{\alpha\beta\gamma}A^\beta$ and use matrix notation to avoid indices. ψ and A are spinor and tensor fields degenerate along representation spaces of some internal symmetry group. The field equations (without indices) are

$$(\Box + m^2)A = j \qquad A(x) = \int \Delta_R(x-y)j(y)$$
$$(\gamma\partial - m)\psi = f \qquad \psi(x) = \int S_R(x-y)f(y)$$

where $j_\beta = \bar{\psi}^\alpha G_{\alpha\beta\gamma}\psi^\gamma$ and $f_\alpha = G_{\alpha\beta\gamma}A^\beta\psi^\gamma$ i.e. $f = \varphi\psi$
The following table should clarify the notation:

Interaction	Representation Space ψ	A	φ
electron-photon	1 dim	1 dim	$e\gamma A$
nucleon-photon	2 dim isospin space	1 dim	$e\frac{1+\tau_3}{2}\gamma A$
quark-meson 1	3 dim SU_3	1 dim	$\rho_s\phi_s + \rho_p\gamma_5\phi_p$
octet-octet	8 dim SU_3	8 dim SU_3	$(af_{\alpha\beta\gamma} + bd_{\alpha\beta\gamma})\phi^\beta$

If λ is a matrix in the representation (generalized isospin-) space of ψ and $\gamma_?$ is a Dirac matrix, then the current constructed from the spinors corresponding to the Kronecker product $\Lambda = \lambda\otimes\gamma_?$ is designated and defined as follows:

$$j_\Lambda(x) = \bar{\psi}(x)\Lambda\psi(x)$$

where it is understood that all renormalization steps necessary to produce finite operators in Quantum electrodynamics are also to be taken here. Up to second order, this amounts to subtracting the vacuum expectation value and multiples of φ, and performing the usual charge renormalization.*)

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*) As is done in [2]

Once the current operators are obtained and their commutators are constructed in the various orders, it is necessary to have a procedure for passing to equal times. In order that this treatment retain as wide a range of applicability, the following procedure (which seems to make the mildest distribution-theoretical requirements and still produce operator functions of x and y *)) will be employed. The commutator will be smeared by the function $f(\vec{\xi},\vec{X})g(X_0)\phi(\xi_0)$ ($\xi = x-y$ and $X = \frac{1}{2}(x+y)$) where f and g remain arbitrary and ϕ will be made to approach a δ-function, i.e. $\tilde{\phi} \to 1$.

The equal time limiting procedure will be illustrated for the case of the free field current commutator which, formed according to the contraction rules for Wick products from $j_\Lambda(x) = i{:}\bar{\psi}\Lambda\psi{:}(x)$, $j_\Sigma(y) = i{:}\bar{\psi}\Sigma\psi{:}(y)$ is

$$i{:}\bar{\psi}(x)\Lambda S(x-y)\Sigma\psi(y){:} - i{:}\bar{\psi}(y)\Sigma S(y-x)\Lambda\psi(x){:} - \sum_{\pm}(\pm)\, S_p\left(\Lambda S^{(\pm)}(x-y)\Sigma S^{(\mp)}(y-x)\right)$$

When the first term is smeared with $f(\vec{\xi},\vec{X})\, g(X_0)\, \phi(\xi_0)$, the result is

$$i\int\tilde{f}(\tfrac{1}{2}(\vec{p}-\vec{q})+\vec{r},\ \vec{p}+\vec{q})\,\tilde{g}(p_0+q_0)\,\tilde{\phi}(\tfrac{1}{2}(p_0-q_0)+r_0)\,{:}\tilde{\bar{\psi}}(p)\Lambda(i\gamma r-m)\Sigma\tilde{\psi}(q){:}\,d\Omega^{(\varepsilon)}(r)\,d^4p,q$$

where the tilde's (twiddles +)) represent Fourier transforms and $d\Omega^{(\varepsilon)}(r) = \varepsilon(r_0)\,\delta(r^2+m^2)\,d^4r$. The limit $\tilde{\phi}\to 1$ can be performed with no trouble and from the antisymmetry of the measure, all terms in $i\gamma r - m$ drop out except $i\gamma_0 r_0$. The integral then becomes

$$i^2\int\tilde{f}(\vec{r},\vec{p}+\vec{q})\,d^3\vec{r}\ \tilde{g}(p_0+q_0)\,{:}\tilde{\bar{\psi}}(p)\Lambda\gamma_0\Sigma\tilde{\psi}(q){:}\,d^4pq$$

The $\vec{r}$-integration of $\tilde{f}$ is the same as smearing f with $\delta(\vec{\xi})$ and with this done, we recognize [3] the defining expression for

$$i^2\int d^4X,\xi\ \ \delta(\vec{\xi})f(\vec{\xi},\vec{X})\,g(X_0)\,{:}\bar{\psi}\Lambda\gamma_0\Sigma\psi{:}(X)$$

With the notation $[\Lambda,\gamma_0,\Sigma] = \Lambda\gamma_0\Sigma - \Sigma\gamma_0\Lambda$ one has the rigorous result

$$[j_\Lambda(x),\ j_\Sigma(y)] - \langle[\cdot\cdot]\rangle_0 \xrightarrow[(x_0-y_0\to 0)]{} i\,\delta(\vec{\xi})\,j_{[\Lambda,\gamma_0,\Sigma]}(X)$$

+) British usage .

*) There are many attempts at rigorous derivations of the AW relations which do not require current operators, but only the matrix elements of I_3 as the end result. These may escape the divergences to come.

The necessity of the time-smearing function g should be noticed. This function alone is sufficient for the existence and convergence of the integrals occurring at all stages of the limit process; its presence will become increasingly important as the distribution theoretical singularities get worse.

The vacuum expectation values $\langle \cdots \rangle_0$ will either vanish or diverge as $\gamma \to 1$.

§3. First Order

The renormalized current is, in first order

$$j^{(1)}_\Lambda(x) = \int d^4y \left[:\bar{\psi}(x)\Lambda S_R(x-y)\varphi(y)\psi(y): + :\bar{\psi}(y)\varphi(y)S_A(y-x)\Lambda\psi(x): \right]$$

and the commutator, in first order, is

$$[j_\Lambda(x), j_\Sigma(y)]^{(1)} = [j^{(0)}_\Lambda(x), j^{(1)}_\Sigma(y)] + [j^{(1)}_\Lambda(x), j^{(0)}_\Sigma(y)]$$

where each term on the right can be obtained from the other by changing the sign and exchanging (x,Λ) with (y,Σ). A calculation of the first gives (where the z-integration is understood)

$$-:\bar{\psi}(x)\Lambda S(x-y)\Sigma S_R(y-z)\varphi(z)\psi(z): + :\bar{\psi}(y)\Sigma S_R(y-z)\varphi(z)S(z-x)\Lambda\psi(x): -$$
$$-:\bar{\psi}(x)\Lambda S(x-z)\varphi(z)S_A(z-y)\Sigma\psi(y): + :\bar{\psi}(z)\varphi(z)S_A(z-y)\Sigma S(y-x)\Lambda\psi(x): -$$
$$-\sum_{\pm}\left[\pm S_p\left(\Lambda S^{(\pm)}(x-y)\Sigma S_R(y-z)\varphi(z)S^{(\mp)}(z-x)\right) \pm S_p\left(S^{(\pm)}(x-z)\varphi(z)S_A(z-y)\Sigma S^{(\mp)}(y-x)\Lambda\right)\right]$$

The treatment used for the free-field currents is applied to the first and fourth terms and their counterparts in the reversed commutator because they contain $S(\xi)$; the result is the expected $i\delta(\vec{\xi})j^{(1)}_{[\Lambda,\gamma,\Sigma]}(X)$. The second term will cancel the counterpart of the third (and vice versa) which together amount to

$$:\bar{\psi}(y)\Sigma S_R(y-z)\varphi(z)S(z-x)\Lambda\psi(x): + :\bar{\psi}(y)\Sigma S(y-z)\varphi(z)S_A(z-x)\Lambda\psi(x):.$$

The formal argument is that since $S_R(x) = -\theta(-x)S(x)$ the θ function at equal times may be moved to the S function to change it to S_A thus making the first term cancel the second exactly. A proof is given in the Appendix for those who might wonder what happens at the origin (i.e. $\vec{y}=\vec{x}$).

The remaining terms, coming from two contractions, give contributions proportional to φ and include those calculated in [1] between vacuum and one φ-particle states (which, of course, is no real restriction). Using the fact that $2S^{(\pm)} = S \pm S^{(1)}$, one can perform the summations and obtain for these terms 1/2 of

$$Sp\left(\Lambda S^{(1)}(x-y)\Sigma\left\{\begin{matrix} S_R(y-z)\varphi(z)S(z-x) \\ + \\ S(y-z)\varphi(z)S_A(z-x)\end{matrix}\right\}\right) - Sp\left(\left\{\begin{matrix} S_R(x-z)\varphi(z)S(z-y) \\ + \\ S(x-z)\varphi(z)S_A(z-y)\end{matrix}\right\}\Sigma S^{(1)}(y-x)\Lambda\right)$$

$$+\, Sp\left(\Lambda S(x-y)\Sigma\left\{\begin{matrix} S^{(1)}(y-z)\varphi(z)S_A(z-x) \\ + \\ S_R(y-z)\varphi(z)S^{(1)}(z-x)\end{matrix}\right\}\right) - Sp\left(\left\{\begin{matrix} S^{(1)}(x-z)\varphi(z)S_A(z-y) \\ + \\ S_R(x-z)\varphi(z)S^{(1)}(z-y)\end{matrix}\right\}\Sigma S(y-x)\Lambda\right)$$

If we allow ourselves a cutoff of the p-space integral of the S and $S^{(1)}$ functions inside the brackets, the equal time limit can be easily performed. The terms with $S^{(1)}(x-y)$ give 0 by the argument in the appendix, and the others amount to

$$i\delta(\vec{\xi})\, S_p\left([\Lambda, \gamma_0, \Sigma]\left\{\hat{S}^{(1)}(x-z)\varphi(z)S_A(z-x) + S_R(x-z)\varphi(z)\hat{S}^{(1)}(z-x)\right\}\right)$$

where the carat designates the cutoff of the S-function: $\hat{S}^{(1)}(x) = c\cdot\int e^{ipx}(i\gamma p - m)\, d\hat{\Omega}(p)$; $d\hat{\Omega}(p) = d\Omega(p)\,\theta(\alpha - \vec{p}^{\,2})$.

For the zeroth components of vector and axial vector currents, this term discriminates between broken and exact symmetries (which it does not do in general, however). For example, if $\Sigma = G\gamma_0$ then $[\Lambda, \gamma_0, \Sigma] = [\lambda, G] \otimes \gamma_?$ and the trace of $[\lambda, G]$ is 0. For $\Sigma = G \otimes \gamma_5\gamma_0$, $\Lambda = \lambda \otimes \gamma_5\gamma_\mu$ the same thing happens. When the symmetry is not broken, the mass matrices (in the S-functions) and the coupling matrix G are multiples of the identity so that the trace in the generalized isospin space makes the whole term 0. However, for the photon*)-nucleon interaction, for example, only the proton mass occurs. With the Furry's theorem trick $Sp(C^{-1}A^TC) = Sp(A)$, the two terms may be seen to be equal if $C^{-1}\gamma_?^TC = -\gamma_?$ and to cancel if a $+$ sign appears. For an example, we choose $\gamma_? = \gamma_5$ and evaluate the cut off trace:

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*) To avoid the infrared problem, we will let the photons have a small mass $0 < \mu < 2m_p$.

$$\int S_p(\gamma_0 S^{(1)}(x-z)\gamma_\nu S_A(z-x))A^\nu(z)\,dz = \int \frac{e^{ikx}}{(q-k)^2+m^2} S_p(\gamma_0(i\gamma q-m)\gamma_\nu(i\gamma(q-k)-m))\tilde{A}^\nu(k)\,d\hat{\Omega}(q)\,d^4k$$

whose Fourier transform is $4\int \frac{(qk)A_0(k)+(qA(k))(2q_0+k_0)}{2qk+\mu^2}\,d\hat{\Omega}(q)$.
If one chooses the point $\vec{k}=0$ i.e. $k_0=i\mu$, the second term in the numerator will vanish because of the symmetry of $d\hat{\Omega}$ under the transformation $q\to -q$. The first, after symmetrization, becomes $16A_0(k)\int \frac{(qk)^2}{(2qk)^2-\mu^4}\,d\hat{\Omega}^{(+)}(q)$ which definitely diverges as the cutoff variable $\alpha\to\infty$ because $(2qk)^2-\mu^4 = (2\sqrt{q^2+m^2}\,\mu)^2-\mu^4 \geq (2m\mu)^2-\mu^4 = \mu^2((2m)^2-\mu^2)>0$.
The same divergence with a minus sign will occur for the 0th components of axial vector currents in this model; and in the model of Johnson and Low, the trouble comes when chiralities are mixed. In this model, many seemingly accidental features conspire in first order to make the extra part vanish, but with the pseudoscalar part of the coupling and the choices $\Lambda = \tau_-\otimes\gamma_5\gamma_0$ $\Sigma = \tau_+\otimes\gamma_0$ one can, with such cutoff techniques as used above, convince himself that this particular commutator should diverge in the equal time limit regardless of the sequence of time-smearing functions.

§4 Some features of second order

As is known, in first order, the only renormalization step necessary is the removal of the 1 particle matrix element, which can also be symbolized as $j_\Lambda(x) = i\bar{\Psi}(x)\Lambda\Psi(x) - C_\Lambda(1)A^\lambda(x)$
However, in second order one runs into the charge renormalization difficulty and the occurence of the anomalous magnetic moment term. The current in second order is

$$j_\Lambda^{(2)}(x) = i\iint d^4y,z \begin{Bmatrix} :\bar{\Psi}(x)\Lambda S_R(x-y)\check{G}\Psi(y)\bar{\Psi}(z)\check{G}\Psi(z): + :\bar{\Psi}(z)\check{G}\Psi(z)\bar{\Psi}(y)G\,S_A(y-x)\Lambda\Psi(x): \\ -:\bar{\Psi}(x)\Lambda S_R(x-y)\varphi(y)S_R(y-z)\varphi(z)\Psi(z): - :\bar{\Psi}(z)\varphi(z)S_A(z-y)\varphi(y)S_A(y-x)\Lambda\Psi(x): \\ -:\bar{\Psi}(y)\varphi(y)S_A(y-x)\Lambda S_R(x-z)\varphi(z)\Psi(z) \end{Bmatrix} + j_\Lambda^{(a)}(x)$$

where $j_\Lambda^{(a)}$ is the analog of the anomalous magnetic moment term in the second order electromagnetic current ([2] pages 293 to 299). When the commutator is formed without taking $j_\Lambda^{(a)}$ into account, all the expected terms are present and all the unwanted terms cancel before passage to the equal time limit except for those with two Ψ's and those with two φ's (and, as usual, the scalar parts). After the maze of terms is reduced by cancellation of the type justified in the appendix, there are two sets of φ-terms left.

One, in the equal time limit reduces to

$$\delta(\vec{\xi})\, Sp\left([\Lambda,\gamma_0,\Sigma]\, S_{(\alpha)}(x-z)\varphi(z) S_{(\beta)}(z-w)\varphi(w) S_{(\gamma)}(w-x)\right) \text{ (sum over } (\alpha\beta\gamma)\in\{(R1A),(1AA),(RR1)\}$$

and the other is

$$Sp\left(\Sigma S^{(1)}(y-x)\Lambda S_{(\alpha)}(x-z)\varphi(z) S_{(\beta)}(z-w)\varphi(w) S_{(\gamma)}(w-y)\right) - \cdots \text{ (sum over } (\alpha\beta\gamma)\in\{(R0A),(RR0),(0AA)\})$$

where the term with (x,Λ) and (y,Σ) exchanged is still to be subtracted. Here 0 stands for the ordinary (local) S-function. This should be expected on formal grounds to cancel since (ROA) = (RAA) - (RRA), and adding this to the last two gives (RRO)-(RRA)+(RAA)+(OAA) = - (RRR) + (AAA); since the beginning of the chain of retarded (or advanced) functions is at the same time as the end in the equal time limit, the terms should individually cancel.

The terms quadratic in ψ containing $S^{(?)}(x-y)$ also fall into two groups; those with S(x-y) have (divergent) equal time limits of

$$-\int :\bar{\psi}(z) K_{[\Lambda,\gamma_0,\Sigma]}(z-x,x-w)\psi(w):$$

where K_Λ is defined by the slight generalization

$$K_\Lambda(z-x,x-w) = -\tfrac{1}{2}\check{G}\left[S^{(1)}(z-x)\Lambda S_R(x-w) D_R(w-z) + S_A(z-x)\Lambda S^{(1)}(x-w) D_R(z-w) + S_A(z-x)\Lambda S_R(x-w) D^{(1)}(z-w)\right]\check{G}$$

of the function connected with the anomalous magnetic moment corrections in second order ([2] eq (31.3)); the inverted carats above the G's indicate contraction over the indices in the middle. Upon renormalization, the function K_μ is decomposed according to covariance properties in q-space ([2] eq.(31.31-4))

$$\tilde{K}_\mu(q,q') = \gamma_\mu R(Q^2) + i\,\frac{q_\mu+q'_\mu}{2m} S(Q^2) = \gamma_\mu R(Q^2) + \tilde{K}'_\mu(q,q') \qquad [Q=q-q']$$

where the integral expression for R is divergent and $j^{(a)}$ is gotten from the second term:

$$j^{(a)}_\mu(x) = \int dz\,dw :\bar{\psi}(z) K'_\mu(z-x,x-w)\psi(w):$$

The rest of the bilinear terms fall into 3 macroscopic groups: the anomalous magnetic moment terms $[j^{(0)}_\Lambda(x), j^{(a)}_\Sigma(y)] - [(x,\Lambda)\leftrightarrow(y,\Sigma)]$, terms in which one ψ is evaluated at x or y and the other at w or z, and a few remaining terms which will be discussed last.

The second group falls into four similar subgroups which are typified by the $\bar{\psi}(x) \cdots \psi(w)$ terms:

$$-\frac{i}{2}(-1)^{\alpha} :\bar{\psi}(x)\Lambda S_R(x-z)\check{G}S_{(\alpha)}(z-y)\Sigma D_{(\beta)}(w-z)S_{(\gamma)}(y-w)\check{G}\psi(w):$$

which is to be summed over $(\alpha\beta\gamma)\in\{(0,1,R),(10R),(0R1),(1A0)\}$. Formally, one writes $(10R)=(1AR)-(1RR)$ In the first term will be $S_R(x-z)D_A(w-z)$ and thus $S_R(y-w)$ can be replaced by $-S(y-w)$, the retardation being provided by the first combination of S_R and D_A. In the $-(1,R,R)$ term, the minus sign cancels the last factor of $(-1)^{\alpha}$ appearing and the R can be removed from $S_R(x-z)$ with a corresponding change of sign. In the first and third terms the R can be transferred from $S_R(x-z)$ to $S(z-y)$ giving $-S_A(z-y)$. The effect of all this is

$$-:\bar{\psi}(x)\Lambda S(x-z)K_{\Sigma}(z-y,\,y-w)\psi(w):$$

which is precisely the negative of one of the terms in $[j^{(0)}_{\Lambda}(x),\,j^{(\infty)}_{\Sigma}(y)]$ where $j^{(\infty)}_{\Sigma}(y)$ is the (infinite) unrenormalized part of the second order current whose survivor is $j^{(a)}_{\Sigma}(y)$. Thus with the exception of the uncompensated infinity, each of the first group cancels with a subgroup of the second. This leaves only the third group to be discussed. Half of the members of this group contain $j^{(0)}_{G}(z)$ and after manipulating give an equal time limit of

$$i\,\delta(\vec{\xi})S_P\left([\Lambda,\gamma_0,\Sigma]\left\{\begin{matrix} S_R(x-w)\check{G}S^{(1)}(w-x) \\ + \\ S^{(1)}(x-w)\check{G}S_A(w-x)\end{matrix}\right\}\right)$$

The last half of this group, arising from contracting both $\bar{\psi}(z)$ and $\psi(z)$ in the first two terms of the current, does not admit a considerable simplification. The expression is

$$\sum_{\pm}(\pm)S_P\left(\Lambda S^{(\pm)}(x-z)\check{G}S^{(\mp)}(z-x)\right)D_R(w-z)\left(:\bar{\psi}(y)\Sigma S_R(y-w)\check{G}\psi(w):+\right.$$
$$\left.+:\bar{\psi}(w)\check{G}S_A(w-y)\Sigma\psi(y):\right)-[(x,\Lambda)\leftrightarrow(y,\Sigma)].$$

These terms seem neither to correspond directly to renormalization steps nor to represent a local contribution to the commutator.

Recalling that the cancellation between the first two groups was not complete and noting that these terms have many features in common, one might hope that the remaining part, which has been seen to have an infinite equal time limit, would be cancelled, at least for spacelike separations, by the last group. That this is not generally true can be seen in the electrodynamics model $G_{\alpha\beta\gamma} = (\cdot\cdot)_{\alpha\gamma}\gamma_\beta$ with pseudovector currents $\Lambda = \gamma_5\gamma_\mu \otimes \lambda$. The latter terms all vanish because

$$S_p\left(C^{-1}\left(\gamma_5\gamma_\mu S^{(\pm)}_{(x-z)}\gamma_\nu S^{(\mp)}_{(z-x)}\right)^T C\right) = S_p\left(-S^{(\pm)}_{(x-z)}(-\gamma_\nu)(-S^{(\mp)}_{(z-x)})(-\gamma_\mu)\gamma_5\right) = -S_p\left(\gamma_5\gamma_\mu S^{(\pm)}_{(x-z)}\gamma_\nu S^{(\mp)}_{(z-x)}\right)$$

but a calculation of $K_{\gamma_\mu\gamma_5}(q,q')$ shows that the divergent integral, resulting from the 0'th degree terms in k , is still present. Thus the infinity resulting from groups one and two will appear in this case uncancelled by group three.

§5 Conclusions

Perhaps the most useful way of interpreting the foregoing results lies in a comparison with the previous work [1] on this topic. Two fundamental differences are to be noticed in the approach. Here only the finite parts of the currentsare considered, and no attempt at pairing the divergences arising from these finite quantities with infinities in the definition of the current is made. If one were, however, seriously to pursue such an endeavour, he could also find counterparts to the so-called terms of Schwinger in zero-th order. If the current were taken as $\frac{i}{2}[\bar\psi(x'), \Lambda\psi(x)] = i:\bar\psi(x')\Lambda\psi(x) - -iS_p(\Lambda S^{(1)}_{(x'-x)})$ for $x'=x$. an additional term of $\delta(\vec\xi)S_p([\Lambda,\gamma_0,\Sigma]S^{(1)}_{(y-x)})$ could be accounted for in the equal time limit. A calculation of the scalar part of the commutator in 0th order gives

$$\sum_{\pm}(\pm)S_p(\Lambda S^{(\pm)}_{(x-y)}\Sigma S^{(\mp)}_{(y-x)}) = -i\left[S_p(\Lambda S(\xi)\Sigma S^{(1)}(y-x)) - S_p(\Lambda S^{(1)}_{(x-y)}\Sigma S(-\xi))\right]$$

which, with the naive substitution $S(\xi) \rightarrow i\gamma_0\delta(\vec\xi)$ for equal times becomes $\delta(\vec\xi)S_p([\Lambda,\gamma_0,\Sigma]S^{(1)}(y-x)) = i\delta(\vec\xi)\langle\frac{1}{2}[\bar\psi(y),[\Lambda,\gamma_0,\Sigma]\psi(x)]\rangle_0$.

The second difference lies in the computational techniques. Although the method employed here lays no claim to ingenuity, it has the advantage of affording a rather direct connection between definitions and results without having to throw away δ-functions at intermediary stages or to make special restrictions on the limit process. A more important aspect of this difference is that the results obtained are not the same; there appears nothing in first order that could be called "extra" in the sense of [1]. All the divergences have their exact parallels in the unrenormalized currents. Of course the terms in $S^{(1)}(\xi)$ may not disappear so nicely if the p-space integration is not cut off, but since this shady feature is common to both calculations, it should not be a source of differences in the results.

The most evident result of this calculation is that in the first three perturbational orders, not only are all parts of the current present, but also all the divergences that had once been so painstakingly removed from the currents have seen fit to reassert themselves on the occasion of forming the equal time commutator. This would be natural to expect if one had an iterative proof of the canonical relations.

Acknowledgements:

This work was performed as a public service under the sponsorship of the National Science Foundation (USA). I would like to thank Prof.H.Lehmann for suggesting this investigation and for several explanations of the renormalization procedure.

APPENDIX

Smearing the term $\int :\bar\psi(y)\Sigma S_R(y-z)\varphi(z)\,S(z-x)\Lambda\psi(x):d^4z$ with $f(\vec\xi,\vec X)\,g(X_0)\,\delta(\xi_0)$ gives

$$\int \tilde f(\vec p+\vec r-\vec q+\vec s,\ \vec p+\vec q+\vec k)\,\tilde g(p_0+q_0+k_0)\,:\tilde{\bar\psi}(p)\Sigma\frac{i\gamma r-m}{r^2+m^2}\,\tilde\varphi(k)\,\delta(-r+k+s)\,(i\gamma s-m)\,\delta(s^2+m^2)\Lambda\tilde\psi(q):d^4r,s,p,q,k$$

where, because of the equal time limiting process, r_0 and S_0 do not appear in the smearing functions. With the identities

$$\frac{1}{r^2+m^2} = \int\frac{d\tau}{\tau}\,\delta((r+\tau e)^2+m^2) \qquad \text{and} \qquad 0 = \int d\tau\,\delta((r+\tau e)^2+m^2)$$

(e is the unit vector $(1,0,0,0)$), the part between Σ and Λ may be written as

$$\int\frac{d\tau}{\tau}\,(i\gamma(r+\tau e)-m)\,\delta((r+\tau e)^2+m^2)\,\tilde\varphi(k)\,\delta(r+k+s)\,(i\gamma s-m)\,\delta(s^2+m^2)$$

The transformations $r\to r-\tau e$, $s\to s-\tau e$, $\tau\to-\tau$ give it the form

$$-\int\frac{d\tau}{\tau}\,(i\gamma r-m)\,\delta(r^2+m^2)\,\tilde\varphi(k)\,\delta(-r+k+s)\,(i\gamma(s+\tau e)-m)\,\delta((s+\tau e)^2+m^2)$$

Again using the two identities, one completes the transformation of the first term into minus the second.

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