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A B S T R A C T

An additional (magnetic) form factor G_2 which violates T-invariance is introduced in the electromagnetic vertex of the deuteron. This additional interaction yields a right-left asymmetry in the scattering of electrons from polarized deuterons. For our numerical evaluations we used values of G_2 which are consistent with the deviations from the impulse approximation found in measurements of the unpolarized cross section. A maximum asymmetry of approximately 40 percent was obtained at $q = 3.5\ f^{-1}$. However, care must be taken in the preparation of the polarized target, as a suitable tensor polarization may also produce asymmetry.

1. Introduction

In connection with CP violation in K_2^o decay the remark has been made [1] that we do not yet have a good experimental verification of the validity or violation of T-invariance in the electromagnetic interaction of strongly interacting particles.[*] In elastic electron-nucleon scattering T-invariance does not yield an additional restriction in the one-photon exchange approximation, since, of course, Lorentz invariance, P-invariance and electromagnetic current conservation are assumed to be valid. This is different for the scattering of electrons from a particle of spin 1: if T-invariance is not assumed to hold in addition to Lorentz invariance and P-invariance, we have one additional form factor in the one-photon exchange interaction [2]. Thus a test of T-invariance should be possible in elastic electron-deuteron scattering. Of course, there will be no violation of T-invariance if the impulse approximation is strictly valid, but recent measurements showed large discrepancies in the magnetic term for higher momentum transfers [3].

Subsequently, we reconsider previous work [4] (hereafter referred to as I) on polarization in elastic electron-deuteron scattering including the additional form factor which violates T-invariance. In particular, it will be shown in Section 2 that violation of T-invariance leads to a right-left asymmetry in the scattering of unpolarized electrons from polarized deuterons (with a vector polarization perpendicular to the scattering plane). Similar considerations have been made independently by V.M. Dubovik and A.A. Cheshkov [5]. These authors do not consider the effect of the tensor polarization of the deuteron which may yield asymmetry effects of the same kind as those which result from a violation of T-invariance. The tensor polarization also enters into the numerical value of the asymmetry resulting purely from a violation of T-invariance.

[*]) Recently, experiments on η-decay have been carried out, [10] and an electroproduction experiment on polarized protons is in preparation [11].

2. Effect of T-Invariance Violation upon the Cross Section

Treating the electron-deuteron interaction in the one-photon exchange approximation we have the matrix element [2)]

$$\langle f \,|R|\, i\rangle = \frac{e}{2\sqrt{P^oP'^o}}\, e_\rho^{(i)*}(P')R_\mu^{\rho\sigma}(P',P)e_\sigma^{(k)}(P)\frac{1}{q^2}\; x$$

$$x\; \frac{e}{2\sqrt{p^o p'^o}}\, \bar{u}(p')\,\gamma^\mu u(p) \qquad (1)$$

where

$$R_\mu^{\rho\sigma} = (P_\mu + P'_\mu)\;(F_1 g^{\rho\sigma} + \frac{1}{2M^2}F_2 q^\rho q^\sigma) +$$

$$+\; G_1\;(q^\rho g_\mu^\sigma - q^\sigma g_\mu^\rho) +$$

$$+\; \frac{i}{M^2}\, G_2\;(q_\mu q^\rho q^\sigma - \frac{q^2}{2}\;(g_\mu^\rho q^\sigma + g_\mu^\sigma q^\rho)). \qquad (2)$$

In these expressions (as in I) P,P' = initial and final four-momentum of the deuteron, p,p' = initial and final fourmomentum of the electron, $q = p-p' = P'-P$ = fourmomentum transfer and M = mass of the deuteron. $e_\sigma^{(k)}(P)$ describes a pure state of polarization of the deuteron and fulfills the conditions

$$e^{*(i)\mu}\, e_\mu^{(j)} = g^{ij},$$

$$e_{(i)\mu}\, e^{*\,(i)}_\nu = g_{\mu\nu} + \frac{1}{M^2}\, P_\mu P_\nu$$

and finally $u(p)$ ist the electron spinor.

The form factors $F_1(q^2)$, $F_2(q^2)$ and $G_1(q^2)$ are related to the charge, quadrupole and magnetic form factors of the deuteron by

$$F_C = F_1 + \frac{2}{3}\eta\,(F_1 - G_1 + F_2(1+\eta))$$

$$F_Q = F_1 - G_1 + F_2(1+\eta)$$

$$F_M = G_1$$

where $\eta = \frac{q^2}{4M^2}$.

The additional term multiplied by the form factor $G_2(q^2)$ violates T-invariance.

The cross section for elastic scattering of unpolarized electrons from polarized deuterons may be written [4)]

$$\frac{d\sigma}{d\Omega} = k({}^1a + {}^3a_k s^k + {}^4a_{ik} s^{ik}),$$

where k is a kinematical factor, ${}^1a$, ${}^3a_k$ and ${}^4a_{ik}$ depend on the momenta of the particles before and after scattering and s^k and s^{ik} describe the deuteron polarization

$$s^i = s^\alpha t_\alpha^{(i)}, \quad s^\alpha P_\alpha = 0,$$

$$s^{ik} = s^{\alpha\beta} t_\alpha^{(i)} t_\beta^{(k)}, \quad s^{\alpha\beta} = s^{\beta\alpha}, \quad s^{\alpha\beta}P_\beta = 0, \quad s^\alpha{}_\alpha = 0.$$

The unit vectors $t^{(\alpha)\mu}(P)$ fulfill

$$t^{(\alpha)\mu}(P)\,t_\mu^{(\beta)}(P) = g^{\alpha\beta},$$

$$t^{(o)\mu} = \frac{1}{M}P^\mu.$$

In the rest system of the deuteron, the vector- and tensor-polarization will be denoted by $\overset{o}{s}{}^\alpha$ and $\overset{o}{s}{}^{\mu\nu}$. The results of our calculation will be given in the laboratory system, where the deuteron is at rest before scattering, and the vectors $t^{(\alpha)\mu}$ are chosen to be

$$t^{(1)\mu} = (0,\vec{k}), \quad t^{(2)\mu} = (0,\vec{\ell}), \quad t^{(3)\mu} = (0,\vec{n}),$$

$$\vec{k} = \frac{\vec{p}}{|\vec{p}|}, \qquad \vec{\ell} = \vec{n} \times \vec{k}, \qquad \vec{n} = \frac{\vec{p} \times \vec{p}\,'}{|\vec{p} \times \vec{p}\,'|}.$$

If PT-invariance is required, we have ${}^3a_k = 0$ in the one-photon exchange approximation. If only P-invariance is valid, ${}^3a_3 \neq 0$ is allowed.

Taking the trace of the density matrix after scattering as has been described in (I) with the interaction (2), for extremely relativistic electrons ($m^2/\vec{p}^{\,2} \ll 1$), ${}^3a_k$ is obtained to be

$$\frac{{}^3a_3}{{}^1a} = \frac{1}{N} G_2 F_Q \eta(\eta - \varepsilon) 2\tan^2(\theta/2) \frac{|\vec{p}||\vec{p}\,'|}{M^2} \sin\theta$$

$${}^3a_1 = {}^3a_2 = 0.$$

In this expression, θ ($0 = \theta = \pi$) is the scattering angle of the electron in the laboratory system, $\eta = q^2/4M^2$, $\varepsilon = |\vec{p}|/M$ and

$$N = F_C^2 + \frac{8}{9}\eta^2 F_Q^2 + \frac{2}{3}\eta(1 + 2(1+\eta)\tan^2(\theta/2)(F_M^2 + 4\eta^2 G_2^2) \equiv$$

$$\equiv A(q^2) + B(q^2)\tan^2(\theta/2)$$

The expressions for ${}^1a$ and ${}^4a_{ik}$ given in (I) turn out to be modified by the substitution

$$F_M^2 \rightarrow F_M^2 + 4\eta^2 G_2^2$$

and are obtained to be:

$${}^1a = \frac{4M^2\vec{p}^{\,2}\cos^2(\theta/2)}{m^2(1 + 2\,\varepsilon\sin^2(\theta/2))} N$$

$$\frac{{}^4a_{11}}{{}^1a} = - \frac{\eta(1+\varepsilon)^2}{\varepsilon^2(1+\eta)N} \left(\frac{4}{3}\eta^2 F_Q^2 + 4\eta F_C F_Q + \right.$$

$$\left. + (F_M^2 + 4\eta^2 G_2^2)(1+\eta)\left(\frac{\varepsilon^2}{(1+\varepsilon)^2} + \eta\tan^2(\theta/2)\right) - \right.$$

$$F_M F_Q \frac{4\eta(\varepsilon - \eta)}{(1 + \varepsilon)} \Big)$$

$$\frac{{}^4a_{21}}{{}^1a} = \frac{{}^4a_{12}}{{}^1a} = \frac{\eta^2(1 + \varepsilon)}{\varepsilon^2(1 + \eta)\tan(\theta/2)N} \Big(\tfrac{4}{3}\eta F_Q^2 + 4F_c F_Q +$$

$$+ (F_M^2 + 4\eta^2 G_2^2)(1 + \eta)\tan^2(\theta/2) +$$

$$+ 2F_M F_Q \frac{(\varepsilon - \eta)}{(1 + \varepsilon)} ((1 + \varepsilon)^2 \tan^2(\theta/2) - 1)\Big)$$

$$\frac{{}^4a_{22}}{{}^1a} = \frac{-\eta}{\varepsilon^2(1 + \eta)\tan^2(\theta/2)N} \Big(\tfrac{4}{3}\eta^2 F_Q^2 + 4\eta F_c F_Q +$$

$$+ 4\eta F_M F_Q \tan^2(\theta/2)(\varepsilon - \eta)(1 + \varepsilon) + (F_M^2 + 4\eta^2 G_2^2) \times$$

$$\times \tan^2(\theta/2)(1 + \eta)(\varepsilon^2 + \eta)\Big)$$

$${}^4a_{33} = {}^4a_{13} = {}^4a_{31} = {}^4a_{23} = {}^4a_{32} = 0.$$

We now compare scattering of electrons towards the left (i.e. $\vec{n} = \vec{n}^L$, where $\vec{n}^L \cdot \overset{o}{\vec{s}} > 0$, $\vec{\ell}^L \equiv \vec{n}^L \times \vec{k}$, $\theta = \theta^L$) with scattering towards the right (i.e. $\vec{n} = \vec{n}^R$, where $\vec{n}^R \cdot \overset{\ell}{\vec{s}} < 0$, $\vec{\ell}^R \equiv \vec{n}^R \times \vec{k}$, $\theta = \theta^R = \theta^L$). The following asymmetry As is obtained

$$As \equiv \frac{d\sigma_L - d\sigma_R}{d\sigma_L + d\sigma_R} = \frac{\frac{{}^3a_3}{{}^1a} \overset{\ell}{s} \cdot \vec{n}^L + 2\frac{{}^4a_{12}}{{}^1a} s_L^{12}}{1 + \frac{{}^4a_{11}}{{}^1a} s^{11} + \frac{{}^4a_{22}}{{}^1a} s^{22}}$$

where

$$s_L^{12} \equiv \overset{o}{s}{}^{ik} \ell_i^L k_k$$

$$s^{22} = \overset{o}{s}{}^{ik} \ell_i^L \ell_k^L = \overset{o}{s}{}^{ik} \ell_i^R \ell_k^R$$

From the expression for As we see that there is an asymmetry even if T-invariance is not violated ($^3a_3 = 0$) unless $S_L^{12} = 0$. $S_L^{12} = 0$ may be achieved if the polarized target is produced in such a way that there is no preferred direction in the scattering plane, for instance by applying a magnetic field strictly perpendicular to the scattering plane. In this case the measured asymmetry is proportional to the form factor G_2 violating T-invariance:

$$As = \frac{\frac{1}{N} G_2 F_Q \eta(\eta - \varepsilon) 2\tan^2(\theta/2) \frac{|\vec{p}||\vec{p}\,'|}{M^2} \sin\theta \; \overset{o}{\vec{s}} \cdot \vec{n}^L}{1 + \frac{^4a_{11}}{^1a} s^{11} + \frac{^4a_{22}}{^1a} s^{22}} .$$

To get a numerical estimate of the magnitude of As which could possibly be expected, we assume values of G_2 which are roughly consistent with current experimental information on the electron-deuteron elastic cross section. A discrepancy ΔB from simple impulse approximation in the magnetic term $B(q^2)$ up to nearly 100 percent is allowed by the experimental values of [3]. The values for the asymmetry As obtained at $q = 3.5\ f^{-1}$ for a totally polarized target, i.e. $S^3 = 1$, $S^{11} = S^{22} = -1/6$, $S^{33} = 1/3$, $S^{ik} = 0$ for $i \neq k$, may be found in Table 1.*)

*) In these calculations the impulse approximation has been used to evaluate the deuteron form factors F_C, F_Q and F_M using the deuteron integrals of [6] (model of Breit, Yale potential). The nucleon form factors have been taken from [7].

G_2	ΔB	|As|		
		$\theta = 45^{\circ}$	$\theta = 90^{\circ}$	$\theta = 135^{\circ}$
0.4	27 %	0.19	0.23	0.28
0.6	60 %	0.28	0.33	0.37
0.8	100 %	0.36	0.41	0.43

Table 1

Asymmetry As obtained in scattering of electrons from polarized deuterons at $q = 3.5\ f^{-1}$ for different values of form factor G_2 violating T-invariance. ΔB is the change of the magnetic term B in the expression for the cross section resulting from the corresponding values of G_2. The change in A due to G_2 is always less than 10 %. θ is the scattering angle of the electron.

3. Other Polarization Effects

A violation of T-invariance also yields a vector polarization of the deuteron perpendicular to the scattering plane in the scattering of electrons from unpolarized deuterons. However, measurements are complicated because the vector polarization appears together with a tensor polarization, which is rather large, even if T-invariance is valid [8).

Finally, we remark that there is no influence of the electron polarization on the cross section in the one-photon exchange approximation in the scattering from unpolarized deuterons, even if T-invariance is violated.

4. Conclusions

Using the one-photon exchange approximation and assuming the existence of a form factor G_2 violating T-invariance in the deuteron electromagnetic vertex, we found a maximum right-left asymmetry of 40 percent in the cross section for scattering of electrons from polarized deuterons. In the numerical evaluation values of G_2 have been taken which are consistent with current experimental information on elastic electron-deuteron scattering. Asymmetry may also occur without violation of T-invariance when the target has a suitable tensor polarization. However, this last possibility may be excluded, provided that the polarization of the target has been produced in such a way that there is no preferred direction in the scattering plane.

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