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by

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Summary:

For many-particle production we study a model which assumes at fixed impact parameter a statistical picture of fireball formation and decay. We obtain in this way on one hand for the usual statistical model an extension which includes angular momentum conservation and allows predictions for all inelastic and corresponding absorptive elastic cross-sections, on the other a "central collision" counterpart of the peripheral model.

The asymptotic predictions for production processes are discussed and compared to those of the usual statistical model and of the multiperipheral model. The resulting form for elastic and total cross-sections, obtained by unitarity, agrees with that given by Van Hove's uncorrelated particle model.

I. Introduction

Both the present lack of a general theory of strong interactions and the inherent difficulties of many-particle problems cause the theoretical description of high energy production processes to rely essentially on models based upon simplified physical pictures. Two opposite points of view, regarding the collision of the two incoming particles as either central or peripheral, have led respectively to the statistical model (SM)[1-5] and the peripheral[6], later the multiperipheral[7,8] model (MPM); the correct description hopefully lies somewhere in between. The search for it is however severely hindered by the incompleteness of the statistical picture: it allows only a few predictions (branching ratios, multiplicities, spectra) about inelastic processes and practically none about the coupled elastic reaction; moreover it does not include all of the kinematics (angular momentum conservation) and thus from the outset precludes the discussion of certain kinematical questions (such as CMS angular distributions). On the other hand, the statistical model, because of its simplicity, is up to now the generally employed tool in the analysis of multiparticle experiments and serves as a reference in the "definition" of "dynamical" effects.

It is the aim of this paper to propose an extension of the SM which makes definite predictions about all aspects of production processes and (apart from phase questions) also about the corresponding elastic reaction. Postponing for the moment both a more detailed mathematical formulation and quantitative numerical comparisons with experiments, we shall here present the physical basis of the model and discuss qualitatively a number of its predictions.

II. The Model

Consider an N-particle ($N \geq 3$) production process (identical scalar particles of mass m). The statistical hypothesis states that it proceeds via an intermediate state (fireball) in which all information about the initial state is lost, except for kinematical conservation laws, and whose decay into the N particles is determined by the available phase space Ω_N, restricted in coordinate space to a small "interaction volume" of radius R to incorporate the short range nature of the interaction. For the transition probability $P_{2\to N}(X)$ from two incoming to N outgoing particles, summed over the momenta of the outgoing particles, but at fixed kinematic quantities X (e.g. total CMS energy, angular momentum of the system) we thus have

$$P_{2\to N}(X) = U_{2\to F}(X)\ V_{F\to N}(X) \qquad N \geq 3 \qquad (1)$$

where $U_{2\to F}(X)$ and $V_{F\to N}(X)$ describe fireball formation and decay, respectively. The conventional statistical model fixes only the total CMS energy W (X=W), and sets for $V_{F\to N}(W)$ the quotient of N particle to entire phase space

$$V_{F\to N}(W) = \Omega_N(W)/\sum_{N=2}^{\infty} \Omega_N(W); \quad \sum_{N=2}^{\infty} V_{F\to N}(W) = 1 \qquad (2)$$

i.e., $V_{F\to N}(W)$ is the fireball decay distribution over N at fixed W. Nothing is said about how the centrality of the collision effects fireball formation, and hence this model can only be expected to lead to predictions in which $V_{F\to N}(W)$ does not enter (ratios), or else further assumptions have to be invoked[4].

Instead of postulating (1) for the transition 2→N at fixed CMS energy only, we shall now require in addition that the total CMS angular momentum ℓ of the system also be fixed[9-12]. Since (classically) the impact parameter of the collision is ℓ/K (where K is the CMS momentum of one of the two incoming particles), this gives us a convenient measure of the centrality of the process[2,13], and moreover allows us to use phase space arguments also in the determination of $U_{2\to F}$, as we shall shortly see. We thus postulate for the transition probability 2→N at fixed ℓ and W

$$P_{2\to N}(\ell,W) = U_{2\to F}(\rho,W)\ V_{F\to N}(\ell,W) \qquad (3)$$

Here $\rho = \ell/KR$ is the dimensionless impact parameter, and $U_{2\to F}(\rho,W)$ gives the distribution of fireball formation over ρ at fixed W +):

$$\int_{o}^{\infty} d\rho^2 \; U_{2\to F}(\rho,W) = 1 \tag{4}$$

The function $V_{F\to N}(\ell,W)$ gives the decay distribution over N of a fireball of fixed mass W and spin ℓ :

$$\sum_{N=2}^{\infty} V_{F\to N}(\ell,W) = 1 \tag{5}$$

We emphasize that (3) is to be an "Ansatz" only for inelastic processes ($N \geq 3$); but conservation of probability then requires

$$\sum_{N=2}^{\infty} P_{2\to N}(\ell,W) = 1 \tag{6}$$

and hence the elastic transition probability

$$P_{2\to 2}(\ell,W) = 1 - \sum_{N=3}^{\infty} P_{2\to N}(\ell,W) \tag{7}$$

is determined as well.

Within this framework the only a priori given "building blocks" to construct both $U_{2\to F}$ and $V_{F\to N}$ are the kinematically available phase spaces, suitably restriced in coordinate space by an interaction volume. The complete classical N-particle phase space integral (i.e., with all ten conserva-

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+): This choice of ρ and the normalization (4) yield as total "fireball cross-section"

$$\sigma_{2\to F} = \frac{\pi}{K^2} \sum_{\ell=o}^{\infty} (2\ell+1) \; U_{2\to F}(\rho,W) = \pi R^2$$

just the geometrical cross-section, R being the effective radius. Elastic scattering is of course only partially covered by $\sigma_{2\to F}$ (shadow scattering!); we return to this point shortly.

tion laws - energy, momentum, angular momentum, center of mass - taken into account) is given by[14)]

$$\Omega_N(\ell^2,W) = \int \prod_{i=1}^{N} \{d^3p_i d^3x_i r(x_i,p_i)\} \times$$
$$\times\ \delta(\Sigma p_{io}-W)\ \delta^{(3)}(\Sigma\vec{p}_i)\ \delta^{(3)}(\Sigma\vec{x}_i\times\vec{p}_i-\vec{\ell})\ \delta^{(3)}(\Sigma p_{io}\vec{x}_i) \tag{8}$$

where the $r(x_i,p_i)$ are to give the (relativistic) restriction of integration to the interaction volume ("size" of the fireball). We shall choose a Gaussian form

$$r(x_i,p_i) = e^{-\frac{1}{R^2}\left[\vec{x}_i^{\,2} + (\vec{x}_i\cdot\vec{p}_i)^2/m^2\right]} \tag{9}$$

with interaction radius R, which leads to the usual Lorentz-contraction and furthermore renders (8) Poincaré-invariant, hence dependent only on total CMS energy W and spin value ℓ[14)]. Quantum effects we shall include only in the form of multiplying (8) by $q^N/N!$, where the N! takes into account the identity of the particles, and $q=(2\pi)^{-3}$ gives with $\hbar=1$ the density of states, as in the simplest version of the usual SM, with only one interaction volume, no isospin, etc.; the inclusion of isospin or different interaction strength for different types of particles would modify this value of q.

For N=2 the integral (8/9) gives

$$\Omega_2(\ell^2,W) = \Omega_2(W)\ F_2(\ell^2,W) \tag{10}$$
$$\Omega_2(W) = q^2/2!\ \left[(\pi/2)^{3/2}\ (2m/W)R^3\ 8\pi K/W^2\right]$$
$$F_2(\ell^2,W) = (2\pi\ell)^{-1}\ (2K^2R^2)^{-1}\ e^{-\ell^2/2K^2R^2}$$

where $\Omega_2(W)$ is essentially the usual energy-conservation phase space, and $F_2(\ell^2,W)$ the angular momentum distribution

$$\int d^3\ell\ F_2(\ell^2,W) = 1 \tag{11}$$

For N > 2 the integral can be evaluated by statistical methods[15,16)] (central limit theorem), and for high energy ($m/W \to 0$) and large N the result is[14)]

$$\Omega_N(\ell^2,W) = \Omega_N(W)\; F_N(\ell^2,W) \tag{12}$$

$$\Omega_N(W) = 2\pi \left[\pi^{5/2} mR^3W^2q\right]^N \Big/ N^{3/2}R^3W^7(N-1)!\,(N-2)!\,(N-3)!$$

$$F_N(\ell^2,W) = \left[N/\pi R^2W^2\right]^{3/2} e^{-N\ell^2/W^2R^2}$$

with $\Omega_N(W)$ and F_N again giving energy phase space and angular momentum distribution. At energies where the mass m is not negligible the above-mentioned approximation methods are also applicable but require some numerical computations.

Let us now return to the statistical picture. The intermediate fireball state has led us to consider two distinct situations: in the formation we ask for the distribution over all possible impact parameters, i.e., we compare the chances of fireball creation at various ρ values; in the decay we have a given fireball of fixed energy and spin - the competing channels are those of different particle number $N \geq 2$. The latter case we take, as in the usual SM, to be determined by phase space (here however with full kinematics) restricted to the interaction volume. We therefore have

$$V_{F\to N}(\ell,W) = \Omega_N(\ell^2,W) \Big/ \sum_{N=2}^{\infty} \Omega_N(\ell^2,W) \tag{13}$$

for the fireball decay distribution into N particles. Now we assume the inverse process $N\to F$ also to be governed by phase space and consider in particular N=2. Since here we want to compare different impact parameter channels, we set for the distribution of fireball formation over ρ

$$U_{2\to F}(\rho,W) = \Omega_2(\rho,W) \Big/ \int_0^\infty d\rho^2\,\Omega_2(\rho,W) = e^{-\rho^2} \tag{14}$$

with $\rho = \ell/\sqrt{2}\,KR$, since here $\sqrt{2}\,R$ is the effective radius. The last equation in (14) follows from (10). Fireball creation is thus most likely to take place in central collisions ($\rho=0$) and falls off as a Gaussian for $\rho>0$. It is at fixed ρ independent of energy. While the particular form (14) is of course a consequence of the Gaussian interaction volume cut-off, any short range restriction there will lead to a short range $U_{2\to F}(\rho,W)$. Here we recall that peripheral processes generally give a linearly exponential fall-off in ρ, so that (14), although allowing some fraction of non-central fireball formation, damps out much quicker as the collision becomes more peripheral. If instead of (14) we had simply chosen $U_{2\to F}(\rho,W) = \delta(\rho)$ and thus allowed fireball formation only for exactly central collisions, we would have obtained the usual SM with only minor modifications. We believe the form (14), obtained from two-particle phase space, to provide a physically reasonable extension of this.

Combining (13) and (14) we finally have for the transition probability $2\to N\geq 3$ at fixed ℓ and W

$$P_{2\to N}\ (\ell,W) = e^{-\ell^2/2K^2R^2}\ \Omega_N(\ell^2,W)/\sum_{N=2}^{\infty}\ \Omega_N(\ell^2,W) \tag{15}$$

We now consider the N-particle production cross-section $\sigma_N(W)$. It is given by

$$\sigma_N(W) = \frac{\pi^2}{KW}\int\prod_{i=1}^{N}\frac{d^3p_i}{2p_{io}}\ \delta^{(4)}(\Sigma p_i-P)\,|\langle p,\ldots p_N|S|q,q_2\rangle|^2 \tag{16}$$

where q_1, q_2 with $W^2 = (q_1 + q_2)^2$ denote the incoming particles.
A generalized partial wave expansion[12)] of the S-matrix element gives

$$\sigma_N(W) = \frac{\pi}{K^2}\sum_{\ell=o}^{\infty}(2\ell+1)\ \sum_{\eta_N}|\langle\eta_N||S(\ell,W)||\rangle|^2 \tag{17}$$

with the reduced matrix element $\langle\eta_N||S(\ell,W)||\rangle$; the set of parameters η_N describes all possible inner configurations of the N particle system leading to total CMS energy W and angular momentum ℓ. The sum of $|\langle\eta_N||S(\ell,W)||\rangle|^2$ over all η_N is just the transition probability for $2\to N$ at fixed ℓ and W:

$$P_{2\to N}(\ell,W) = \sum_{\eta_N}|\langle\eta_N||S(\ell,W)||\rangle|^2 \tag{18}$$

so that (15) and (17) determine $\sigma_N(W)$. The sum over all $N \geq 3$ of (18) gives the contribution of all inelastic channels

$$f(\ell,W) \equiv \sum_{N=3}^{\infty} \sum_{n_N} |\langle n_N \| S(\ell,W) \| \rangle|^2 = \sum_{N=3}^{\infty} P_{2\to N}(\ell,W) \qquad (19)$$

$$0 \leq f(\ell,W) \leq 1$$

from which we determine by unitarity (equivalently to (6/7))

$$\sum_{N=2}^{\infty} \sum_{n_N} |\langle n_N \| S(\ell,W) \| \rangle|^2 = 1 \qquad (20)$$

the magnitude of the elastic partial wave S-matrix element

$$|S(\ell,W)|^2 = 1 - f(\ell,W) \qquad (21)$$

Introducing the (real) phase $\delta_\ell(W)$

$$S(\ell,W) = |S(\ell,W)| \, e^{2i\delta_\ell(W)} \qquad (22)$$

we obtain from the partial wave scattering amplitude $t(\ell,W) = (1-s(\ell,W))i/2$ the elastic cross-section

$$\sigma_{el}(W) = \frac{\pi}{K^2} \sum_{\ell=0}^{\infty} (2\ell+1) \left| 1 - |S(\ell,W)| e^{2i\delta_\ell(W)} \right|^2 \qquad (23)$$

It can thus only be predicted by our model if additional information about the phase $\delta_\ell(W)$ is invoked.

III. Predictions and Comparisons

A. Inelastic Reactions:

The cross-section $\sigma_N(W)$ for the reaction $2 \to N \geq 3$ at high energy, where $m/W \to 0$ and the partial wave summation may be replaced by integration, is from (15/17/18) given by

$$\sigma_N(W) = \frac{4\pi}{W^2} \int_0^\infty d\ell^2 \, e^{-2\ell^2/W^2R^2} \, \Omega_N(\ell^2,W) \Big/ \sum_{N=2}^{\infty} \Omega_N(\ell^2,W) \qquad (24)$$

Using (12), the phase space sum over $N \geq 3$ is found to be

$$\Omega(\ell^2,W) \equiv \sum_{N=3}^{\infty} \Omega_N(\ell^2,W) = \frac{\pi^7}{W^4} q(mR)^3 \left(\frac{x}{y}\right)^3 {}_0F_2(2,3;x) \qquad (25)$$

$$x = y\, e^{-\ell^2/W^2R^2} \quad ; \quad y = \pi^{5/2} m R^3 W^2 q$$

where ${}_0F_2(2,3;x)$ is a generalized hypergeometric function[17] with the limiting behaviour

$$ {}_0F_2(2,3;x) = 1 + \frac{x}{6} + O(x^2) \qquad x \ll 1 \qquad (26)$$

$$ e^{3x^{1/3}} / 3^{1/2} \pi x^{4/3} \left[1 + O(x^{-1/3})\right] \quad x \gg 1$$

With its help $\sigma_N(W)$ may be calculated in closed form for the limits $W^{2/3} \gg N$ (asymptotic region) and $W \geq N \gg W^{2/3}$ (threshold region distorted by $m/W \to 0$). In the asymptotic region we find as leading term

$$\sigma_N^{as}(W) = \frac{12}{q^2\pi^4} N^5 \Big/ m^2 (WR)^4 \qquad (27)$$

i.e., $\sigma_N(W)$ vanishes at high energies with W^{-4}. In the threshold region we get

$$\sigma_N(W) = \frac{\frac{\text{const.}}{m^2}\left[\pi^{5/2}W^3R_c^3q/2\right]^{N-1} e^{-3\left[\pi^{5/2}W^3R_c^3q/2\right]^{1/3}}}{N!(N-2)!(N-3)!} \qquad (28)$$

with $R_c^3 = R^3(2m/W)$ as Lorentz-contracted interaction volume. The maximum value of $\sigma_N(W)$, attained in an intermediate region, decreases with increasing N approximately as $N^{-1/2}$. While the increase in the threshold region (apart from the $m = 0$ distortion due to our calculation) is given by the usual phase space, as expected, the W^{-4} fall-off at high energies is in contrast to the usual SM[+)], which yields an exponentially vanishing transition probability (exp-const. $W^{2/3}$).

[+)]: We always refer to the "covariant" version[3)], but everything said holds with slight modifications also for the Fermi version[1)].

On the other hand the multipheripheral model for $W \to \infty$ with

$$\sigma_N^{MPM}(W) = \text{const.}\ (\lambda \log W)^N/N!W^4 \ ; \ \lambda = \text{const.} \qquad (29)$$

gives a similar but somewhat weaker fall-off than our model.

In Fig. 1 we illustrate our form of $\sigma_N(W)$, calculated numerically for non-vanishing identical masses. As an indication of the experimental situation we show in Fig. 2 the cross-sections for some π-production reactions in p-$\bar{p}$ annihilation[18]; while they seem to be qualitatively compatible with our predicted behaviour, numerical calculations including isospins, different masses etc. are necessary to give conclusive information.

The branching ratio $Q(N/\bar{N}) = \sigma_N/\sigma_{\bar{N}}$ in our model approaches asymptotically the constant value

$$Q^{as}(N/\bar{N}) = (N/\bar{N})^5 \qquad (30)$$

while the usual SM gives

$$Q^{as}_{SM}(N/\bar{N}) = \frac{Z^{N-\bar{N}}\ \bar{N}!(\bar{N}-1)!(\bar{N}-2)!}{N!(N-1)!(N-2)!} \ ; \quad Z = \pi m R^3 W^2 \qquad (31)$$

The MPM leads to

$$Q^{as}_{MPM}(N/\bar{N}) = (\lambda \log W)^{N-\bar{N}}\ \ \bar{N}!/N! \qquad (32)$$

which for $\bar{N} > N$ gives a fall-off in W, but very much weaker than that of the usual SM.

In Fig. 3 we show the experimental ratios from the above mentioned p-$\bar{p}$ experiments; they seem to support strongly a constant or at most very slowly decreasing asymptotic value (the smallest particle numbers should exhibit the most asymptotic behaviour). A comparison of our result (30) including isospin is seen to give quite good agreement; note that our $Q^{as}(N/\bar{N})$ is independent of the value of the interaction radius R.

The average multiplicity of inelastic processes

$$\langle N \rangle = \sum_{N=3}^{\infty} N\sigma_N \Big/ \sum_{N=3}^{\infty} \sigma_N \tag{33}$$

is given by

$$\langle N \rangle^{as} = \frac{6}{7} \left(\pi^{5/2} qmR^3W^2\right)^{1/3} + \text{const.} \tag{34}$$

as $W \to \infty$. This result exhibits the same energy= and R-dependence as the usual ("covariant") SM; the Fermi version with Lorentz-contraction gives $W^{1/2}$ instead. In the MPM the multiplicity increases logarithmically with energy. Present experimental data make it diffucult to exclude any of these predictions, although Ref. 18d) finds in p-$\bar{p}$ annihilation good agreement with a $W^{2/3}$ dependence.

The total inelastic cross-section

$$\sigma_{in}(W) = \sum_{N=3}^{\infty} \sigma_N(W)$$

$$= \frac{4\pi}{W^2} \int_0^{\infty} d\ell^2 e^{-2\ell^2/W^2R^2} \left[1 - \frac{\Omega_2(\ell^2,W)}{\sum_{N=2}^{\infty} \Omega_N(\ell^2,W)}\right] \tag{35}$$

is found to become asymptotically

$$\sigma_{in}^{as}(W) = 2\pi R^2 \left[1 - O(W^{-3})\right] \tag{36}$$

which is the familiar result from the optical model, since here $\sqrt{2}\,R$ is the effective radius. The MPM also gives a constant high energy value for σ_{in}, however with correction terms $O(1/\log W)$.

Our model as well as the usual SM, in contrast to the MPM, do not allow the possibility of outgoing particles "remembering" their origin as incoming - since the incoming particles are assumed to give up their character as such in the formation of a fireball with phase space governed decay. Hence these models by construction always lead to an asymptotic inelasticity (the fraction of available energy going into production) of 100 %, while according to cosmic ray evidence it seems to be less than 50 % even at highest energies[8,19]. This has led to proposals[19] that the incoming particles only give up part of their kinetic energy to cause fireball formation, while they themselves do not "submerge" in the fireball.

Finally a word about single-particle spectra of the emitted particles. The introduction of angular momentum leads to non-isotropic CMS angular distributions[10], though by incoherence arguments forward-backward symmetry relative to the beam axis is retained. Since the angular momentum $\vec{\ell} \neq 0$ is perpendicular to the beam axis and since due fo our form of fireball formation distribution over ρ higher angular momenta do contribute to the production process, the emitted particles will favor a certain "bundeling" around the beam axis, thus giving a restriction on transverse momenta of secondaries.

B. Elastic Scattering:

Elastic scattering is in our model governed by the unitarity relation (21) with (15/19)

$$|s(\ell,W)|^2 = \left[\frac{\Omega_2(\ell^2,W)}{\sum\limits_{N=2}^{\infty} \Omega_N(\ell^2,W)}\right] e^{-2\ell^2/W^2R^2} + \left[1-e^{-2\ell^2/W^2R^2}\right] \qquad (37)$$

Here the first term corresponds to "compound elastic" reactions via fireball, the second to shadow scattering. At high energy and not too large values of the impact parameter the large number of inelastic channels almost totally suppress the decay of the fireball into two particles; the first term then becomes vanishingly small and only shadow scattering remains.

Consider now the partial wave amplitude

$$t(\ell,W) =$$
$$\frac{i}{2}\left[1 - \sqrt{1-e^{-2\ell^2/W^2R^2\left(1-\Omega_2(\ell^2,W)/\sum_{N=2}^{\infty}\Omega_N(\ell^2,W)\right)}}\,e^{2i\delta_\ell(W)}\right] \quad (38)$$

In a large range of ℓ (ℓ < WR/4) the value of the S-matrix element (the square root in (38)) is practically zero, so that $t(\ell,W)=i/2$, independent of the phase. For very large ℓ causality requires $\delta_\ell(W)$ to be zero: since at large distances there should be no more interaction, the magnitude of $s(\ell,W)$ should approach unity (as in fact it does in our model) and the phase should vanish. We shall now assume that in the region $\ell \gtrsim WR/4$ the phase has already become so small that we may approximate it by zero+). We then obtain that the scattering amplitude

$$T(W,\cos\theta) = \frac{2W}{\pi K}\sum_{e=0}^{\infty}(2\ell+1)\;P_\ell(\cos\theta)\;t(\ell,W) \quad (39)$$

is purely imaginary at high energy.

From (38/39) and the optical theorem we then have for the total cross-section

$$\sigma_{tot}(W) =$$
$$\frac{8\pi}{W^2}\int_0^{\infty}d\ell^2\left[1-\left(1-e^{-2\ell^2/W^2R^2\left\{1-\Omega_2(\ell^2,W)/\sum_{N=2}^{\infty}\Omega_N(\ell^2,W)\right\}}\right)^{1/2}\right] \quad (40)$$

which with $W \to \infty$ yields

$$\sigma_{tot}^{as}(W) = 4\pi R^2 \times 2(1-\log 2) = .614 \times 4\pi R^2 \quad (41)$$

With (36) this gives

$$\sigma_{el}^{as}(W) = .228 \times 2\pi R^2 \quad (42)$$

+): The behaviour of large angle (~90°) elastic scattering will presumably depend quite crucially on the validity of this assumption.

and hence

$$\sigma_{el}^{as}/\sigma_{tot}^{as} = .185 \tag{43}$$

The numerical values of (41)-(43) are determined essentially by the fireball formation distribution $\exp-(2\ell^2/W^2R^2)$, which in turn is given us from two-particle phase space with Gaussian interaction volume cut-off.

The high energy limits of σ_{el} and σ_{tot} thus found are the same as those obtained from Van Hove's uncorrelated particle model[20,21]. While Van Hove only made use of some quite general assumptions about inelastic reaction (primarily uncorrelatedness of secondaries) to obtain a Gaussian form in ℓ/KR for the inelastic contribution to unitarity ("overlap function"), we have considered here a specific model for the description of inelastic processes, based on the statistical picture; this model, in accord with Van Hove's more general results, also leads to a Gaussian partial wave overlap function at high energies, and hence to the same asymptotic form for elastic and total cross-sections. We can, however, because of our specific "Ansatz" make definite predictions e.g. for $\sigma_N(W)$ or for branching ratios, which are left open in Van Hove's treatment; these predictions should be compared in form and parameter values to the experimental data for inelastic reactions, which then leaves us with definite quantitative results for elastic scattering.

The MPM also yields a constant total cross-section, but predicts that σ_{el} and hence σ_{el}/σ_{tot} vanish logarithmically at high energies. At present energies there seems to be very little experimental evidence for such a behaviour.

For small angle elastic scattering (diffraction peak) we have, with the Bessel function approximation of the Legendre polynomials,

$$\frac{d\sigma_{el}}{dt} = \frac{4\pi}{W^2} \left| \int_0^\infty d\ell^2 \; J_o\left(\frac{2\ell}{W}\sqrt{-t}\right) \left[1 - |s(\ell,W)|\right] \right|^2 \; ; \qquad t = -2K^2(1-\cos\theta) \tag{44}$$

which upon expansion of the square root gives, as in Van Hove's treatment,

$$\frac{d\sigma_{el}}{dt} = \left[\frac{d\sigma_{el}}{dt}\right]_{t=o} e^{-.92R^2|t| + .0095 R^4 t^2} \tag{45}$$

The MPM gives with $\exp(a \times t \times \log s)$ the well-known Regge shrinkage, which for elastic reactions at present energies does not seem to occur.

Our results on diffraction and σ_{el}, σ_{tot} are, as known from Van Hove's and subsequent work[21,22], quite compatible with experiments on high energy p-p and π-p elastic scattering, provided a suitable R ($\sim 1/2m_\pi$) is chosen. The essential question here is to what extent such a value of R is also compatible with the one needed for a good quantitative description of inelastic processes. Since both production and elastic scattering predictions depend quite sensitively on the precise value of R, this question can be settled only by detailed numerical comparison with specific experiments.

IV. Concluding Remarks

We have shown that a statistical model at fixed angular momentum or impact parameter provides a scheme leading to definite predictions for all inelastic and coupled absorptive elastic reactions. The qualitative behaviour of these predictions is quite compatible with experiment as far as integral inelastic quantities (σ_N, $Q^{as}(N/\bar{N})$, $\langle N \rangle$) are concerned; the elastic predictions coincide with the (at least qualitatively) quite well confirmed results of the Van Hove model. In the sense of "bracketing the correct description between two extremes"[1] we find a remarkable similarity between our results for inelastic quantitities and those of the multiperipheral model, giving perhaps an indication of some quite general multiparticle features. Not obtainable in our model are an asymptotic inelasticity less than 100 % and forward-backward anisotropy in secondary distributions. Both are due to non-statistical mechanisms and thus point to extensions of the model in the direction of including partial transmission of "memory" from initial to final state (as e.g. asummed in some multi-fireball models[19]).

We have in all considerations neglected finite and different masses, isospins, etc., and all quantum effects except for the factor $q^N/N!$. While we believe that quantum effects will not greatly alter our results[23], masses and isospins will of course have to be included in actual calculations to be quantitatively compard with experiments (in progress).

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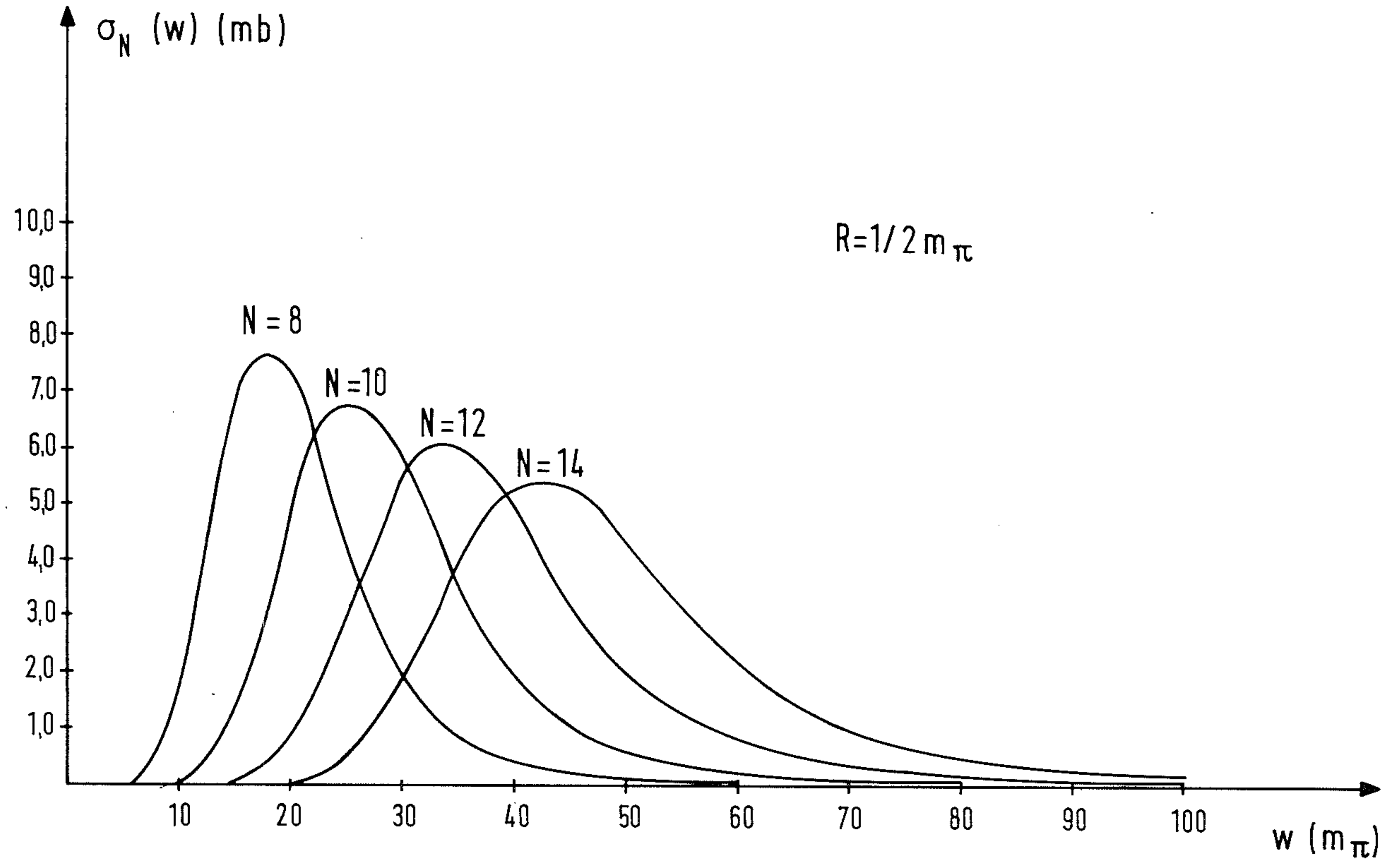
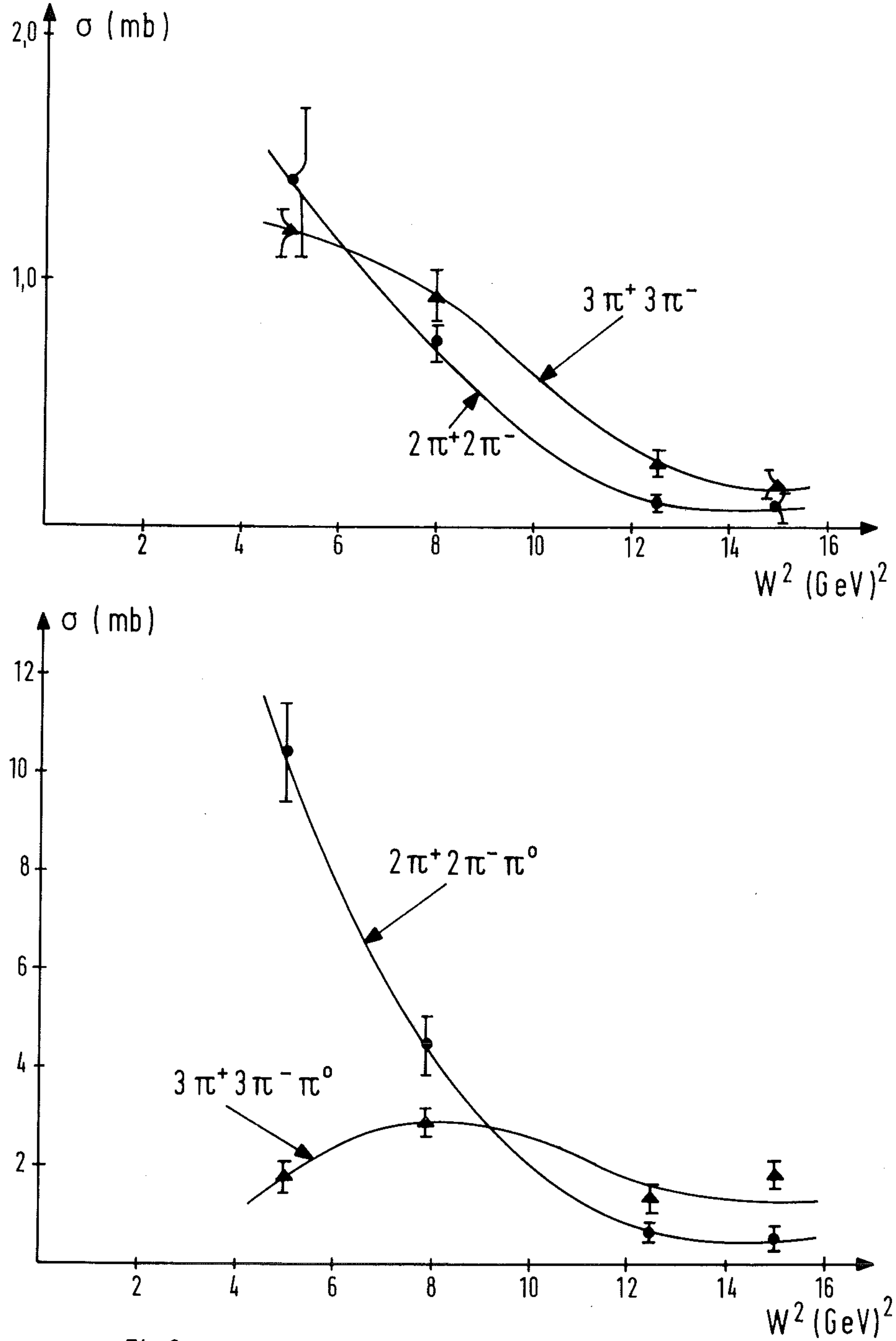

σN (w) (mb)
10,0
9,0
8,0
7,0
6,0
5,0
4,0
3,0
2,0
1,0
R=1/2 mπ
N = 8
N =10
N =12
N = 14
10
20
30
40
50
60
70
80
90
100
w (mπ)

Fig. 1



σ (mb)
2,0
1,0
$3\pi^+3\pi^-$
$2\pi^+2\pi^-$
2
4
6
8
10
12
14
16
W^2 (GeV)^2
σ (mb)
12
10
8
6
4
2
$2\pi^+2\pi^-\pi^o$
$3\pi^+3\pi^-\pi^o$
2
4
6
8
10
12
14
16
W^2 (GeV)^2

Fig. 2

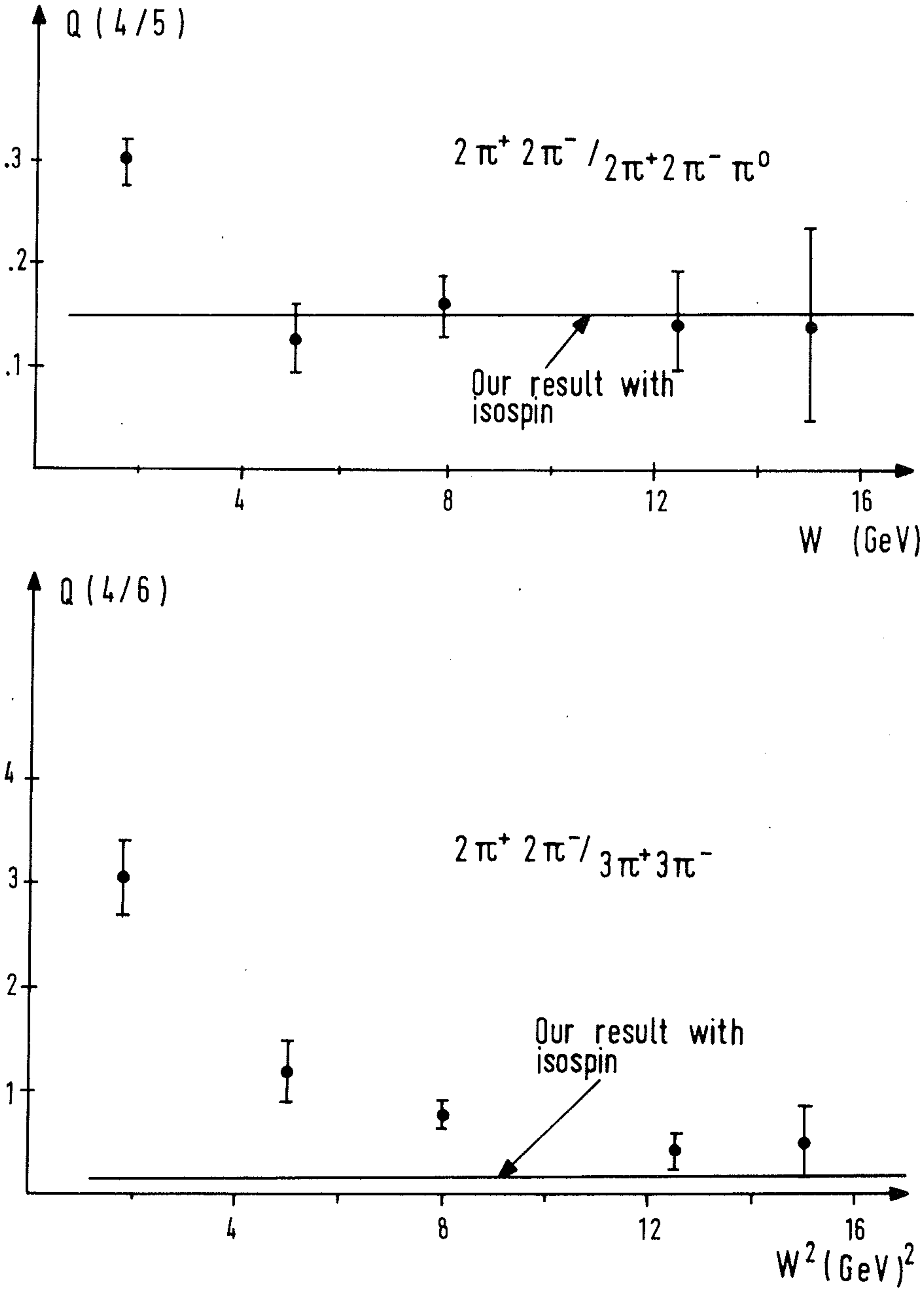

Q (4 / 5)
.3
.2
.1
2π⁺ 2π⁻ / 2π⁺ 2π⁻ π⁰
Our result with isospin
4
8
12
16
W (GeV)
Q (4 / 6)
4
3
2
1
2π⁺ 2π⁻ / 3π⁺ 3π⁻
Our result with isospin
4
8
12
16
W² (GeV)²

Fig. 3