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DETERMINATION OF THE ELLIPTICITY OF LIGHT AND OF OPTICAL CONSTANTS BY USE OF TWO REFLECTION POLARIZERS

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A method is described which allows to determine the ellipticity of light (i.e. the degree of polarization as well as the phase difference of the mutually perpendicular electric field components) by intensity measurements behind two reflection polarizers. The method yields simultaneously the complex reflection coefficient of the first polarizer, and thus its optical constants if it consists of only one mirror. The method is especially suited for the extreme ultraviolet where neither transmission polarizers nor simple phase shifters are available.

1. Introduction

For optical experiments it is of great importance to analyse the polarization of light before and after interaction with the sample under investigation. The experimentalist is interested in measuring the degree of polarization $P = \left| |\tilde{E}_x|^2 - |\tilde{E}_y|^2 \right| (|\tilde{E}_x|^2 + |\tilde{E}_y|^2)^{-1}$ as well as the phase difference between the mutually perpendicular electric field components $\tilde{E}_x$ and $\tilde{E}_y$, that means the ellipticity of light.

The ellipticity of visible and near ultraviolet radiation can easily be determined with transmission polarizers and phase shifters, for example $\lambda/4$-plates. In the vacuum ultraviolet (vuv) a lack of transmitting materials complicates this task. In order to produce polarized light or analyse the polarization of light in this spectral range one has to use reflection polarizers.

Hamm, MacRae, and Arakawa[1] have proposed a method for the determination of the degree of polarization with two reflection polarizers which are rotated about the axis of the beam. Intensity measurements behind the polarizers at four special positions yield the degree of polarization together with the ratio of the reflectivity for parallel (p) and perpendicular (s) polarized light for both polarizers. This ratio does not necessarily have to be known before the experiment and Abelès' relation is not used; the use of this relation is necessary if the degree of polarization using only one mirror[2,3] be determined. Surface layers on the mirrors do not affect the practicability of the method of Hamm et al.

Rosenbaum and coworkers[4] have used the method of Hamm et al. to determine the degree of polarization of extreme ultraviolet light emerging from a monochromator with synchrotron radiation as the light source. They applied two reflection polarizers, each consisting of four mirrors, in order to avoid rotation of the beam profile with respect to the detector which would have caused errors due to inhomogeneities in the cathode sensitivity.

In this work we would like to describe a method of determining both the degree of polarization and the phase difference between the mutually perpendicular electric field components by intensity measurements behind two reflection polarizers. The method requires two polarizers mounted on angle dividers as in the method of Hamm et al. If a detector with spatially homogeneous sensitivity is at hand a polarizer can, in principle, consist of one mirror under oblique incidence. Its actual properties do not have to be known in advance; they are determined in the experiment. The method permits the evaluation of the complex ratio of the p- and s-reflection coefficient for the first polarizer and from this its optical constants in a way well known from ellipsometry[5].

2. The formula for the intensity behind two reflection polarizers

In this section we will derive a general formula to calculate the intensity behind two reflection polarizers for any rotation angle of the polarizers, if the polarization of the incident beam and the reflection properties of the polarizers be known. The experimental arrangement is shown in Fig. 1: Radiation of a fixed wavelength with the electric field components $\tilde{E}_x$ and $\tilde{E}_y$[6] is incident on two successive reflection polarizers P_1 and P_2 mounted on angle dividers, followed by a detector D. The rectangular right-handed co-ordinate system x, y, z is fixed to the laboratory, the x_1, y_1, z_1- and x_2, y_2, z_2-systems to the polarizers P_1 and P_2; z, z_1 and z_2 are the directions of light propagation not necessarily parallel to each other. The first and second polarizer can be rotated together about the z-axis. This angle of rotation is denoted by ψ. The second polarizer can be rotated with respect to the first about the z_1-axis. The angle of this rotation is ϕ (cf. Fig. 1). Each polarizer may consist of one or more mirrors. Their planes of incidence are assumed to all lie in the y_1-z_1-plane and the y_2-z_2-plane for P_1 and P_2 respectively. The reflection of the first and second polarizer are characterized by generalized complex reflection coefficients $\tilde{r}_{s1}$, $\tilde{r}_{p1}$, $\tilde{r}_{s2}$ and $\tilde{r}_{p2}$ for perpendicular (s) and parallel (p) polarized light. The ratios $\tilde{r}_{p1}/\tilde{r}_{s1}$ and $\tilde{r}_{p2}/\tilde{r}_{s2}$ are defined as $\tilde{\rho}_1$ and $\tilde{\rho}_2$, respectively. In the special case that a polarizer consists of n mirrors with the same angle of incidence $\tilde{\rho}_i = \tilde{\rho}^n$ (i=1,2), where $\tilde{\rho}$ is the reflection coefficient of a single mirror.

The intensity $|\tilde{E}_{x_2}|^2+|\tilde{E}_{y_2}|^2$ behind the polarizers as a function of ψ and ϕ can be calculated after determination of the field components $\tilde{E}_{x_2}$ and $\tilde{E}_{y_2}$ in the x_2, y_2-system using matrix formalism:

$$\begin{pmatrix} \tilde{E}_{x_2} \\ \tilde{E}_{y_2} \end{pmatrix} = \tilde{r}_{s_1} \tilde{r}_{s_2} \begin{pmatrix} 1 & 0 \\ 0 & \tilde{\rho}_2 \end{pmatrix} \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & \tilde{\rho}_1 \end{pmatrix} \begin{pmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} \tilde{E}_x \\ \tilde{E}_y \end{pmatrix} \qquad (1)$$

From the right to the left the matrices in Eq. (1) correspond to a rotation by ψ, a reflection at the first polarizer, a rotation by ϕ and a reflection at the second polarizer.

In general, amplitude and phase of the electric field vectors $\tilde{E}_x$ and $\tilde{E}_y$ vary over the cross section F of the incident beam. Evaluation of Eq. (1) and integration over the cross section yield the intensity J recorded by the detector:

$$J_{\psi,\phi} = \int_F \left(|\tilde{E}_{x_2}|^2 + |\tilde{E}_{y_2}|^2\right) dF = K \sum_{i=1}^{9} A_i B_i C_i \qquad (2)$$

where

$$\begin{array}{lll}
A_1=\cos^2\psi\ \cos^2\phi & B_1=1+U|\tilde{\rho}_1|^2|\tilde{\rho}_2|^2 & C_1=C_2=C_3=C_4=1 \\
A_2=\sin^2\psi\ \cos^2\phi & B_2=U+|\tilde{\rho}_1|^2|\tilde{\rho}_2|^2 & \\
A_3=\cos^2\psi\ \sin^2\phi & B_3=|\tilde{\rho}_2|^2+U|\tilde{\rho}_1|^2 & \\
A_4=\sin^2\psi\ \sin^2\phi & B_4=|\tilde{\rho}_1|^2+U|\tilde{\rho}_2|^2 & \\
A_5=\sin2\psi\ \sin2\phi & B_5=\frac{1}{2}|\tilde{\rho}_1|(|\tilde{\rho}_2|^2-1)(1-U) & C_5=\cos\delta_1 \\
A_6=\sin2\psi\ \cos^2\phi & B_6=1-|\tilde{\rho}_1|^2|\tilde{\rho}_2|^2 & C_6=C_7=V \\
A_7=\sin2\psi\ \sin^2\phi & B_7=|\tilde{\rho}_2|^2-|\tilde{\rho}_1|^2 & \\
A_8=\cos2\psi\ \sin2\phi & B_8=|\tilde{\rho}_1|(1-|\tilde{\rho}_2|^2) & C_8=V\ \cos\delta_1 \\
A_9=\sin2\phi & B_9=|\tilde{\rho}_1|(|\tilde{\rho}_2|^2-1) & C_9=W\ \sin\delta_1
\end{array}$$

and

$$N = \int_F |\tilde{E}_x|^2 \, dF \ , \quad U = N^{-1} \int_F |\tilde{E}_y|^2 \, dF$$

$$V = N^{-1} \int_F |\tilde{E}_x| |\tilde{E}_y| \cos\theta \, dF \quad , \quad W = N^{-1} \int_F |\tilde{E}_x| |\tilde{E}_y| \sin\theta \, dF$$

and the common factor

$$K = N |\tilde{r}_{s_1} \tilde{r}_{s_2}|^2$$

We have assumed that at a general point (x,y) of the cross section $\tilde{E}_x = |\tilde{E}_x|(x,y)\ e^{i\theta_1(x,y)}, \tilde{E}_y = |\tilde{E}_y|(x,y)\ e^{i\theta_2(x,y)}$ so that $\theta(x,y) = \theta_2 - \theta_1$ represents the phase difference between $\tilde{E}_x$ and $\tilde{E}_y$. The phase difference δ_1 between the reflectivity for p and s light for the first polarizer, is defined by $\tilde{\rho}_1 = |\tilde{\rho}_1| e^{i\delta_1}$. The A_i's depend only on the rotation angles ψ and ϕ, the B_i's on $|\tilde{\rho}_1|, |\tilde{\rho}_2|$ and U and the C_i's are determined by V, W, $\cos\delta_1$ and $\sin\delta_1$ including the phase angles θ and δ_1.

Apart from the common factor K and the experimental parameters ψ and ϕ the intensity J depends on U,V,W characterizing the ellipticity of the incident radiation as well as on the properties of the polarizers $|\tilde{\rho}_1|, |\tilde{\rho}_2|$ and δ_1:

$$J_{\psi,\phi} = Kf(\psi,\phi,\ U,V,W,\ |\tilde{\rho}_1|, |\tilde{\rho}_2|,\ \delta_1) \qquad (2a)$$

Equation (2) shows the dependence of the intensity on the six quantities U, V, W, $|\tilde{\rho}_1|$, $|\tilde{\rho}_2|$ and δ_1. It will be used in sections

4 - 6 to determine these quantities by variation of ψ and ϕ. This can be achieved by measurement of the intensity for at least seven different and independent (ψ,ϕ)-pairs. One pair is needed for normalization to eliminate the seventh unknown factor K.

3. Special cases of the intensity formula

In this section we will illustrate Eq. (2) by considering some special cases. Equation (2) can be used as long as two successive optical units, P_1 and P_2, can be specified by complex transmission coefficients $\tilde{\rho}_1$ and $\tilde{\rho}_2$. P_1 and P_2 have not necessarily to be reflection polarizers. For general transmission polarizers and phase shifters for instance Eq. (2) holds as well.

a) homogeneous beam

The meaning of the quantities U, V, W can be best demonstrated in the case of a homogeneous beam, i.e. $|\tilde{E}_x|, |\tilde{E}_y|$ and θ do not depend on the co-ordinate (x,y) in the beam. In this case the expressions for U, V, W are reduced to:

$$U = |\tilde{E}_x|^{-2}\ |\tilde{E}_y|^2$$

$$V = |\tilde{E}_x|^{-1}\ |\tilde{E}_y|\ \cos\theta = \sqrt{U}\ \cos\theta$$

$$W = |E_x|^{-1}\ |\tilde{E}_y|\ \sin\theta = \sqrt{U}\ \sin\theta$$

Knowledge of the quantities U, V, W is equivalent to the knowledge of the ellipticity of the light, that is the position of the oscillation ellipse with respect to the laboratory system, the ratio of the lengths of the main axes, and the sense of rotation of the electric vector (cf. ref. 8). θ can be assumed to lie in the interval $0 \leq \theta < 2\pi$. If $\sin\theta > 0$ (<0) the electric vector rotates performing a left (right) - handed helix with respect to the direction of propagation. For $\theta = 0, \pi$ the light is linearly polarized; for $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ the light is circularly polarized if U = 1. Since in most of the practical cases the beam will be inhomogeneous, the more general integrals U, V, W have to be used to specify the ellipticity.

b) linear polarization ($\tilde{E}_y = 0$)

If only the $\tilde{E}_x$ component is present, U=V=W=0 and the degree of polarization $P = |1-U|(1+U)^{-1} = 1$. The factors B_1, B_2, B_3 and B_4 consist of only one term and C_6, C_7, C_8 and C_9 vanish. Equation (2) is considerably simplified to a form which has been used by the authors[9] to determine $|\tilde{\rho}_1|$ and δ_1 and the optical constants of gold and glass which served as a first polarizer in the extreme ultraviolet (xuv), see section 6.

c) synchrotron radiation

It has been predicted by the theory of synchrotron radiation[10,11] and verified experimentally in the visible[12] that circulating relativistic electrons emit elliptically polarized light in a narrow cone around their velocity vector. The state of polarization depends on the elevation angle χ between the electron orbit and the observer. The main component $\tilde{E}_x$ oscillates parallel to the synchrotron plane. For $\chi = 0$ only this component is present, so that for $\chi \to 0$ U,V,W $\to 0$. With increasing elevation angle the y-component $\tilde{E}_y$ increases. $|\tilde{E}_x|$ and $|\tilde{E}_y|$ only depend on the absolute value $|\chi|$. The phase difference θ is $\pm\,\pi/2$ depending on observation above or below the synchrotron plane. $\theta = \pm\,\pi/2$ means that V becomes zero. If the light is incident symmetrically to the synchrotron plane the contribution to W from the upper part of the beam is cancelled by the same contribution of the corresponding lower part since $\sin(\pi/2) + \sin(-\frac{\pi}{2}) = 0$. Thus W vanishes together with V and so do the terms C_6, C_7, C_8, and C_9 of Eq. (2).

d) perfect polarizers

Perfect polarizers which are available for the visible and near ultraviolet, for example a Nicol prism or a lossless material at Brewster's angle, are characterized by $|\tilde{\rho}_i| = 0$. If both polarizers are perfect Eq. (2) yields $J = K \cos^2\phi(\cos^2\psi + U \sin^2\psi + V \sin 2\psi)$ by which U and V can be determined using different sets of (ψ,ϕ), cf. section 4. The $\cos^2\phi$ - dependence is well known.

If only the second polarizer is perfect $|\tilde{\rho}_2| = 0$, but $|\tilde{\rho}_1| \neq 0$, and if the light is totally polarized ($\tilde{E}_y = 0$) Eq. (2) yields:

$$J = K (\cos^2\psi \cos^2\phi + |\tilde{\rho}_1|^2 \sin^2\psi \sin^2\phi - \frac{1}{2} |\tilde{\rho}_1| \cos\delta_1 \sin 2\psi \sin 2\phi).$$

The last formula is equivalent to that given by Conn and Eaton[7] for determination of $|\tilde{\rho}_1|$ and $\cos\delta_1$.

e) quarter wave plate and perfect polarizer

A quarter wave plate with a subsequent perfect polarizer is frequently used to determine the ellipticity of light in the visible. In terms of the variables of Eq. (2) the quarter wave plate is described by $\tilde{\rho}_1 = e^{\pm i\pi/2}$ and the polarizer by $\tilde{\rho}_2 = 0$. The condition for zero intensity specifying the ellipticity can easily be calculated from Eq. (2).

4. Determination of U, V, W, $|\tilde{\rho}_1|$, $|\tilde{\rho}_2|$ and δ_1 by intensity ratios

We now turn to the problem of how to determine U, V, W, $|\tilde{\rho}_1|$, $|\tilde{\rho}_2|$ and δ_1 with the help of Eq. (2). The first step is to calculate U, $|\tilde{\rho}_1|$ and $|\tilde{\rho}_2|$. These quantities can be obtained by four intensity measurements. For $\psi = 0, \pi/2$ and $\phi = 0, \pi/2$ Eq. (2) yields the following, since $A_i = 0$ for i=5-9:

$$\begin{aligned} J_{o,o} &= K\ (1+U|\tilde{\rho}_1|^2|\tilde{\rho}_2|^2) \\ J_{\pi/2,o} &= K\ (U+|\tilde{\rho}_1|^2|\tilde{\rho}_2|^2) \\ J_{o,\pi/2} &= K\ (|\tilde{\rho}_2|^2+U|\tilde{\rho}_1|^2) \\ J_{\pi/2,\pi/2} &= K\ (|\tilde{\rho}_1|^2+U|\tilde{\rho}_2|^2) \end{aligned} \tag{3}$$

From these four intensities U, $|\tilde{\rho}_1|$ and $|\tilde{\rho}_2|$ can be determined by three intensity ratios, for example

$$\begin{aligned} R_1 &\equiv J_{o,o}^{-1}\ J_{\pi/2,o} = (U+|\tilde{\rho}_1|^2|\tilde{\rho}_2|^2)(1+U|\tilde{\rho}_1|^2|\tilde{\rho}_2|^2)^{-1} \\ R_2 &\equiv J_{o,o}^{-1}\ J_{o,\pi/2} = (|\tilde{\rho}_2|^2+U|\tilde{\rho}_1|^2)(1+U|\tilde{\rho}_1|^2|\tilde{\rho}_2|^2)^{-1} \\ R_3 &\equiv J_{o,o}^{-1}\ J_{\pi/2,\pi/2} = (|\tilde{\rho}_1|^2+U|\tilde{\rho}_2|^2)(1+U|\tilde{\rho}_1|^2|\tilde{\rho}_2|^2)^{-1} \end{aligned} \tag{4}$$

Here $J_{o,o}$ is used for normalization. Equation (4) can be solved for U, $|\tilde{\rho}_1|$ and $|\tilde{\rho}_2|$:

$$\begin{aligned} U &= A\pm(A^2-1)^{1/2} \\ |\tilde{\rho}_1|^2 &= a\pm(a^2-1)^{1/2} \\ |\tilde{\rho}_2|^2 &= \bar{a}\pm(\bar{a}^2-1)^{1/2} \end{aligned} \tag{5}$$

where

$$\begin{aligned} A &= \tfrac{1}{2}(1+R_1^2-R_2^2-R_3^2)\ \ (R_1-R_2R_3)^{-1} \\ a &= \tfrac{1}{2}(1-R_1^2-R_2^2+R_3^2)\ \ (R_3-R_1R_2)^{-1} \\ \bar{a} &= \tfrac{1}{2}(1-R_1^2+R_2^2-R_3^2)\ \ (R_2-R_1R_3)^{-1} \end{aligned}$$

The procedure described in Eqs. (3), (4), (5) for determination of U has been proposed by Hamm et al.[1] It was used by Rosenbaum et al.[4] to study the degree of polarization of extreme ultraviolet synchrotron radiation.

The next step is to determine the other unknowns V, W, $\cos\delta_1$ and $\sin\delta_1$, which include the phase factors θ and δ_1. This can be done, for example, as follows:
We assume that U, $|\tilde{\rho}_1|$ and $|\tilde{\rho}_2|$ have been calculated with Eq. (5). Then V can be evaluated from

$$J_{\psi,o} = K\{(1+U|\tilde{\rho}_1|^2|\tilde{\rho}_2|^2)\cos^2\psi + (U+|\tilde{\rho}_1|^2|\tilde{\rho}_2|^2)\sin^2\psi + V(1-|\tilde{\rho}_1|^2|\tilde{\rho}_2|^2)\sin 2\psi\} \tag{6}$$

which yields for $\psi = \pi/4$

$$J_{\pi/4,o} = K\{\tfrac{1}{2}(|\tilde{\rho}_1|^2|\tilde{\rho}_2|^2+1)(1+U) + V(1-|\tilde{\rho}_1|^2|\tilde{\rho}_2|^2)\}$$

As before K is eliminated by normalizing with $J_{o,o}$. The expressions $J_{\psi,\pi/2}$ and $J_{\pi/4,\pi/2}$ yield V in a similar way.

The products $V\cos\delta_1$ and $W\sin\delta_1$ can be derived from $J_{o,\phi}$ and $J_{\pi/2,\phi}$:

$$J_{o,\phi} = K\{(1+U|\tilde{\rho}_1|^2|\tilde{\rho}_2|^2)\cos^2\phi + (|\tilde{\rho}_2|^2+U|\tilde{\rho}_1|^2)\sin^2\phi + (W\sin\delta_1 - V\cos\delta_1)\,|\tilde{\rho}_1|\,(|\tilde{\rho}_2|^2-1)\sin 2\phi\} \tag{7}$$

and

$$J_{\pi/2,\phi} = K\{(U+|\tilde{\rho}_1|^2|\tilde{\rho}_2|^2)\cos^2\phi + (|\tilde{\rho}_1|^2+U|\tilde{\rho}_2|^2)\sin^2\phi + (W\sin\delta_1 + V\cos\delta_1)\,|\tilde{\rho}_1|\,(|\tilde{\rho}_2|^2-1)\sin 2\phi\} \tag{7a}$$

$V \cos\delta_1$ is calculated by the difference $J_{o,\phi}-J_{\pi/2,\phi}$ and $W \sin\delta_1$ by $J_{o,\phi} + J_{\pi/2,\phi}$. It follows that V, $\cos\delta_1$ and $W \sin\delta_1$ are now known from the described procedure Eqs. (6), (7). If the first polarizer consists of only one mirror δ_1 can be assumed to lie between 0 and π, i.e. $0 \leq \delta_1 < \pi$ (or $- \pi < \delta \leq 0$ depending on the phase convention in the definition of $\tilde{\rho}_1$). δ_1 and W can then be calculated explicitly.

The derivation given above is only one of several possible ways of calculating the 6 unknown quantities. Values of the angles ψ and ϕ other than 0, $\pi/4$, and $\pi/2$ can be used. In general one could think about solving Eq. (2) for the unknowns by a least square fit with a computer. In such a fit the values obtained by the procedure described above can be used as first values in the iteration process.

5. The derivative of the intensity $J_{\psi,\phi}$ with respect to ϕ and the extrema of $J_{\psi,\phi}$

In our experiments (section 6) we have measured the intensity $J_{\psi,\phi}$ continuously varying the angle ϕ, while ψ was used as parameter. In order to study how these experimental curves are influenced by U, V, W, $|\tilde{\rho}_1|, |\tilde{\rho}_2|$ and δ_1, we differentiate Eq. (2) with respect to ϕ. This yields:

$$J' = \frac{\partial J_{\psi,\phi}}{\partial \phi} = K\ (|\tilde{\rho}_2|^2 - 1) \sum_{i=1}^{6} A_i' B_i' C_i' \qquad (8)$$

where K is the same factor as in Eq. (2) and

$A_1' = \cos^2\psi\ \sin 2\phi \qquad B_1' = 1 - U|\tilde{\rho}_1|^2 \qquad C_1' = C_2' = 1$

$A_2' = \sin^2\psi\ \sin 2\phi \qquad B_2' = U - |\tilde{\rho}_1|^2$

$A_3' = \sin 2\psi\ \cos 2\phi \qquad B_3' = |\tilde{\rho}_1|(1-U) \qquad C_3' = \cos\delta_1$

$A_4' = \sin 2\psi\ \sin 2\phi \qquad B_4' = |\tilde{\rho}_1|^2 + 1 \qquad C_4' = V$

$A_5' = \cos 2\psi\ \cos 2\phi \qquad B_5' = -2|\tilde{\rho}_1| \qquad C_5' = V \cos\delta_1$

$A_6' = \cos 2\phi \qquad B_6' = 2|\tilde{\rho}_1| \qquad C_6' = W \sin\delta_1$

Equation (8) may be used to evaluate the unknown quantities from the derivatives of the intensity for different (ψ,ϕ)-pairs in a way similar to that described in section 4 for the intensities themselves. Such a procedure will not be discussed here in detail.

The position ϕ_m of the extrema can be calculated explicitly from the condition $J'(\phi_m) = 0$ which gives:

$$\mathrm{tg}\ 2\phi_m = ZM^{-1} \equiv f_m(\psi, U, V, W, |\tilde{\rho}_1|, \delta_1) \qquad (9)$$

where

$$Z = |\tilde{\rho}_1| \ (U-1) \ \cos\delta_1 \ \sin 2\psi + 2|\tilde{\rho}_1| \ V \cos\delta_1 \ \cos 2\psi - 2|\tilde{\rho}_1| \ W \sin\delta_1$$

$$M = (1-U|\tilde{\rho}_1|^2) \ \cos^2\psi + (U-|\tilde{\rho}_1|^2) \ \sin^2\psi + (|\tilde{\rho}_1|^2+1) \ V \sin 2\psi$$

Equation (9) shows that ϕ_m is independent of the properties of the second polarizer. Equation (9) yields for $\psi = 0$ and $\pi/2$:

$$\begin{aligned} \psi = 0: \ \mathrm{tg}\ 2\phi_m &= 2|\tilde{\rho}_1|(1-U|\tilde{\rho}_1|^2)^{-1} \ (V \cos\delta_1 - W \sin\delta_1) \\ \psi = \tfrac{\pi}{2}: \ \mathrm{tg}\ 2\phi_m &= -2|\tilde{\rho}_1|(U-|\tilde{\rho}_1|^2)^{-1} \ (V \cos\delta_1 + W \sin\delta_1) \end{aligned} \qquad (10)$$

These equations demonstrate how the quantities $V \cos\delta_1$ and $W \sin\delta_1$ containing the phase factors θ and δ_1 affect the position of the maxima of $J_{\psi,\phi}$.

If we assume that U and $|\tilde{\rho}_1|$ are known, for example from Eqs. (3), (4), (5), $V \cos\delta_1$ and $W \sin\delta_1$ can be determined by addition and subtraction of the two equations (10) after ϕ_m has been measured for $\psi = 0$ and $\pi/2$. In a similar way $\cos\delta_1$ and V can then be derived from tg $2\phi_m$ for $\psi = \pm\pi/4$. Thus V, W and δ_1 are known. In principle one can calculate all unknowns U, V, W $|\tilde{\rho}_1|$ and δ_1 by a least square fit for different ψ's for which according to Eq. (9) ϕ_m has been measured. The advantage of using ϕ_m's at different ψ's as compared to the ratio method is that the ϕ_m - method only needs the correct ϕ_m from the measured $J(\phi)$-curve. ϕ_m is more easily obtained than correct intensity ratios when problems arise due to detector inhomogeneity with variation of ψ or long time instabilities of the light source.

Two special cases of Eq. (9) should be mentioned: For $\tilde{E}_y = 0$ (U=V=W=0) Eq. (9) reduces to

$$\mathrm{tg}\ 2\phi_m = 2|\tilde{\rho}_1|\ \cos\delta_1\ (|\tilde{\rho}_1|^2\ \mathrm{tg}\ \psi - \mathrm{ctg}\ \psi)^{-1} \qquad (11)$$

In the case of symmetric synchrotron radiation (cf. sec. 3) we obtain with V=W=0

$$\mathrm{tg}2\phi_m = 2|\tilde{\rho}_1|\ \cos\delta_1\ (U-1)\ \{(U-|\tilde{\rho}_1|^2)\,\mathrm{tg}\psi + (1-U|\tilde{\rho}_1|^2)\ \mathrm{ctg}\psi\}^{-1} \qquad (12)$$

6. Application. The influence of U, V and W on ϕ_m and determination of optical constants

Figure 2 shows a photograph of the double polarizer used for polarization studies in the extreme ultravilet below 1000 Å wavelength. It has all the degrees of freedom of the system sketched in Fig. 1. The system was developed as a part of an ultrahigh vacuum reflectometer described earlier[13]. The light is incident from the lower left through the ball-bearing. It falls on the first polarizer P_1, a single mirror, and is reflected towards a four mirror-polarizer P_2. Behind the latter a detector is mounted, e.g. an open magnetic photomultiplier Bendix M 306. Both polarizers and the multiplier are fastened to a ring which can be turned about the axis of the incident beam by means of two ball-bearings and a feedthrough attached to the second ball-bearing on the right side. The angle of incidence of the first polarizer and the adequate position of the second polarizer and detector can be adjusted from outside by two other feedthroughs which are coupled to the two gears on the ring seen in front of the photograph. The polarizers are rotated by motors. The positions of the rotation angles are indicated by built-in potentiometers operating under vacuum which can also be seen in the photograph. The equipment was used in a modified commercial vacuum system[13] at a pressure of 10^{-7} Torr behind the exit slit of a Wadsworth-monochromator[14]. Synchrotron radiation was used as light source.

From the angle positions $\psi,\phi = 0,\pi/2$ we found using Eqs. (3), (4), (5) that the light emerging from the monochromator had a degree of polarization P = 0.92 (U = 0.042) at λ = 700 Å. During the course of our experiments we extensively studied the relation for ϕ_m (Eq. (10)). We had adjusted the polarizer-system symmetrically with respect to the synchrotron plane so that V and W were expected to have no influence on ϕ_m, presuming the radiation at the exit slit behaved as predicted by theory (Eq. (12)). The first measurements of ϕ_m, however, gave for $\psi = 0$ and $\psi = \pi/2$ ϕ_m-values different from 0 or π. The conclusion was that the light did not behave as in the symmetric case of synchrotron radiation with V = W = 0. Obviously the grating of the monochromator had changed phase and intensity over the whole cross section of the beam. In a second experiment, during which the beam profile was measured, the intensity distribution was actually shown to be not synchrotron-radiation-like. From the position of ϕ_m for $\psi = 0, \pi/2$ we calculated V=-0.011 and W=0.112 by using Eq. (10) with previously determined values for $|\tilde{\rho}_1|$, δ_1 of glass at an angle of incidence 45° at λ = 700 Å, where U = 0.042.

By means of a prepolarizer consisting of 3 mirrors, which was inserted before the double polarizer system, the polarization was enhanced to about P = 0.996. Then within the accuracy (±1°) with which ϕ_m could be read, the ϕ_m's were at 0 and π as expected from Eq. (11). This equation was used to derive $|\tilde{\rho}_1|$ and δ_1 from the position of the extrema ϕ_m while ψ was varied between 0 and $\pi/2$.[9] The results for n and k, calculated from $|\tilde{\rho}_1|$ and δ_1 were in good agreement with the results of other authors who used the reflectance vs. angle of incidence method[15].

The determination of U, V, W $|\tilde{\rho}_1|$ and δ_1 by intensity ratios (Section 4) or derivatives (Section 5) works properly if a detector with a homogeneous cathode sensitivity is used. Inhomogeneous detectors might cause errors in the ratios due to rotation of the beam profile on the cathode when the angles ψ and ϕ are changed. If a four mirror polarizer is used, as we did for P_2, such effects are excluded. The ϕ_m-method (Section 5) has the advantage that no intensity ratios are needed, the position of the extrema can often be more safely determined experimentally.

7. Summary

A method was described which allows to determine the ellipticity of light by intensity measurements behind two reflection polarizers together with the optical constants of the first polarizer. The quantities U, V, and W characterizing the ellipticity of the incident light and $|\tilde{\rho}_1|$ and δ_1 characterizing the reflection properties of the first polarizer can be calculated from intensity measurements at different sets of the rotation angles of the polarizers. The unknowns can be derived from pure intensity measurements, from the derivative of the intensity with respect to the rotation angle of the second polarizer ϕ and by the position of the extrema ϕ_m.

The reflection coefficients of the polarizersdo not have to be known in advance. Surface layers on the mirrors of the polarizers do not affect the accuracy with which the ellipticity can be determined. They do, however, affect the values of $|\tilde{\rho}_1|$ and δ_1 so that the actual optical constants of the mirror material can not be calculated in such a case. The method described here does not require high quality reflection polarizers with $|\rho| \sim 0$, since the equations given above can be solved for the unknowns without this assumption. In principle one reflecting surface is sufficient for each polarizer so that the intensity of light which has passed the polarizers remains relatively high.

The method is useful in spectral ranges where no transmission polarizers are available, for example in the extreme ultraviolet. The double reflection - polarizer system corresponds to the combination of a phase shifter and a transmission polarizer

generally applied in the visible and near ultraviolet. The first reflection polarizer takes over the role of the phase shifter.

A double polarizer system suitable for the described analysis was constructed. As yet it has been applied in the extreme ultraviolet to study the influence of U, V, and W on the position of the extrema ϕ_m and to determine the optical constants of the material (gold and glass) used as the first polarizer.

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Figure Captions

Fig. 1 Sketch of the double polarizer system, P_1 first polarizer, P_2 second polarizer, D detector. The x,y,z-, x_1,y_1,z_1- and x_2,y_2,z_2-systems are fixed to the laboratory, P_1 and P_2 respectively, z,z_1,z_2 being the direction of light propagation and simultaneously the axes of rotation of P_1 and P_2. The y_1-z_1-plane and y_2-z_2-plane are the planes of incidence of the mirrors used in the construction of P_1 and P_2. The definition of the rotation angles ψ and ϕ is shown in the lower part.

Fig. 2 Photograph of the double polarizer system constructed for polarization studies in the extreme ultraviolet. Light is incident from the lower left through the ball-bearing. It is reflected at a plate (P_1) and passes through a four-mirror-polarizer (P_2) before being detected by a multiplier.

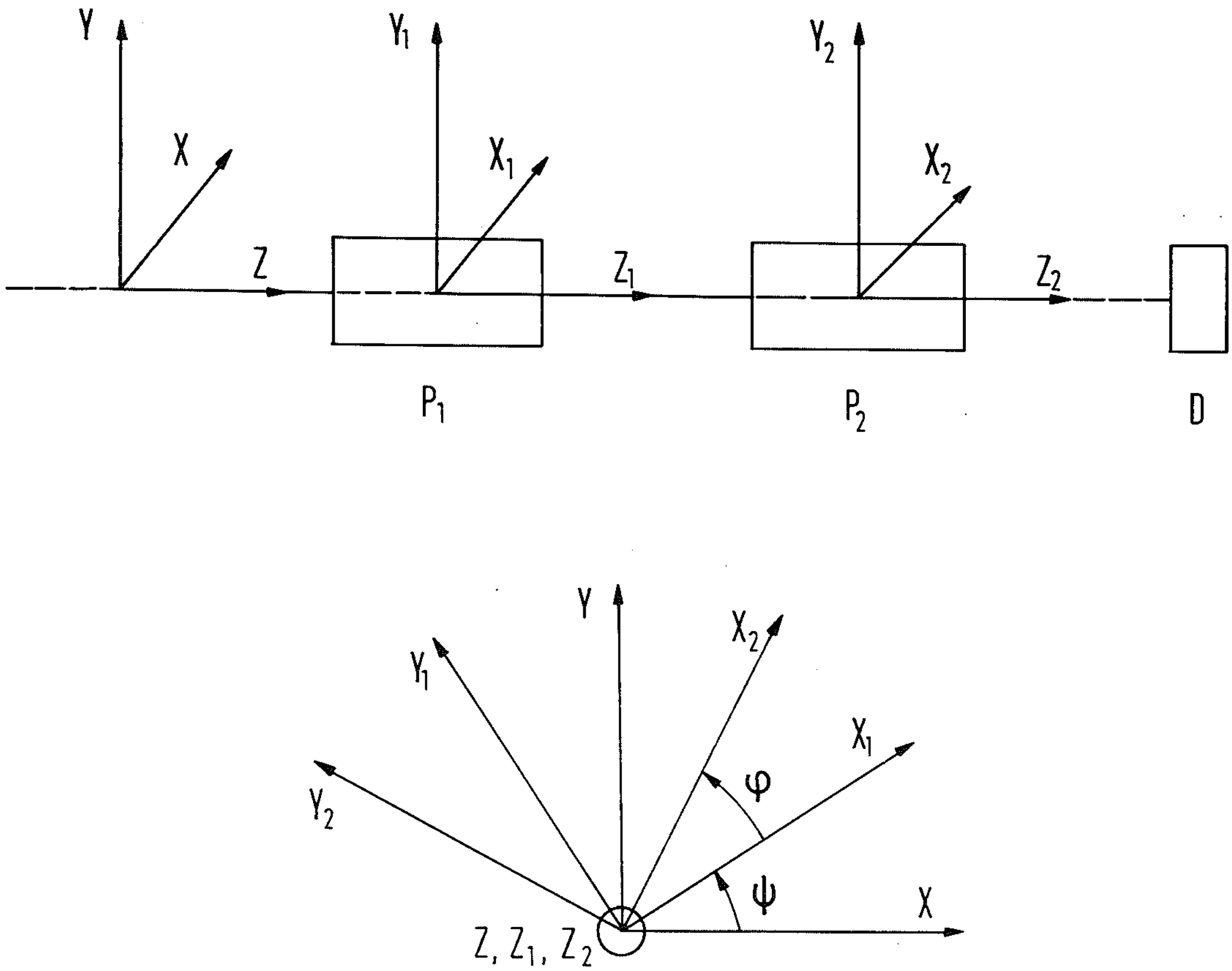

Fig. 1