DEUTSCHES ELEKTRONEN-SYNCHROTRON DESY

DESY 70/49
September 1970

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[+]Work supported in part by the Deutsche Forschungsgemeinschaft

Abstract

Nucleon-antinucleon ($N\bar{N}$) bound states are calculated in the framework of relativistic quantum mechanics. The potential is taken from a previous treatment of nucleon-nucleon (NN) interaction with parameters adjusted to elastic NN phase shifts and the deuteron. The OBE potential is given by the field theoretic Born terms of π , η , σ , ε , ρ and ω exchange contributions which are supplied with form factors. The NN and $N\bar{N}$ channels are related by crossing. In case of $N\bar{N}$ the π and ω exchange contribution to the potential change sign compared to the NN case. This way the ω exchange gives rise to a repulsive core for NN and to short-range attractive forces for $N\bar{N}$. In contrast to similar calculations by other authors we obtain the input mesons as bound states below 1000 MeV in the $N\bar{N}$ channels. In addition there are bound states, mostly between 1 GeV and $N\bar{N}$ threshold, corresponding to ϕ , ρ', δ , A_1 , A_2 , f , f' and a B for both $T = 1$ and $T = 0$. The uncoupled partial waves with $L = J$ contain at most one bound state for each isospin. The coupled $L = J \pm 1$ partial waves $^3S_1 - {}^3D_1$ and $^3P_2 - {}^3F_2$ contain two $T = 0$ bound states which reflect the mixed singlet-octet mesons ω , ϕ and f , f' . Most of the calculated bound states are close to experimentally observed mesons, but the bound states corresponding to the ω and the fictitious σ mesons appear at negative energies. The bound state energies are particularly sensitive to the choice of the ω and scalar meson coupling constants and to the form factor parameters.

I. Introduction

In 1949 Fermi and Yang [1] first proposed that the π meson is a nucleon-antinucleon $(N\bar{N})$ bound state bound by short-range vector and tensor forces. Later, in Heisenberg's nonlinear spinor theory [2] bosons were also constructed as $N\bar{N}$ states, and reasonable values for the masses and coupling constants of the π and η mesons were computed with only one free parameter. In the literature there are many other attemps to construct bosons from four-fermion interactions among which we only mention the work of Nambu and Jona-Lasinio [3].

We are here interested in the one-boson exchange (OBE) type models [4] which have been extensively studied for elastic NN interaction, but only scarcely for the related $N\bar{N}$ case. The connection between NN and $N\bar{N}$ interactions as well as the assumption that mesons are $N\bar{N}$ bound states is in the OBE model closely connected with crossing. The NN as well as $N\bar{N}$ forces are constructed from the crossed channel absorptive part and, in the simplest approximation, they are saturated by certain meson poles. If the dynamics is sufficiently described by the pole terms including certain form factors, $N\bar{N}$ bound states should appear at the input meson masses and their residues should coincide with the coupling constants. Quantitative OBE calculations for the $N\bar{N}$ case are first done by Ball, Scotti and Wong [5], who saturate partial wave dispersion relations by single meson poles and require an additional cutoff. For the parameters which were obtained from NN interactions, only the $N\bar{N}$ S-waves contain bound states, and except for one, all appear above 1.5 GeV. Recently, Dalkarov, Mandelzweig and Shapiro [6] obtained bound states in the same region for the S- P- and D-waves. They solved the Schrödinger equation for a non-relativistic OBE potential which was adapted to elastic NN interactions. The SU(3) symmetric baryon-antibaryon interaction including vector meson exchange contributions was qualitatively discussed by H. Sugawara and von Hippel [7] without going into detailed numerical calculations. None of the previous OBE model calculations were able to account for the low lying mesons in the $N\bar{N}$ channels. We present an improved relativistic OBE model which includes off-shell contributions. The model was previously developed by one of us [8],[9] to account for elastic NN interactions. Using the parameters obtained from fitting the NN phase shifts and the deuteron we calculate here the $N\bar{N}$ bound states.

In Section II we give the OBE potentials including the form factors, the eigenvalue equations and the method of computing the bound states numerically. In

Section III we present the results of the calculation and discuss the variation of the bound state energies for different NN adjustments. In addition, (in the figures) the essential input parameters were varied independently to show their influence on the calculated bound state energies. In Section IV the $N\bar{N}$ bound states are compared with the physical meson spectrum. In the last section we compare the results of calculations with previous models and we discuss limitations and possible extensions of our model towards a self consistent solution.

II. OBE Potential and Dynamical Equations

One may believe that the potential concept of relativistic quantum mechanics [10] provides an adequate approach to strong interaction physics.* For the case of the NN interaction an OBE model based on this approach has already been developed in Refs.[8],[9], whose results give a fairly good description of elastic NN interactions. The only thing that must be changed for the $N\bar{N}$ case is the sign of the π and ω exchange contributions to the potential.

The basic assumption inherent in the OBE model is that NN (as well as $N\bar{N}$) forces can be saturated by the exchange of elementary mesons. However, in order to account for the finite size of the nucleon and to avoid divergence difficulties additional form factors must be introduced. These form factors are assumed to be different for each meson exchange diagramm. They are expected to represent the correlated multimeson exchange contribution belonging to the respective channel.

For the case of NN as well as $N\bar{N}$ interactions the potentials are given by the field theoretic Born terms including the chosen form factors as follows:**

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* Relativistic quantum mechanics was also successfully applied to the multichannel cases of SU(3) symmetric meson-meson [11] and meson baryon [12] interactions.

** The notation follows Ref.[8].

(α) Scalar meson

$$v^{T}_{jj'_3,\,jj_3}(\vec{k}',\vec{k}) = a^{T}_{I}\cdot b_{G} \sum_{\bar{s}_1^3 \bar{s}_2^3 s_1^3 s_2^3} C^{\bar{s}_1^3 \bar{s}_2^3 s_1^3 s_2^3}_{jj'_3 jj_3} \{\tilde{w}^{\bar{s}_1^3}(\vec{k}')\, g_S G_S(\vec{k}',\vec{k})\, w^{s_1^3}(\vec{k})\} \tag{2.1}$$

$$\times \{\tilde{w}^{\bar{s}_2^3}(-\vec{k}')\, g_S G_S(\vec{k}',\vec{k})\, w^{s_2^3}(-\vec{k})\}\ \tilde{\Delta}_S(t)$$

(β) Pseudoscalar meson

$$v^{T}_{jj'_3,\,jj_3}(\vec{k}',\vec{k}) = a^{T}_{I}\cdot b_{G} \sum_{\bar{s}_1^3 \bar{s}_2^3 s_1^3 s_2^3} C^{\bar{s}_1^3 \bar{s}_2^3 s_1^3 s_2^3}_{jj'_3 jj_3} \{\tilde{w}^{\bar{s}_1^3}(\vec{k}')\, g_P G_P(\vec{k}',\vec{k})\, \gamma_5\, w^{s_1^3}(\vec{k})\} \tag{2.2}$$

$$\times \{\tilde{w}^{\bar{s}_2^3}(-\vec{k}')\, g_P G_P(\vec{k}',\vec{k})\, \gamma_5\, w^{s_2^3}(-\vec{k})\}\ \tilde{\Delta}_P(t)$$

(γ) Vector meson

$$v^{T}_{jj'_3,\,jj_3}(\vec{k}',\vec{k}) = a^{T}_{I}\cdot b_{G} \sum_{\bar{s}_1^3 \bar{s}_2^3 s_1^3 s_2^3} C^{\bar{s}_1^3 \bar{s}_2^3 s_1^3 s_2^3}_{jj'_3 jj_3}$$

$$\times \{\tilde{w}^{\bar{s}_1^3}(\vec{k}')\,(g_{V_1} G_{V_1}(\vec{k}',\vec{k})\,\gamma^{\mu} - g_{V_2} G_{V_2}(\vec{k}',\vec{k})\,\frac{1}{4m}[\gamma^{\mu},\gamma^{\nu}]\,\Delta^{+}_{\nu})\, w^{s_1^3}(\vec{k})\}$$

$$\times \{\tilde{w}^{\bar{s}_2^3}(-\vec{k}')\,(g_{V_1} G_{V_1}(\vec{k}',\vec{k})\,\gamma^{\sigma} - g_{V_2} G_{V_2}(\vec{k}',\vec{k})\,\frac{1}{4m}[\gamma^{\sigma},\gamma^{\rho}]\,\Delta^{-}_{\rho})\, w^{s_2^3}(-\vec{k})\} \tag{2.3}$$

$$\times \left(-g_{\mu\sigma} + \frac{1}{m_V^2}\,\Delta^{+}_{\mu}\Delta^{-}_{\sigma}\right)\ \tilde{\Delta}_V(t)\ ,$$

where

$$\tilde{\Delta}_{S,P,V}(t) = \frac{1}{t - m^2_{S,P,V}} \tag{2.4}$$

and I denotes the isospin of the exchanged meson. T denotes the isospin of the NN or $N\bar{N}$ system. The isospin factor a^T_I is given by $a^0_1 = -3$ and $a^T_I = 1$ elsewhere. G denotes the G-parity of the exchanged mesons with $b_G = 1$ for NN and $b_G = G$ for $N\bar{N}$ respectively.

Furthermore

$$\mathcal{C}^{\bar s_1^3 \bar s_2^3 s_1^3 s_2^3}_{j j_3' j j_3} = \frac{1}{(2\pi)^3}\,\frac{m^2}{(m^2+k'^2)^{\frac12}(m^2+k^2)^{\frac12}}\;\langle \tfrac12 \bar s_1^3 ; \tfrac12 \bar s_2^3 | j j_3' \rangle \langle \tfrac12 s_1^3 ; \tfrac12 s_2^3 | j j_3 \rangle ,$$

$$w^s(\vec k) = \left(\frac{(m^2+k^2)^{\frac12}+m}{2m}\right)^{\frac12}\left(1+\frac{\vec\alpha\,\vec k}{(m^2+k^2)^{\frac12}+m}\right)\chi^s , \quad \chi^{+\frac12} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}, \chi^{-\frac12} = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}, \tag{2.5}$$

$$\Delta^{\pm} = \left((m^2+k'^2)^{\frac12}, \pm\vec k'\right) - \left((m^2+k^2)^{\frac12}, \pm\vec k\right) , \quad t = (\Delta^{\pm})^2 .$$

The form factors are taken as in Ref.[8], where $G_{V1} = G_{V2} = G_V$ was assumed. Defining $F_{S,P,V} = G^2_{S,P,V}$ they are

$$F_{S,P}(\vec k', \vec k) = \frac{m^2_{S,P} - \Lambda_{S,P}}{t - \Lambda_{S,P}} \tag{2.6}$$

and

$$F_V(\vec k', \vec k) = \exp\{(\alpha_V(0)-1)\bar\xi_V\}\;\exp\{\alpha_V'(0)\,t\,\bar\xi_V\} , \tag{2.7}$$

where

$$\bar\xi_V = \operatorname{arc\,cosh}\sqrt{\frac{4(m^2+k'^2)}{4m^2-t_0}} + \operatorname{arc\,cosh}\sqrt{\frac{4(m^2+k^2)}{4m^2-t_0}} \tag{2.8}$$

and $t_0 = (2m_\pi)^2$ $(t_0 = (3m_\pi)^2)$ for $V \to \rho$ $(V \to \omega)$.

In the partial-wave representation the eigenvalue equations giving rise to bound states $|\vec{p}\, m_B J J_3\rangle$ of mass m_B and spin J are as follows:

$$(m_B - E_k)\,\chi(kJ[\ell j]) = \sum_{\ell'} \int dk'\, k'^2\, v^{J,T}_{\ell j,\ell' j}(k,k')\,\chi(k'J[\ell' j]) , \tag{2.9}$$

where we have written

$$\langle \vec{K}\, k\, J' J_3' [\ell j] | \vec{p}\, m_B J J_3 \rangle = 2 K_0^{\frac{1}{2}} p_0^{\frac{1}{2}}\, \delta(\vec{K}-\vec{p})\, \delta_{J'J}\, \delta_{J_3'J_3}\, \chi(kJ[\ell j]) . \tag{2.10}$$

Eq.(2.9) is equivalent to

$$\tilde{\chi}(kJ[\ell j]) = \sum_{\ell'} \int dk'\, k'^2\, K^{J,T}_{\ell j,\ell' j}(k,k')\, \tilde{\chi}(k'J[\ell' j]) , \tag{2.11}$$

$$K^{J,T}_{\ell j,\ell' j}(k,k') = -(E_k - m_B)^{-\frac{1}{2}}\, v^{J,T}_{\ell j,\ell' j}(k,k')\, (E_{k'} - m_B)^{-\frac{1}{2}} .$$

It can be shown that $K^{J,T}_{\ell j,\ell' j}(k,k')$ is of Hilbert-Schmidt type provided that form factors like (2.6) and (2.7) are assumed. Hence it is completely continuous and leads to a number of discrete eigenvalues depending on m_B and the potential parameters. The implicit eigenvalue equation (2.11) is solved by Gaussian quadrature and Jacobi's method.

III. Results of Calculation

The results of our calculations are presented in Tables I and II and in Figs.1-7. Table I contains the input parameters of the OBE model. Set A is taken from an adjustment of the deuteron and the elastic NN phase shifts which is described in Ref.[9]. Set B is a different set of parameters which also gives a comparatively good fit to the NN data. Set C corresponds to the parameters of Ref.[8], where only the proton-proton phase shifts were adapted. The calculated bound states of Sets A and C are rather close to each other except for those in the $^1S_0$ partial wave.

For Set A the $T = 0$ and $T = 1$ $^1S_0$ bound states appear roughly at about the mass of the π, while for Set B they both appear at about the mass of the η.

For Set C the mass of the $T = 0$ $^1S_0$ bound state becomes negative whereas the mass of the $T = 1$ $^1S_0$ bound state is shifted to 800 MeV. The lower $T = 0$ bound state in the partial waves $^3S_1 - {}^3D_1$ appears for Sets A-C at negative energies which means that the ω meson is too strongly bound. The same occurs for the lower $T = 0$ bound state in the partial wave $^3P_0$. The lower $T = 1$ bound state in the partial waves $^3S_1 - {}^3D_1$ has negative mass for Sets A and C, whereas it has positive mass for Set B. In the higher partial waves bound states appear between 500 MeV and threshold. Most of the bound states in Set B are considerably different from those of Set A, although the NN phase shifts are about the same. The reason for this is primarily that the ω and ε contributions to the potential cancel to some extent in the case of NN , whereas they add in the case of $N\bar{N}$. Hence the association of the $N\bar{N}$ bound states with the physical mesons (see next section) allows a much more restrictive limitation on the OBE parameters than the adjustment to the NN data.

We were looking for a different choice of parameters which fit the low-lying bound states at the meson masses better than the parameters of Sets A, B and C with the restriction that the NN adjustment is not altered considerably. Without further improvements of our model (see Section 5) a quantitative fit is not possible. For parameters derived from NN calculations the ω-meson, for instance, appears always at negative energies. However, it was found possible to fit the $T = 1$ S-wave bound states to the masses of the π and ρ mesons. This is arranged by changing the form factor parameters of the vector mesons and the coupling constant of the η meson in Set A. These parameters are not very sensitive to the NN results. We obtain $\alpha_{\rho,\omega}(0) = 0.5$, $\alpha'_{\rho,\omega}(0) = 1\ \mathrm{GeV}^{-2}$ and $g_\eta^2/4\pi = 2$. This we call Set D. It may be interesting to note that the parameters of Set D are more reasonable than those obtained by the NN adaption. Namely, the Regge trajectories of the vector mesons are the common ones [13], and $g_\eta^2/4\pi = 2$ fits better to the SU(6) models than $g_\eta^2/4\pi = 7$. The results of calculations for Set D are listed in the last column of Table I and II. The overall fit is somewhat better in this case. We are only left with one negative energy bound state corresponding to the ω-meson which now appears at $-$ 380 MeV. Furthermore, the σ and f mesons, which are too strongly bound in Sets A, B and C, are here obtained at approximately the correct energies, but some other mesons as η and ϕ are bound too weakly.

In Fig. 1a-6a we vary separately the coupling constants $g_\varepsilon^2/4\pi$, $g_{\omega_1}^2/4\pi$, $g_{\rho_2}^2/4\pi$ and $g_\eta^2/4\pi$ between 3 and 15, leaving the other parameters fixed at their values of Set A. The same is done in Figs. 1b-6b for the form factor parameters Λ_π ,

$\alpha_\rho = \alpha_\omega$ and $\alpha'_\rho = \alpha'_\omega$. From these figures we see that, with the exception of the ω-Meson, there is nearly no influence of the coupling constant of a given meson on its own bound state position. Even the π meson does not depend sensitively on its own coupling constant, as can be seen from Fig.1a. The dependence of the bound state positions on the η coupling constant is negligible except for the S-waves and $^3P_0$, which require a somewhat lower value than in the tables. Except for the form factor parameters which are relevant in each partial wave the essential parameters are only $g^2_{\omega_1}/4\pi$ and $g^2_\varepsilon/4\pi$.

As already mentioned some of the lower bound states occur at negative energies. We do not take this to be a severe disadvantage of our model. In Fig.7 we show that there are no more negative energy bound states for somewhat lower coupling constants of the ω and ε mesons. The variation of the coupling constants g_{ω_1} and g_ε in Fig.7 was made in such a way that the ratio $g^2_{\omega_1}/g^2_\varepsilon$ remains constant. Under this variation the NN adjustment should not change considerably. We note that a somewhat lower coupling constant g_{ω_1} is also predicted in SU(6) models. The large coupling constant g_{ω_1} derived from NN calculations stems to some extent from the parametrization of the $\pi\pi$-S wave contribution. We remark that this contribution is not well known and that in our parametrization some sort of double counting cannot be excluded.

IV. Discussion: $N\bar{N}$ Bound States in Relation to the Physical Meson Spectrum

In this section we associate the calculated $N\bar{N}$ bound states to the physical meson spectrum. We are, however, not going to discuss this comparison in greater detail, since in view of Figs.1-7 and Table II a quantitative fit is not possible within the limitations of our present model (we believe that this can be arranged in an extended model as will be proposed in the next section). The qualitative features of the $N\bar{N}$ bound state spectrum interpreted as mesons are as follows.

The exchanged mesons π , η , ρ , ω and the fictitious σ come out as $N\bar{N}$ bound states below 1000 MeV which did not occur in previous on-shell [5] or nonrelativistic [6] calculations. The scalar resonance ε appears at about 1.5 GeV. There are several bound states above 1 GeV which correspond to experimentally observed mesons and which are not exchanged in our model, namely, the mesons ϕ, δ, A_1, A_2, f, f' and B mesons for T = 1 and T = 0. There is no A_1 for T = 0. A ρ' as well as two D-wave bound states appear just below threshold for the Sets A and C and are absent for the Sets B and D. The F-waves are no longer bound. We obtain at most one bound state for the uncoupled L = J partial waves. This is in agreement with experiment except in the T = 0 $^1S_0$ partial wave where, besides the η , there seem to be one or even two more resonances present.* For T = 0 the coupled partial waves $^3S_1 - {}^3D_1$ and $^3P_2 - {}^3F_2$ contain two bound states, the lower one is predominantly in the state L = J - 1 , the higher one in the state L = J + 1 . This reflects the well established experimental situation that the vector and tensor mesons for T = 0 are mixed singlet-octet states, namely ω, ϕ and f, f' . The D- and F-state probability of the $^3S_1 - {}^3D_1$ and $^3P_2 - {}^3F_2$ bound states are given in Table III. For T = 0 the lower bound states (ω and f) have a D- and F-state admixture of 10-30%,whereas the higher states (ϕ and f') have a D- and F-state probability of 50-60%. For T = 1 the D- and F-state probability is very small so that the ρ ,ρ' and A_2 mesons are almost pure $^3S_1$ and $^3P_2$ states. In the $^3P_2 - {}^3F_2$ T = 1 partial wave there is no second pole present which could correspond to a split A_2 meson that has been indicated in some experiments.*

For the $^3S_1 - {}^3D_1$ bound states we have calculated the quadrupol moment neglecting virtual meson contributions. The quadrupol moment is in the range of $10^{-29} - 10^{-27}$ cm^2 depending on the bound state masses. It is positive for ρ and ρ' (cigar shape) and negative for ω and ϕ (pancake shape).

Hence, with the two exception just mentioned, our model accounts for all well established non-strange mesons. It indicates the existence of a T = 1 scalar resonance, for which there are the candidates $\delta(960)$ and/or $\pi_N(1016)$. A $0^-(1^+)$ bound state occurs in our model as in other models based on SU(6) which does not correspond to an observed meson. The T = 0 partner of the A_1 does not appear in our model and in fact has not been experimentally observed.

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*Experimentally the situation of the pseudoscalar nonet resonance and the splitting of the A_2 is still slightly in doubt. Both X^o and A_{2H} may have a higher value of the spin. In our model there is no bound state in the higher partial wave. However, there is some indication that a SU(3) symmetric $B_8\bar{B}_8$ model may account for a pseudoscalar nonet resonance [17] and for a second A_2 meson [7].

So far we have only payed attention to the multiplicity of the bound states which is quite well predicted. With the comparison of the actual energies the situation is not as good. In particular, the ω and σ mesons are bound so strongly that they appear at negative energies. In the last section we showed (see Fig.7) that for lower values of the ω and ε coupling constants, the ω and σ bound states are shifted to positive energies. However, most of the other bound states appear then at too high energies. In addition to this the f meson is generally bound too strongly, whereas the ε, δ, A_1 and f' mesons generally appear at too high energies.

V. Comparison, Limitation and Possible Extensions of the Model

An essential feature of our model is that it is off-shell in comparison to that of Ball, Scotti and Wong [5]. By this we mean that multimeson exchanges are included in our model. In both models relativistic effects are taken into account. In the case of proton-proton interaction the influence of off-shell effects can be seen from Fig.2 of Ref.[8]. In almost all partial waves off-shell effects cause an additional attraction which is very strong in the lower partial waves. The same holds for $N\bar{N}$. Even in this case off-shell effects give rise to a considerable additional attraction, irrespective to the different sign of the ω exchange contribution. For this reason the on-shell calculations do not give sufficiently strong attraction in the S-waves which is required to obtain the bound states at the position of the lower mesons.

The relation of our model to the calculations using a Schrödinger equation with a nonrelativistic OBE potential [6] can easily be understood as follows. It is well known [7] that for S-waves the nonrelativistic approximation to vector meson exchange leads to negligible contributions for NN as well as for $N\bar{N}$ interactions. In fact, in Ref.[6] the pseudoscalar and vector mesons appear at energies of about 1.5 GeV, whereas the D-wave $N\bar{N}$ bound states have positions similar to those in our model.

As already pointed out in Ref.[9] the basic equation of our model is equivalent to the Blankenbecler-Sugar approximation [15] of the Bethe-Salpeter (BS) equation, i.e. our equation and the Blankenbecler-Sugar equation are equivalent approximations to the BS equation. However, in order to arrive at the correct analytic structure of the physical amplitudes predicted by the BS equation further improvements on the potential must be done. In the case of two spinless particles interacting by exchange of scalar mesons the BS solution has been compared to the solution of Blankenbecler-Sugar type equations by several authors [19], [20]. It is found that there is a difference only for strongly bound states as in our case the π meson. In this situation pair, multimeson and retardation effects cause a readjustment of the effective coupling constants $g_S^2/4\pi$ of about 35% in order to obtain the bound state at the same mass.* In the more realistic case of nucleons interacting by exchange of vector mesons, form factors are required, as already mentioned. We want to point out again that it is the vector meson exchange contribution which gives rise to strongly bound states. For energies where second order contributions, due to pair and multimeson effects, become non-negligible the effect caused by form factors is much more important.

The relation of our model to meson-meson bootstrap calculations [11],[14] can be seen from Fig.8. There we have drawn the corresponding duality diagram which in our model accounts for the meson-meson bootstrap. The ingoing and outgoing $N\bar{N}$ pairs represent the mesons under consideration. In those calculations the t-channel absorptive part is constructed from single meson poles corresponding to $N\bar{N}$ bound states. That means for instance that the meson structure is neglected. If consistency is achieved in this simple approximation the meson resonances then appear at the bound state positions of the intermediate $N\bar{N}$ states.

We believe that the deviations in our calculations from the physical meson spectrum are partially due to the neglect of P-wave meson exchanges. In fact, the mesons ϕ, δ, A_1, A_2, f, f' and B which were obtained in our calculations as $N\bar{N}$ bound states should also be exchanged. Another improvement of our model should come from a more realistic treatment of the $\pi\pi$-S-wave contribution.

So far strange baryons and mesons are not included. The effect of those channels containing strange particles is easily estimated. They always give rise to additional attraction. H. Sugawara and von Hippel [7] and Peaslee [18] considered the SU(3) symmetric $B_8\bar{B}_8$ interaction with vector meson exchange. In their calculations as well as in Ref.[17] and for meson-meson bootstrap calculations [11] the lower mesons appear in the representations 1 and 8 as in the quark model.

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*In our calculation retardation effects are included.

Acknowledgement

We wish to thank Professor G. Kramer for valuable discussions. We are also indebted to Dr. L. D. Landau for the assistance in preparing the text. One of us (S.W.) would like to thank Professor R. Haag for his kind hospitality at the II. Institute and Dr. M. Glück for various conversations. The numerical calculations were done on the IBM 360/75 and 360/65 at DESY. Computation time is gratefully acknowledged. Thanks are due to Dr. M. Böhm for pointing our attention to Ref.[20].

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H. Cohen and A. Pagnamenta; Phys. Rev. 181, 2098 (1969).

TABLE I: Coupling Constants and Form Factor Parameters for the Exchanged Mesons π , η , σ , ε , ρ and ω

The masses of the mesons are as in Refs. [8],[9]. The parameters of the fictitious σ are adjusted to the uncorrelated $\pi\pi$-S-wave contribution [8]. Set A fits the deuteron and the NN phase shifts [9]. Set B gives a comparatively good fit to the deuteron and NN phase shifts. Set C corresponds to the adjustment of pp-phase shifts only [8]. Set D fits the T = 1 S-wave bound states to the masses of the π and ρ mesons.

TABLE II: Calculated $N\bar{N}$ Bound States vs. Experimentally Observed Mesons

The calculated $N\bar{N}$ bound state energies are listed together with the experimentally observed mesons for the different $N\bar{N}$ partial waves.

TABLE III: D- and F-State Probabilities

Calculated D- and F-state probabilities for parameters of Set A.

TABLE I:

Input Parameters	Fit of Deuteron and NN Phase Shifts Set A	 Set B	Fit of Proton-Proton Elastic Phase Shifts Set C	Fit of π and ρ Meson as NN Bound States Set D
$g_\pi^2/4\pi$	14.4	14.4	14.4	14.4
Λ_π	0.82 GeV^2 = $(6.5\ m_\pi)^2$	0.72 GeV^2 = $(6.1\ m_\pi)^2$	0.25 GeV^2 = $(4.3\ m_\pi)^2$	0.82 GeV^2 = $(6.5\ m_\pi)^2$
$g_\eta^2/4\pi$	7.3	6.5	9.9	2.0
Λ_η	$(13m_\pi)^2 \approx (2m)^2$	$(13m_\pi)^2 \approx (2m)^2$	$(10m_\pi)^2$	$(13m_\pi)^2 \approx (2m)^2$
$g_\sigma^2/4\pi$	1.40	1.40	1.40	1.40
Λ_σ	m_ε^2	m_ε^2	$(10m_\pi)^2$	m_ε^2
$g_\varepsilon^2/4\pi$	6.8	5.7	5.4	7.0
Λ_ε	$(13m_\pi)^2 \approx (2m)^2$	$(13m_\pi)^2 \approx (2m)^2$	$(10m_\pi)^2$	$(13m_\pi)^2 \approx (2m)^2$
$g_{\rho_1}^2/4\pi$	0.605	0.605	0.750	0.605
$g_{\rho_2}^2/4\pi$	13.8	13.8	15.0	13.8
α_ρ	0.8	0.7	0.92	0.5
α'_ρ $[GeV^{-2}]$	0.66	0.6	0.72	1.0
$g_{\omega_1}^2/4\pi$	9.05	8.25	8.50	9.05
$g_{\omega_2}^2/4\pi$	0.09	0.09	0.084	0.09
α_ω	0.8	0.7	0.9	0.5
α'_ω $[GeV^{-2}]$	0.66	0.6	0.68	1.0

TABLE II :

$N\bar{N}$ States	$T^G(J^P)C_n$	Experimental Mesons	Energies of $N\bar{N}$ Bound States in MeV Set A	Set B	Set C	Set D
$^1S_0$	$1^-\ (0^-)+$	$\pi(139)$	240	680	800	140
	$0^+\ (0^-)+$	$\eta(549)$	220	520	- 590	1270
		$X^o(958)$	none	none	none	none
$^3S_1-^3D_1$	$1^+\ (1^-)-$	$\rho(765)$	-30	300	- 170	760
		$\rho'(1660)$	1850	none	1850	none
	$0^-\ (1^-)-$	$\omega(784)$	- 1460	- 720	- 990	- 380
		$\phi(1010)$	940	1360	930	1500
$^1P_1$	$1^+\ (1^+)-$	B(1235)	1450	1780	1530	1740
	$0^-\ (1^+)-$		1150	1500	640	1840
$^3P_0$	$1^-\ (0^+)+$	$\delta(960)$ $\pi_N(1016)$	1600	none	1300	none
	$0^+\ (0^+)+$	$\sigma(400)$	-600	-100	-200	+500
		$\varepsilon(700)$	1480	1200	1450	1780
$^3P_1$	$1^-\ (1^+)+$	$A_1(1070)$	1500	1670	1520	1740
	$0^+\ (1^+)+$		none	none	none	none
$^3P_2-^3F_2$	$1^-\ (2^+)+$	$A_{2_L}(1280)$	1380	1640	1210	1760
		$A_{2_H}(1320)$	none	none	none	none
	$0^+\ (2^+)+$	f (1260)	475	850	480	1220
		f'(1540)	1740	none	1650	none
$^1D_2$	$1^+\ (2^-)+$		none	none	none	none
	$0^-\ (2^-)+$		1840	none	1440	none
$^3D_2$	$1^+\ (2^-)+$		1820	none	1800	none
	$0^-\ (2^-)-$		none	none	none	none

TABLE III:

$N\bar{N}$ States	T	Meson	D- and F-State Probability in %	
			Set A	Set D
$^3S_1 - {}^3D_1$	1	ρ	0.109	0.486
	1	ρ'	9.620	-
	0	ω	18.21	9.28
	0	ϕ	59.76	54.18
$^3P_2 - {}^3F_2$	1	A_2	0.425	1.22
	0	f	28.74	15.06
	0	f'	49.30	-

Figs.1a-6a: <u>Energies of S- and P-Wave $N\bar{N}$ Bound States vs. Values of Meson-Nucleon Coupling Constants $g^2/4\pi$</u>

The coupling constants $g_\varepsilon^2/4\pi$, $g_{\omega_1}^2/4\pi$, $g_{\rho_2}^2/4\pi$ and $g_\eta^2/4\pi$ are varied from 3 to 15 leaving the others and the form factor parameters fixed at their values chosen in Set A. The values of Set A are marked by dots. In addition $g_\pi^2/4\pi$ is varied for the $^1S_0$ partial wave in Fig.1a. The masses of the experimentally observed mesons are marked by arrows.

Figs.1b-6b: <u>Energies of S- and P-Wave $N\bar{N}$ Bound States vs. Values of Form Factor Parameters</u>

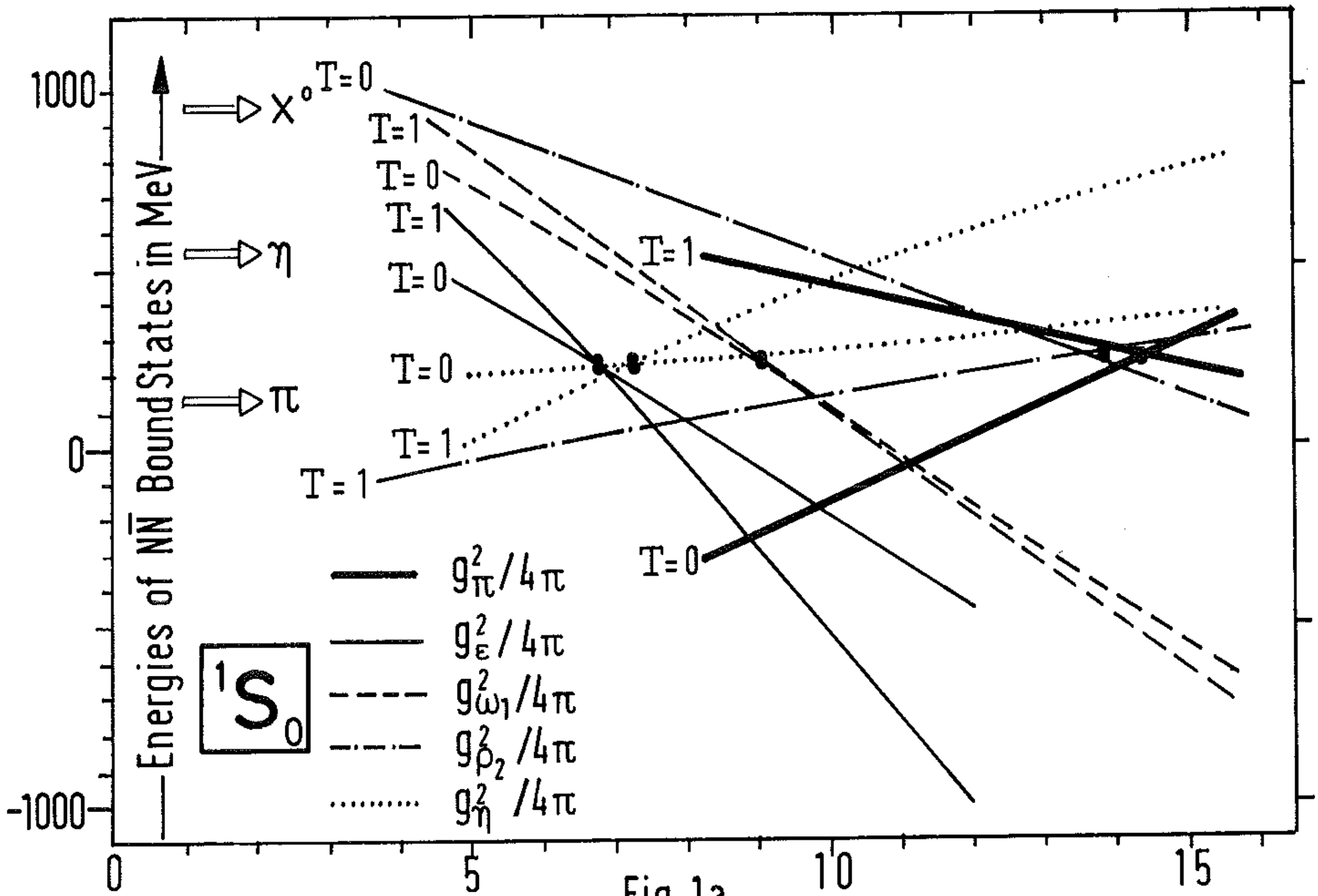
The form factor parameters Λ_π, $\alpha_\omega = \alpha_\rho$ and $\alpha'_\omega = \alpha'_\rho$ are varied leaving the others and the coupling constants fixed at their values chosen in Set A. The values of Set A are marked by dots. The experimentally observed mesons are marked by arrows.

Fig.7: <u>Energies of $I = 0$ $N\bar{N}$ Bound States vs. Values of $g_{\omega_1}^2/4\pi$ and $g_\varepsilon^2/4\pi$</u>

The coupling constants $g_{\omega_1}^2$ and g_ε^2 are varied leaving their ratio fixed at $g_{\omega_1}^2/g_\varepsilon^2 = 1.33$. This value as well as the remaining parameters are taken from Set A. The NN adaption is not very sensitive to this variation.

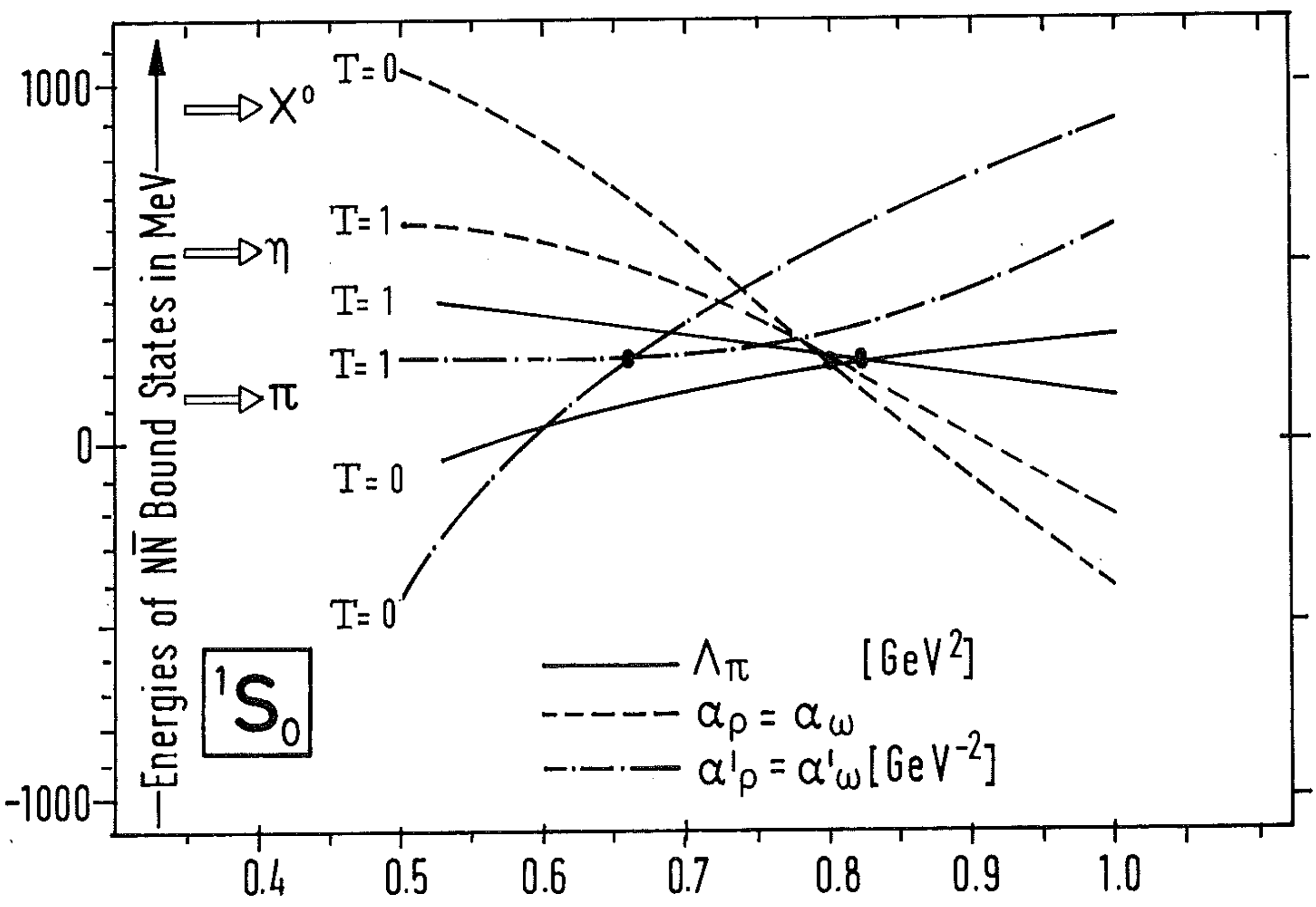
Fig.8: <u>Duality Diagram for Meson-Meson Interactions</u>

Solid lines represent nucleons and antinucleons respectively.



Energies of $N\bar{N}$ Bound States in MeV
1000
0
-1000
X^0
η
π
T=0
T=1
$^1S_0$
$g_\pi^2/4\pi$
$g_\varepsilon^2/4\pi$
$g_{\omega_1}^2/4\pi$
$g_{\rho_2}^2/4\pi$
$g_\eta^2/4\pi$
0
5
10
15

Fig. 1a



Energies of $N\bar{N}$ Bound States in MeV
1000
0
-1000
X^0
η
π
T=0
T=1
$^1S_0$
Λ_π [GeV^2]
$\alpha_\rho = \alpha_\omega$
$\alpha'_\rho = \alpha'_\omega$ [GeV^{-2}]
0.4
0.5
0.6
0.7
0.8
0.9
1.0

Fig. 1b

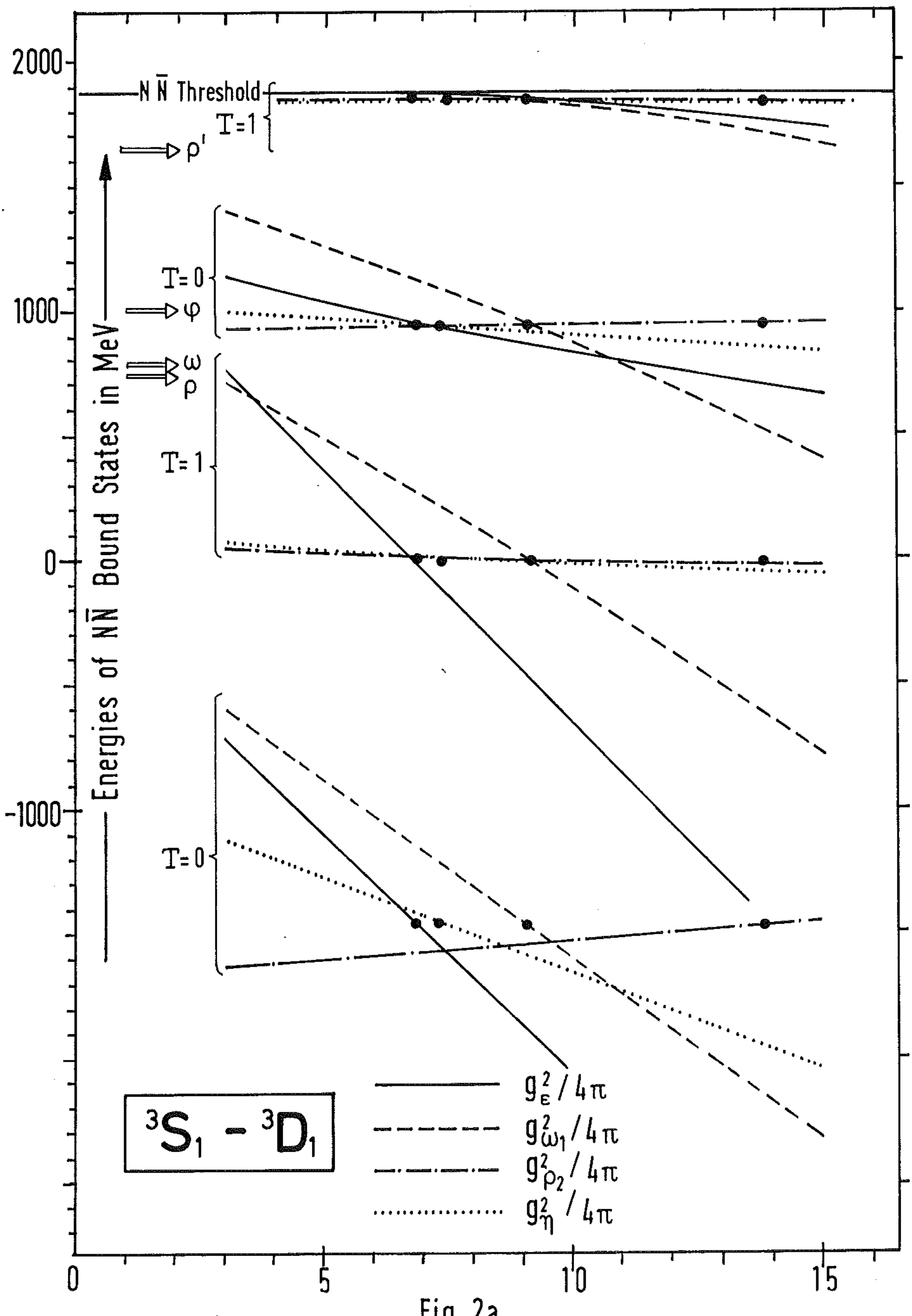
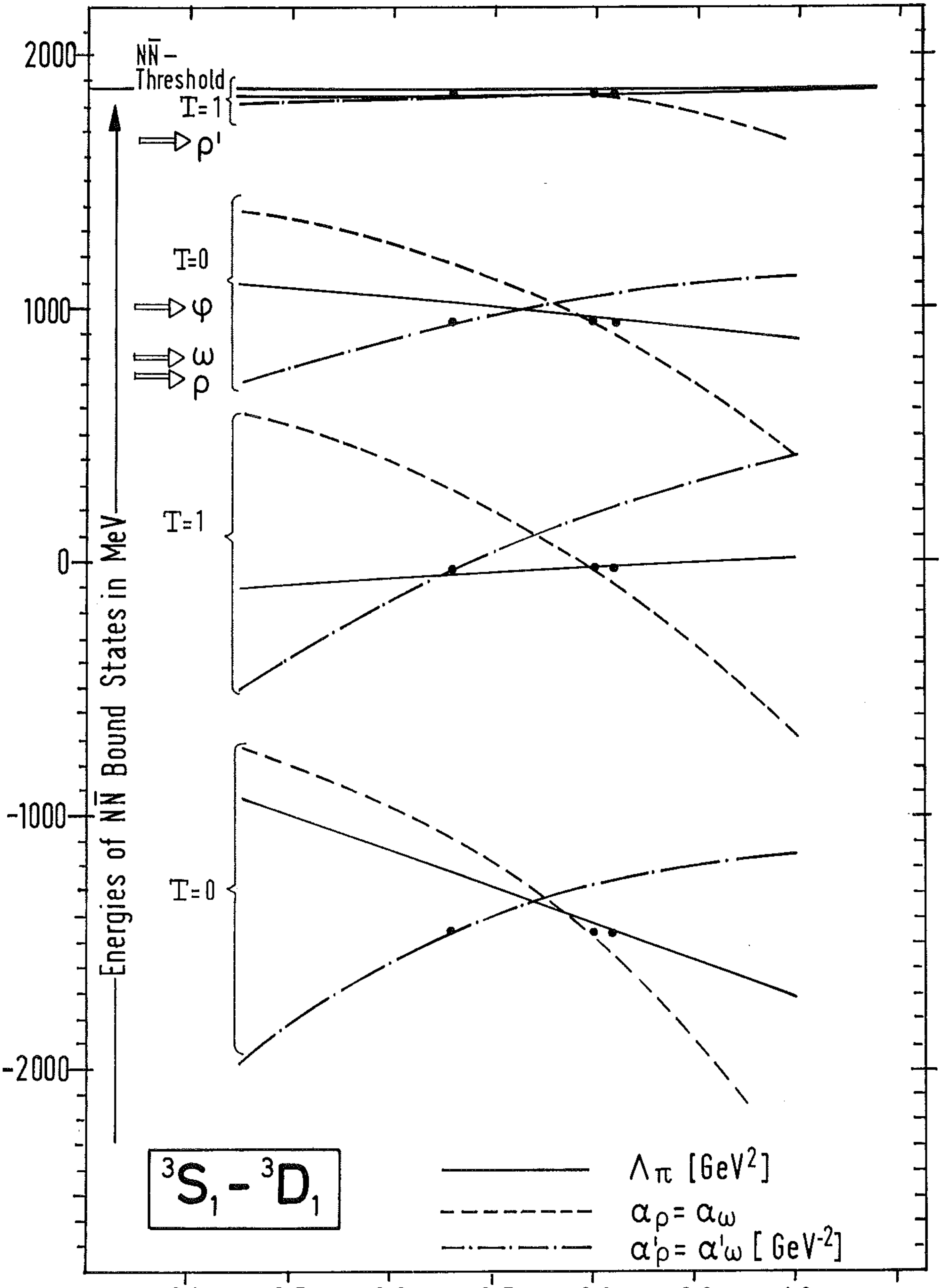
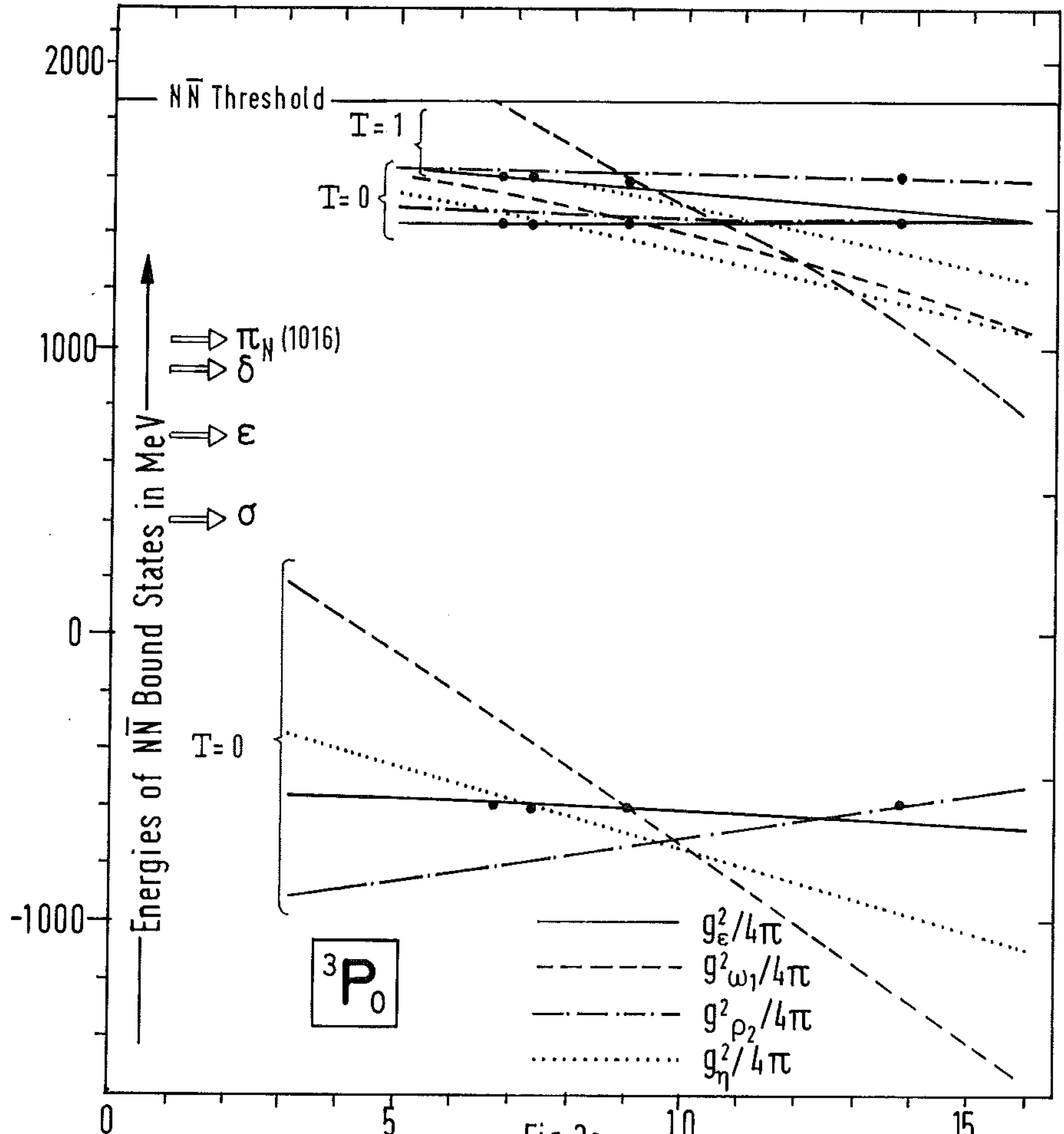

2000
1000
0
-1000
N N̄ Threshold
T=1
ρ'
T=0
φ
ω
ρ
T=1
T=0
Energies of NN̄ Bound States in MeV
$^3S_1 - {}^3D_1$
$g_\varepsilon^2/4\pi$
$g_{\omega_1}^2/4\pi$
$g_{\rho_2}^2/4\pi$
$g_\eta^2/4\pi$
0
5
10
15

Fig. 2a



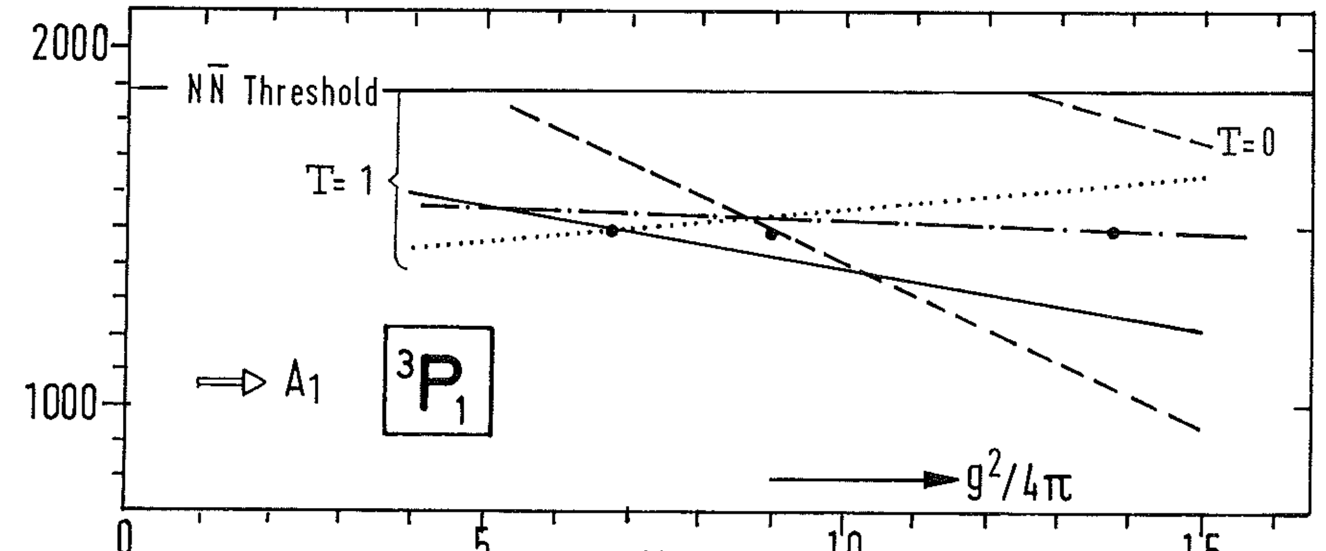
Energies of $N\bar{N}$ Bound States in MeV
2000
1000
0
-1000
-2000
$N\bar{N}$ - Threshold
T=1
ρ'
T=0
φ
ω
ρ
T=1
T=0
$^3S_1 - {}^3D_1$
Λ_π [GeV^2]
$\alpha_\rho = \alpha_\omega$
$\alpha'_\rho = \alpha'_\omega$ [GeV^{-2}]
0.4
0.5
0.6
0.7
0.8
0.9
1.0

Fig. 2b



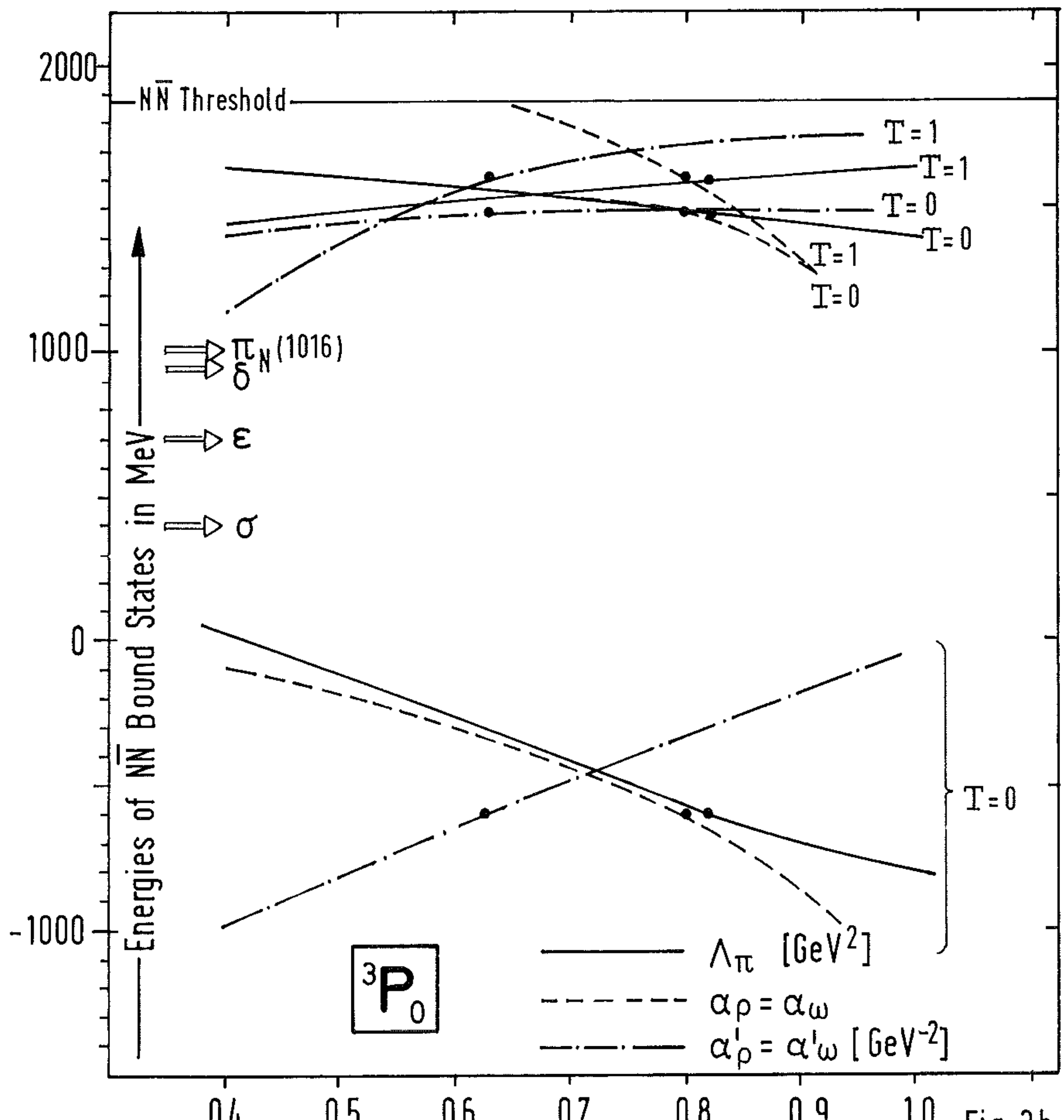
2000
1000
0
-1000
NN̄ Threshold
T= 1
T= 0
T= 0
Energies of NN̄ Bound States in MeV
π_N (1016)
δ
ε
σ
$g_\varepsilon^2/4\pi$
$g^2_{\omega_1}/4\pi$
$g^2_{\rho_2}/4\pi$
$g_\eta^2/4\pi$
$^3P_0$
0
5
10
15

Fig.3a



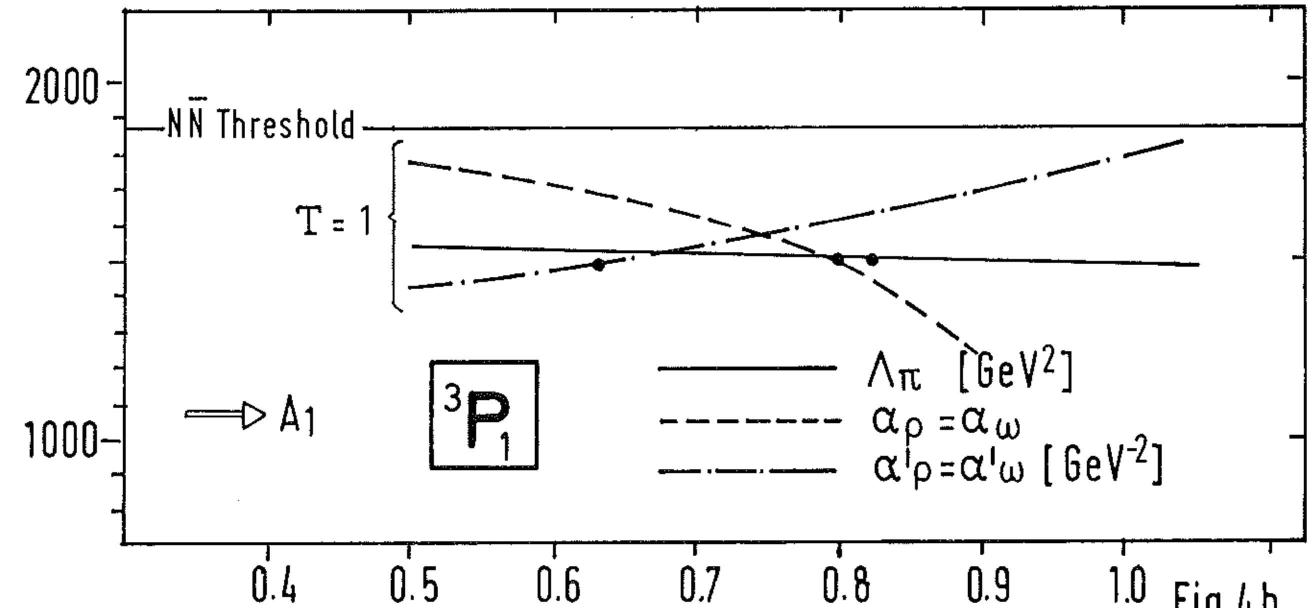
2000
1000
NN̄ Threshold
T= 1
T= 0
A1
$^3P_1$
$g^2/4\pi$
0
5
10
15

Fig.4a



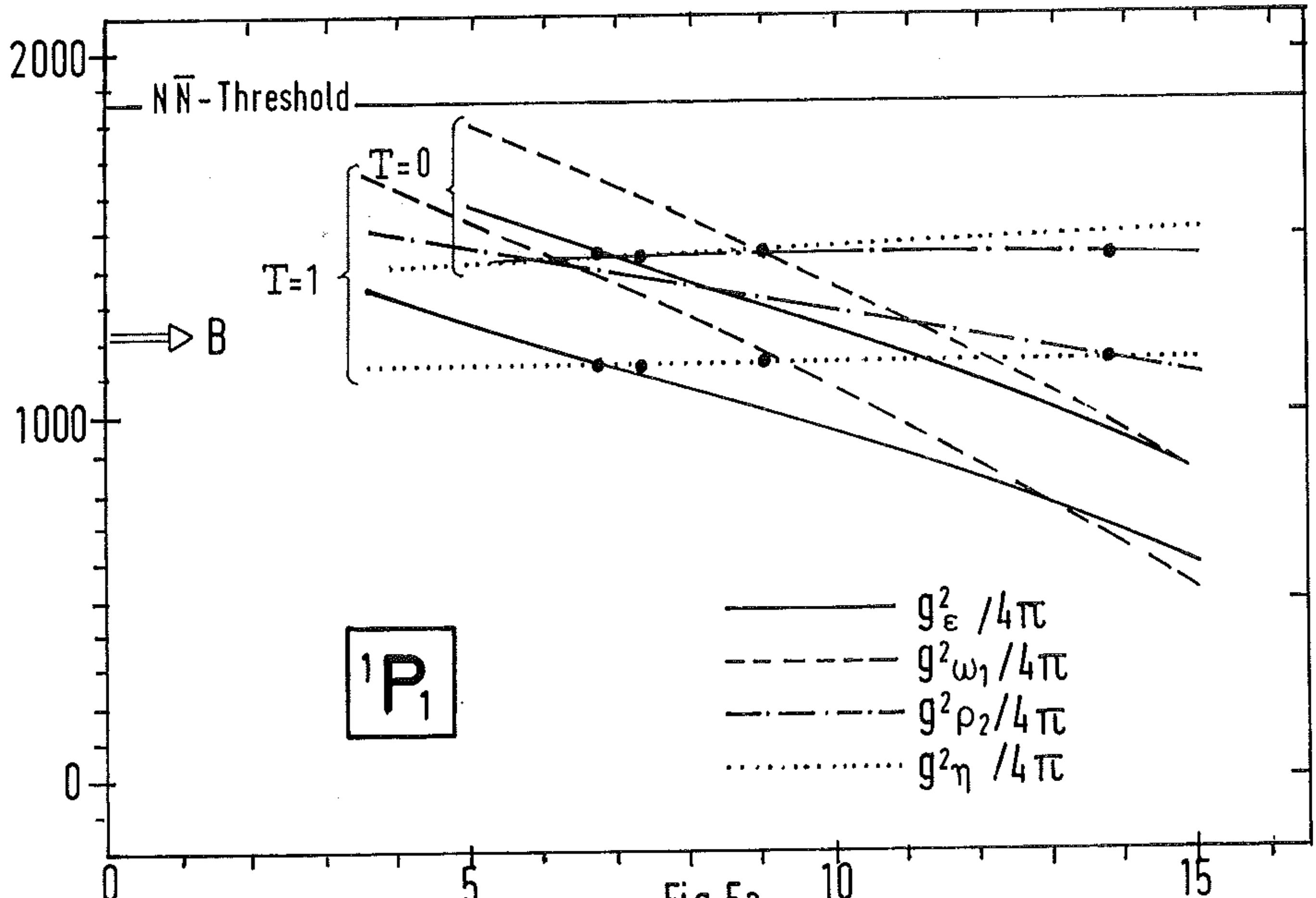
2000
1000
0
-1000
NN̄ Threshold
T=1
T=1
T=0
T=0
T=1
T=0
πN (1016)
δ
ε
σ
Energies of NN̄ Bound States in MeV
T=0
$^3P_0$
Λ_π [GeV^2]
$\alpha_\rho = \alpha_\omega$
$\alpha'_\rho = \alpha'_\omega$ [GeV^{-2}]
0.4
0.5
0.6
0.7
0.8
0.9
1.0

Fig. 3b



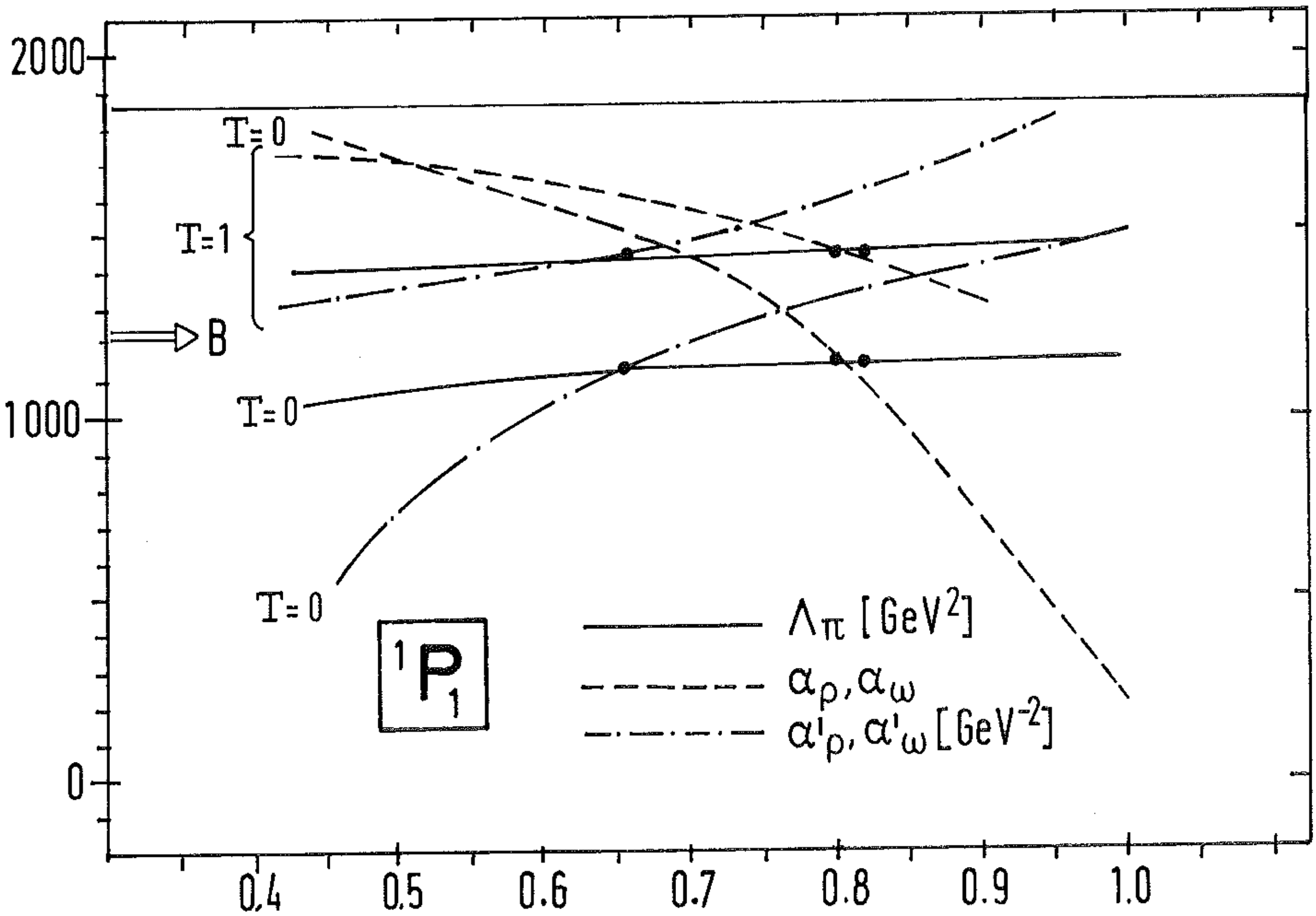
2000
1000
NN̄ Threshold
T = 1
A1
$^3P_1$
Λ_π [GeV^2]
$\alpha_\rho = \alpha_\omega$
$\alpha'_\rho = \alpha'_\omega$ [GeV^{-2}]
0.4
0.5
0.6
0.7
0.8
0.9
1.0

Fig. 4b



2000
NN̄ - Threshold
T=0
T=1
B
1000
0
$^1P_1$
$g^2_\varepsilon/4\pi$
$g^2\omega_1/4\pi$
$g^2\rho_2/4\pi$
$g^2\eta/4\pi$
0
5
10
15

Fig.5a



2000
T=0
T=1
B
T=0
1000
T=0
0
$^1P_1$
Λ_π [GeV^2]
$\alpha_\rho, \alpha_\omega$
$\alpha'_\rho, \alpha'_\omega$ [GeV^{-2}]
0.4
0.5
0.6
0.7
0.8
0.9
1.0

Fig.5b

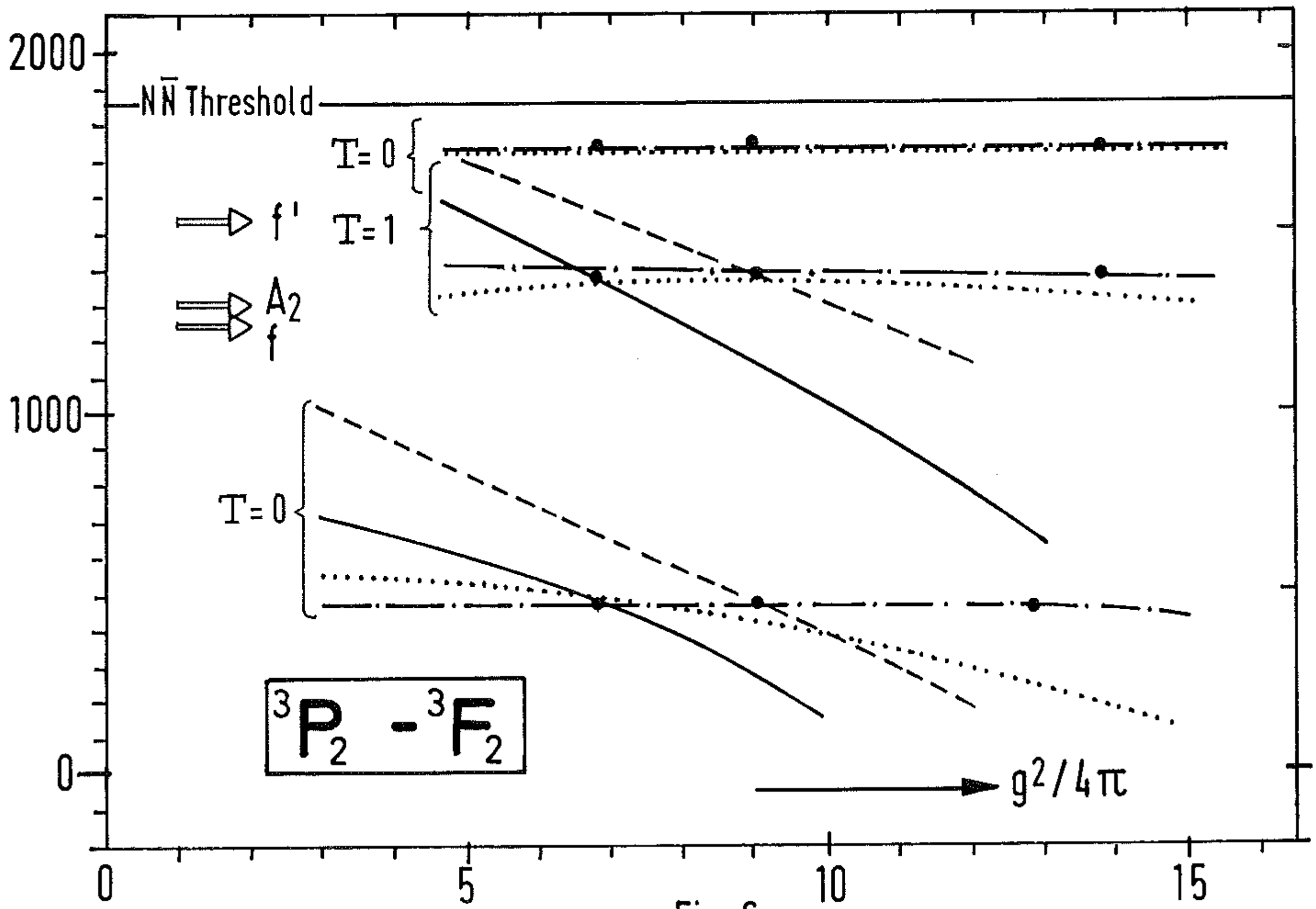

2000
1000
0
$N\bar{N}$ Threshold
T= 0
T= 1
T= 0
f'
A_2
f
$^3P_2$ - $^3F_2$
$g^2/4\pi$
0
5
10
15

Fig. 6 a

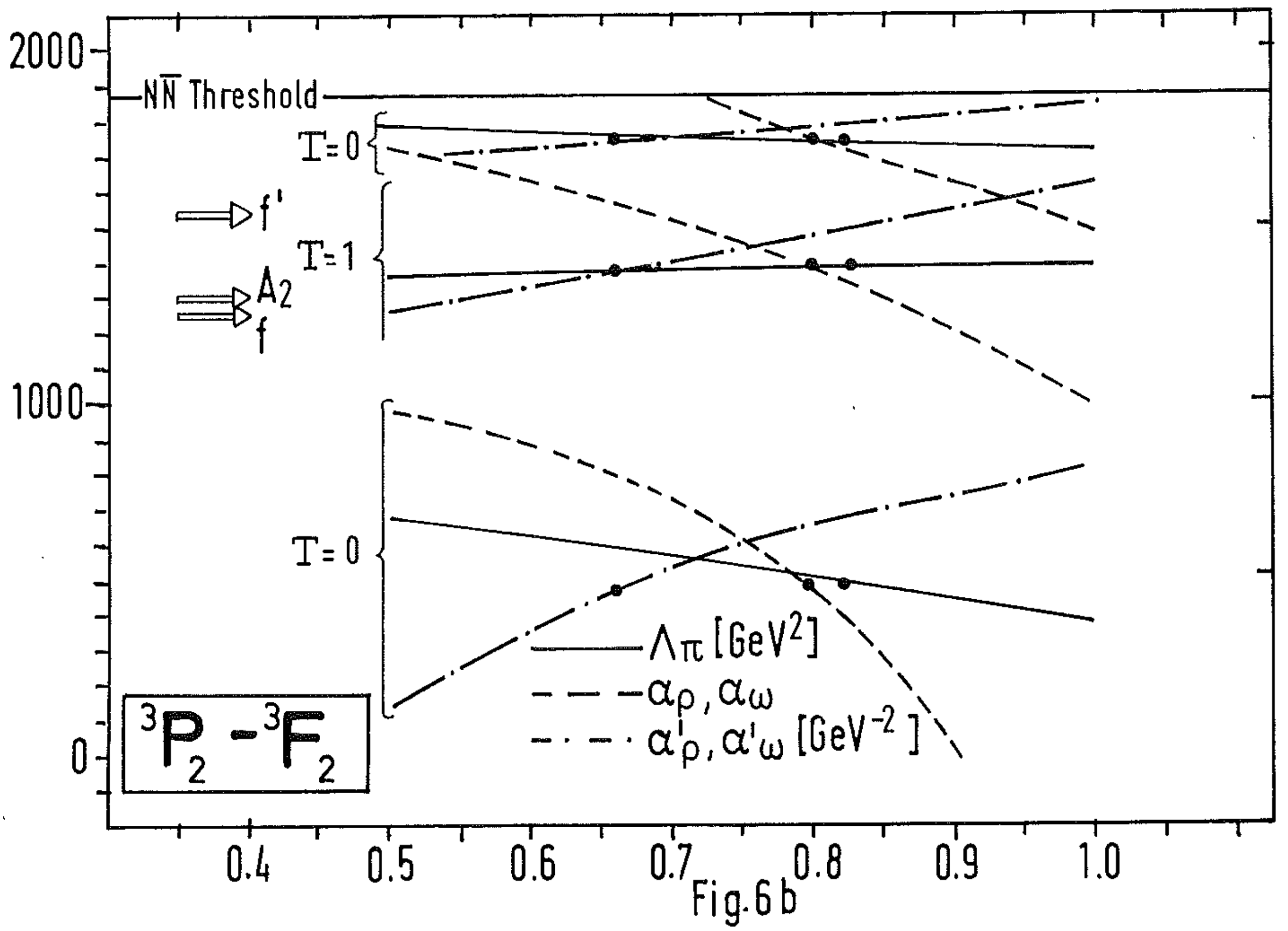
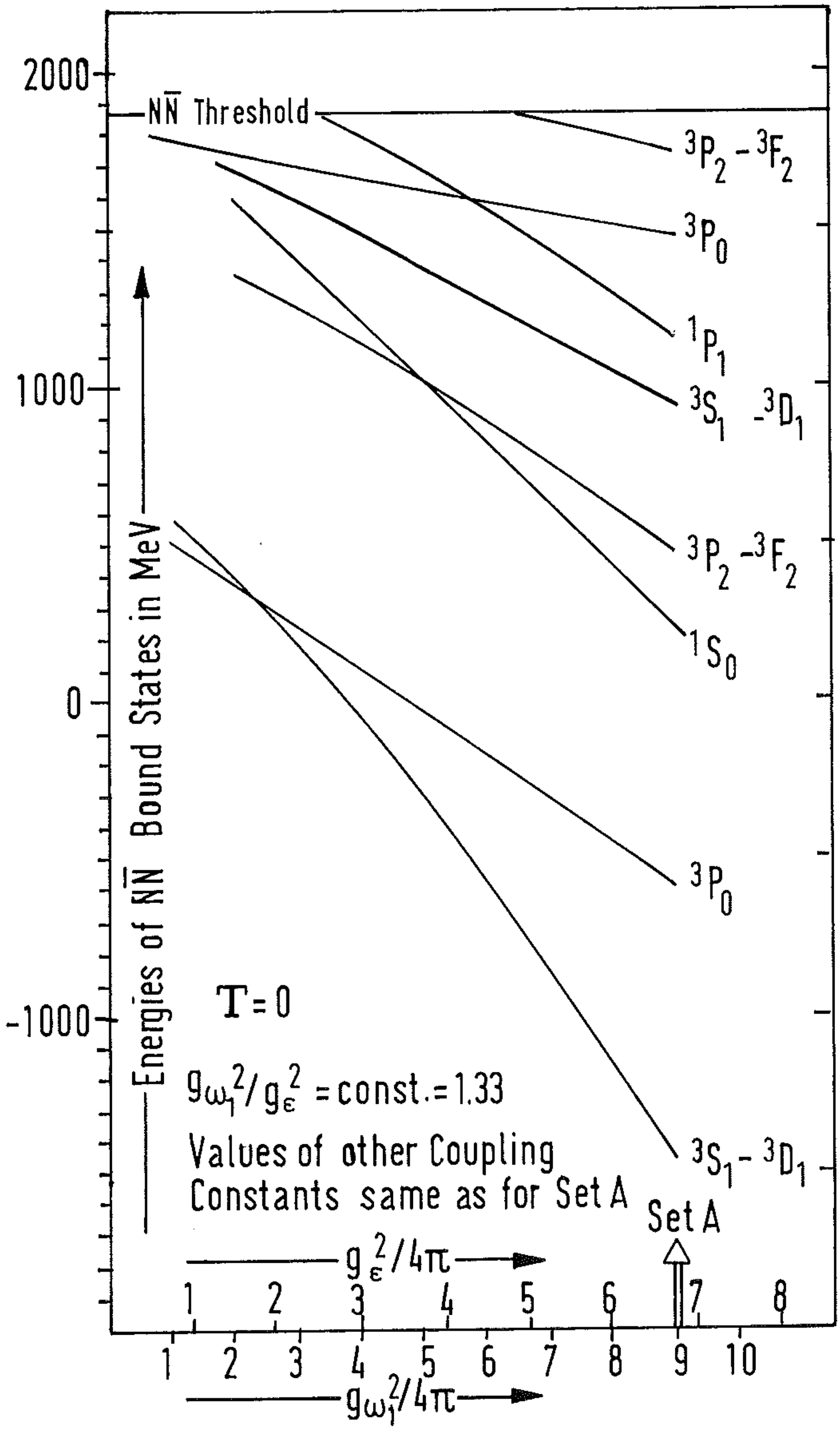

2000
1000
0
$N\bar{N}$ Threshold
T= 0
T= 1
T= 0
f'
A_2
f
$^3P_2$ - $^3F_2$
Λ_π [GeV^2]
$\alpha_\rho, \alpha_\omega$
$\alpha'_\rho, \alpha'_\omega$ [GeV^{-2}]
0.4
0.5
0.6
0.7
0.8
0.9
1.0

Fig. 6 b



2000
1000
0
-1000
NN̄ Threshold
$^3P_2 - {}^3F_2$
$^3P_0$
$^1P_1$
$^3S_1$ $-{}^3D_1$
$^3P_2 - {}^3F_2$
$^1S_0$
$^3P_0$
$^3S_1 - {}^3D_1$
Energies of NN̄ Bound States in MeV
T = 0
$g_{\omega_1}^2/g_\varepsilon^2$ = const. = 1.33
Values of other Coupling
Constants same as for Set A
Set A
$g_\varepsilon^2/4\pi$
1 2 3 4 5 6 7 8
1 2 3 4 5 6 7 8 9 10
$g_{\omega_1}^2/4\pi$

Fig. 7

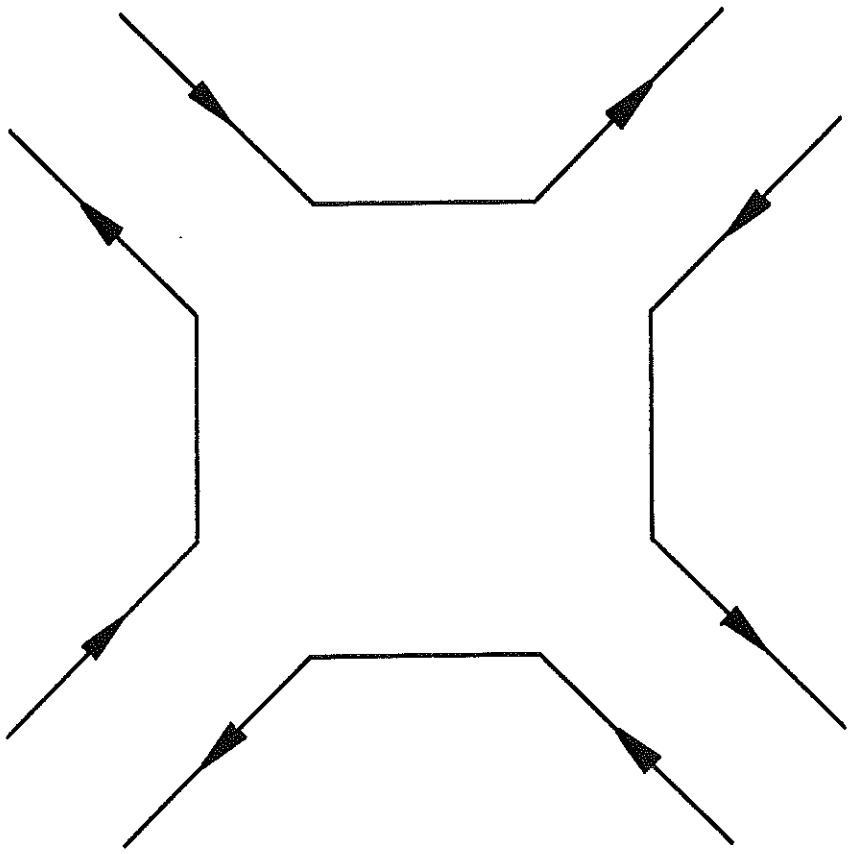


Fig. 8