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Abstract

A model for the electroproduction of pions at high energies is discussed and applied to the recent data from DESY. With respect to the isovector amplitudes the model represents the electric Born term model, which includes the pion exchange and the pion form factor. The model is supplemented by absorption effects, which are partly derived from fixed-t-dispersion relations or fitted to the data. The isoscalar amplitudes are calculated under the assumption of ρ-exchange. The pion form factor is new determined by a fit to the data. According to this fit the pion form factor is larger than predicted by ρ-dominance for photon masses in the range of the DESY experiment. This is a consequence of the unexpected large values for the longitudinal cross-sections. A comparison is also made with the recent results from CEA and NINA.

I. Introduction

Several experiments are now in progress to explore at high energies the production of hadronic states by scattering electrons off nucleons. Electroproduction of pions is part of this general program. This reaction is of interest by itself, since it yields information about the pion formfactor $F_\pi(k^2)$ in the spacelike region ($k^2 < 0$) if the experiment is done in suitable kinematic regions. This condition applies for the recent data from DESY[1] where W = 2.2 GeV, t varying between t_{min} and -0.1 $(GeV/c)^2$ and k^2 between -0.18 and -0.825 GeV^2/c^2; here W is the total c.m. energy of the hadronic final state, t the momentum transfer squared to the nucleon k^2 the mass squared of virtual photon (Fig. 1a).

In the one photon exchange approximation the pion electroproduction cross-section can be written in terms of four structure functions σ_U, σ_L, σ_T, σ_I depending on $s = W^2$, t and k^2

$$\frac{d\sigma^4}{ds\,dk^2\,dt\,d\varphi_{\pi k}} = \tilde{\Gamma}\left\{\sigma_U + \varepsilon\sigma_L + \varepsilon\sigma_T\cos 2\varphi_{\pi k} + \sqrt{2\varepsilon(\varepsilon+1)}\,\sigma_I\cos\varphi_{\pi k}\right\} \tag{1}$$

$$= \tilde{\Gamma}\left\{\rho^u_{11} + \varepsilon\rho^u_{00} - \varepsilon\rho^u_{1-1}\cos 2\varphi_{\pi k} + \sqrt{\varepsilon(\varepsilon+1)}\,2\,\mathrm{Re}\,\rho^u_{10}\cos\varphi_{\pi k}\right\}$$

The azimuthal angle $\varphi_{\pi k}$ and the functions $\tilde{\Gamma}$ and ε are conventional[3]; they depend only on the electron variables.

The first form of (1) is common usage. The second choice $\rho^u_{ik}(s,t,k^2)$ of writing the structure functions is useful in the context of vector meson dominance because the current field identity (and time reversal invariance) relates the ρ's directly to the (unnormalized) density matrix elements $\rho^{u,V}_{ik}$ of the vector mesons $V(\rho, \omega, \varphi)$ produced by pions in the reaction $N\pi \to N'V$.[4]

The somewhat striking feature of the recent electroproduction data from DESY[1] is the behavior of the longitudinal contribution σ_L which is unexpectedly large. We think that this fact has consequences for the pion formfactor.

In the DESY-experiment σ_L dominates over all other contributions σ_U, σ_T, σ_I, which all involve transverse amplitudes. The dominance of σ_L is in contrast to the general feature of the total inelastic electron scattering cross section

σ^{tot} , where $\sigma_L^{tot} < \sigma_U^{tot}$ [5] . But no contradiction arises since the electroproduction cross-section is only a few per cent of the total inelastic electron scattering cross-section. This somewhat peculiar behavior of σ_L in pion-electroproduction is well understood in the framework of the different versions of the electric Born term model and was expected[4,6].
From the point of view of Vector Meson Dominance it is in line with the large longitudinal contribution $\rho_{oo}\, d\sigma/dt$ in the reaction $\pi^- p \to \rho^o n$ [7] which is related to ρ_{oo}^u . The present experiment yields actually only the sum $\sigma_U + \varepsilon\sigma_L$, with ε being typically 0.7....0.8. In Ref. (1) the seperation was performed under the assumption that $\sigma_U \approx |\sigma_T|$ which relation is true in photoproduction for the t-range considered. For $k^2 \neq 0$ the relation $\sigma_U \approx |\sigma_T|$ is confirmed also in the model, which we present, so that one can assume that σ_L is well determined from a qualitative point of view.

The physical origin for the strong increase of σ_L around $k^2 = 0$ is the direct coupling of the virtual photon to the pion in the one pion exchange contribution (Fig. 1b). For $t \sim -m_\pi^2$ this diagram dominates over all known other effects at present energies. The strength of this coupling of the photon is determined by the pion formfactor $F_\pi(k^2)$. One can therfore expect that the present experiment is well suited for a determination of this important structure function. Previous determinations of $F_\pi(k^2)$ in electroproduction used experiments in the $\Delta(1236)$ resonance region[8]. But in this case more theoretical efforts are necessary and the theoretical uncertainties are still large at present. On the other hand for the DESY high energy experiment a very simple dynamical model applies. This model can be motivated for the isovector amplitudes from fixed-t-dispersion and is essentially the electric Born term model[4,6] supplemented by absorption effects. We believe, that the new experiments yields new information about the pion formfactor $F_\pi(k^2)$ when analysed in terms of the model.

Stated qualitatively our result for the pion formfactor $F_\pi(k^2)$ is that for $-0.18\ \text{GeV}^2 > k^2 > -0.85\ \text{GeV}^2$ $F_\pi(k^2)$ is larger than the Vector Meson Dominance law $(1 - k^2/m_\rho^2)^{-1}$ predicts. To state the result more quantitatively we write

$$F_\pi(k^2)\cdot(1 - k^2/m_\rho^2) = R(k^2) = 1 - \frac{c_1 k^2}{m_\rho^2}\left(1 + \frac{c_2 k^2}{m_\rho^2}\right) \qquad (2)$$

Then the fit to the data yields $c_1 = 0.85$ and $c_2 = 0.65$. The errors of c_1, c_2

due to the statistical errors of the data are negligibly small (≈ 2 %) compared to the incertainty introduced by the model, with which the data are analyzed. With the quoted values for $c_{1,2}$ one obtaines R = 1.21, 1.29, 1.32, 1.30, 1.15 in the k^2-range of the experiment, i.e. for $-k^2$ = 1/3, 1/2, 2/3, 1 and 4/3 m_ρ^2 (= 0.585 GeV^2). The deviations of R from the value 1 are somewhat unexpected. They are a consequence at the unexpected large values of σ_L . Possible sources for systematic errors will be discussed in Section IIc. But at the moment we do not see indications that improvements of the model will yield R ≈ 1. These values of R would be more in agreement with the previous determinations[8] of $F_\pi(k^2)$.

Due to the results of these analyses[8] one very often used the assumptions that $F_\pi(k^2) \approx F_1(k^2)$ or even $F_\pi(k^2) = G_e^P(k^2)$, where $F_1(k^2)$ is the electric isovector Dirac-Pauli form factor of the nucleon and G_e^P is the electric Sachs form factor of the proton. We show in Fig. 2 the behaviour of the different versions of the pion formfactor including the present result.

II. The Model

II.a The Isovector Amplitudes

We define the model, with which we analyze the data, in terms of Dennery's set $A_i(s,t,k^2)$ of invariant functions[9]. But as in Ref. (4) we replace A_2 and A_5 by $(\mu^2 = m_\pi^2)$

$$D_2 = (t-\mu^2) A_2 \quad , \quad D_5 = \frac{s-u}{2} A_2 + k^2 A_5 \qquad (3)$$

The amplitudes D_2 and D_5 have the advantage of having no kinematic singularities at $t=\mu^2$ but are dependent at $k^2 = 0$. For the dominant isovector amplitudes A_2^- we write as in Ref. (4) unsubtracted dispersion relations for $A_{1,3,4,6}$ and D_2 and a subtracted dispersion relation for D_5 (subtracted at threshold). It is assumed that these dispersion integrals are dominated by the low energy contributions. In practice only the $\Delta(1236)$ contribution will be kept. The real parts A_i^- are then given by the following relations.

$$\mathrm{Re}\, A_1^- = -\frac{e_r}{2} F_1(k^2) g_r \left(\frac{1}{M^2-s} - \frac{1}{M^2-u}\right) + DP_1 \qquad (4a)$$

$$\mathrm{Re}\, D_2^- = e_r F_1(k^2) g_r \left(\frac{1}{M^2-s} - \frac{1}{M^2-u}\right) + DP_2 \qquad (4b)$$

$$\mathrm{Re}\, D_5^- = \frac{1}{2} e_r F_1(k^2) g_r \left(\frac{1}{M^2-s} + \frac{1}{M^2-u} + DP_5\right.$$

$$- 2 e_r F_\pi(k^2) g_r \frac{1}{t-\mu^2} \left(1 + \frac{a}{2} t\right) \qquad (4c)$$

$$\mathrm{Re}\, A_3^- = \frac{1}{2} e_r F_2(k^2) g_r \left(\frac{1}{M^2-s} + \frac{1}{M^2-\mu}\right) + DP_3 \qquad (4d)$$

$$\mathrm{Re}\, A_4^- = \frac{1}{2} e_r F_2(k^2) g_r \left(\frac{1}{M^2-s} + \frac{1}{M^2-u} - \frac{2}{s}\right) + DP'_4 \qquad (4e)$$

$$\mathrm{Re}\, A_6^- = DP_6 \qquad (4f)$$

Here $F_1(k^2) = F_1^p(k^2) - F_1^n(k^2)$, $F_2(k^2) = (1.793\, F_2^p(k^2)$

$+ 1.913\, F_2^n(k^2)) / (2M)$, $e_r^2 = 4\pi/137$, $g_r = 2Mf_r/\mu$, $f_r^2 = 4\pi\, 0.080$, M, μ nucleon, pion mass, respectively.

The DP_i's represent dispersion integral contributions, which we do not give explicitly here. They follow e.g. from the formulae in Ref. (4). The DP_i's are linear in t due to the $\Delta(1236)$ resonance approximation and the weak dependence of u on t in our applications: $DP_i \approx a_i + b_i t$. The b_i-terms reduce the almost constant pole terms (and a_i) for $t < 0$. In this sense they represent absorption effects. Since we keep only terms linear in t, our approximation is limited to small values of t.

The last term in $\mathrm{Re}D_5^-$ represents the subtraction function for D_5^-. It is chosen in our ansatz in such a way that the residue at $t = \mu^2$ determined by the pion form factor $F_\pi(k^2)$ is correct. The factor $(1 + \frac{a}{2} t)$ allows for an additional t-dependence and represents absorption effects as long as quadratic terms in t can be neglected. The constant a is unknown and may in principle even depend on k^2. We shall consider a as constant and fit it together with the constants c_1, c_2 of the pion formfactor (2) to the data. The last three amplitudes $A_{3,4,6}$ in (4 d-f) play at present energies no significant role. For $s \to \infty$ they vanish as s^{-2}. For A_4 this is true because we applied as in Ref. (4) the superconvergence relation

$$\frac{1}{2} F_2(k^2)\, g_r = -\frac{1}{\pi} \int_{(M+\mu)^2}^{\infty} ds \, \mathrm{Im}\, A_4^-(s,t,k^2) \qquad (5)$$

Due to this relation the asymptotic term in the original pole term of A_4^- is cancelled, so that both anomalous magnetic terms do not contribute to the leading $1/s$ terms of the amplitudes A_i^-. One recovers the pole terms of Fubini et al.[10) by putting in Equs. (4a-f) $a = DP_i = 0$, $i = 1 \ldots 6$. DP_4 is DP_4' plus the contribution arising from (5).

At the energies considered we neglect the imaginary parts of the amplitudes, when we calculate cross-sections, where the imaginary parts do not infere with the real parts. Even if the imaginary parts amount 50 % of the real parts, their contribution to the cross section is 25 %. A 25 % increase of the cross-section can be simulated by a 12,5 % change of the dominant real part D_5.

II.b The Isoscalar Amplitudes

The isoscalar contributions A_i^0 to the real parts deserve a more careful consideration than the imaginary parts. The elemantary ρ-exchange gives the following contributions

$$A_1^o = A\ t/2,\ A_3^o = 0,\ A_4^o = A(MF_1^\rho/(F_2^\rho\kappa^\rho)),\ A_6^o = 0$$

$$D_2^o = \frac{-A}{2}(k^2+t-\mu^2),\quad D_5^o = A\frac{u-s}{4} \tag{6}$$

$$\text{with } A = \frac{1}{t-m_\rho^2}\ \frac{\kappa^\rho}{M}\ F_2^\rho\ \lambda_\rho(k^2) \tag{7}$$

Here $\lambda_\rho(k^2)$ is the coupling constant of the ρππ-vertex and F_1^ρ, $F_2^\rho\ \kappa^\rho$ the coupling constants of the ρNN-vertex. We have not explicitly reggeized the ρ-exchange, since we consider only a small t-range.

Instead we write for A (from an exploratory point of view)

$$A = \bar{A}\ (1 - k^2/m_\omega^2)^{-1} \tag{8}$$

and treat $\bar{A}$ as a constant for fixed energy into which all effects from the Reggeization and the coupling constants are lumped together in the considered small t-range. The factor $(1 - k^2/m_\omega^2)^{-1}$ represents the formfactor dependence of λ_ρ. The parameter $\bar{A}$ is fitted to the π^-/π^+ ratio in photoproduction[11]. At $s = 4.84$ GeV^2 its value is $\bar{A} = -\frac{1}{m_\rho^2}\ \frac{4.7}{M}\ \frac{0.39}{m_\rho}$ in units such that $m_\pi = \hbar = c = 1$. Near $t = 0$ the energy dependence of $\bar{A}$ is roughly $s^{\alpha_\rho(0)-1}$ as expected. We compare the order of magnitude of the different physical effects by calculating the ratio r of the ρ to π-exchange in ReD_5. According to (4c) and (6) we have at $t = 0$

$$r = \bar{A}\ \frac{s-M^2}{2}\left(\frac{s-M^2}{s_0 \cdot M^2}\right)^{\alpha_\rho(0)} \frac{m_\pi^2}{2\, e_r\, g_r} \tag{9}$$

where it is assumed that $F_\pi(k^2) \approx (1 - k^2/m_\omega^2)^{-1}$, $s-u = 2(s-M^2)+t - k^2 \cdot m_\pi^2 \approx 2(s-M^2)$. With the quoted value for $\bar{A}$ we obtain at $s = 4.84$ GeV^2 for the ratio r 2 %. The t-dependence of r will be roughly $(m_\pi^2 - t)$, so that at $t = -4m_\pi^2 = -0.08(GeV/c)^2$ we can expect a 10 % effect in ReD_5 arising from the isoscalar contribution. Nevertheless the ρ-exchange treated this way yields in σ_L only a 5 % effect for the π^-/π^+ ratio (at $s = 4.84$ GeV^2, $t = -0.1$ (GeV/c)^2) but in σ_U a 40 % effect [11]. That the effect in σ_L is so small is a consequence of the fact that in the two helicity amplitudes the ρ-exchange works in different directions, so that the effect cancels in the sum for σ_L . At high energies the situation becomes different.

II.c Some Incertainties of the Analysis and their Elimination

For an absolute determination of $F_\pi(k^2)$ it is necessary to separate the one pion exchange effect in σ_L . The measurement of the ratio σ_T/σ_U yields $F_\pi(k^2)$ relative to $F_1(k^2)$ as already discussed [12]. But the determination of σ_T/σ_U with reasonable accuracy needs improved experimental techniques. - We shall mention now five steps of a possible program to eliminate some incertainties of the present analysis based on the knowledge of $\sigma_U + \varepsilon\sigma_L$.

1.) Although $\sigma_U < \sigma_L$ in the kinematic range of the present experiment a separation of σ_U and σ_L by varying ε eliminates one source for a systematic error.

2.) To reduce the influence of the isoscalar terms the sum $0.5\,(\sigma_L^{\pi^+} + \sigma_L^{\pi^-})$ should be known, where $\sigma_L^{\pi\pm}$ is the σ_L for π^+ or π^- electroproduction. In this sum the isoscalar and isovector terms no longer interfere. But one should keep in mind that the influence of the isoscalar terms is small in σ_L according to the estimate in Section II.b and larger in σ_U .

3.) The apsorption effects become stronger with increasing values of -t(see e.g. the a/2 t term in the factor (1 + a/2 t) in D_5).The concentration of the measurement at the t-range around $t = -m_\pi^2$ (where σ_L has also a maximum in the t-distribution) reduces therfore the influence of the absorption effects. In principle "a" is dependent on k^2. For example a fit to the data at 11 GeV[13] of the reaction yields a ≈ 9.5 GeV^{-2}, whereas the fit the π^+-electroproduction data yields a ≈ 5 GeV^{-2} (see the discussion in the following Section III). The contribution of the higher resonances in the dispersion integrals affects the b_i's stronger than the a_i's, where $DP_i = a_i + b_i t$.

4.) A systematic study of the s-dependence may prove useful. According to the model the pion exchange effect is independent of s in $s^2\sigma_L$; $s \to \infty$. The present "intermediate" energies do not exclude the possible influence of resonances.

5.) So far nothing is known about the imaginary part of the amplitudes and their possible influence in σ_L . Measurements with polarized targets are necessary to obtain information about the imaginary parts.

III. Numerical Results and Remarks Concerning Vector Meson Dominance

III. a Results

In Fig. 3a - 3e the numerical results are presented of the best fit to $\sigma_U + \varepsilon\sigma_L$ [1] with the model amplitudes (4) and (6) of Section II. Free parameters were the constants $c_{1,2}$ of (2) for the pion formfactor and the absorption constant "a" of the subtraction function in D_5 (4e). The constant "a" was treated independent of k^2. The errors of the fitted parameters $c_{1,2}$ and "a" due to the statistical errors of the experiment are of the order 1 %. These errors are presumably negligible with respect to the systematic errors of the model and the experiment. Since the statistical errors of σ_T and σ_I are quite large we present for these functions only the results with the parameters $c_{1,2}$ and a, which were found form the fit to $\sigma_U + \varepsilon\sigma_L$. The agreement is quite good. For σ_I there are in some cases systematic deviations which remain unresolved for the moment. The zero of σ_I is primarily due to the vanishing of the transverse amplitudes for the parallel cross-section $\sigma_{\parallel} = 0.5\,(\sigma_U - \sigma_T)$. The position of this zero is partly dependent on t_{min}. The transverse nucleon helicity non flip amplitudes shift at the present energy this zero to smaller values of -t. At higher energies these amplitudes die out so that than the zeros of σ_I and $\sigma_{\parallel}$ are strictly correlated. For comparison we show also the results for $\sigma_U + \varepsilon\sigma_L$ with R=1 (eq.2) i.e. with the strict ρ-dominance formfactor $F_\pi(k^2) = (1 - k^2/m_\rho^2)^{-1}$. It is clear that the model and the experiment are incompatible under this assumption.

III. b Vector Meson Dominance (= VMD)

Two analyses of the present electroproduction data have been performed already under the hypothesis of VMD[14, 15]. Qualitative agreement is found by and large. The VMD analysis is at the present energy involved by a variety of effects which are mostly kinematical:

a) There is first of all the principal difficulty that the formfactors $F_1(k^2)$ and $F_\pi(k^2)$ may deviate from the strict ρ-dominance law $(1 - k^2/m_\rho^2)^{-1}$.

b) Secondly, there are kinematical effects due to the present low energies. For example the longitudinal amplitude with nucleon helicity flip contains the overall factor $(t - t_{min}(k^2))^{1/2}$ due to angular momentum conservation in forward direction. This leads to a correction factor C for this amplitude of the form

$$C^2 = (t-t_{min} (k^2)) / (t - t_{min}(k^2 = m_\rho{}^2)) \qquad (10)$$

At $s = 4.84\ GeV^2$ and $t = -2\mu^2$ the factor C varies in the present experiment in the limits: $0.49 = C\ (k^2 = -0.825\ GeV^2) \leq C\ (k^2) \leq C\ (k^2 = -0.180\ GeV^2) = 2.34$. Only at $k^2 = -m_\rho{}^2 = -0.585\ GeV^2$ $C = 1$. It is worthwhile to note that the VMD predictions in Refs. (14, 15) agree best at this special value of k^2, which is certainly not fortuitous.
At the present energies both the longitudinal helicity flip and non flip amplitude (which does not contain the overall factor $(t-t_{min})^{1/2}$) are for $-t < -t_{min}$ of equal order. Therefore one cannot apply directly the correction factor C^2 to the experimental quantity σ_L. Of course at higher energies, where t_{min} is much smaller this typical difficulty disappears.

c.) Finally, even at high energies there remains the ambiguity of the extrapolation through the point $k^2 = 0$ where the longitudinal amplitudes have to vanish due to gauge invariance. The prescription for σ_L given by Fraas and Schildknecht[14] applies in the context of the electric Born term model only for $s \rightarrow \infty$.

In both VMD analyses the authors try to avoid some of the difficulties by using information from pion photoproduction instead of the data for $\pi^- p \rightarrow \rho^0 n$. So in Ref. (15) the asymmetry ratio $A = \rho_{1-1}/\rho_{11}$ is taken from photoproduction, since it is known that A varies with k^2 in the timelike region $k^2 > 0$ since it depends critically on the ratio $R = F_\pi(k^2)/F_1(k^2)$ [12]. But since R may also vary strongly in the spacelike region $k^2 < 0$ as it obviously does in the timelike region, the asymmetry A has to be considered essentially unknown for small values of t. In Ref. (14) $d\sigma/dt\ (\pi^- p \rightarrow \rho^0 n)$ has been taken according to the prescription

$$d\sigma/dt\ (\pi^- p \rightarrow \rho^0 n) = \frac{\rho_{00}}{\rho_{11}}\ \frac{d\sigma}{dt}\ (\gamma^V p \rightarrow \pi^+ n) \qquad (11)$$

where ρ_{00} , ρ_{11} are the ρ-production density matrix elements. At present, the main difficulty here is that $\rho_{00}\ /\ \rho_{11}$ is essentially unknown at small values of t and low energies, so that the authors had to extrapolate from the recent 15 GeV experiment at SLAC[16)] under the assumption of energy independence of ρ_{oo} / ρ_{11}. In the framework of the model presented there is a strong energydependence of ρ_{oo} / ρ_{11} for very small values of t, which is still in complete agreement with experiment due to the large errors at low energies.

We conclude therfore from the experience with the present model that no straightforward quantitative VMD prescription applies for the electroproduction data of Ref. (1). One needs a more detailed dynamical model for the interpretation of the data at least at the present low energies. From a quantitative point of view VMD says in the present situation not much more than that the same dynamics applies for the amplitudes, which are connected by the current field identity. More specifically one may say that the same types of Feynman-diagrams have to be included in the calculation of those amplitudes.

d.) Comparison with other data[(2)] and with recent work based on dispersion relations[(17)].

After having completed our analysis we received also the data from CEA and NINA[(2)] and the two reports on the dispersion relation work[(17)]. In Fig. 4 we show therefore some data from CEA and our results with the fitted pion formfactor (2) and with the assumption $F_\pi = F_1$, which is favored by the authors. Unfortunately both results do not fit the data. With the new pion formfactor (2) the cross-sections are generally overestimated, whereas with the other assumption they are underestimated. Plotted is the quantity 0.5 $(d\sigma(\varphi = 0)/d\Omega + d\sigma(\varphi = 180^\circ)/d\Omega)$ in order to eliminate the badly predicted σ_I -contribution (and to avoid the ambiguity how φ is defined!).

The CEA-authors compare then their results with the electroproduction model of Behrends[(18)] and find good agreement for small values of (-t) and the data of Fig. 4a under the assumption that $F_\pi = F_1 = 0.566$ at $k^2 = -0.396$ GeV^2. The assumption $F_\pi = F_1$ would work with our model only if F_1 is larger. We would like to point out that according to Fig. 2 $F_1 = 0.525$ at $k^2 = -0.396$, i.e. 5 % smaller than in the CEA calculation. We assumed: $F_1 = (\chi G_M^V + G_E^V)/(1 + \chi)$,

$\chi = -k^2/4M^2$; $G_E^V = G_E^P - G_E^n$, $G_M^V = 4.7\ G_E^P$,

$G_E^n = \chi\ G_E^P$, $G_E^P = 1/(1-k^2/0.71)^2$).

The 5 % larger formfactor brings the cross-section up by 10 %, i.e. half the difference. On the other hand our model includes the ρ-contribution (6), which in this case shifts the cross-section already by 3-5 %, so that part of this effect is taken care off. The remaining discrepancy must be seeked for in the model and is presumably connected with the different treatment of the A_2 and A_5 amplitudes, which in our case are replaced by D_2 and D_5[3]. (For details and motivation see Ref. 4). In the dispersion integrals we have included the Δ(1236) multipoles $M_{1+}^{3/2}$ as well as $E_{1+}^{3/2}$, $S_{1+}^{3/2}$ taken from von Gehlen[19]. Behrends does not include $E_{1+}^{3/2}$, $S_{1+}^{3/2}$, but this does not explain the difference, seen from our model. The difference between both models is typically of the order of the systematic errors to be expected and should not be too much emphasized. The major difficulty is that according to the DESY data we would have expected larger values for the CEA points by about 10 %.

Devenish and Lyth[17] have done calculations with the same model for the isovector amplitudes as we use, except that they do not include the absorption effect in the subtraction function for D_5 (the constant "a" in (4c)1). They agree with our main conclusion that for small $k^2 \approx -0.3$ GeV^2 the present model gives too small cross-sections if $F_\pi(k^2) = (1 - k^2/m_\rho^2)^{-1}$. Devenish and Lyth concentrate mainly on the NINA data [2] for $k^2 = -0.7$ GeV^2. At this large value of $-k^2$ the difference between the ρ-dominated pion formfactor and our fit are less serious (see Fig. 2).

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C.N. Brown et al. Phys. Rev. Letters, 26, 991 (1971)

18) F. Behrends, Phys. Rev. D 1 , 2590 (1970)

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Fig. Captions

1. a) electroproduction of pions
   b) pion exchange

2. Different versions of the pion formfactor $F_\pi(k^2)$. Fit to the data with $c_1 = 0.85$ $c_2 = 0.65$ (Equ. 2): full line.
   Three assumptions for the pion form factor as indicated: broken line.

3. Electroproduction cross-sections Equ. (1) at $s = 4.84\ GeV^2$. Data Ref. (1). Full line: model with fitted formfactor Equ.(2). Broken line (Figs. 3a - 3c): model with ρ-dominance formfactor $F_\pi = (1 - k^2/m^2\rho)^{-1}$.
   t-dependence at $k^2 = -0.26, -0.55, -0.75\ GeV^2/c^2$, Figs. 3a - 3c. k^2-dependence at $t = -0.037, -0.075\ GeV^2/c^2$, Figs. 3d, 3e. In Figs. 3d, 3e the ε-values of $\sigma_u + \varepsilon\sigma_L$ vary from point to point and are given in the figure.

4. Comparison with the results of CEA (Ref. 2); plotted is the quantity 0.5 $(d\sigma(\varphi=0°)/d\Omega + d\sigma(\varphi=180°)/d\Omega)$.
   Full line: Model with F_π from fit DESY data (Ref. 1). Broken line: Model with $F_\pi = F_1$.

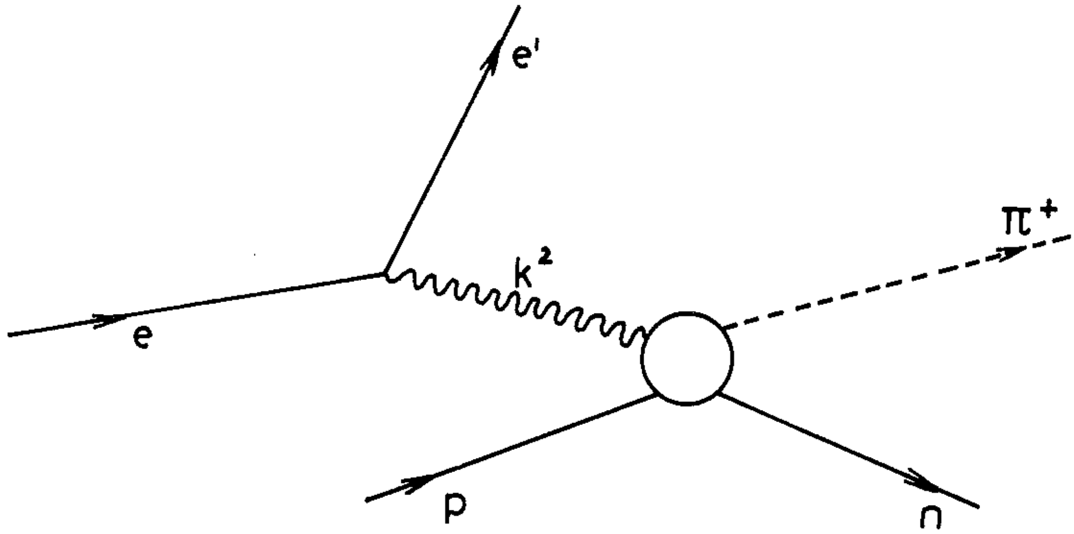


Fig. 1a

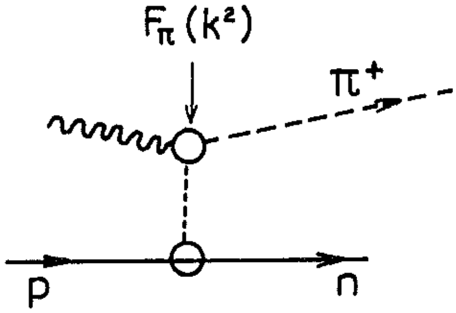


Fig. 1b

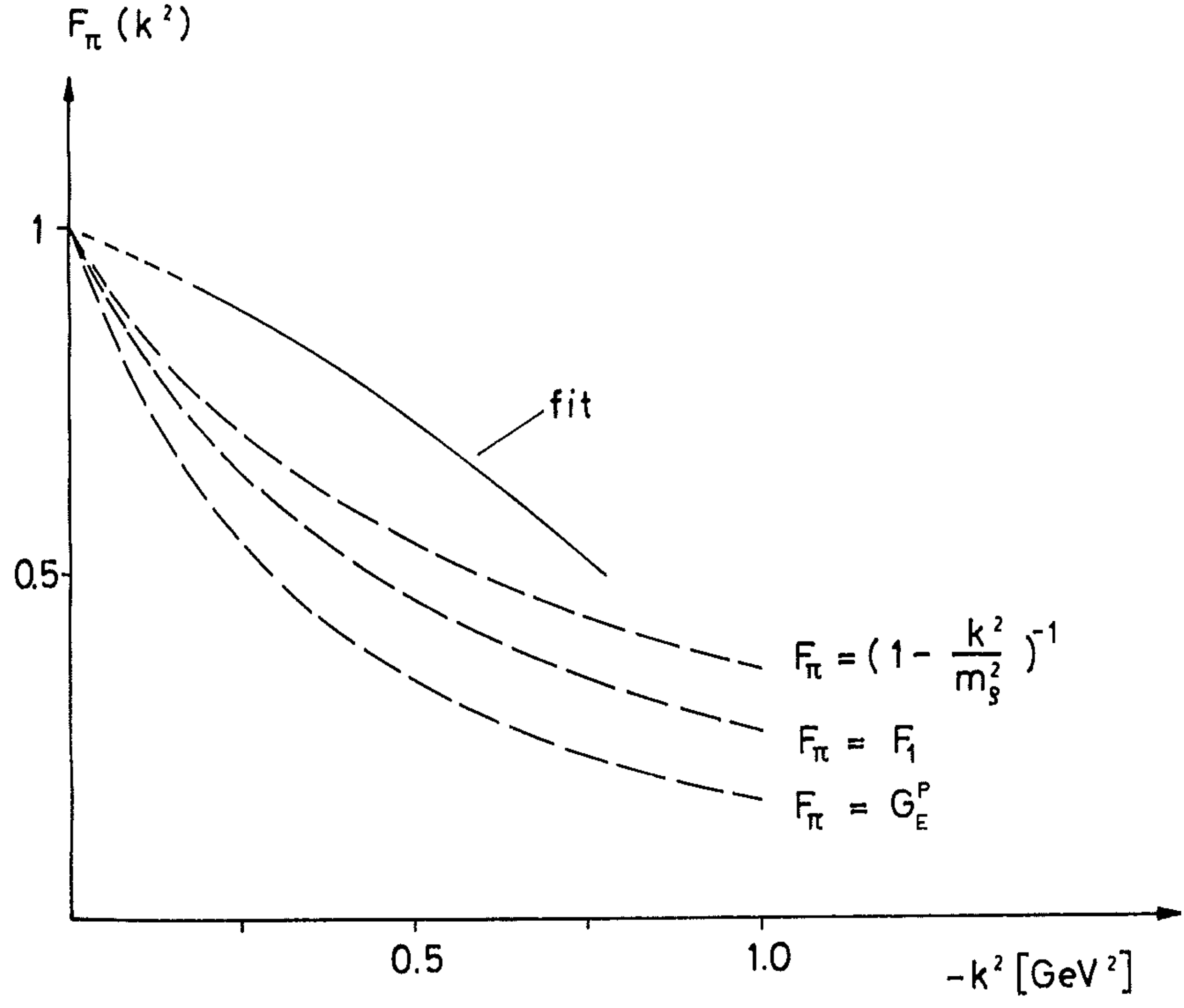
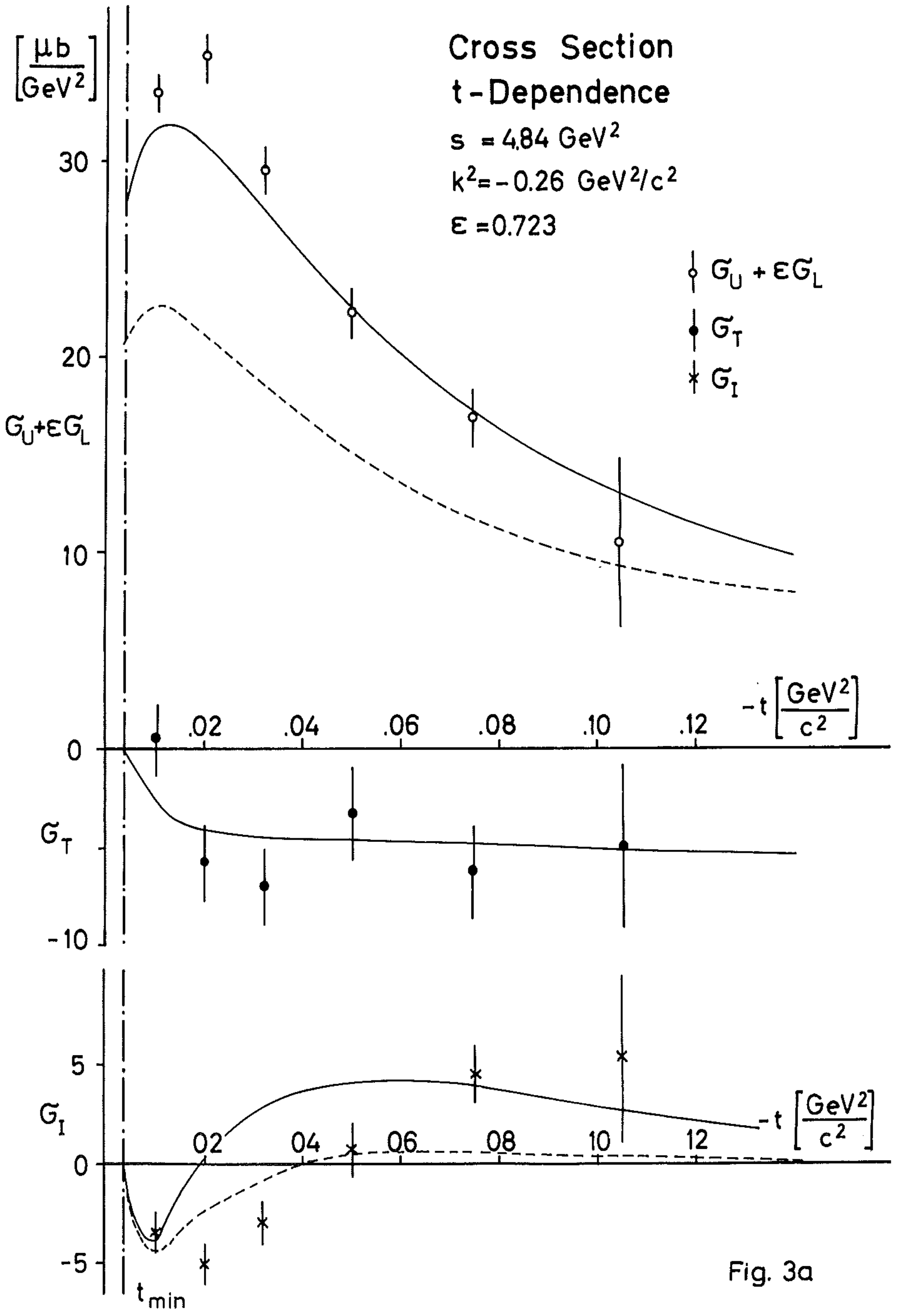


Fig. 2



Cross Section
t - Dependence
s = 4.84 GeV²
k² = -0.26 GeV²/c²
ε = 0.723
σ_U + εσ_L
σ_T
σ_I
[μb/GeV²]
30
20
10
0
-10
5
0
-5
σ_U+εσ_L
σ_T
σ_I
.02
.04
.06
.08
.10
.12
-t [GeV²/c²]
t_min

Fig. 3a

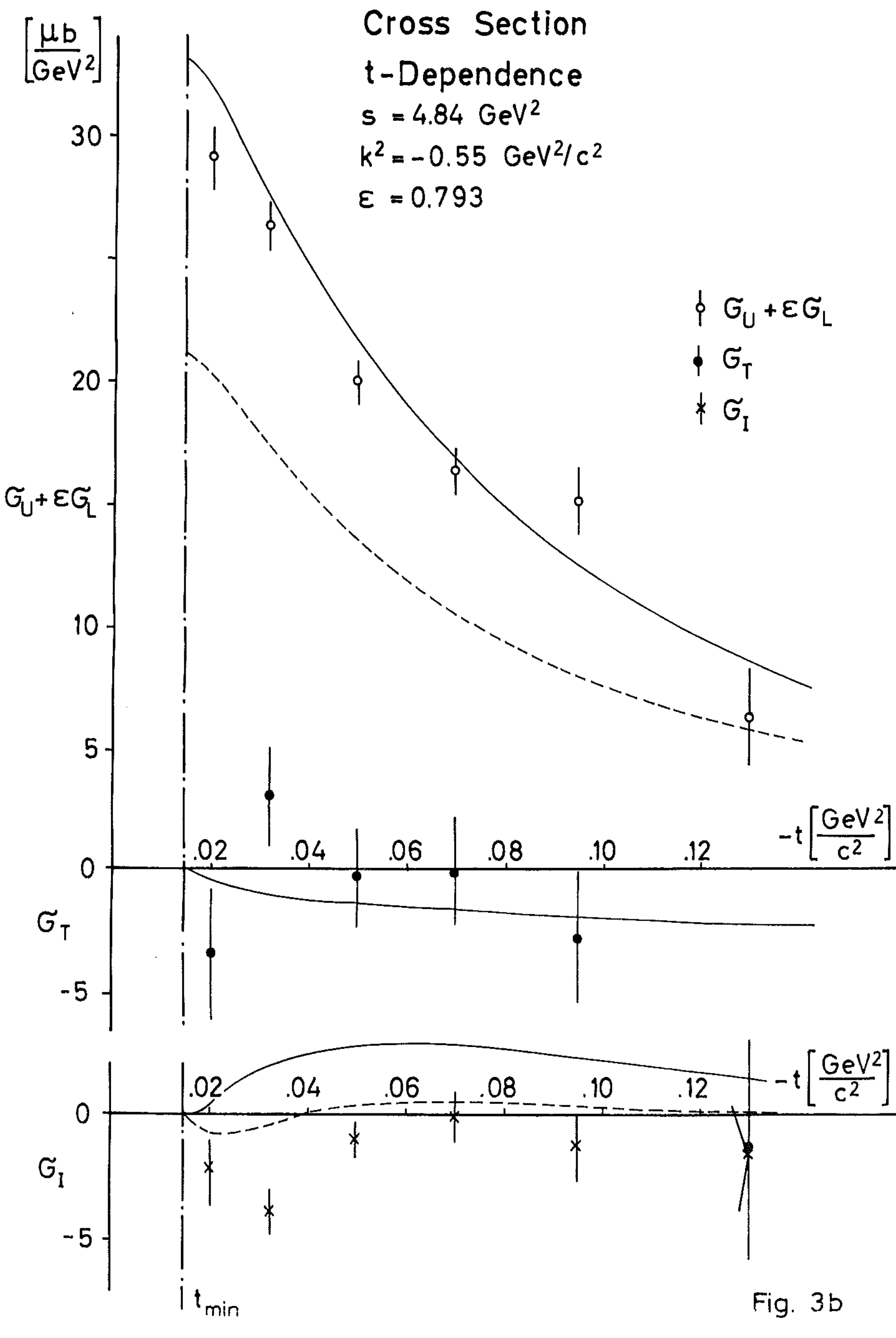
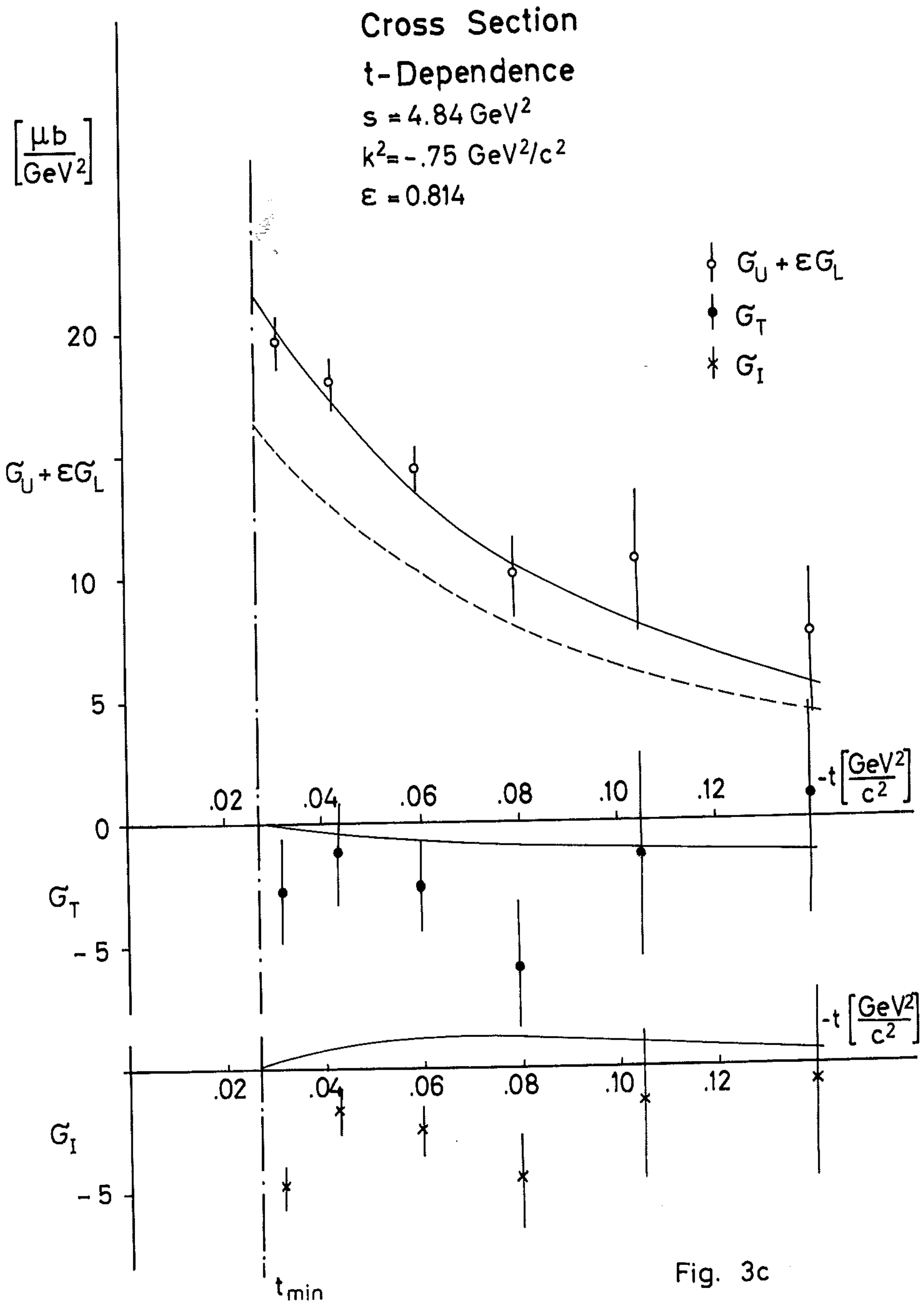

Cross Section
t-Dependence
s = 4.84 GeV²
k² = -0.55 GeV²/c²
ε = 0.793
[μb/GeV²]
30
20
10
5
0
-5
G_U + εG_L
G_T
G_I
.02
.04
.06
.08
.10
.12
-t [GeV²/c²]
t_min

Fig. 3b



Cross Section
t-Dependence
s = 4.84 GeV²
k² = -.75 GeV²/c²
ε = 0.814
[μb/GeV²]
σ_U + εσ_L
σ_T
σ_I
20
10
5
0
- 5
.02
.04
.06
.08
.10
.12
-t [GeV²/c²]
t_min

Fig. 3c

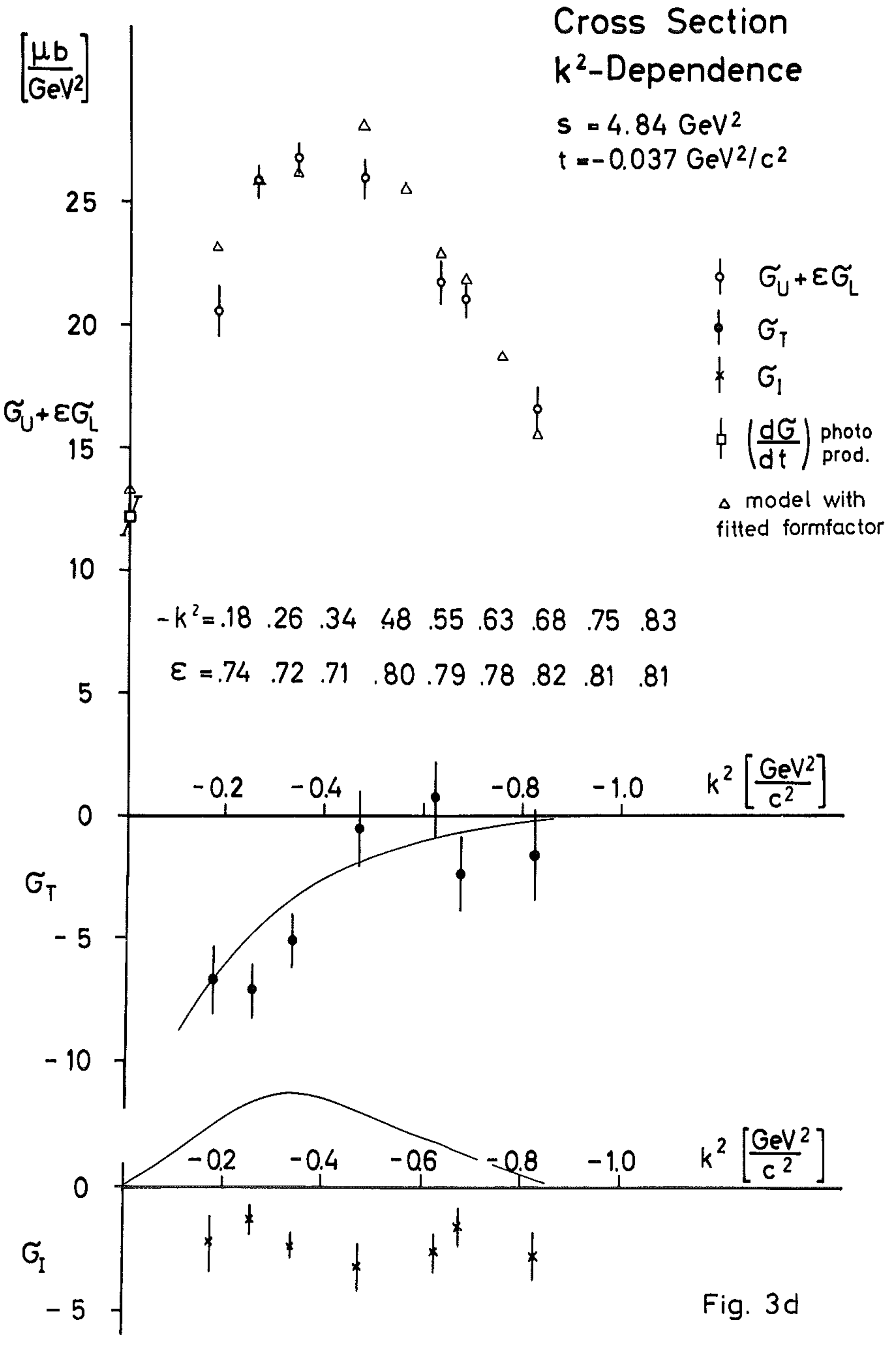

Cross Section
k²-Dependence
s = 4.84 GeV²
t = -0.037 GeV²/c²
[μb/GeV²]
25
20
15
10
5
0
-5
-10
σ_U + εσ_L
σ_T
σ_I
σ_U + εσ_L
σ_T
σ_I
(dσ/dt) photo prod.
model with fitted formfactor
-k² = .18 .26 .34 .48 .55 .63 .68 .75 .83
ε = .74 .72 .71 .80 .79 .78 .82 .81 .81
-0.2 -0.4 -0.8 -1.0
k² [GeV²/c²]
-0.2 -0.4 -0.6 -0.8 -1.0
k² [GeV²/c²]

Fig. 3d

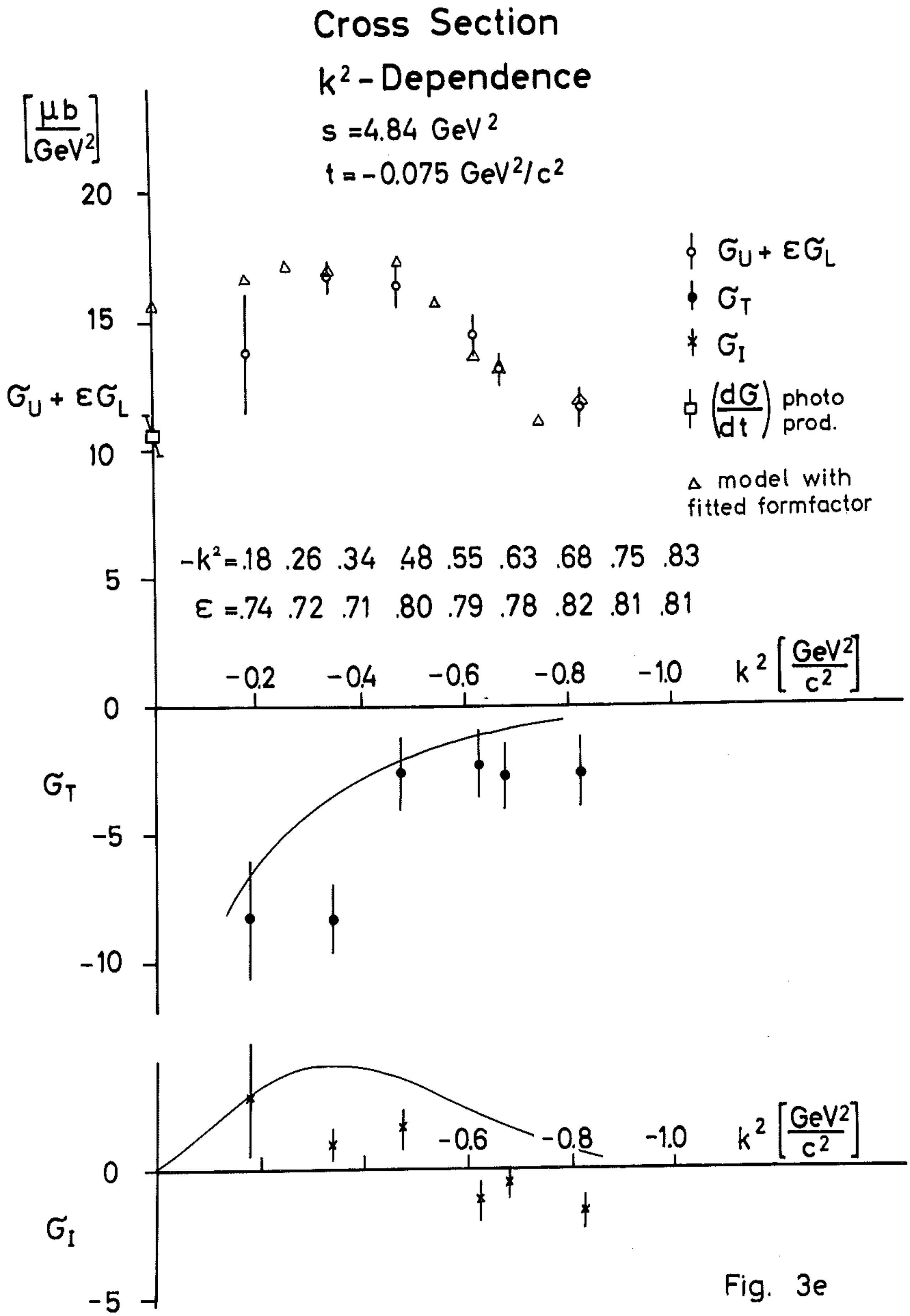

Cross Section
k² - Dependence
s = 4.84 GeV²
t = -0.075 GeV²/c²
[μb / GeV²]
20
15
σU + εσL
10
5
0
-5
-10
σT
σI
σU + εσL
σT
σI
(dσ/dt) photo prod.
model with fitted formfactor
-k² = .18 .26 .34 .48 .55 .63 .68 .75 .83
ε = .74 .72 .71 .80 .79 .78 .82 .81 .81
-0.2 -0.4 -0.6 -0.8 -1.0
k² [GeV²/c²]
-0.6 -0.8 -1.0
k² [GeV²/c²]

Fig. 3e

$$\frac{1}{2}\left(\frac{d\sigma}{d\Omega}\,(\varphi=0°) + \frac{d\sigma}{d\Omega}\,(\varphi=180°)\right)$$

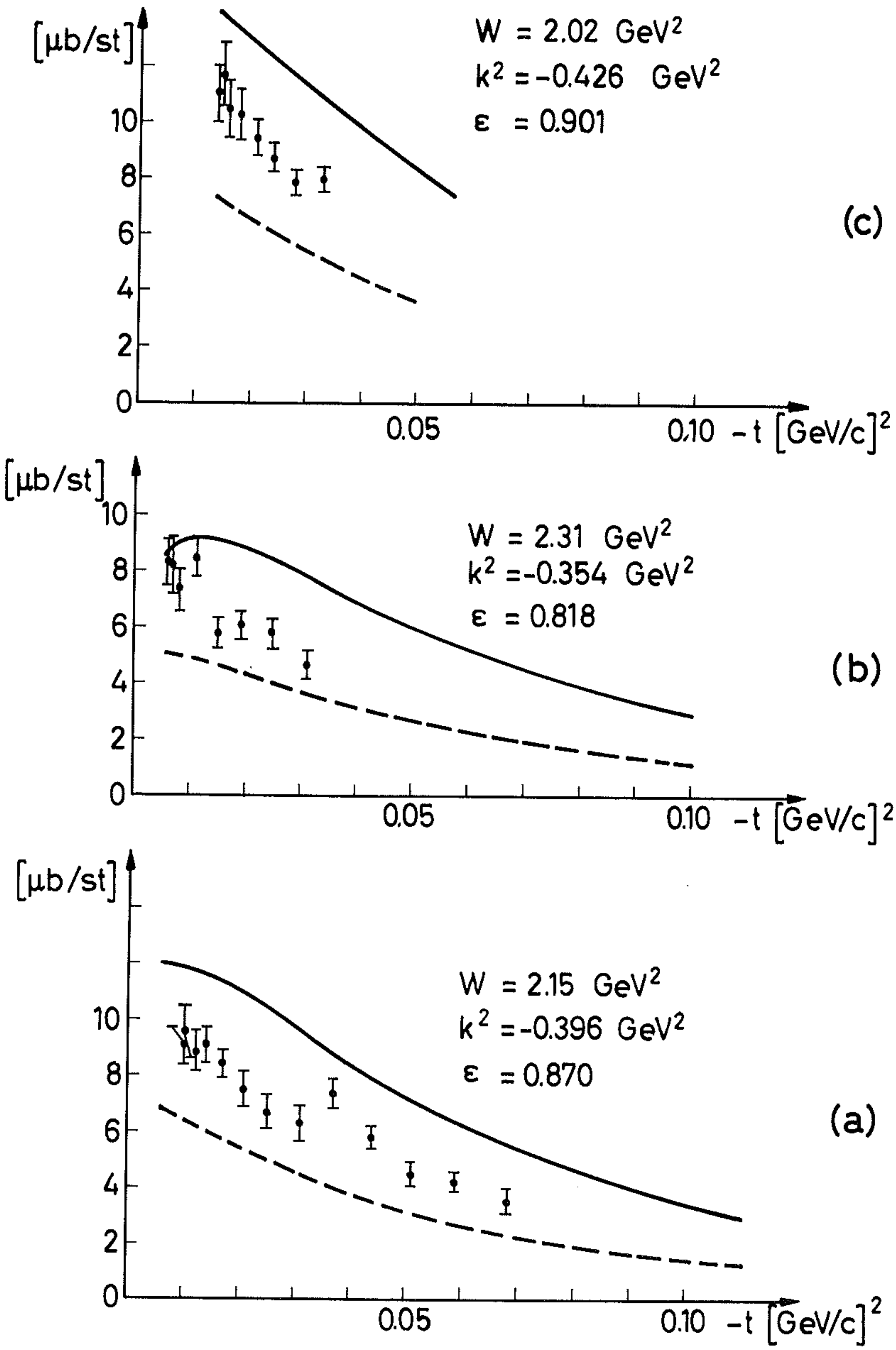


Fig. 4