

DESY 72/74  
December 1972

DESY 72/74  
23. Okt. 1972

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from the Ward Identity

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Abstract

A previously proposed Bethe-Salpeter model for  $\pi\pi$  P-wave scattering containing a nucleon loop as force is improved in two respects: First of all the connection between the electromagnetic pion vertex and the pion propagator given by the Ward identity is used to renormalize the propagator. Secondly the influence of the  $\omega\pi$ -channel via its strongly energy dependent coupling to the  $\pi\pi$  channel on the  $\rho$ -width is investigated. The results show that the propagator corrections have very strong influence, and reasonable assumptions on the  $\omega\pi\pi$  coupling constant lead to a  $\rho$ -width of 200 MeV.

## I. Introduction

In a previous paper I <sup>(1)</sup> we have studied a model for the  $\rho$ -meson, which contained pion pairs and nucleon-antinucleon pairs as the constituents of the  $\rho$ . Only an "off-diagonal" coupling between these two channels, given by nucleon exchange, has been taken into account, and this is equivalent to treating the nucleon loop as a force in the  $\pi\pi$ -channel. In order to reduce the strength of the resulting attraction a cut-off in the nucleon loop is necessary, but then the solution of the Bethe-Salpeter equation (BSE) in the  $\pi\pi$ -channel showed up a very large width similar to many bootstrap calculations <sup>(2)</sup>. We then proposed in I to include pion propagator corrections due to self-energy effects, which is easy, but also necessary in the Bethe-Salpeter approach, in order to ensure two particle unitarity <sup>(3)</sup>. We took the order of magnitude of the propagator correction from perturbation theory with nucleon loops, but since this leads to drastic changes in the propagator for off-shell pions, a perturbation argument is not convincing. Therefore we shall derive in this paper the pion propagator by an application of Ward's identity <sup>(4)</sup> in the following way: Given a model for the  $\rho$ -meson the electromagnetic vertex of the pion with both pions off-shell can be calculated <sup>(5)</sup> from the diagrams of Fig. 1. Since it turns out that the  $\rho$ -meson dominates the vertex at  $k^2 = 0$  ( $k$  = photon four momentum), which is equivalent to  $Z_1 \ll 1$  ( $Z_1$  = pion vertex renormalization constant), we expect the pion electromagnetic vertex at  $k^2 = 0$  to have a similar decrease in the pion off-shell momenta as determined for the  $\rho$ -meson vertex by the BSE. There the scale of variation is given by nucleon masses, and therefore the Ward identity requires changes in the pion propagator, compared to the unrenormalized one, in the same scale. By inserting the propagator thus determined into the BSE for the  $\pi\pi$ -amplitude, we can find a selfconsistent model.

The resulting propagator modification is slightly smaller than that derived from perturbation theory <sup>(1)</sup>, and it would lead to a large  $\rho$ -width of 500 MeV (after including the  $K\bar{K}$  channel besides the  $\pi\pi$ -channel). We therefore investigate, again with the BSE,

the influence of the admixture of the  $\pi\omega$ -channel in the  $\rho$ -wave function. This channel is especially important since the  $\pi\pi \rightarrow \pi\omega$  transition matrix element has, via the magnetic character of the spin coupling, an extra factor of  $^*) W = \sqrt{s}$ , and an attractive force, increasing with energy, will clearly make any resonance narrower. We shall derive the  $\pi\pi \rightarrow \pi\omega$  amplitude again from a nucleon loop diagram, and calculate the  $\rho$ -width as a function of the not too well known  $\omega NN$ -coupling constant. We shall find that values of  $g_{\omega NN}^2/4\pi$  within the presently discussed range lead to a reasonable  $\rho$ -width.

In Section II we present the calculation for the  $\pi\pi \rightarrow \pi\omega$ -channel, and in Section III we determine the renormalized propagator  $\Delta'_\pi(q^2)$  selfconsistently. Section IV contains numerical results, and we give a summary in Section V.

## II. The nucleon loop contribution to $\pi\pi \rightarrow \pi\omega$

The diagram of Fig. 2 with two directions for the nucleon momenta gives the following invariant amplitude for  $\pi\pi \rightarrow \pi\omega$   $^*)$ :

$$M_{\pi\omega}^{\text{loop}}(k_i, e_\mu) = \frac{g^3 g_{\omega NN}}{(4\pi)^2} \frac{4}{\pi^2} \phi_3^* \cdot (\phi_1 \times \phi_2) \times e_\mu K^\mu, \quad (1)$$

with  $g^2/4\pi = 14.6$ , and

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$^*)$  See Appendix and Fig. 2 for definitions of momenta etc.

$$\begin{aligned}
 K_{\mu} &= i \int d^4 q \frac{\gamma_5 (\not{P}_1 + M) \gamma_5 (\not{P}_2 + M) \gamma_5 (\not{P}_3 + M) \not{P}_4 (\not{P}_4 + M)}{\prod_{i=1,4} (\not{P}_i^2 - M^2)} \\
 &= 4iM \epsilon_{\mu\nu\rho\sigma} k_1^\nu k_2^\rho k_3^\sigma i \int \frac{d^4 q}{\prod_{i=1,4} (\not{P}_i^2 - M^2)} .
 \end{aligned} \tag{2}$$

The two normalized amplitudes with helicity  $\sigma = \pm 1$ , spin 1, l-spin = 1 are therefore

$$\begin{aligned}
 M_{\sigma, \pi\omega}^{\text{loop}}(k_i^2, s) &\approx \pm i \frac{g^3 g_{\omega NN}}{(4\pi)^2} \frac{16\sqrt{2}}{3\pi^2} WM |\vec{k}_1| |\vec{k}_3| \times \\
 &\times \frac{1}{2} \int_{-1}^{+1} dX i \int \frac{d^4 q}{\prod_{i=1,4} (\not{P}_i^2 - M^2)}
 \end{aligned} \tag{3}$$

where  $X$  is the cosine of the CMS scattering angle between  $k_1$  and  $k_3$ . In eq. (3) we neglected D-wave contributions in the last integral, which is a very good approximation.

After the Wick rotation we have

$$\begin{aligned}
 M_{\sigma, \pi\omega}^{\text{loop}}(k_i^2, s) &= \mp i \frac{g^3 g_{\omega NN}}{(4\pi)^2} \frac{64\sqrt{2}}{3\pi} WM |\vec{k}_1| |\vec{k}_3| \times \\
 &\times \int_{-\infty}^{+\infty} d\tau \int_0^{\infty} \frac{d|\vec{q}| |\vec{q}|^2}{(\not{P}_1^2 - M^2)(\not{P}_3^2 - M^2)} \frac{Q_0(z) Q_0(z')}{4|\vec{k}_1| |\vec{k}_3| |\vec{q}|^2},
 \end{aligned} \tag{4}$$

$$z = (M^2 - k_1^2 - P_1^2 + 2P_{10} k_{10}) / 2|\vec{k}_1| |\vec{q}|,$$

$$z' = (M^2 - k_3^2 - P_1^2 + 2P_{10} k_{30}) / 2|\vec{k}_3| |\vec{q}|,$$

$Q_\ell(z)$  = Legendre-function of second kind.

Eq. (3) has to be compared with our result for the  $\pi\pi$  amplitude (I)

$$M_{\pi\pi}^{\text{loop}}(k_i^2, s) = -\left(\frac{g^2}{4\pi}\right)^2 \frac{64}{3\pi} \int_{-\infty}^{+\infty} dt \int_0^{\infty} \frac{d|\vec{q}| |\vec{q}|^2}{(P_1^2 - M^2)(P_3^2 - M^2)} \frac{1}{4|\vec{k}_1||\vec{k}_2||\vec{q}|^2} \times \left\{ (s - P_1^2 - P_3^2 + 2M^2) |\vec{k}_1| |\vec{k}_3| Q_0(z) Q_0(z') + \right. \quad (5)$$

$$\left. + \text{correction terms} \right\}$$

The correction terms in (5) are given by

$$(s - P_1^2 - P_3^2 + 2M^2) \left[ \frac{3}{2} (k_1^2 + k_3^2 - 2k_{10}k_{30}) Q_1(z) Q_1(z') + 2 |\vec{k}_1| |\vec{k}_3| Q_2(z) Q_2(z') \right]$$

$$- \frac{3}{2} \left[ k_1^2 k_4^2 + k_2^2 k_3^2 (P_1^2 - M^2)(P_3^2 - M^2) - (k_1^2 + k_3^2)(P_3^2 - M^2) - (k_2^2 + k_4^2)(P_1^2 - M^2) \right] Q_1(z) Q_1(z') \quad (6)$$

We shall estimate the relative contribution of the  $\pi\omega$  intermediate state in the following way: The ratio of the transition amplitude  $M_{\pi\omega}^{\text{loop}}$  to the elastic scattering amplitude is approximately at the  $\rho$ -mass (including a total factor 2 for the two helicity states and the two possible  $\pi\omega$  intermediate diagrams)

$$R = \frac{M_{\pi\omega}^{\text{loop}}}{M_{\pi\pi}^{\text{loop}}} \approx \frac{g_{\omega NN} \bar{N}}{g} \frac{2\sqrt{2} M W}{s + 2M^2 - \bar{P}_1^2 - \bar{P}_3^2} \quad (7)$$

rather independently of  $k_i^2$ ,  $i = 1 \dots 4$ .

Here  $\bar{P}_1^2$ ,  $\bar{P}_3^2$  are the mean values of  $p_1^2$  and  $p_3^2$  in the integration of eq. (5). Taking into account the correction terms in (5), numerical calculations show that

$$R_{W=m_p} \approx \frac{g_{\omega NN}}{g} \frac{2\sqrt{2}}{3} . \quad (8)$$

If  $g_{\omega NN} < g$  and if the additional suppression of the  $\pi\omega$  state due to the higher threshold is considered, we can treat the  $\pi\omega$ -channel as a perturbation compared to the  $\pi\pi$ -channel. This permits solving a modified one-channel BSE instead of a two-channel BSE: If the  $\pi\pi$ -BSE reads in compact form

$$M_{\pi\pi} = M_{\pi\pi}^{\text{loop}} + M_{\pi\pi}^{\text{loop}} \Delta'_{\pi}(1) \Delta'_{\pi}(2) M_{\pi\pi} , \quad (9)$$

we solve instead of (9)

$$M_{\pi\pi} = M_{\pi\pi}^{\text{loop}} + M_{\pi\pi}^{\text{loop}} (\Delta'_{\pi}(1) \Delta'_{\pi}(2) + R^2 \Delta'_{\pi}(1) \Delta'_{\omega}(2)) M_{\pi\pi} . \quad (10)$$

This simplification leads to a fictitious  $\pi\omega \rightarrow \pi\omega$  coupling, which is very small however for  $R < 0.5$ .

Similarly we include the  $K\bar{K}$ -channel, the relative strength of which is determined by SU(3) alone, so that our final BSE will be

$$M_{\pi\pi} = M_{\pi\pi}^{\text{loop}} (\Delta'_{\pi}(1) \Delta'_{\pi}(2) + R^2 \Delta'_{\pi}(1) \Delta'_{\omega}(2) + .5 \Delta'_{K}(1) \Delta'_{K}(2)) M_{\pi\pi} . \quad (11)$$

Here we have assumed that the renormalized K- and  $\omega$ -propagators are the same as the pion propagator. While for the K this is

reasonable within SU(3) symmetry, for the  $\omega$  it is not obvious since the  $\omega$  is in our picture a composite particle as the  $\rho$  for which self-energy corrections have no clear meaning. Our assumption is motivated by the fact that besides the  $\omega$ -pole contribution we also have to consider uncorrelated  $K\bar{K}$  states in relative p-waves etc., which probably have a slower decrease in momentum space than the  $\omega$ -propagator.

We now turn to a determination of the renormalized propagator and to the discussion of the necessary cut-off.

### III. Vertex and Propagator corrections

Our model requires a reduction of the nucleon loop diagram  $M_{\pi\pi}^{\text{loop}}$  by a cut-off procedure, since the iteration of the unmodified loop diverges badly (I). Since we want to separate vertex and propagator effects more carefully as in I, we introduce a (spin independent) vertex function for the  $\pi N\bar{N}$ -coupling, depending on all three momenta symmetrically:

$$g\gamma_5 \longrightarrow \frac{g\gamma_5}{1 - (P_i^2 + P_j^2 + k_k^2 - 2M^2 - \mu^2)/\Lambda^2} \quad (12)$$

The cut-off mass  $\Lambda$  is the most important parameter in our model, and it will be determined by requiring the resonance to occur at the correct position <sup>\*</sup>).

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<sup>\*</sup>) Its relation to other BSE-calculations is discussed in I.



In analogy to the pion propagator we assume that also the nucleon propagators in the box diagram are modified by self-energy corrections. For simplicity we take these again as spin independent and determine them <sup>\*\*</sup>) by lowest order perturbation theory <sup>(6)</sup> (see Fig. 3):

$$\frac{1}{\not{p} - M} \Rightarrow \frac{1}{\not{p} - M} \left\{ 1 - \frac{3}{8} \frac{g^2}{4\pi} \frac{\not{p}^2 - M^2}{\pi} \int_0^\infty \frac{ds}{(M+\mu)^2 s^2 (s - \not{p}^2)} \times \right. \\ \left. \times \sqrt{(s - (M-\mu)^2)(s - (M+\mu)^2)} \left( 1 - \frac{\mu^2 (s + M^2)}{(s - M^2)^2} \right) \right\} \quad (13)$$

The pion propagator  $\Delta_\pi(q^2)$  will now be derived from the e.m. pion vertex  $\Gamma_\mu(q, k)$ ,

$$\Gamma_\mu(q, k) = 2q_\mu F_\pi(q^2, k^2, q \cdot k) + k_\mu G_\pi(q^2, k^2, q \cdot k) \quad (14)$$

where the form factor  $G_\pi(q^2, k^2, q \cdot k)$  vanishes for equal pion masses, i.e. for  $q \cdot k = 0$ .

Then Ward's identity <sup>(4)</sup> gives us, at  $s = k^2 = 0$

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<sup>\*\*</sup>) Actually we have cut-off the integral in (13) at  $7 M^2$  and approximated it by the following simple pole term:

$$\frac{1}{\not{p} - M} \left\{ 1 - 2 \frac{\not{p}^2 - M^2}{\not{p}^2 - 4M^2} \right\}$$

$$\frac{d}{dq^2} \Delta'^{-1}(q^2) = F_\pi(q^2, 0, 0) \quad (15)$$

The pion vertex function  $F_\pi(q^2, 0, 0)$  will be calculated within our model from the assumption that the photon couples only to pions and kaons, but not to nucleons<sup>\*</sup>). Thus we get (5), assuming the same vertex renormalization constant  $Z_1$  for  $\pi$  and  $K$ ,

$$F_\pi(q^2, 0, 0) = Z_1 \left\{ 1 + \frac{i}{\pi^2} \int_{-\infty}^{+\infty} d\tau \int_0^\infty d|\vec{q}'| |\vec{q}'|^2 \frac{|\vec{q}'|}{|\vec{q}|} M_\pi(q', q', q^2, q^2, s) \times \right. \\ \left. \times (\Delta_\pi'^2(q'^2) + .5 \Delta_K'^2(q'^2)) \right\} \quad (16)$$

Here  $q'^2 = -(\tau^2 + \vec{q}'^2)$ ,  $q^2 = -\vec{q}^2$ , and the vertex renormalization constant  $Z_1$  is determined by

$$F_\pi(\mu^2, 0, 0) = 1. \quad (17)$$

The vertex function  $F_\pi(q^2, 0, 0)$  is a smoothly decreasing function for increasing negative values of  $q^2$ , and it can be very well approximated by

$$\bar{F}_\pi(q^2, 0, 0) \approx Z_1 + \frac{1 - Z_1}{\left(1 - \frac{q^2 - \mu^2}{C^2}\right)^n}, \quad (18)$$

$$n \approx 2.$$

\* ) Before we have included the nucleon electromagnetic vertex, we cannot calculate the nucleon vertex renormalization constant  $Z_N$ . It has been checked however, that the last diagram in Fig. 1) leads to a similar pion vertex function as the pion loop, because of  $\rho$ -dominance. There are no cancellations between the pion and nucleon contributions because of  $Z_1 \geq 0$ ,  $Z_N \geq 0$ .

From (15) we then obtain

$$\Delta'(q^2) \approx \left\{ Z_1(q^2 - \mu^2) - C^2 \frac{1-Z_1}{n-1} \left( 1 - \frac{1}{\left( 1 - \frac{q^2 - \mu^2}{C^2} \right)^{n-1}} \right) \right\}^{-1} \quad (19)$$

Unfortunately  $Z_1$  is, in our model, not well determined. The integral for  $F_\pi(q^2, 0, 0)$  in (16) receives important contributions from  $-q^2 > 3 M^2$ , since it does not contain, as compared to the BSE, the decreasing kernel  $M_{\pi\pi}^{\text{loop}}(q'^2, q'^2, q^2, q^2, s)$ . At large  $-q^2$  however,  $\Delta'_\pi(q^2)$  is given by

$$\Delta'_\pi(q^2) \xrightarrow{q^2 \rightarrow \infty} \frac{1}{Z_1 q^2} \quad (20)$$

Thus the determination of  $Z_1$  by eq. (16), (17) and (19) is a nonlinear problem, which depends strongly on the asymptotic behavior of  $M_{\pi\pi}^{\text{loop}}(k_i^2, s)$  for  $-k_i^2 \rightarrow \infty$ . With our parametrization (12) for the  $\pi N\bar{N}$  vertex we obtain selfconsistency for  $Z_1 \approx 0.1$ . The inclusion of the  $\gamma N\bar{N}$  coupling would decrease this value. The following results, however, will be based on the assumption

$$Z_1 = 0, \quad (21)$$

which allows for an asymptotic decreasing pion on-shell form factor, and which follows from conformal invariance arguments (7). Our results would not change drastically, if we set  $Z_1 = 0.1$ . Thus, with  $Z_1 = 0$  and assumed values for  $C$  and  $n$  in eq. (15), we solve eq. (11) and vary the cut-off mass  $\Lambda$  in (12) until the  $\pi\pi$  p-wave phase  $\delta_1$  satisfies  $\delta_1(m_\rho) = \frac{\pi}{2}$ . Then the solution of (11) is inserted into eq. (16) in order to calculate  $F_\pi(q^2, 0, 0)$ , which determines new values of  $C$  and  $n$  in eq. (19). We shall present the results of this procedure in the next section.

#### IV. Numerical results

The iterative procedure outlined before converges extremely rapidly, since the "output" values for  $C^2$  and  $n$  depend very weakly on the input values, on  $\Lambda$  and on  $\lambda \equiv g_{\omega NN}^2/4\pi$ . We shall use the values

$$\begin{aligned} C^2 &= 1,95 \text{ GeV}^2 \\ n &= 2, \end{aligned} \tag{22}$$

which were the result of the iteration with  $\lambda = 12$ , for all values of  $\lambda$ . In Fig. 3) we show how the width  $\Gamma_\rho$  of the  $\rho$ -meson depends on  $\lambda$ . If we ignore the self-energy contributions to the pion propagator and set  $\lambda = 0$ , we obtain a width of  $\Gamma_\rho = 920 \text{ MeV}$ .

We see from Fig. 3) that we need  $\lambda > 10$  in order to obtain a value for  $\Gamma_\rho$  below 200 MeV. This coupling constant is considerably larger than what one would expect from  $\omega$ -dominance for the isoscalar nucleon charge combined with the colliding beam result (8) for the  $\gamma\omega$ -coupling constant, which would give

$$\lambda = 4.6 \pm 0.5 \tag{23}$$

However, it should be kept in mind, that in addition to the  $\pi\omega$ -channel the 4 different  $K\bar{K}^*$  channels together contribute, forgetting mass differences, as much as the  $\pi\omega$  state, and there is also the  $\eta\rho$  state whose influence depends on the  $\eta\chi^0$  mixing angle. We therefore consider  $\lambda \approx 10$  as a reasonable effective coupling for the sum of vector-pseudoscalar meson contribution.

One attractive aspect of our model is that the  $\pi\omega$  intermediate state is still not dominant in the wave function of the  $\rho$ -meson.

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\*)  $\Gamma_\rho$  is defined as in I.

A convenient way of expressing this is to consider the  $\pi\omega$ -contribution in the second order iteration of (11), formally written as

$$M_{\pi\pi}^{(2)} = M_{\pi\pi}^{\text{loop}} \left\{ \Delta'_{\pi}(1) \Delta'_{\pi}(2) + R^2 \Delta'_{\pi}(1) \Delta'_{\omega}(2) + .5 \Delta'_{\kappa}(1) \Delta'_{\kappa}(2) \right\} M_{\pi\pi}^{\text{loop}} \quad (24)$$

Taking  $\lambda = 12$  we obtain for the ratio of  $M_{\pi\pi}^{(2)}$  to the same quantity with  $R^2 = 0$

$$\frac{M_{\pi\pi}^{(2)}(\lambda = 12)}{M_{\pi\pi}^{(2)}(\lambda = 0)} = 1.42 \quad (25)$$

which indicates that the  $\pi\pi$ -state is still dominant. On the other hand the  $\pi\omega$  admixture is so large that our simplification of the two-channel problem, as given by (11), is no longer justified. The fictitious  $\pi\omega \rightarrow \pi\omega$  amplitude will be small up to  $\lambda \leq 8$ , so that the results for the width  $\Gamma_{\rho}$  in Fig. 3) beyond that value are subject to minor modifications in a correct two-channel calculation.

Another number of interest is the additional attraction given by the propagator correction terms as compared to the bare propagators ( $Z_1 = 1$ ). Proceeding as before we obtain for the ratio of the second order contribution with a fixed cut-off mass  $\Lambda^2 = 3 M^2$

$$\frac{M_{\pi\pi}^{(2)}(Z_1 = 0, \lambda = 0)}{M_{\pi\pi}^{(2)}(Z_1 = 1, \lambda = 0)} = 2.6, \quad (26)$$

which indicates the strong influence of the self-energy contributions in the  $\rho$ -wave function.

## V. Summary and Discussion

We have constructed a completely relativistic model which incorporates the coupling of the  $\rho$ -meson to 4 channels, namely  $\pi\pi$ ,  $K\bar{K}$ ,  $\pi\omega$  and  $N\bar{N}$  and which produces, with reasonable assumptions on  $g_{\omega NN}^2/4\pi$ , a width  $\Gamma_\rho \approx 200$  MeV. Since we have included self-energy corrections into the meson and nucleon propagators, we expect that we now can perform rather realistic calculations for electromagnetic form factors. Especially the vertex renormalization constant  $Z_1$  can be taken equal zero, so that form factors decreasing as some power for  $k^2 \rightarrow -\infty$  are likely to emerge. This is not possible within the bare propagator ladder approximation, if the "potential" is smooth, since then the integral in (16) is well convergent.

The relation of this approach to previous work on  $\pi\pi$ -scattering is not simple. In the bootstrap method the large  $\rho$ -width obtained usually <sup>(2)</sup> seems to be a consequence of two facts: First of all with cut-off masses in the GeV-range there are in the  $\pi\pi$  channel no really short range forces present, and secondly the  $\rho$  coupling in the crossed channel has to be large in order to provide enough attraction. These facts are no special features of N/D-calculations, but persist also in the relativistic Schrödinger equation <sup>(9)</sup>. The inclusion of the  $\pi\omega$ -channel does not change the situation drastically <sup>(9,10)</sup> as long as the  $\pi\rho\omega$ -coupling is kept at its physical value, since the admixture of the  $\pi\omega$ -state is not very large and one still needs the strong attraction of  $\rho$ -exchange with a large coupling constant  $g_{\rho\pi\pi}$ . In our model the effective attraction is strongly enhanced by the propagator corrections, as indicated in eq. (26), and we can allow for a corresponding weakening of the on-shell  $\pi\pi$ -potential by lowering the cut-off mass  $\Lambda$ . A small  $\rho$  width can be obtained in the relativistic Schrödinger equation <sup>(11)</sup>, if the necessary cut-off ( $\rho$ -exchange provides a singular potential) is chosen around 10 M. Although we are far from understanding our parameter  $\Lambda$ , such high cut-off masses seem difficult to interpret as vertex corrections alone within the BSE.

A very narrow resonance ( $\Gamma_\rho \sim 35$  MeV) obtained by a second order calculation within the  $\sigma$ -model<sup>(12)</sup>. The reason for this is, that in first order the  $\pi\pi$ -amplitude is small, and so is the absorptive part in second order. The real part in second order however receives important contributions from 4  $\pi$  intermediate states, which are larger than the ladder-type iterations of the first order. Whether the small width would persist in higher order calculations, is not clear.

As stated in I, we do not think that our model is fundamentally different from the Fermi-Yang model<sup>(13)</sup> for the vector mesons, since the  $\pi\pi$  and  $\pi\omega$  annihilation diagrams in the nucleon anti-nucleon channel may be regarded as the short range part of meson exchange in the nucleon t-channel.

#### Acknowledgement

This work was stimulated by calculations on nucleon form factors done in collaboration with Dr. J. Fleischer. The author is indebted to Prof. K. Symanzik to helpful comments for the case  $Z_1=0$ . Discussions with Dr. M. Krammer on the possible SU(3) extension were very useful.

## Appendix

### List of Notation

$M$	=	nucleon mass
$\mu$	=	pion mass
$\Delta'(q^2)$	=	renormalized pion propagator
$\phi_i$	=	pion isospin wave functions
$p_{i,\mu}$	=	nucleon loop momenta (see Fig. 2)
$\epsilon_\mu$	=	$\omega$ polarization four vector
$k_\mu$	=	photon four momentum
$s$	=	$W^2 = k^2$
$k_{i,\mu}$	=	pion ( $\omega$ ) four momenta (see Fig. 2)
$g^2/4\pi$	=	14.5



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## Figure Captions

- Fig. 1: Diagrams for the pion electromagnetic vertex.  
 $Z_1$  is the pion (kaon) vertex renormalization constant,  
 $Z_N$  is the same for the nucleon.
- Fig. 2: Momenta for the baryon loop diagram, describing the  
 $\pi\pi \rightarrow \pi\omega$  amplitude. The total four momentum is  $k$ , and  
 $\epsilon_\mu$  denotes the  $\omega$  polarization vector. The  $\omega N\bar{N}$  coupling  
is taken as a pure  $\gamma_\mu$  vertex.
- Fig. 3: Results for the width of the  $\rho$  meson as a function of  
 $\lambda = g_{\omega N\bar{N}}^2/4\pi$ . Also indicated is the variation of the  
cut-off mass  $\Lambda^2$  (eq. 12) with  $\lambda$ .

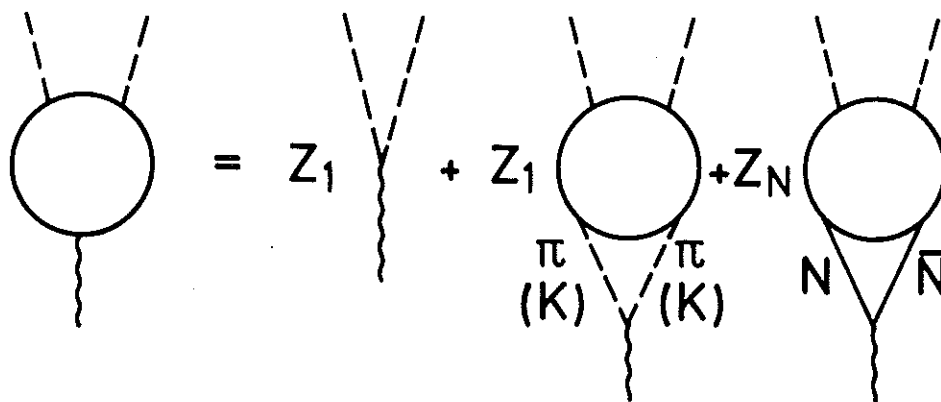


Fig. 1

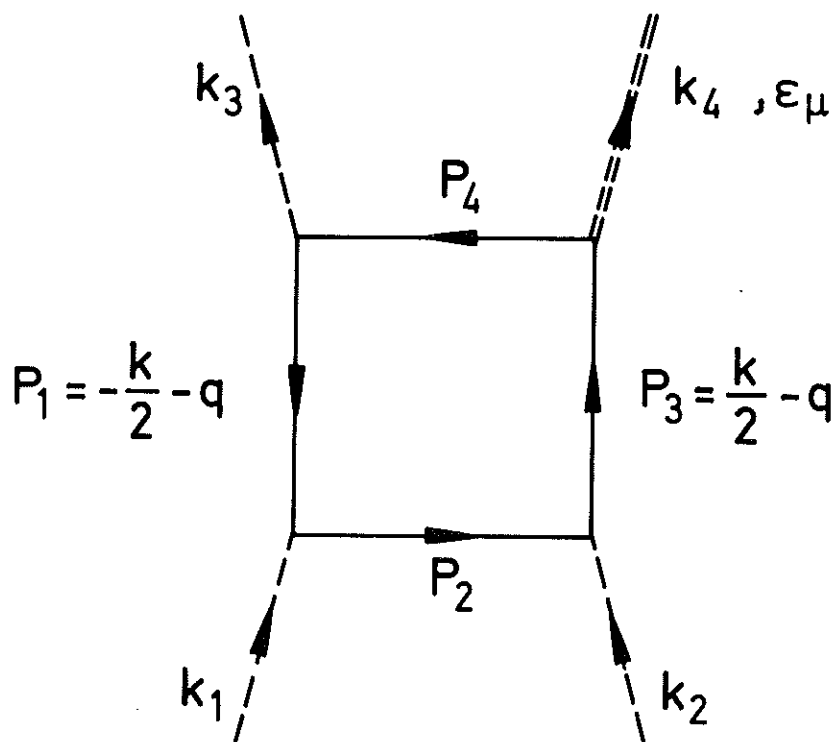


Fig. 2

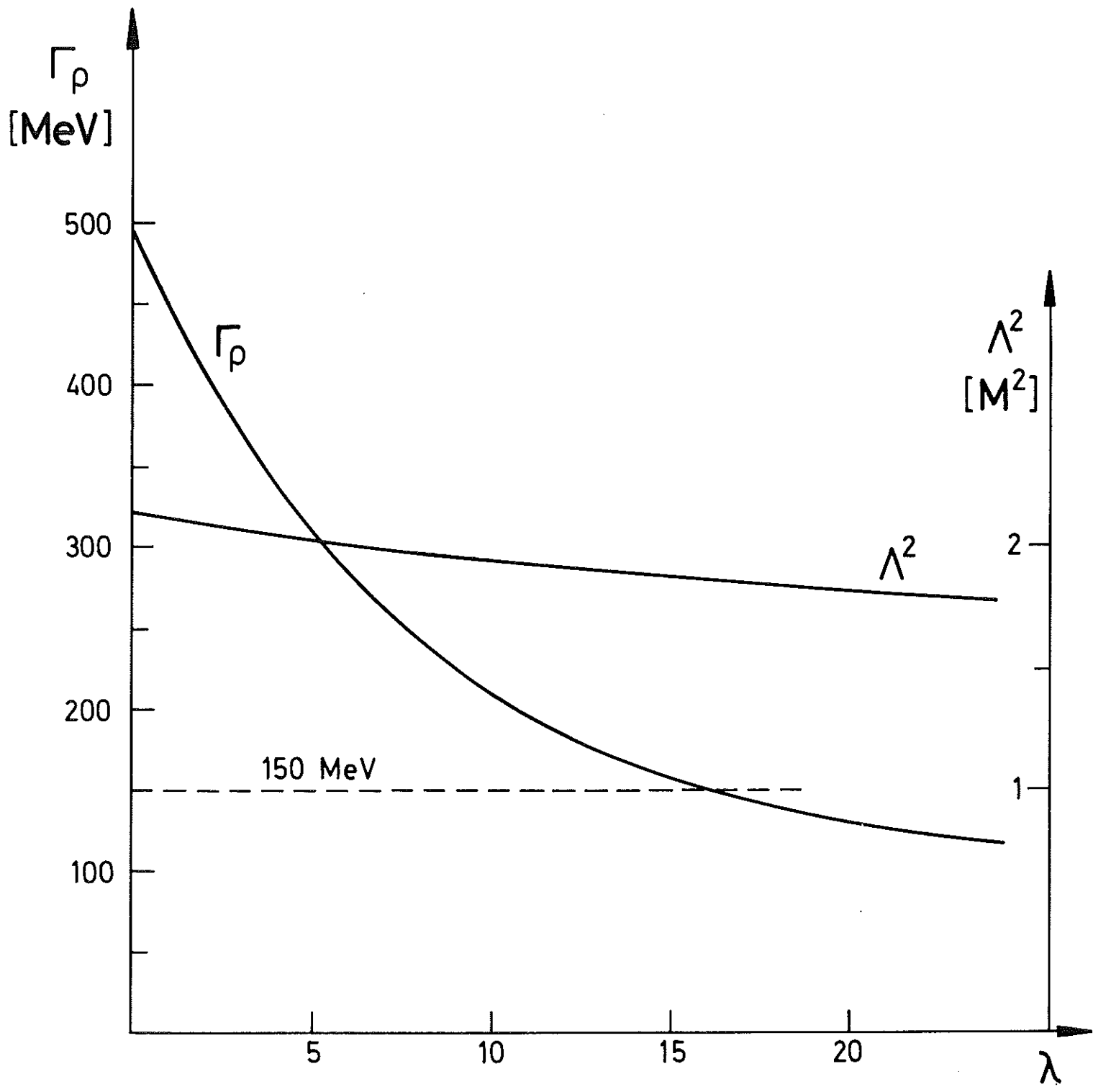


Fig. 3