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HEAVY QUARK SYMMETRY AT LARGE RECOIL: THE CASE OF BARYONS

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Abstract

We analyze the large recoil behaviour of heavy baryon transition form factors in semi-leptonic decays. We use a generalized Brodsky- Lepage hard scattering formalism where diquarks are considered as quasi-elementary constituents of baryons. In the limit of infinitely heavy quark masses the large recoil form factors exhibit a new model-independent heavy quark symmetry which is reminiscent but not identical to the Isgur-Wise symmetry at low recoil.

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1. INTRODUCTION

In a recent paper we have discussed a new type of heavy quark symmetry in semi-leptonic decays of mesons at large recoil [1]. We have shown that in this kinematical region the heavy quark effective theory (HQET) results, originally derived for the low recoil region, also hold once the transition form factors are divided by the meson decay constants. That is, in the formal limit of infinite quark masses, the 6 hadronic form factors describing the $\bar{B} \to D, D^*$ transitions, can be expressed in terms of a single universal function of the velocity transfer variable ω, the Isgur-Wise function $\xi(\omega)$ at low recoil [2] or the Brodsky-Lepage function $\xi_{BL}(\omega)$ at large recoil. In other words, the form factors exhibit heavy quark symmetries in the two limiting regions. The two symmetries are similar to each other but not identical. The corrections for finite quark masses have also been calculated by us. To a certain, very reliable approximation (the peaking approximation) they have the same general $1/M_Q$ structure as found in the HQET[3,4]. But in general the corrections depend on the non-perturbative structure of the meson wave functions and do not possess the low recoil $1/M_Q$ structure. One thus has a picture for the transition form factors of heavy mesons whose limiting behaviour is known at zero recoil [2] and at large recoil. It will be certainly of utmost interest to find a theoretical description of these form factors which interpolates between the two regions.

In this paper we are going to extend our investigations to transition form factors of heavy baryons such as $\Lambda_b \to \Lambda_c$, $\Sigma_b \to \Sigma_c(\Sigma_c^*)$. To calculate the form factors at large recoil we use a variant of the Brodsky-Lepage picture for exclusive reactions [5] in which heavy baryons are assumed to consist of a heavy quark and a light diquark. This is a rather natural picture in the HQET. The new point, however, is that the diquark, being a cluster of the two light valence quarks and a certain amount of glue and see quark pairs, is regarded as a quasi-elementary constituent which partly survives medium hard collisions. The composite nature of the diquarks is taken into account by diquark form factors which are parametrized such that the pure quark picture of Brodsky and Lepage emerges for asymptotically large velocity transfer.

The quark-diquark model of baryons has turned out to work rather well for exclusive reactions [6]-[8]. It is particularly well suited for moderately large momentum transfer since the diquark picture models non-perturbative effects which are known to play an important role in that kinematical region. With a common set of parameters specifying the diquarks, and process independent wave functions (or distribution amplitudes, DA's) for the hadrons a good description of a large number of exclusive reactions has been accomplished by now. Among these reactions are the electromagnetic form factors of baryons, Compton scattering of protons and photoproduction of mesons. We would like to mention that diquarks have also been introduced in other investigations, as for example in non-leptonic weak interactions where a diquark picture naturally explains the famous $\Delta I = 1/2$ rule [9].

The paper is organized as follows: In Sect.2 we present in some detail the application of the diquark model to the $\Lambda_b \to \Lambda_c$ transition form factors. Analytical and numerical results are presented in Sect.3 and compared with those obtained from the HQET at small recoil. Sect.4 is devoted to the Σ-type transitions. The paper ends with a few concluding remarks (Sect.5).

2. CALCULATION OF $\Lambda_i \to \Lambda_f$ FORM FACTORS

We are interested in semi-leptonic decays of heavy baryons where we concentrate on Λ-type baryons in this section. The momenta and masses of the initial and final state Λ's are denoted by p_i, M_i and p_f, M_f, respectively. Since we are interested in the so-called heavy quark limit $p \to \infty$, $M \to \infty$, but p/M kept fixed, it is convenient to work in terms of the velocities of the heavy baryons, v_i and v_f.
The momentum transfer to the leptonic system is denoted by $q(= p_i - p_f)$. Instead of the invariant momentum transfer q^2 it is more convenient to make use of the invariant velocity transfer variable $\omega = v_i v_f$ which is related to the momentum transfer by

$$\omega = \frac{M_i^2 + M_f^2 - q^2}{2M_i M_f} \tag{2.1}$$

In the decay processes ω varies between 1, the point of zero recoil, and

$$\omega_{max} = \frac{M_i^2 + M_f^2}{2M_i M_f} \tag{2.2}$$

where q^2 is zero. Larger values of ω can be reached in principle in the scattering reaction $\nu\Lambda_i \to e\Lambda_f$ where q^2 is now space-like ($q^2 \leq 0$).
Following Isgur and Wise[10] we write the matrix elements of the weak current as

$$\begin{aligned} < \Lambda_f \mid j^\mu \mid \Lambda_i > &= \bar{u}(v_f)[F_1\gamma^\mu + F_2 v_i^\mu + F_3 v_f^\mu \\ &+ G_1\gamma^\mu\gamma_5 + G_2 v_i^\mu\gamma_5 + G_3 v_f^\mu\gamma_5]u(v_i) \end{aligned} \tag{2.3}$$

We use old-fashioned E/m state normalization as in Ref.[10]. The spinors are normalized such that $\bar{u}u = 1$. The invariant form factors F_j and G_j, j=1,2,3, are functions of q^2, or equivalently of ω.
We are going to calculate the form factors appearing in Eq.(2.3) within the framework of the generalized hard scattering scheme. In the spirit of this model we write a state of a heavy Λ for a given helicity as

$$\mid \Lambda_j, \lambda_j >= \Psi_j(x_1, k_\perp) \mid q_j, \lambda_j; S > \tag{2.4}$$

where q_j is the heavy quark (j=c,b,t) and S is a spin 0, isospin 0 diquark (with [ud] quantum numbers). Colour indices are being omitted for convenience. The ansatz (2.4) is similar to that made for example by Isgur et al[10], the only, albeit important, difference is that here we treat the scalar diquark as a quasi- elementary constituent whereas in Ref.[10] it merely denotes the total quantum numbers of the cluster of the light degrees of freedom (gluons, sea and valence quarks). The ansatz (2.4) is more

general than the valence quark approximation since the diquark, as a bound object, contains a certain amount of glue and sea quarks besides the u and d valence quarks. In other words, the state (2.4) can be considered to be a certain, however unspecified, superposition of quark-gluon Fock states.
Ψ in Eq.(2.4) is the hadronic (light cone) wave function which depends on the longitudinal momentum fraction x_1 carried by the heavy quark and $k_\perp$ is the heavy quark's transverse momentum with respect to its parent hadron. The mean $k_\perp$ is of the order of a few 100 MeV. Since higher Fock states are ignored the wave function is normalized to 1. The neglect of other Fock states than (2.4) is a reasonable assumption for heavy hadrons.
Actually, in the calculation of form factors along the lines of the Brodsky-Lepage picture one only needs the wave function integrated over $k_\perp$, the so-called distribution amplitude (DA)

$$f_j\Phi_j(x_1) = \int \frac{d^2k_\perp}{16\pi^3}\Psi_j(x_1, k_\perp) \tag{2.5}$$

where, by convention, Φ is normalized such that

$$\int_0^1 dx_1 \Phi_j(x_1) = 1 \tag{2.6}$$

The constant f_j, the wave function at the origin in configuration space, is proportional to the decay constant of the hadron. It is determined by the $k_\perp$ dependence of the wave function.
The DA for a heavy hadron is known to exhibit a maximum which becomes more pronounced as the mass of the heavy quark increases, see Fig.1. The position of the peak is approximately at

$$x_{j0} = 1 - \varepsilon/M_j \tag{2.7}$$

and its width shrinks with increasing mass (proportional to $1/M_j$ in present models). The parameter ε is the difference between hadron and heavy quark mass

$$\varepsilon = M_j - m_j \tag{2.8}$$

and is about equal to the mass of the diquark (zero binding approximation). ε is flavour independent and has a value of about $0.6 - 1.0 GeV$. This behaviour of the DA's for the heavy baryons parallels the theoretically expected and experimentally confirmed behaviour of heavy baryon and meson fragmentation functions.
Forced by QCD evolution the DA's also depend on ω, in fact logarithmically. This fact is ignored here.
Several wave functions or DA's for heavy hadrons can be found in the literature [11,12,13]. Most appropriate for our purpose is the wave function

$$\begin{aligned} \Psi_j(x_1, k_\perp) &= N_j\Phi_j(x_1) exp[-b^2k_\perp^2] \\ \Phi_j(x_1) &= \tilde{N}_j x_1 x_2^3 exp[-b^2M_j^2(x_1 - x_{j0})^2] \end{aligned} \tag{2.9}$$

which is an adaptation of the meson wave function of Ref.[11] to the quark-diquark case. This meson wave function has been proven to be useful in weak decays. $x_2 = 1 - x_1$ is the fraction of the hadron's momentum carried by the diquark. The DA's obtained from this wave function are shown in Fig.1 for various Λ's. Another example of such a wave function is

$$\begin{aligned} \Psi_j(x_1, k_\perp) &= N_j \frac{1}{x_1 x_2} \Phi_j(x_1) exp[-b^2 \frac{k_\perp^2}{x_1 x_2}] \\ \Phi_j(x_1) &= \hat{N}_j x_1 x_2^3 exp[-b^2 (\frac{m_j^2}{x_1} + \frac{\varepsilon^2}{x_2})] \\ &= \hat{N}_j x_1 x_2^3 exp[-b^2 M_j^2 \frac{(x_1 - x_{j0})^2}{x_1 x_2}] \end{aligned} \tag{2.10}$$

which has been proven to be quite successful in other applications of the diquark model. The above wave function is a suitable adaptation of the corresponding meson wave function obtained by transforming the harmonic oscillator wave function to the light cone [12]. For both the above wave functions the oscillator parameter b is adjusted such that realistic mean $k_\perp$'s are obtained.

Both the wave functions (2.9) and (2.10) can be taken as reasonable parametrizations for real baryons. In fact, although the two wave functions look rather different, they actually lead to rather similar results in many cases as for instance the form factors for the transition $\Lambda_b \to \Lambda_c$ (see below). However, Eq.(2.10) has the theoretical deficiency that the constant f_j does not behave as $M_j^{-1/2}$ for $M_j \to \infty$ as expected. Rather f_j calculated from Eq.(2.10) behaves as $M_j^{-5/4}$ whereas f_j calculated from Eq.(2.9) has the appropriate mass scaling dependence. Related to that fact is the behaviour of the baryon radius. Eq.(2.9) obviously leads to a mass-independent radius whereas for the other wave function the radius shrinks with increasing mass. In the limit $M_j \to \infty$ the light degrees of freedom only see a static colour field leading to a constant baryon radius. By virtue of these theoretical considerations we favour the wave function (2.9). The wave function (2.10) is kept for the sake of comparison.

In the hard scattering scheme the current matrix element is expressed by a convolution of the DA's with amplitudes T for the elementary subprocess $q_i S \to q_f S W^*$ calculated in the collinear approximation within perturbative QCD. In a collinear situation, i.e. in a situation in which intrinsic transverse momenta can be neglected and all constituents of a hadron have momenta parallel to each other and parallel to the momentum of ther parent hadron, one can write the Λ_j-state of Eq.(2.4) as

$$| \Lambda_j, \lambda_j >= \Phi_j(x_1) u(v_j, \lambda_j) \tag{2.11}$$

where u is the baryon's spinor. This is the simplest case of a covariant spin wave function [14,15]. In the hard scattering scheme the current matrix element reads

$$< \Lambda_f \lambda_f \mid j^\mu \mid \Lambda_i \lambda_i >= \bar{u}(v_f, \lambda_f) \int dx_1 dy_1 f_f \Phi_f(y_1) T^\mu(x_1, y_1, \omega) f_i \Phi_i(x_1) u(v_i, \lambda_i) \tag{2.12}$$

The two diagrams that contribute to the hard scattering amplitude are shown in Fig.2. Following previous work (see e.g. Ref. [6]) the diquark-gluon vertex is defined as

$$SgS : -ig_s t^a (q_1 + q_2)^\mu F_S(Q^2) \tag{2.13}$$

g_S is the usual coupling constant of QCD and $t(= \lambda/2)$ the Gell-Mann colour matrix. This is the most general coupling of a scalar particle to a gluon. F_S represents a diquark form factor which takes into account the composite nature of the diquarks. It is parameterized in such a way that asymptotically, i.e. for $Q^2 = -(q_1 - q_2)^2 \to \infty$ ($\omega \to \infty$), the diquark model evolves into the pure quark model of Brodsky-Lepage, i.e. the usual dimensional counting rules emerge. Bearing this in mind the diquark form factor is parameterized as

$$F_S(Q^2) = \frac{Q_S^2}{Q_S^2 + Q^2} \tag{2.14}$$

For the parameter Q_S^2 we take the value $3.22 GeV^2$ as in previous applications of the scalar diquark [6]-[8]. This value is consistent with the higher twist term found by the EMC in the structure functions of deep inelastic μp scattering[16]. With this remark the diquark model is fully specified. The DA's and all the parameters are taken as in previous studies of various reactions as for instance the electromagnetic form factors of the nucleon or Compton scattering. No new parameters have been introduced here. For more details on the diquark model we refer to Refs.[6]- [8].

The decays of the B^0 into two pseudoscalar mesons has been investigated in Ref.[13]. In many respects that calculation is similar to ours and we can use analogous techniques. Working out the elementary amplitudes and comparing Eq.(2.3) with (2.12) and properly identifying the various covariants one arrives at the following expressions for the transition form factors

$$\begin{aligned} F_1 &= +f_f f_i [\omega (\chi_1 + \chi_2) + \chi_3 + \chi_4] \\ G_1 &= -f_f f_i [\omega (\chi_1 + \chi_2) + \chi_3 + \chi_4 \\ &\quad -\varepsilon/M_f (\chi_1 + \chi_3) - \varepsilon/M_i (\chi_2 + \chi_4)] \\ F_2 &= -G_2 = -f_f f_i \varepsilon/M_f (\chi_1 + \chi_3) \\ F_3 &= +G_3 = -f_f f_i \varepsilon/M_i (\chi_2 + \chi_4) \end{aligned} \tag{2.15}$$

where we have introduced the four independent form factor integrals

$$\begin{aligned} \chi_1 &= \int dx_1 dy_1 \Phi_f(y_1) \Phi_i(x_1) \frac{\rho_S x_2 M_i M_f}{q_G^2 (q_f^2 - m_f^2)} \\ \chi_2 &= \int dx_1 dy_1 \Phi_f(y_1) \Phi_i(x_1) \frac{\rho_S y_2 M_f M_i}{q_G^2 (q_i^2 - m_i^2)} \\ \chi_3 &= \int dx_1 dy_1 \Phi_f(y_1) \Phi_i(x_1) \frac{\rho_S y_2 M_f^2}{q_G^2 (q_f^2 - m_f^2)} \\ \chi_4 &= \int dx_1 dy_1 \Phi_f(y_1) \Phi_i(x_1) \frac{\rho_S x_2 M_i^2}{q_G^2 (q_i^2 - m_i^2)} \end{aligned} \tag{2.16}$$

with

$$\rho_S = -8\pi\alpha_S(-q_G^2)\, c_F F_S(-q_G^2) \tag{2.17}$$

$c_F(=4/3)$ is a colour factor. q_G, q_f, and q_i are the momenta of the internal gluons and quarks respectively (see Fig.2). In the course of this calculation we have made use of the usual parton model relation between quark and hadron masses

$$m_f = y_1 M_f \qquad\qquad m_i = x_1 M_i \tag{2.18}$$

This assumption implies equal velocities for the constituents. It is particularly safe in the case of heavy baryon form factors since, due to the pronounced peaks the DA's exhibit, the main contributions to the form factors come from the immediate vicinity of the position of the peaks at which the relation (2.18) is exact. At these positions, i.e. $x_1 = x_{i0}$ and $y_1 = y_{f0}$, one has $x_2 = \varepsilon/M_i$ and $y_2 = \varepsilon/M_f$. We have used these relations to simplify the propagators. As far as possible we have kept only first order differences between ε and $x_2 M_i$, $y_2 M_f$, neglecting quadratic terms. Thus, we have written the quark propagator appearing in the diagram on the right hand side of Fig.2 as

$$\begin{aligned} \not{q}_f + m_f &= M_f(\not{p}_f + 1) - \varepsilon(\not{p}_i + 1) + O(x_2 M_i - \varepsilon) \\ q_f^2 - m_f^2 &= 2\varepsilon M_f(1 - \frac{x_2 M_i}{\varepsilon}\omega) + O(x_2 M_i - \varepsilon) \end{aligned} \tag{2.19}$$

Analogous expressions hold for the other quark propagator. Thus, the spin sum numerators of the internal quarks can approximately be written in the rather convenient form of positive energy projectors.

3. DISCUSSION OF THE RESULTS

Before computing the form factors (2.15) it is instructive to study the δ-function-like limit, i.e. the case where the x and y dependence of the elementary amplitude is ignored relative to that of the DA's (evaluating the hard scattering amplitude T at the positions of the maxima of the two DA's). Pulling T out of the integrals, these simplify to the normalization integrals (2.6). In view of the sharply peaked DA's this is an acceptable approximation. This approximation (referred to as the peaking approximation) allows one to discuss the main features of our results in a simple fashion as well as certain limitations of our approach. In the peaking approximation the $\Lambda_i \to \Lambda_f$ form factors read:

$$\begin{aligned} F_1/f_f f_i &= \xi_{BL} \\ G_1/f_f f_i &= -\xi_{BL}[1 - \frac{\varepsilon}{1+\omega}(1/M_f + 1/M_i)] \\ F_2/f_f f_i &= -\xi_{BL}\frac{\varepsilon}{M_f}\frac{1}{1+\omega} \\ F_2 = -G_2 &= \frac{M_i}{M_f}F_3 = \frac{M_i}{M_f}G_3 \end{aligned} \tag{3.1}$$

where

$$\xi_{BL}(\omega) = \frac{\rho_S}{q_G^2}\frac{1+\omega}{1-\omega} \tag{3.2}$$

is a mass scale independent universal function which we shall refer to as the Brodsky-Lepage form factor function.
Several remarks are in order:
a) The virtuality of the gluon is

$$q_G^2 = 2\varepsilon^2(1-\omega) \tag{3.3}$$

For the decay process $\Lambda_b \to \Lambda_c$ the maximum value of ω is 1.43, hence for any reasonable value of ε the virtuality is smaller than $1 GeV^2$. However, since we are mainly interested in the general structure of the form factors we may consider larger values of ω. Thus for ω larger than say 1.5 or 2 (depending on the value of ε) q_G^2 is large enough to justify the application of perturbation theory. Such values of ω can be reached in principle in the scattering reaction $\nu\Lambda_b \to e\Lambda_c$ or in the decay $\Lambda_t \to \Lambda_b$. For several reasons we have computed ratios of form factors down to fairly small values of ω, in fact down to 1.3. The first reason is that this allows us to test the quality of the peaking approximation. The second reason is that the low-ω trends of the form factor ratios may be reliable although the form factors themselves are affected by soft, non-perturbative physics. This is so because we found the various ratios to be very stable against variations of the parameters and the cut-off in the full calculation (see below). The form factors themselves are, on the contrary, sensitive to these variations.
b) In the limit of infinitely large masses only one independent function remains, the Brodsky-Lepage function $\xi_{BL}(\omega)$, which is reminiscent of the Isgur-Wise function at low recoil. This function multiplies the covariant $\gamma^\mu(1-\gamma_5)$. As we have discussed above the DA factors f_j scale as $M_j^{-1/2}$. After removal of these factors one remains with universal form factors satisfying the same relations as have been shown to hold at low recoil [10,14,17]. We have made the analogous observation in the case of mesons [1].
c) The mass corrections $1/M_f$ in Eq.(3.1) have exactly the same structure as has been derived by Georgi et al [18] using the HQET (but we have the $1/M_i$ corrections as well). The reason for our results is that the integrals χ_i are related to each other in the peaking approximation by

$$\chi_1 = \chi_2 = \chi_3 = \chi_4 = \frac{1}{2(1+\omega)}\xi_{BL} \tag{3.4}$$

It is quite remarkable that, after removing the factors f_j, also the mass corrections have the same structure in both the regions, namely at low recoil where the HQET applies and at large ω where perturbative QCD is at work.
d) The large ω limit of our predictions is in agreement with dimensional counting and with the predictions of the Brodsky-Lepage picture [5] for form factors of baryons:

$$F_1, G_1 \sim \omega^{-2}, \quad F_2, F_3, G_2, G_3 \sim \omega^{-3} \tag{3.5}$$

This can easily be deduced from Eqs.(2.14), (3.1) and (3.2). The behaviour (3.5) is not restricted to the peaking approximation, it is valid in general. Note that the mass corrections disappear with $\omega \to \infty$.

e) It has been pointed out by Bjorken [19] that the universal form factor is related to the heavy quark component of the electromagnetic form factor of the Λ_i in the heavy quark limit. This holds true in our explicit model. For the relevant form factor F_1 one merely has to replace f_f by f_i in Eq.(2.15) or (3.1).

The results obtained in the peaking approximation can be generalized. Suppose the following limits exist for $M_i, M_f \to \infty$:

$$\begin{aligned} \chi_1 &\to \xi_1(\omega) + O(1/M_i, 1/M_f) \\ \chi_3 &\to \xi_3(\omega) + O(1/M_i, 1/M_f) \end{aligned} \tag{3.6}$$

with $\xi_1, \xi_3 \neq 0$. As is obvious from Eq.(2.16) χ_2 has the same limit as χ_1, and χ_4 the same as χ_3. Under these circumstances one arrives at similar expressions for the form factors as in Eq.(3.1) whereby the Brodsky-Lepage function is now defined as

$$\xi_{BL}(\omega) = 2[\omega\xi_1(\omega) + \xi_3(\omega)] \tag{3.7}$$

The mass corrections are, however, somewhat more complicated than those appearing in Eq.(3.1). The factor $1/(1+\omega)$ in Eq.(3.1) has to be replaced by $(\xi_1+\xi_3)/(2(\omega\xi_1+\xi_3))$. I.e. in general the mass corrections depend on the wave function. It can also been shown that a treatment of the propagators different from Eq.(2.19) will change the mass correction but not the leading terms.

Inserting the DA's (2.9) and (2.10) into the integrals (2.16) one may easily convince oneself that the limits (3.6) exist in these cases. The limits still exist when the functions multiplying the exponentials are moderately modified (using another power behaviour for example). These results can be proven by employing for instance substitutions $z_1 = x_2 M_i$ and $z_2 = y_2 M_f$ in the case of DA's of the type (2.9) or by using the saddle point method in the case of DA's of the type (2.10) (the leading term of the saddle point expansion is the peaking approximation). Finally we note, that for the DA used in Ref.[13], the limits (3.6) also exist. However, Ref.[13] refers to a mesonic DA. We do not believe that this DA is appropriate for a heavy baryon made of a quark and a diquark. We emphasize: the observation that the $\Lambda_i \to \Lambda_f$ transition form factors are controlled by one universal function in the limit of infinitely heavy masses holds for a rather large class of DA's. All DA's presently used for the description of heavy hadrons belong to this class. The mass corrections, on the other hand, depend on the details of the model assumptions.

Let us now turn to the numerical evaluation of the form factors obtained by integrating over the DA's (2.9,2.10). The results for ratios of form factors are compared to those obtained in the peaking approximation in Fig.3. Following Szczepaniak et al. [13] the integrals have been cut-off at $y_2(x_2) = \epsilon/(\omega M_f(M_i)) + \Delta y(x)$. The end-point regions

are controlled by soft physics. Perturbative QCD is not readily applicable there. The quarks and diquarks are far off-shell and a relation like (2.18) is probably a very bad approximation. Fortunately, these regions are strongly suppressed by the DA's [3]. An additional suppression is expected from Sudakov form factors in these regions. Thus one can be confident that the cut-off procedure does not distort the results much.

We have computed the form factors for various cut-off values $\Delta x = \Delta y$ and for α_S fixed and running ($\Lambda_{QCD} = 200 MeV$). The ratios of form factors are extremely stable under these variations. In fact the ratios vary by about 15% at $\omega = 1.3$ and less for larger values of ω. This is certainly tolerable and therefore we believe that the trends of our predictions based on perturbative QCD are reliable for the ratios down to a ω-value as small as 1.3.

From Fig.3 one sees that the results for the peaking approximation and for the full calculation using actual DA's agree quite well whith each other. Deviations between the two predictions are only substantial near $\omega = 1.3$. Utilizing the DA (2.10) one finds results which are practically indistinguishable from those obtained with the DA (2.9). The predictions for the form factors itself do not show the same stability against the variations mentioned above. They depend strongly on the DA chosen and, for a given DA, on the position of the cut-off. We estimate the uncertainties of our predictions for form factors to amount to about $30-40\%$ at $\omega = 1.3$ and still to about $10-15\%$ at $\omega = 10$ for a given value of the product $f_i f_f$. Keeping all other parameters fixed the difference between the results of the full calculation and the peaking approximation never exceeds $\simeq 20\%$ for the form factors themselves for any ω (for $\omega \gtrsim 2$ the difference is less than 3%).

At large space-like ω, however, we are able to make absolute predictions. In Ref.[8] it has been shown that the diquark model successfully describes the electromagnetic form factors of the nucleon. In fact the parameters of the model used here have been fixed in Ref.[8]. Since the calculation presented here is of the same type we believe that the predictions for the transition form factors of heavy baryons are on rather safe grounds at large ω. In order to give an impression of the magnitude of the form factors we quote an example. Using the values of the parameters as quoted above, we find a value of $2.65 f_i f_f GeV^{-2}$ for $F_1(\Lambda_b \to \Lambda_c)$ at $\omega = 4$ in the peaking approximation. A similar value is obtained from the full calculation.

Although it is not really within the spirit of the heavy quark physics we have also calculated the form factors for the process $\Lambda_c \to \Lambda_s$. Within the diquark model the calculation is identical to the case of $\Lambda_b \to \Lambda_c$. The only difference is that we do not expect the Fock state (2.4) to be the dominant one. Rather based on experience with an investigation of the reaction $p\bar{p} \to Y\bar{Y}$ as carried out by the authors of Ref.[7], we think that other quark-diquark states including such with strange diquarks, are equally important. Assuming the relative strength of these various quark-diquark states to follow SU(6), one gets an additional suppression factor $3^{-1/2}$ for the sS(ud) Fock state [7]. This is the only state of interest for the decay under consideration because the Λ_c

[3] Note that in the baryon case the sensitivity to the cut-off is less severe than for mesons [13]. This is due the factor x_2^3 in the DA's that we use.

and the Λ_s have in common only the scalar diquark with the [ud] quantum numbers. In other words, the value for f_s has to be divided by that factor. With this in mind we arrive at the same expressions for the case of $\Lambda_c \to \Lambda_s$ as before (Eqs.(2.15,3.1)). The numerical results for ratios of form factors are also shown in Fig.3. For uncertainties of these predictions the above remarks made for the other decay apply as well.

4. $\Sigma_i \to \Sigma_f$ FORM FACTORS

The matrix elements for vector and axial vector currents are decomposed in the same way as for the Λ's. The 6 form factors have to be calculated within the diquark model along the same line as we did in Sect.2. The quantum numbers of the Σ's imply that the cluster of light degrees of freedom is in a spin 1, isospin 1 state. This cluster is treated as a vector diquark. As for the case of the $\Lambda's$ we again rely on the equal velocity assumption in order to calculate the transition form factors at large recoil. A heavy Σ baryon is then represented by the following covariant spin wave function [14,15]

$$| \Sigma_j, \lambda_j >= \frac{1}{\sqrt{3}}\Phi_j(x_1)\,[\,\gamma^\alpha + v_j^\alpha\,]\gamma_5 u(v_j,\lambda_j) \tag{4.1}$$

where u is the spinor of the hadron. For the DA the same functions, Eqs.(2.9) or (2.10) are used as for the Λ's. The mass of the V diquark is taken to be somewhat larger than that of the scalar one ($m_V = \varepsilon = 770 MeV$ [20]). This is in accordance with the experience made in Refs.[6]- [8].
The gluon vector diquark vertex is defined as

$$\begin{aligned} VgV: \quad & -ig_s t^a\{(q_1+q_2)^\mu g^{\lambda\nu} - ((1+\kappa_V)q_2^\lambda - \kappa_V q_1^\lambda)g^{\mu\nu} \\ & -((1+\kappa_V)q_1^\nu - \kappa_V q_2^\nu)g^{\lambda\mu}\}F_V(Q^2) \end{aligned} \tag{4.2}$$

F_V is the form factor of the vector diquark which is parametrized as

$$F_V(Q^2) = (\frac{Q_V^2}{Q_V^2+Q^2})^2 \tag{4.3}$$

in order to be compatible with the asymptotic pure quark model. κ_V is the anomalous magnetic moment of the V diquark. In the non-relativistic quark model its value should be 1. The best fit values of κ_V and Q_V^2 are 1.16 and 1.58 GeV^2 respectively [8]. In principle there is a third coupling in Eq.(4.3). According to Refs.[6]-[8] this coupling only plays a minor role and is, therefore, neglected here. We have however checked that the inclusion of this coupling would not change our conclusions concerning the heavy quark limiting structure.
Working out the two Feynman diagrams shown in Fig.2 for vector diquarks and inserting the resulting elementary amplitudes into Eq. (2.12) one arrives at expressions for the six form factors similar to those given in eq.(2.15). Since these formulae are very lengthy we do not present them here but list only those obtained in the peaking

approximation:

$$\begin{aligned} F_1 &= \frac{f_f f_i}{3}\xi_V[(\omega+1)a] \\ G_1 &= \frac{f_f f_i}{3}\xi_V[-(\omega+1)a + \varepsilon(1/M_f + 1/M_i)(2a-\omega)] \\ F_2 &= \frac{f_f f_i}{3}\xi_V[-2(a+1) + \varepsilon/M_f(2-2\omega+3a)] \\ G_2 &= \frac{f_f f_i}{3}\xi_V[-2(\omega+1)\kappa_V + \varepsilon/M_i(1+3\kappa_V) + \varepsilon/M_f(2+(2+\omega)\kappa_V)] \\ F_3 &= F_2(M_f \to M_i) \\ G_3 &= -G_2(M_f \longleftrightarrow M_i) \end{aligned} \tag{4.4}$$

In Eq.(4.4) we have introduced the following abbreviations

$$a = 1 - \kappa_V(\omega - 1) \tag{4.5}$$

and the universal heavy mass independent function

$$\xi_V = \frac{\rho_V}{q_G^2}\frac{1}{1-\omega} \tag{4.6}$$

ρ_V is equal to ρ_S (see Eq.(2.17)) with the scalar diquark form factor F_S replaced by the vector one, F_V.
Obviously, in the limit of infinitely heavy quark masses, the six form factors are determined by two independent functions which can be shown to originate from longitudinal-longitudinal and transverse-transverse transitions of the vector diquarks. The other transitions are either zero or contribute to the mass correction terms only. For these two mass scale independent universal functions one may use

$$\xi'_{BL1} = -\xi_V(1+\omega) \qquad \xi'_{BL2} = -\xi_V(1+\kappa_V) \tag{4.7}$$

in accordance with the definitions proposed by Boyd and Brahm [21]. These functions play the same role as the one given in Eq.(3.2) and are, therefore, again referred to as the Brodsky-Lepage functions. After removal of the DA factors f_j the relations between the form factors and the Brodsky -Lepage functions are identical to those found by Boyd and Brahm at low recoil (see also Refs.[10,17]). On the other hand the $1/M_f$ correction terms do not show the general pattern found by Boyd and Brahm. This observation has to be contrasted with our conclusions for $B \to D(D^*)$ and $\Lambda_i \to \Lambda_f$ transitions for which the general structure of the $1/M_f$ terms is identical at low and at large recoil, at least in the peaking approximation.
For $\omega \to \infty$ the form factors behave in accordance with the dimensional counting rules (cf. Eq.(3.5)); the mass corrections disappear. In this limit only the covariant $\gamma_\mu(1-\gamma_5)$ remains as in the Λ case. Again the form factor F_1 is related to the heavy quark component of the electromagnetic form factor in this limit.

Some of the form factors in Eq.(4.4) develop zeros in the space-like region, namely $F_1 = 0$ at $\omega = 1 + 1/\kappa_V = 1.86$ and $F_2 = 0$ at $\omega = 1 + 2/\kappa_V$ (ignoring mass corrections). Therefore, in Fig.4 we plot the ratios of the Σ form factors with respect to G_2 which does not have a zero. We have also evaluated these form factors in the full calculation not using the peaking approximation. Again the results are very similar to those obtained in the peaking aproximation. Therefore, we refrain from displaying them in Fig.4. Limitations and uncertainties of the calculation are similar to those in Λ case. Therefore we do not repeat the corresponding discussion. We have also computed the form factors for the process $\Sigma_c \to \Sigma_s$ (cf.Fig.4).
As for the case of the Λ's the results obtained in the peaking approximation can be generalized. When the peaking approximation is not made the $\Sigma_i \to \Sigma_f$ form factors can again be expressed in terms of the four integrals (2.16) (with ρ_S replaced by ρ_V). It can then be seen that for all DA's for which the limits (3.6) exist, the form factors are controlled by two universal functions in the heavy quark limit. It is noteworthy that in the Σ-case the two functions ξ_1 and ξ_3 in Eq.(3.6) enter separately whereas in the Λ-case there is only one relevant linear combination. The mass corrections depend on details such as the DA used or on the treatment of the propagators.
Identical conclusions hold for the transitions $\Omega_b \to \Omega_c$ where now the diquark is a $\{ss\}$ state. In fact, from the physics point of view a study of the $\Omega_b \to \Omega_c$ transition may be more relevant since the Ω_b is a weakly decaying particle whereas the Σ_b is expected to decay strongly by one pion emission in most present estimates of the level structure of bottom baryons.
In the heavy quark limit the two independent functions (4.7) that determine the Σ form factors also fix the form factors for the process $\Sigma_i \to \Sigma^*_f$. For low recoil this has already been shown by the authors of Refs.[10,17,21]. For completeness we also examine these form factors at large recoil but we refrain from presenting details. Rather we just quote the results for the form factors in the peaking approximation.
The current matrix element for the $\Sigma_i \to \Sigma^*_f$ transition is decomposed as follows

$$\begin{aligned} < \Sigma^*_f \mid j^\mu \mid \Sigma_i > \; &= \; \bar{u}_\alpha(v_f)[\, N_1 v_i^\alpha \gamma^\mu \gamma_5 + N_2 v_i^\alpha v_i^\mu \gamma_5 + N_3 v_i^\alpha v_f^\mu \gamma_5 + N_4 g^{\alpha\mu}\gamma_5 \\ &\quad + K_1 v_i^\alpha \gamma^\mu + K_2 v_i^\alpha v_i^\mu + K_3 v_i^\alpha v_f^\mu + K_4 g^{\alpha\mu}\,] u(v_i) \end{aligned} \tag{4.8}$$

The spin 3/2 Σ^*_f is again written in a covariant fashion

$$\mid \Sigma^*_f, \lambda_f >= \Phi_f(x_1) u_\alpha(v_f, \lambda_f) \tag{4.9}$$

Making use of the peaking approximation, we find for the transition form factors

$$\begin{aligned} N_1 &= \frac{f_f f_i}{\sqrt{3}} \xi_V (1 + a) \\ K_1 &= \frac{f_f f_i}{\sqrt{3}} \xi_V [\, (1+\omega)\kappa_V + \frac{\varepsilon}{2M_f}(1-\kappa_V) - \frac{\varepsilon}{2M_i}(1+3\kappa_V)\,] \\ N_2 &= \frac{f_f f_i}{\sqrt{3}} \xi_V \frac{\varepsilon}{2M_f}(1+3\kappa_V) \\ K_2 &= \frac{f_f f_i}{\sqrt{3}} \xi_V \frac{\varepsilon}{2M_f}(3+\kappa_V) \\ N_3 &= K_3 = \frac{f_f f_i}{\sqrt{3}} \xi_V [\, -2(1+\kappa_V) + \frac{\varepsilon}{2M_i}(1+3\kappa_V)\,] \\ N_4 &= \frac{f_f f_i}{\sqrt{3}} \xi_V 2(1+\omega) \\ K_4 &= \frac{f_f f_i}{\sqrt{3}} \xi_V [\, 2(1+\omega) - 2\frac{\varepsilon}{M_f} - \frac{\varepsilon}{M_i}(2-\omega+a)\,] \end{aligned} \tag{4.10}$$

In the limit of infinitely large quark masses the 14 form factors are determined by two Brodsky-Lepage functions as at low recoil [10,17,21]. With the notation of Boyd and Brahm, one has

$$\xi''_{BL1} = \xi'_{BL1} \qquad \xi''_{BL2} = \xi'_{BL2} \tag{4.11}$$

Again in that limit, as for all the other cases we investigated, the relations between the $\Sigma_i \to \Sigma^*_f$ form factors are the same as at low recoil after removal of the DA factors f_j. Due to the relations (4.11) the new heavy quark symmetry leads to even stronger restrictions at large recoil than at low recoil. Finally, we remark that the mass corrections do not follow the low recoil pattern derived by Boyd and Brahm.

5. CONCLUDING REMARKS

We have analyzed the large recoil behaviour of the transtion form factors for semi-leptonic baryon decay processes such as $\Lambda_b \to \Lambda_c$ or $\Sigma_b \to \Sigma_c(\Sigma^*_c)$. The investigation was based on a generalized Brodsky- Lepage hard scattering formalism in which diquarks are assumed to exist as quasi-elementary constituents of baryons. The parameters of the diquark model are taken from Refs.[6,7,8] where this model has been applied to a large number of exclusive reactions.
For a rather large class of heavy hadron DA's to which in particular all presently used DA's belong to we have shown that the Λ and Σ transition form factors exhibit a spin-flavour symmetry in the formal limit of infinite quark masses. After having removed certain mass scale dependent wave function factors (proportional to the baryon decay constants) this symmetry is identical to that one found by Isgur and Wise [10] to hold at low recoil. The mass corrections to that limit depend on the details of the model assumptions such as the choice of the DA or the treatment of propagators. In general the pattern of the $1/M$ terms is not the same as found by the authors of Refs.[18,21] at low recoil. It is quite remarkable that the diquark model as a particular variant of the Brodsky-Lepage hard scattering formalism leads to the same spin-flavour symmetry of the baryon form factors at large recoil as was found for the meson form factors [1].
We have also computed the explicit ω-dependence of the various form factors making use of the peaking approximation and also in the full calculation by integrating over actual DA's. For values of ω which are not too small both the methods provide results which agree quite well with each other. Deviations are only noticeable when the gluon

virtuality is small. The numerical results for ratios of form factors presented in our figures mainly serve the purpose to demonstrate the quality of the peaking approximation. The low ω-trends of the results give an indication of the actual behaviour of the form factors. This may not be of much phenomenological importance though.
Finally, we emphasize that the purpose of this paper is the investigation of the general structure of the transition form factors at large recoil and the study of the spin-flavour symmetry in that region. Phenomenological implications for baryon transition form factors may be regarded as a side aspect of our work. From the theoretical point of view it is nevertheless interesting to consider very large values of the velocity transfer which are well outside the kinematical region of the semi-leptonic Λ_b or Σ_b decays. Such values of ω may be reached in principle in semi-leptonic decays of top baryons (provided they exist) and in scattering reactions such as $\nu\Lambda_b \rightarrow e\Lambda_c$.

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FIGURE CAPTIONS

Fig.1 The distribution amplitude for various Λ_j according to Eq.(2.9).

Fig.2 Diagrams contributing to the weak transition form factors of baryons. The upper lines represent the heavy quarks, the lower ones the diquarks.

Fig.3 Various ratios of weak transition form factors vs. ω. Shown are the results for the two processes $\Lambda_b \rightarrow \Lambda_c$ and $\Lambda_c \rightarrow \Lambda_s$. Solid (dashed) lines represent the results of the peaking approximation (full calculation). The latter calculation has been carried out using the DA (2.9), $\epsilon = 0.6GeV$, $\Lambda_{QCD} = 200MeV$, $\Delta x = \Delta y = 0.01$ and the nominal mass values: $M_c = 2.285GeV$ and $M_b = 5.6GeV$.

Fig.4 Ratios of weak transition form factors for the two processes $\Sigma_b \rightarrow \Sigma_c$ and $\Sigma_c \rightarrow \Sigma_s$ in the peaking approximation ($M_c = 2.455GeV$; $M_b = 5.76GeV$).

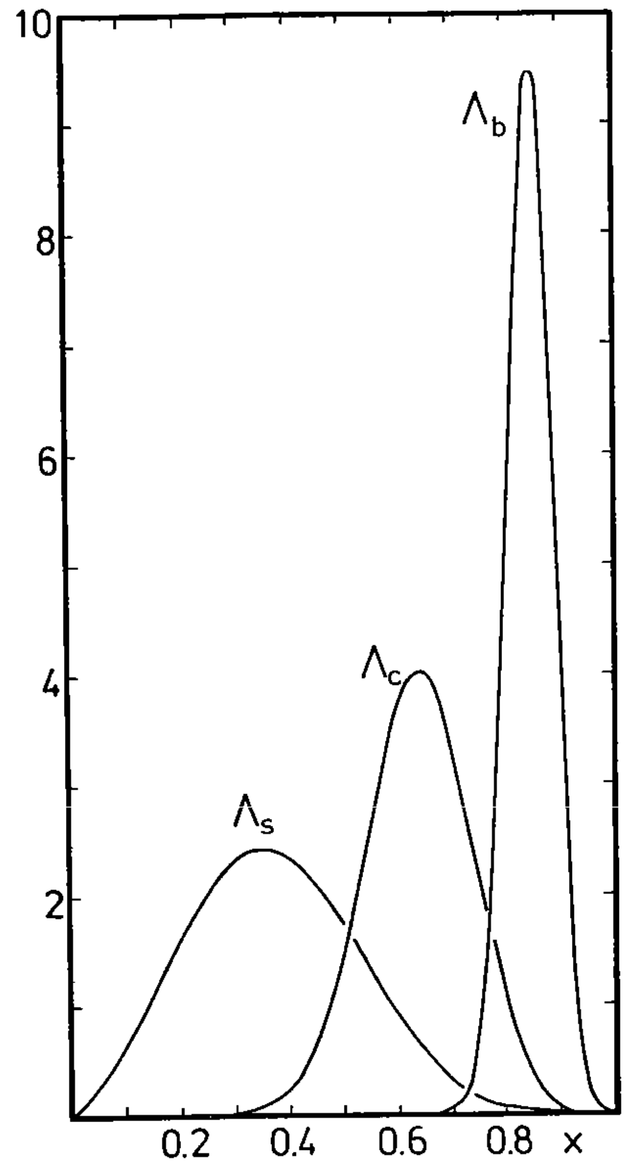

10
8
6
4
2
Λ_b
Λ_c
Λ_s
0.2
0.4
0.6
0.8
x

Fig. 1

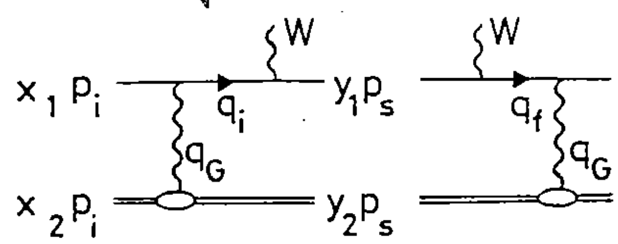
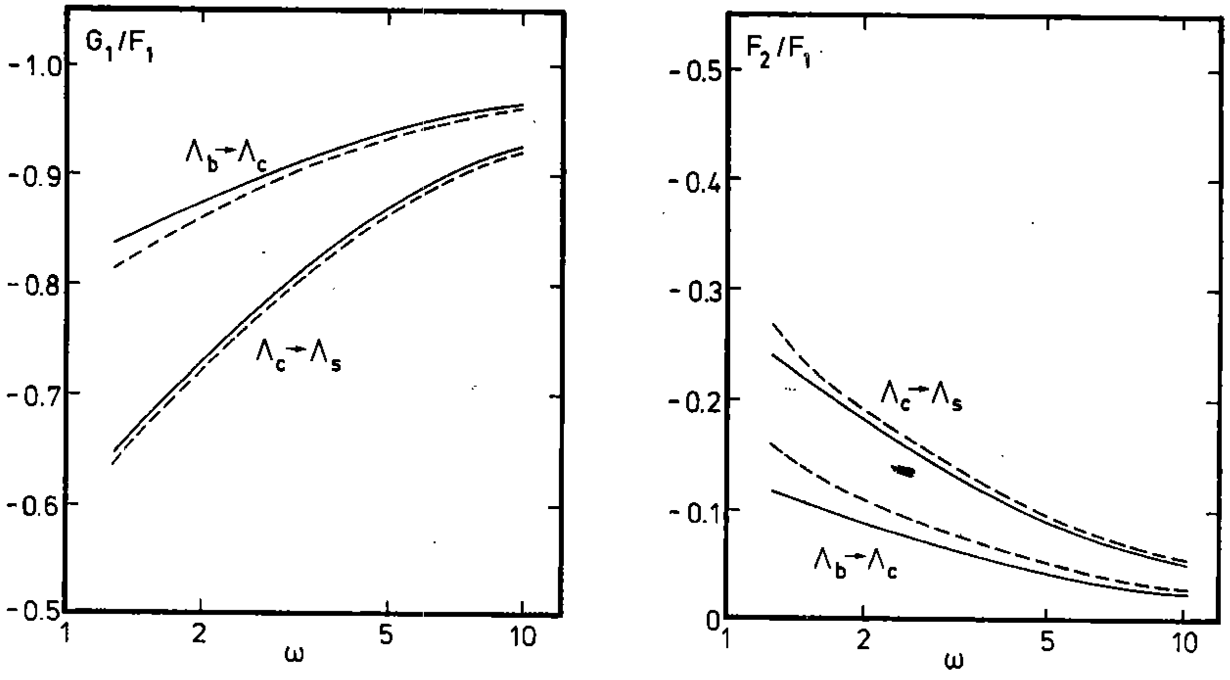
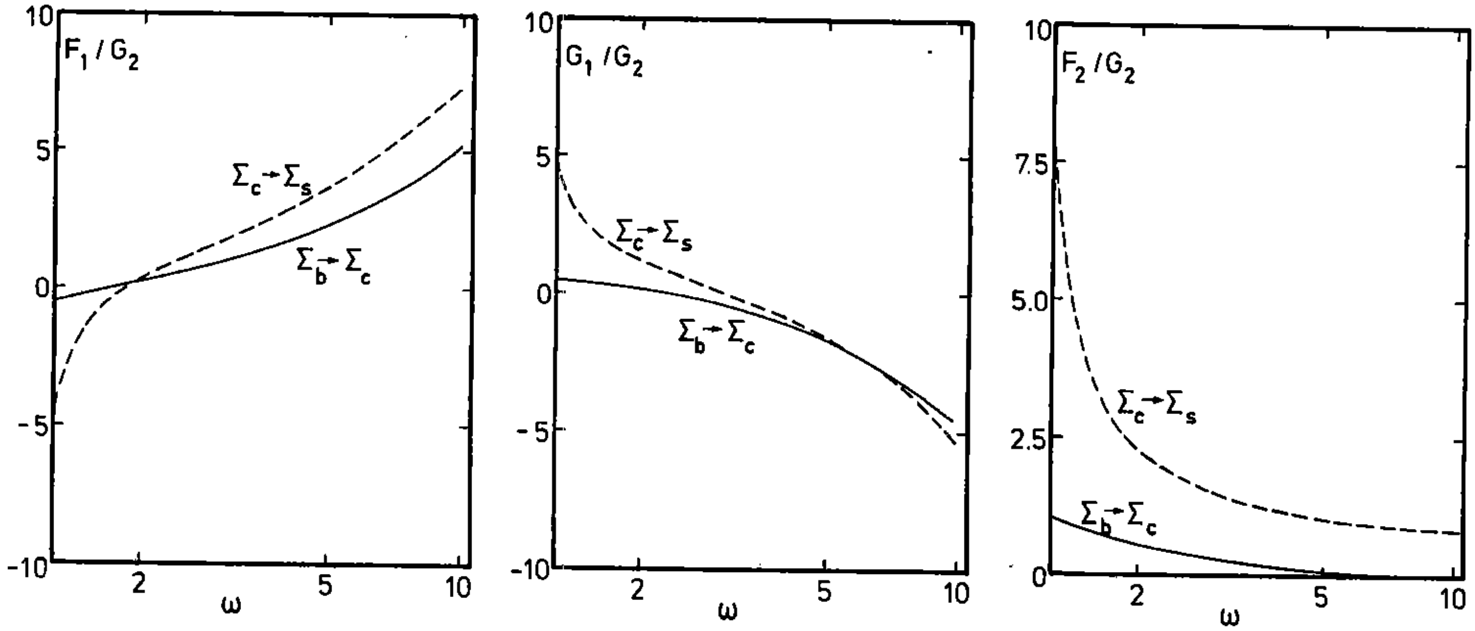

W
W
$x_1 p_i$
q_i
$y_1 p_s$
q_f
q_G
q_G
$x_2 p_i$
$y_2 p_s$

Fig. 2



G1/F1
-1.0
-0.9
-0.8
-0.7
-0.6
-0.5
Λb→Λc
Λc→Λs
1
2
5
10
ω
F2/F1
-0.5
-0.4
-0.3
-0.2
-0.1
0
Λc→Λs
Λb→Λc
1
2
5
10
ω

Fig. 3



F1/G2
10
5
0
-5
-10
Σc→Σs
Σb→Σc
2
5
10
ω
G1/G2
10
5
0
-5
-10
Σc→Σs
Σb→Σc
2
5
10
ω
F2/G2
10
7.5
5.0
2.5
0
Σc→Σs
Σb→Σc
2
5
10
ω

Fig. 4