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Aspects of Two–Photon Physics at Linear e^+e^- Colliders

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Abstract

We discuss various reactions at future e^+e^- and $\gamma\gamma$ colliders involving real (beamstrahlung or backscattered laser) or quasi–real (bremsstrahlung) photons in the initial state and hadrons in the final state. The production of two central jets with large transverse momentum p_T is described in some detail; we give distributions for the rapidity and p_T of the jets as well as the di–jet invariant mass, and discuss the relative importance of various initial state configurations and the uncertainties that arise from the at present rather poor knowledge of the parton content of the photon. We also present results for 'mono–jet' production where one jet goes down a beam pipe, for the production of charm, bottom and top quarks, and for single production of W and Z bosons. Where appropriate, the two–photon processes are compared with annihilation reactions leading to similar final states. We also argue that the behaviour of the total inelastic $\gamma\gamma$ cross section at high energies will probably have little impact on the severity of background problems caused by soft and semi–hard ('minijet') two–photon reactions. We find very large differences in cross sections for all two–photon processes between existing designs for future e^+e^- colliders, due to the different beamstrahlung spectra; in particular, both designs with $\ll 1$ and $\gg 1$ events per bunch crossing exist. The number of hadronic two–photon events is expected to rise quickly with the beam energy. Hadronic backgrounds will be even worse if the e^+e^- collider is converted into a $\gamma\gamma$ collider.

1. Introduction

In recent years an increasing amount of effort has been devoted [1, 2] to the study of the physics potential and design problems of future e^+e^- colliders. There is great physics interest in such machines, since it is quite likely that the planned pp supercolliders [3] will not be very effective for searches for many hypothetical new particles (supersymmetric sleptons and gauginos; non–standard Higgs bosons; heavy leptons; ...) which interact only weakly and/or lack the distinct signatures necessary for their discovery at hadron colliders. Moreover, now that the construction of TRISTAN, the first phase of LEP and the SLC has been completed, the time is ripe to develop specific plans for the next generation of e^+e^- colliders.

Traditionally e^+e^- colliders have offered a very clean experimental environment, allowing for the detailed study of particles that may have been discovered previously at a hadron collider; a recent example is the Z boson, which is now being studied in great detail at the SLC and LEP. This is the second major physics motivation for pushing the energy frontier of e^+e^- colliders to higher values.

However, as the collision energy $\sqrt{s}$ is increased, the cross section for the annihilation events whose study will be the main purpose of any future collider decreases like $1/s$, or at best like $\log s/s$. At the same time, the cross section for the simplest hard two–photon process, $e^+e^- \to e^+e^-q\overline{q}$, *increases* like $\log^3 s$, for fixed transverse momentum of the quarks or fixed invariant mass of the $q\overline{q}$ pair. Moreover, the hadronic structure of the photon also plays an increasingly important role at higher energies. It can be described by introducing [4] quark and gluon densities "inside" the photon. These give rise [5, 6] to processes where the partons "in" the photon, rather than the photons themselves, undergo hard scattering. The cross section for these "resolved photon" processes are predicted [7] to rise almost linearly with the e^+e^- cms energy. This rapid increase has recently been confirmed by the AMY group [8] in the PETRA to TRISTAN energy range, 30 GeV $\leq \sqrt{s} \leq$ 60 GeV. These considerations imply that at future e^+e^- colliders, hard two–photon events will outnumber annihilation events by an increasingly wide margin.

The number of two–photon events is further boosted by beamstrahlung [9], which could increase [10] the two–photon luminosity by as much as a factor of 100 already at $\sqrt{s} = 500$ GeV. As well known, synchrotron radiation makes it prohibitively expensive to build e^+e^- storage rings with energies significantly beyond that of the second stage of LEP, $\sqrt{s} \simeq 200$ GeV. At linear colliders any given bunch of electrons or positrons crosses the interaction point only once, as compared to approximately 10^8 times at LEP; moreover, the luminosity has to increase proportional to s in order to maintain a constant rate of annihilation events. These considerations imply that a very high luminosity per bunch crossing has to be achieved at e^+e^- linacs. This forces one to use small, dense bunches; the particles in each bunch are therefore subject to strong electromagnetic fields just before and during the bunch collisions. The resulting forces on the particles in the bunches lead to their rapid acceleration; beamstrahlung is the radiation emitted by the accelerating electrons and positrons.

In a recent Letter [11] we pointed out that the combination of the rapid increase of the cross section for resolved photon processes and the enhanced photon flux due to beamstrahlung can lead to severe hadronic backgrounds at e^+e^- supercolliders. We

demonstrated this using the design of ref.[12] for a collider operating at $\sqrt{s} = 1$ TeV, which is characterized by a hard beamstrahlung spectrum. Under this assumption, two–photon events dominate total dijet production for $p_T \leq 200$ GeV. What is worse, one might have to expect $\mathcal{O}(10)$ semi–hard two–photon events at *each* bunch crossing; this would give rise to an "underlying event" depositing as much as 100 GeV of transverse energy in the detector.

Since then, more realistic designs for e^+e^- colliders operating at $\sqrt{s} = 500$ GeV have been put forward [2]. Unlike the example of ref.[12], these designs all foresee flat beams; furthermore, they split the bunch into a bunch train consisting of several micro–bunches, which reduces the necessary luminosity per bunch crossing. Both modifications reduce beamstrahlung. In this paper we study representative examples of these recent designs. We find very large differences between them, as far as the severity of two–photon induced backgrounds are concerned. While for one proposed design the situation is almost as problematic as for the "theoretical" collider of ref.[12], other designs, most noteably one using superconducting cavities, are almost free of beamstrahlung–induced hadronic backgrounds.

It has been suggested [13] to convert future e^+e^- colliders into $\gamma\gamma$ colliders. This can be achieved by bathing the incoming e^+ and e^- beams in intense laser light. The incoming electrons would then transmit most of their energy to the photons by inverse Compton scattering. The resulting photon spectrum is very hard, and the achievable $\gamma\gamma$ luminosity is similar to the original e^+e^- luminosity. However, we will show that the hadronic backgrounds at such a $\gamma\gamma$ collider are much *larger* than at e^+e^- colliders operating at the same energy. The cross section for the "direct" process $\gamma\gamma \to q\overline{q}$ now falls with energy; however, as mentioned above, the cross section for resolved photon processes increases with the incident $\gamma\gamma$ energy. A harder photon spectrum therefore always implies larger hadronic backgrounds.

In ref.[11] we used the production of two central jets as benchmark for hadronic two–photon reactions. Here we present a much more detailed description of this reaction, including rapidity distributions and invariant mass spectra. While this is the most common of all hard two–photon reactions, it is usually not the most important background to new physics searches, nor the worst obstacle to precision measurements. In this paper we therefore also give results for a more complete list of hard two–photon reactions, including events with only one central jet and one forward jet (mono–jets), heavy quark production, and Drell–Yan production of W and Z bosons in resolved photon reactions. We find that, at e^+e^- colliders with $\sqrt{s} \leq 500$ GeV, the production of central $c\overline{c}$ and $b\overline{b}$ pairs is always dominated by the direct contribution, except perhaps at very small transverse momenta, $p_T \leq 5$ GeV. Moreover, the total $t\overline{t}$ cross section at such colliders will be dominated by the annihilation process. Our calculations of the annihilation contribution include effects due to initial–state radiation as well as beamstrahlung; at $\sqrt{s} = 500$ GeV, these effects increase (decrease) the annihilation contribution if $m_t < (>)$ 155 GeV.

We also expand our previous discussion [11] of the semi–hard "minijet" background. In its simplest form, leading order QCD extrapolated down to transverse momenta between approximately 1 and 3 GeV predicts an almost linear increase of the total inelastic $\gamma\gamma$ cross section with energy. Of course, this behaviour cannot persist indefinetely. However, we will present arguments showing that it is at least possible that the mechanism which

ultimately "unitarizes" the cross section (e.g., eikonalization) will become effective only at energies beyond the reach of the next generation of e^+e^- colliders. Moreover, we will argue that even an early flattening off of the cross section need not lead to a sizeable reduction of the total E_T in the underlying event, which is a good measure for the "messiness" of the environment.

While most of our numerical results will be given for colliders operating at $\sqrt{s} = 500$ GeV, which is now envisioned as the likely energy for the next e^+e^- collider [2], we also try to extrapolate to higher energies. We argue that for the so–called mainstream designs utilizing X–band microwave cavities the occurence of an underlying event, i.e. of multiple interactions per collision, seems unavoidable at $\sqrt{s} \geq 1$ TeV, unless the bunch structure can be modified considerably. On the other hand, superconducting designs might be able to provide a clean environment up to $\sqrt{s} \simeq 2$ TeV.

The rest of this paper is organized as follows. In Sec.2 we present the necessary formalism. In particular, various parametrizations of the parton content of the photon are briefly discussed. We also describe the photon and electron spectra that we use in our calculations. In Sec.3 results for hard two–photon reactions are presented. We devote different subsections to the production of di–jets (3a), mono–jets (3b), heavy quarks (3c) and single W and Z bosons (3d). Sec.4 contains a discussion of the soft and semi–hard background. Finally, in Sec.5 we summarize our results and present some conclusions.

2. Formalism and distribution functions

In this section we describe the techniques necessary to derive the results of secs. 3 and 4. We will employ the structure function formalism even when estimating annihilation cross sections. In this formalism, the cross section for the production of a given final state X is expressed as a product of the functions $f_1(x_1)$ and $f_2(x_2)$, describing the probabilities to find particles 1 and 2 with fractional momenta x_1 and x_2 in the incident beams, and the hard $1 + 2 \to X$ scattering cross section $d\hat{\sigma}$:

$$d\sigma = f_1(x_1) f_2(x_2) d\hat{\sigma}(\hat{s}). \tag{1}$$

Here $\hat{s} = x_1 x_2 s$ is the invariant mass of the system of particles 1 and 2, and $\sqrt{s}$ is the nominal e^+e^- machine energy.

In case of two–photon processes, particles 1 and 2 are either photons, or quarks or gluons inside a photon. We use the terminology of ref.[7] to classify the various two–photon processes. In "direct" processes, particle s 1 and 2 are both photons. The only process of this kind which is of interest to us is the production of a pair of massive or massless quarks, $\gamma\gamma \to q\overline{q}$; the corresponding hard cross section can for instance be found in ref.[14]. In "once resolved" processes ("1–res" for short) particle 1 is a photon, while particle 2 is a quark or gluon. The relevant subprocesses are $\gamma q \to gq$ and $\gamma g \to q\overline{q}$; their cross sections are listed in ref.[15]. Finally, in "twice resolved" or "2–res" processes particles 1 and 2 are both colored partons. The eight subprocesses contributing to the production of massless parton jets and their cross sections are given, e.g., in ref.[16]. The cross sections necessary to compute the production of massive $Q\overline{Q}$ pairs in once and twice resolved processes can be found in refs. [17] and [18], respectively.

We remind the reader at this point that resolved photon processes are characterized by spectator jets going essentially into the direction of the incoming photons. These spectator or remnant jets are the result of the hadronization of the colored system that is produced when a quark or gluon is taken out of a photon. Although the axes of these jets almost coincide with the beam directions, at least their outer fringes can emerge at substantial angles, due to nontrivial color flow between spectator jets and hard jets as well as the boost from the $\gamma\gamma$ centre–of–mass system to the lab frame. For instance, according to the AMY Monte Carlo simulation of their data on jet production in $\gamma\gamma$ scattering [8], about 50% of the particles that originate from the spectator jets emerge at angles $\theta > 20^\circ$.

We now turn to a discussion of the various probability or distribution functions f_i. In general there are two different contributions to the photon flux $f_{\gamma|e}$ in an electron beam. The first contribution is actually an approximation of the complete two–photon process $e^+e^- \to e^+e^-X$. The corresponding cross section can be cast in the form of eq.(1) using the effective photon (EPA) or Weizsäcker–Williams (WW) approximation [19]. We use the expression of ref.[20] to describe the spectrum of photons that interact directly:

$$f_{\gamma|e}^{\mathrm{EPA,dir.}}(x) = \frac{\alpha_{\mathrm{em}}}{2\pi x}\left\{\left[1+(1-x)^2\right]\left(\ln\frac{E^2}{m_e^2}-1\right)\right.$$
$$\left. +x^2\left[\ln\frac{2\,(1-x)}{x}+1\right]+(2-x)^2\ln\frac{2\,(1-x)}{2-x}\right\}, \qquad (2)$$

where $E = \sqrt{s}/2$ is the nominal electron beam energy and m_e the electron mass. This expression has been shown [21] to reproduce exact results for both differential and total cross sections for the two–photon production of scalars and spin–1/2 fermions to relative accuracy of 10% or better. However, it has been derived by integrating the virtuality $-P^2$ of the exchanged photon over its full kinematically allowed range. On the other hand, it is known [22] that the parton content of highly off–shell photons is reduced compared to that of real photons. If the scale Q^2 at which the photon is probed is less than P^2, the concept of partons residing "in" this photon is no longer applicable; in this kinematical regime the formalism of deep inelastic scattering should be used, where the charactersistic scale would be given by P^2 rather than Q^2. We have conservatively ignored contributions with $P^2 > Q^2$ altogether. Furthermore we introduce a numerical suppression factor of 0.85, estimated from results of Rossi [22], in order to approximate the suppression of virtual photon structure functions in the region $\Lambda^2_{\mathrm{QCD}} < Q^2 < P^2$. Altogether we thus have for the effective spectrum of resolved photons:

$$f_{\gamma|e}^{\mathrm{EPA,res.}}(x) = 0.85\frac{\alpha_{\mathrm{em}}}{2\pi x}\left[1+(1-x)^2\right]\ln\frac{Q^2}{m_e^2}. \qquad (3)$$

We remark that this numerical suppression factor should *not* be introduced if anti–tagging of the scattered electrons already implies $P^2 \leq \Lambda^2_{\mathrm{QCD}}$.

The second major contribution to the photon flux at e^+e^- linacs comes from beamstrahlung [9]. As already mentioned in the Introduction, beamstrahlung is produced when particles in one bunch undergo rapid acceleration upon entering the electromagnetic field of the opposite bunch. The intensity and spectrum of beamstrahlung therefore depend on the strength and extension of this field, which in turn are determined by the size and shape

of the bunches. Unlike the machine–independent bremsstrahlung (EPA) contribution described above, the beamstrahlung contribution to $f_{\gamma|e}$ therefore depends sensitively on the bunch parameters of the collider under discussion. In general the relationship between the photon spectrum and the machine parameters is highly nontrivial [9, 23]. Fortunately, Chen [24] has recently been able to derive approximate expressions, which accurately reproduce the exact spectra as long as the fields produced by the bunches are not too strong; this criterium is always fulfilled for our examples.

In this approximate treatment the beamstrahlung spectrum is determined by three parameters: The electron beam energy E; the bunch length σ_z (for a Gaussian longitudinal bunch profile); and the beamstrahlung parameter Υ, which is proportional to the effective magnetic field of the bunches. For Gaussian beams, the effective or mean value of Υ can be estimated from [24]

$$\Upsilon = \frac{5 r_e^2 E N}{6 \alpha_{\rm em} \sigma_z \left(\sigma_x + \sigma_y\right) m_e}. \tag{4}$$

Here, N is the number of electrons or positrons in a bunch, σ_x and σ_y are the transverse bunch dimensions, and $r_e = 2.818 \cdot 10^{-12}\ mm$ is the classical electron radius. Notice that Υ decreases like $\sqrt{\sigma_y/\sigma_x}$ if $\sigma_x \gg \sigma_y$ with constant $\sigma_x \cdot \sigma_y$. Moreover, for given luminosity and bunch dimensions, N is inversely proportional to the square root of the number of bunch collisions per second; beamstrahlung can therefore be reduced by introducing more bunches.

In terms of these parameters, the beamstrahlung spectrum can be written as [24]

$$f_{\gamma|e}^{\rm beam}(x) = \frac{\kappa^{1/3}}{\Gamma(1/3)} x^{-2/3} \left(1-x\right)^{-1/3} e^{-\kappa x/(1-x)} \left\{ \frac{1-w}{\tilde{g}(x)} \left[1 - \frac{1}{\tilde{g}(x) N_\gamma} \left(1 - e^{-N_\gamma \tilde{g}(x)} \right) \right] + w \left[1 - \frac{1}{N_\gamma} \left(1 - e^{-N_\gamma} \right) \right] \right\}, \tag{5}$$

with

$$\tilde{g}(x) = 1 - \frac{1}{2} \left(1-x\right)^{2/3} \left[1 - x + (1+x) \sqrt{1 + \Upsilon^{2/3}} \right] \tag{6}$$

and $\kappa = 2/(3\Upsilon)$, $w = 1/\left(6\sqrt{\kappa}\right)$. Finally, the average number of photons per electron N_γ is given by

$$N_\gamma = \frac{5 \alpha_{\rm em}^2 \sigma_z m_e}{2 r_e E} \frac{\Upsilon}{\sqrt{1 + \Upsilon^{2/3}}}. \tag{7}$$

Eqs.(5) – (7) are valid as long as Υ is not much larger than one, practically for $\Upsilon \leq 5$ or so.

Notice that the flux of soft photons with $\kappa x \leq 1 - x$ actually decreases slowly with increasing Υ; in contrast, the flux of hard photons is exponentially suppressed if $\Upsilon \ll x/(1-x)$. Furthermore, we see from eqs.(4) and (7) that N_γ is approximately independent of $\sqrt{s}$ and σ_z, while $\Upsilon \propto \sqrt{s}/\sigma_z$; notice that $f_{\gamma|e}^{\rm beam}$ grows almost linearly with N_γ as long as $\tilde{g}(x) N_\gamma \leq 1$. Increasing the bunch length thus strongly suppresses the hard part of the beamstrahlung spectrum, but increases the soft part of the spectrum.

Parameters of some recent designs [2] of e^+e^- linacs are listed in Table 1. In addition to the nominal centre–of–mass energy, the beamstrahlung parameter and the bunch length,

for future reference we also give the luminosity per bunch crossing $\hat{\mathcal{L}}$, the number of bunches per bunch train N, the temporal separation between two consecutive bunches in a train Δt, and the total luminosity $\mathcal{L}$.

The Palmer G and Palmer F designs, first proposed in ref.[25], as well as the proposal for the Japan Linear Collider (JLC) [26] all foresee the use of X–band microwave cavities; these offer strong accelerating fields, and thus allow to construct relatively short accelerators. In contrast, the DESY–Darmstadt designs [27, 28] use larger S–band cavities; this technology is better understood, but the accelerating fields are smaller. Finally, the TeV Superconducting Linear Accelerator (TESLA) design [29] employs superconducting cavities. This allows to store the microwave energy almost indefinetely, which in turn makes it possible to use a very large number of well separated bunches with low luminosity per bunch crossing; we have already seen that this reduces beamstrahlung. On the other hand, this design is technologically most demanding.

The corresponding photon spectra, computed from eqs.(5) – (7), are shown in Figs. 1a,b. As expected from the above discussion, the TESLA beamstrahlung spectrum is very soft; its contribution to the total photon spectrum is negligible for fractional momentum $x \geq 0.05$. In contrast, the beamstrahlung spectrum of the Palmer G design is quite hard, dominating the total photon spectrum out to $x \simeq 0.6$. The other designs fall in between these two extremes. Notice the cross–over of the beamstrahlung spectra of the wide band beam (wbb) DESY–Darmstadt and Palmer F designs; the former uses longer bunches and thus has more soft beamstrahlung, while the larger Υ parameter of the latter leads to enhanced hard beamstrahlung. Since the hard contribution to the total photon spectrum is in both cases dominated by the bremsstrahlung (EPA) contribution, we can expect larger two–photon cross sections at the DESY–Darmstadt (wbb) collider. The narrow band beam (nbb) version of this design has a beamstrahlung spectrum which is almost as soft as that of the TESLA; on the other hand, it has the smallest luminosity of the designs we studied.

Fig. 1b shows the evolution of the beamstrahlung spectrum at the JLC as the energy is increased from 0.5 to 1.5 TeV. We see from eq.(4) that, everything else remaining constant, Υ increases linearly with energy. However, higher energies also necessitate higher luminosities. If this is achieved by reducing the transverse bunch dimensions σ_x and $\sigma_y \propto 1/\sqrt{s}$, or by increasing the number of particles per bunch $N \propto \sqrt{s}$, Υ will grow like s, not like $\sqrt{s}$. Table 1 shows that the first planned extension of the JLC, from $\sqrt{s} = 0.5$ to 1 TeV, indeed leads to an almost fourfold increase of Υ. The difference between the corresponding spectra is therefore quite pronounced. In the second step, from $\sqrt{s} = 1$ to 1.5 TeV, Υ only grows linearly with $\sqrt{s}$. The difference in the spectra, when shown as a function of the scaling variable x, is therefore not very large. This slow increase of Υ, which has been achieved by increasing the aspect ratio σ_x/σ_y from about 120 to 200, implies that the average number of beamstrahlung photons per electron even decreases [26] in this second extension, from 1.8 to 1.45; quantum effects, which lead to the suppression factor $1/\sqrt{1+\Upsilon^{2/3}}$ in eq.(7), also contribute to the reduction of N_γ.

As already mentioned in the Introduction, it has been suggested [13] to convert future e^+e^- colliders into $\gamma\gamma$ colliders by backscattering laser light off the incident electron and positron beams. If these beams are not polarized, the resulting photon spectrum depends only on the electron energy and the frequency of the laser. The laser photons should not

be too energetic, since otherwise a backscattered photon and a laser photon could combine to form an e^+e^- pair, which would drastically reduce the electron to photon conversion efficiency. Here we will assume that the laser frequency is chosen such that one stays just below this threshold. The spectrum of backscattered photons is then given by [13]

$$f_{\gamma|e}^{\rm laser}(x) = \frac{-0.544x^3 + 2.17x^2 - 2.63x + 1.09}{(1-x)^2} \cdot \theta(0.828 - x). \tag{8}$$

This spectrum is shown by the long-dashed dotted curve in Fig. 1a; it slowly rises towards its kinematical cut–off. Notice that the spectrum (8) expressed in terms of the fractional photon energy x is *independent* of the beam energy; this is a consequence of our assumption that the energy of the original laser photons decreases like $1/\sqrt{s}$, as described above. Since (almost) the total electron beam energy will be passed on to the photons, there is (almost) no bremsstrahlung contribution to the photon flux at a $\gamma\gamma$ collider. We will assume that at these machines eq.(8) describes the total spectrum.

Of course, in order to compute cross sections for resolved photon processes one also needs to know the distribution functions $\vec{q}^{\gamma}(x,Q^2) = (q_i^{\gamma}, G^{\gamma})(x,Q^2)$ of partons inside the photon, in addition to the photon spectrum. The f_i of eq.(1) are then given by convolutions of these distribution functions:

$$f_{\vec{q}|e}(x,Q^2) = \int_x^1 \frac{dz}{z} f_{\gamma|e}(z) \vec{q}^{\gamma}(\frac{x}{z}, Q^2). \tag{9}$$

Unfortunately, there is not (yet) much experimental information about $\vec{q}^{\gamma}(x,Q^2)$. The combination $F_2^{\gamma} = x\sum_i e_i^2 q_i^{\gamma}$ (up to higher order corrections) has been measured [30] for $x \geq 0.05$ with a precision of typically 10 – 20 %; however, these measurements tell us practically nothing about the flavour structure of the photon, the quark distributions at low x, or the gluon distribution at any x. This last point was demonstrated explicitly in ref.[31], where it was shown that very different ansätze for $G^{\gamma}(x,Q_0^2)$ can lead to almost equally good descriptions of existing data on $F_2^{\gamma}(x, Q^2 \geq 4\ {\rm GeV}^2)$.

We try to give a feeling for the resulting uncertainties by presenting results for different parametrizations of $\vec{q}^{\gamma}(x,Q^2)$. Our "standard" choice will be the "DG" parametrization of ref.[32]. It is free of unphysical $x \to 0$ divergencies; it also fits the data on F_2^{γ} quite well [31]. The most important feature of this parametrization relevant for phenomenological applications is that it assumes that gluons are only created radiatively in the photon; this leads to a rather soft shape of $G^{\gamma}(x)$, as well as a small total gluon content of the photon.

Our second choice is based on the asymptotic "DO" parametrization of ref.[15]. As discussed in ref.[7], it has to be augmented by a "hadronic" contribution in order to fit data on F_2^{γ} with $\Lambda_{\rm QCD} = 0.4$ GeV; we estimate [7] this component using Vector Meson Dominance (VMD) ideas. We use this parametrization mostly to demonstrate the effects of a relatively hard, truely intrinsic contribution to G^{γ}. However, the DO parametrization should not be used at very small values of x, because it suffers from even worse divergencies than the $x^{-1.6}$ behaviour predicted [4] by the leading order asymptotic calculation. Since this region is important for the accelerators we are discussing, we have modified the DO parametrization for small x, somewhat arbitrarily defined as $x \leq 0.05$:

$$q_i^{\gamma,{\rm mod.DO}}(x,Q^2) = c_i x^{-1.6} \ln \frac{Q^2}{\Lambda_{\rm QCD}^2}, \tag{10}$$

where the c_i are chosen to give smooth transitions at $x = 0.05$; a similar ansatz has been used for the gluon density. We call the result the "modified DO+VMD" parametrization.

Both the DG and the DO+VMD parametrization are able [8] to describe the AMY data on jet production quite well, if the minimal partonic transverse momentum is adjusted properly; we will come back to this point in sec.4. In contrast, the third parametreization of ref.[31] ("LAC3") has been found [33] to over–estimate the resolved photon contribution by a large factor. This is due to the extremely hard gluon density used in this parametrization, $G^\gamma(x, Q_0^2) \propto x^6$, which looks quite unnatural.* The other two parametrizations of ref.[31] are quite similar to each other; we will show some results for the "LAC2" parametrization. However, we again find it necessary to slightly modify the original parametrization. It gives separate and different distribution functions for u, d, s and c quarks; we find that at small x it usually predicts $c^\gamma(x) > u^\gamma(x)$, $s^\gamma(x) > d^\gamma(x)$, opposite to the expectation that the contribution of heavier quarks should be suppressed. We therefore define effective distribution functions for charge 2/3 and 1/3 quarks:

$$u_{\rm eff}^{\gamma,\rm LAC}(x,Q^2) = \frac{1}{2}\left(u^{\gamma,\rm LAC} + c^{\gamma,\rm LAC}\right)(x,Q^2); \tag{11a}$$

$$d_{\rm eff}^{\gamma,\rm LAC}(x,Q^2) = \frac{1}{2}\left(d^{\gamma,\rm LAC} + s^{\gamma,\rm LAC}\right)(x,Q^2). \tag{11b}$$

Note that the DG and DO+VMD parametrizations also assume $s^\gamma = d^\gamma$ and $c^\gamma = u^\gamma$ (for $N_f \geq 4$).

Very recently, two more sets of parametrizations of $\vec{q}^\gamma$ have been proposed. In ref.[34] Glück et al. give a parametrization of their "dynamical" prediction for the photon structure function [35]. They assume a hard, valence–like gluon distribution at a very low input scale $Q_0 = 300$ MeV. As a result their G^γ resembles the DO+VMD parametrization at median and large Bjorken–x and low Q^2, but becomes more similar to the DG parametrization for low x and/or high Q^2. In contrast, Gordon and Storrow [36] use a rather high input scale; their input is a sum of a VMD part and a "pointlike" part estimated from the quark–parton model. They give two parametrizations, depending on whether gluon radiation from the "pointlike" part of the quark densities is included. At median and large x and low Q^2 their gluon densities lie between those of the DG and DO+VMD parametrizations. At low x and low Q^2 it falls even below the DG prediction for G^γ; however, this is mostly due to their choice of a rather high value of Q_0. In fact, their parametrization cannot be used for $Q^2 < 5.3$ GeV^2, so that it cannot predict total $c\overline{c}$ (sec. 3c) or minijet (sec. 4) cross sections. In any case, by comparing predictions from the DG, modified DO+VMD and LAC2 parametrizations we still span the whole range of existing parametrizations for $\vec{q}^\gamma$, with the parametrizations of refs.[34, 36] falling somewhere in between.

We will give results for processes characterized by momentum scales between a few and a few hundred GeV. In between, two flavor thresholds are crossed. The problem of heavy quark distribution functions in the photon still awaits a rigorous treatment [37]. For simplicity we will assume $N_f = 3$ massless flavors in the photon if the momentum scale $Q^2 < 50$ GeV^2, $N_f = 4$ for 50 GeV$^2 \leq Q^2 \leq 500$ GeV^2, and $N_f = 5$ for $Q^2 > 500$ GeV^2.

*The fact that this ansatz reproduces existing data on F_2^γ demonstrates once again that these data give very little information about G^γ.

We use the interpolating expression of ref.[38] for α_s, with $m_c = 1.5$ GeV, $m_b = 5$ GeV and $m_t = 100$ GeV.[†] When using the DG or modified DO+VMD parametrizations we assume $\Lambda_{\rm QCD} = 0.4$ GeV, while the LAC parametrizations have to be used with $\Lambda_{\rm QCD} = 0.2$ GeV.

As mentioned at the beginning of this section, we will employ the structure function formalism of eq.(1) also to compute annihilation cross sections. In this case we will use it to include the effects of initial state radiation and beamstrahlung. Both effects smear out the electron distribution function $f_{e|e}(x)$ from the ideal δ–function at $x = 1$. Initial state radiation (ISR) is described by (to one loop order) [39]

$$f_{e|e}^{\rm ISR}(x) = \frac{\beta}{2}\,(1-x)^{\beta/2-1}\left(1+\frac{3}{8}\beta\right) - \frac{1}{4}\beta\,(1+x)\,, \tag{12}$$

where

$$\beta = \frac{2\alpha_{\rm em}}{\pi}\left(\ln\frac{s}{m_e^2} - 1\right).$$

The first term in eq.(12) re–sums leading logarithms near $x = 1$, i.e. includes soft photon exponentiation. Numerically, $\beta = 0.124$ at $\sqrt{s}$= 500 GeV. Even at this high energy the electron spectrum (12) is strongly peaked at $x = 1$, with 79% (26%) of all electrons satisfying $x > 0.99$ (0.999). Notice that eq.(12) satisfies the charge conservation constraint $\int_0^1 f_{e|e}(x) = 1$ exactly.

We again use the approximate formalism of ref.[24] in order to describe the effects of beamstrahlung. Here the electron spectrum is given by the function ψ:

$$\psi(x) = \frac{1}{N_\gamma}\left\{\left(1-e^{-N_\gamma}\right)\delta(1-x) + \frac{e^{-\eta(x)}}{1-x}\sum_{n=1}^{\infty}\left[(1-x)+x\sqrt{1+\Upsilon^{2/3}}\right]^n \frac{\eta(x)^{n/3}}{n!\Gamma(n/3)}\gamma(n+1,N_\gamma)\right\}, \tag{13}$$

where $\eta(x) = \kappa\,(1/x-1)$, and $\gamma(a,b)$ is the incomplete γ–function, for which we use a power expansion [40]. The parameters N_γ and κ have already been introduced in the discussion of the beamstrahlung photon spectrum. Notice that $\psi(x)$ is approximately, but not exactly normalized to 1. The fact that the deviation is always less than 10% for the machines we are considering gives us some confidence that the formalism of ref.[24] is indeed applicable to them. Nevertheless one would like to achieve better than 20% precision at least for annihilation cross sections. We thus write

$$f_{e|e}^{\rm beam}(x) = \frac{\psi(x)}{\int_0^1 \psi(z)dz}, \tag{14}$$

which satisfies charge conservation exactly. Another possibility would have been to adjust the coefficient of the δ–function in eq.(13) such that charge is conserved exactly. Since most of the electron spectrum is concentrated at and just below $x = 1$, the difference between these two procedures is very small.

[†]The variation of α_s with m_t is always negligible for our processes.

Beamstrahlung and initial state radiation are characterized by quite different time or length scales. The final electron spectrum can therefore to very good approximation be obtained by simply convoluting eqs.(12) and (14):

$$f_{e|e}(x) = \int_x^1 \frac{dz}{z} f_{e|e}^{\rm ISR}(z) f_{e|e}^{\rm beam}(\frac{x}{z}). \tag{15}$$

Since the calculation of $\psi(x)$, eq.(13), is numerically quite costly, we found it convenient to use a cubic spline interpolation for $f_{e|e}(x)$.

The resulting electron spectra for some designs of e^+e^- colliders operating at $\sqrt{s} = 500$ GeV are shown in fig. 2. For comparison we also show a curve where beamstrahlung is not included, so that the spectrum is simply given by eq.(12) (dotted line). We see that at the TESLA design, beamstrahlung affects the spectrum only in the region $x \geq 0.95$; while this may still have some impact on the study of new thresholds, it is unimportant for our purposes, since our cross sections do not depend very sensitively on the incident electron energy. In contrast, at the Palmer G design beamstrahlung modifies the electron spectrum at *all* values of x; it increases $f_{e|e}(x = 0.5)$ by more than an order of magnitude. For all the other designs for colliders operating at this energy the low energy end of the electron spectrum is essentially given by the bremsstrahlung contribution alone. This is true even for the JLC1 design, which has the second largest beamstrahlung contribution of the designs we studied. Notice that for machines with little or no beamstrahlung, a substantial part of the integral over $f_{e|e}$ comes from the region $x > 0.99$, which is not shown in fig. 2.

Finally, we mention that we used a running electromagnetic coupling constant when computing annihilation cross sections:

$$\alpha_{\rm em}^{-1} = 128 \left(1 - \frac{20}{9\pi} \frac{1}{128} \ln \frac{\hat{s}}{m_Z^2} \right). \tag{16}$$

This expression includes contributions from all light fermions, including b–quarks, but no t or W loops. Numerically, $\alpha_{\rm em}(1\ {\rm TeV}) = 1/124.6$. Of course, we include both γ and Z exchange contributions to the annihilation cross section. We do, however, not include QCD corrections, since this would be quite nontrivial in case of the two–photon cross sections.[‡] The distinction between direct and resolved contributions becomes blurred in higher orders; e.g., QCD corrections to the direct process contain collinear divergencies which have to be absorbed in the parton distribution functions [41]. Our results for annihilation cross sections should therefore be precise to about 5 to 10%.

We are now in a position to present our numerical results. We start with a discussion of various hard two–photon induced backgrounds.

3. Hard two–photon reactions

In this section results for "hard" two–photon processes are given, the cross sections of which can in principle be calculated unambiguously from perturbative QCD once the

[‡]Of course, in many cases the QCD corrections to annihilation cross sections can be estimated by simply multplying the cross section with $1 + \alpha_s/\pi$, leading to a 3 to 5% increase of the cross section.

parton densities inside the photon are known. By far the most common of these processes is the production of two high–p_T jets. If both jets are produced at large angles, this process leads to a di–jet final states, which we discuss in sec. 3a; in sec. 3b we present results for the case that one of the two jets emerges at a very small angle, which leads to mono–jet events. The production of heavy quarks ($c\overline{c}$, $b\overline{b}$ and $t\overline{t}$) is discussed in sec. 3c. In sec. 3d the production of single W and Z bosons is studied; these events are comparatively rare, but offer quite striking signatures if the gauge bosons decay leptonically.

3a. Di–jet production

This reaction offers the largest rates of all hard, hadronic two–photon processes; it is also the only one for which experimental data have been analyzed [42, 8]. As already described in sec. 2, all three classes of two–photon production mechanisms (direct, once resolved and twice resolved) contribute here. Recall that the once (twice) resolved contributions are characterized by one (two) spectator jets in addition to the high–p_T jets. However, since the axes of these jets coincide essentially with the beam pipes, it will most likely not be possible to measure their energy on an event-by-event basis.* The only inclusive observables are thus the transverse momenta and angles or rapidities of the two high–p_T jets. In the leading logarithmic approximation of eq.(1), the transverse momenta of both jets are equal and opposite; this is exactly true for on–shell (beamstrahlung) photons, and should still hold to good approximation for bremsstrahlung photons, due to the $1/Q^2$ behaviour of the photon propagator. Any given event can thus be characterized by the three variables p_T, y_1 and y_2, where y_i denotes the rapidity of the i–th jet. On the parton level, these three observables are related to the fractional momenta x_i of eq.(1) via

$$x_1 = \frac{x_T}{2}\left(e^{y_1} + e^{y_2}\right); \tag{17a}$$

$$x_2 = \frac{x_T}{2}\left(e^{-y_1} + e^{-y_2}\right), \tag{17b}$$

where

$$x_T = \frac{2\sqrt{p_T^2 + m^2}}{\sqrt{s}} \tag{18}$$

is an "average" or "typical" value for the x_i. The Mandelstam variables $\hat{t}$ and $\hat{u}$ of the $2 \to 2$ subprocess are given by

$$\hat{t},\ \hat{u} = m^2 + \frac{\hat{s}}{2}\left(-1 \pm \sqrt{1 - x_T^2}\right). \tag{19}$$

For future reference we have allowed for a finite (equal) mass m of the two produced partons; in this and the next subsection we will be concerned with the case $m = 0$.

In this subsection we require both jets to be produced centrally. In this context it is important to realize that detectors at future e^+e^- linacs will almost certainly have

*This does not contradict our previous claim that perhaps as many as 50% of all particles originating from those jets will emerge at large angles. The average transverse momentum of particles from the spectator jets will be a few hundred MeV, while their longitudinal momentum can be many GeV; the most energetic particles will therefore emerge at the smallest angle, and thus escape detection.

substantial dead areas around the beam pipes, i.e. will not be very hermetic. The reason is that beamstrahlung also gives rise to enormous numbers of e^+e^- pairs [43, 23]. For instance, at the Palmer G design, one might have to expect [10] about 500,000 such pairs per bunch crossing. Fortunately, a large majority of these electrons will be produced at small angles and with small transverse momentum; the central part of the detector should therefore remain relatively free of these electrons, although a few central pairs per bunch crossing might still have to be expected [23, 44]. However, the large electron flux at small angles will make it almost impossible to extend the detector close to the beam pipe. We will assume that the angular coverage for jets only extends out to $\theta = 15°$, which corresponds to

$$|y_{1,2}| \leq 2. \tag{20}$$

In order to give a first idea of the magnitude of hard two–photon cross sections, we show in fig. 3a the total cross section for the production of two central jets with $p_T \geq p_{T,min}$, as a function of $p_{T,min}$, for the first four machines of table 1 as well as the $\gamma\gamma$ collider. For fig. 3 and the remaining figures of this subsection we have used $Q^2 = \hat{s}/4$ as the scale in the parton densities, including the bremsstrahlung spectrum of resolved photons (eq.(3)), as well as in α_s; choosing $Q^2 = p_T^2$ instead would have changed the results only by about 10%, but would have lead to even more pronounced kinks at $p_T = \sqrt{50}$ and $\sqrt{500}$ GeV, where the number of participating flavors is changed, as described in sec. 2. The results of fig. 3 have been obtained using the DG parametrization; as will be discussed in more detail later, the other parametrizations mentioned in sec. 2 would lead to even larger cross sections. Notice that the QED point cross section $4\pi\alpha_{\rm em}^2/(3s)$ only amounts to 0.4 pb at $\sqrt{s} = 500$ GeV. The two–photon cross section even for quite hard jets ($p_T > 10$ GeV) is between 50 and 1,000 times larger, where the smaller (larger) number refers to the TESLA (Palmer G) design. The luminosity per year of 10^7 seconds varies between 14 fb^{-1} at Palmer F and 60 fb^{-1} at Palmer G. The two–photon contribution to the total di–jet rate should therefore in principle be measurable out to $p_T = 150$ GeV at least. More importantly, one expects between 4 (at TESLA) and 250 (at Palmer G) *million* two–photon events per year with $p_T > 5$ GeV.

We see from fig. 1 that at very large p_T all e^+e^- colliders must have the same two–photon cross sections, since $f_{\gamma|e}(x)$ is dominated by the bremsstrahlung (EPA) contribution as $x \to 1$. Indeed, at the TESLA and DESY-Darmstadt (nbb) colliders beamstrahlung increases the total di–jet production with $p_T > 20$ GeV only by 20% or less; for the DESY–Darmstadt (wbb), Palmer F and JLC machines this is true only for $p_T > 75$ to 100 GeV. Finally, at the Palmer G design, the beamstrahlung contribution remains sizeable at all values of p_T where the di–jet cross section is measureable. For all e^+e^- colliders the cross section falls quite rapidly with $p_{T,min}$. In contrast, the very hard photon spectrum of the $\gamma\gamma$ collider leads to a relatively flat p_T spectrum once $p_T > 50$ GeV or so; here the total rate is dominated by the direct process $\gamma\gamma \to q\overline{q}$.

In fig. 3b we show the integrated di–jet cross section for the three stages of the JLC. We also give a first indication of the relative importance of the various contributing processes by showing separate curves for the direct process (dashed) and the total cross section (solid). The evolution of the direct cross section with energy closely follows that of the photon spectrum, see fig. 1b: At small p_T, corresponding to small x, the cross section decreases with energy, due to the depletion of soft photons when Υ is increased;

the cross section at high p_T increases quite rapidly with energy, since lager Υ lead to a rapid increase of the flux of hard photons.

However, fig. 3b also shows that at small p_T, the total cross section is dominated by resolved photon contributions. Recall that their cross sections increase [7] with the $\gamma\gamma$ centre–of–mass energy $W_{\gamma\gamma}$. Therefore events with large $W_{\gamma\gamma}$ make sizeable contributions even in the region of rather small p_T, in spite of the decrease of the photon flux with $W_{\gamma\gamma}$. As a result, the total di–jet rate increases monotonously with energy for all p_T. Moreover, the region where resolved photon processes dominate increases with increasing energy, even when this region is expressed in terms of the scaling variable x_T of eq.(18).

This discussion shows that harder photon spectra favour resolved photon processes compared to the direct process. For instance, at the TESLA collider with its very soft beamstrahlung photon spectrum, resolved photon contributions dominate [44] total di–jet production only for $p_T \leq 5$ GeV, while at the $\gamma\gamma$ collider they remain dominant up to $p_T \simeq 50$ GeV. We also find that the once resolved contribution exceeds the twice resolved one only for those values of transverse momentum where the total rate is already dominated by the direct process. This is because the twice resolved contribution gets a dynamical enhancement factor [7] $\hat{s}/\hat{t} \propto \hat{s}/p_T^2$ compared to both the direct and the once resolved contributions; the former can proceed via gluon exchange in the t channel, leading to a $1/\hat{t}^2$ pole in the matrix elements [16], while the latter only have $1/\hat{t}$ poles in the matrix elements, originating from t channel quark exchange. However, we will see below that the once resolved contribution *can* be dominant in certain kinematical configurations.

In fig. 4 we give more details about the final state composition of the once (fig. 4a) and twice (fig. 4b) resolved contribution; in these and the following figures, q stands for a quark or antiquark of any flavour. Not surprisingly, we see that final states that require a gluon in the initial state (qq in the 1–res contribution, and qg and gg[†] for the 2–res contribution) have a steeper p_T spectrum than those that originate from purely quarkonic initial states; we have already seen in sec. 2 that all reasonable parametrizations of the parton densities inside the photon predict $G^\gamma(x)$ to be much softer than the $q_i^\gamma(x)$. Notice that the qg final state makes an important contribution over a wide range pf p_T values. The hard quark distribution functions allow to probe the gluon density at quite small x, where it is large. Moreover, the hard $qg \to qg$ matrix element is dynamically enhanced [16] by a color factor of 9/4 compared to $qq \to qq$ matrix elements.

Note that the relative importance of the various final states depends on the photon spectrum in a nontrivial way. We have already seen that harder photon spectra generally favour more resolved processes. Since harder photons allow to probe the parton densities inside the photon at small Bjorken–x, see eq.(9), they also favour gluon–initiated processes over quark–initiated ones. One must realize, however, that the addition of even a relatively hard beamstrahlung spectrum, like the one at the JLC for which the results of fig. 4 have been obtained, can lead to an effectively softer shape for the total photon spectrum. This is because bremsstrahlung always dominates in the limit $x \to 1$; beamstrahlung can only add to the (comparatively) soft part of the photon spectrum. Moreover, as discussed above, the three classes of processes (direct, 1–res and 2–res) get

[†] We also include the contribution from $q\overline{q} \to gg$, but it is always very small.

contributions from quite different parts of the photon spectrum, for a given value of p_T. For instance, at the TESLA collider gluon–induced processes never dominate the total single resolved contribution, which are dominated by events with quite small $W_{\gamma\gamma}$, where the TESLA photon spectrum is soft, due to the very soft beamstrahlung spectrum. However, the twice resolved processes, especially those involving gluons in the initial state, are dominated by events with much larger $W_{\gamma\gamma}$, where the beamstrahlung contribution is already negligible at TESLA; the total photon spectrum in this region is dominated by the hard bremsstrahlung contribution of eq.(3). As a result, the cross–over between the qq and qg final states within the 2–res contributions occurs at *larger* p_T at TESLA (28 GeV) than at the first stage of the JLC (18 GeV) or even Palmer G (20 GeV). Of course, the cross sections for all final states increase quite rapidly when going from TESLA over JLC1 to Palmer G, as shown in fig. 1; however, the above discussion shows that the cross sections for the various subclasses of contributions increase at quite different rates when the beamstrahlung spectrum is made harder. Finally, at the $\gamma\gamma$ collider with its very hard photon spectrum, the cross–over between quark–initiated and gluon–initiated processes only occurs [44] at $p_T \simeq 45$ GeV.

Of course, the relative importance of the various initial and final state configurations also depends on the parton distribution functions $\vec{q}^{\gamma}(x, Q^2)$. We mentioned already in sec. 2 that the DO+VMD parametrization predicts quite similar quark distribution functions inside the photon as the DG parametrization does[‡], while its G^{γ} exceeds that of the DG parametrization by roughly a factor of 2. It thus predicts [44] approximately two times larger rates for the 1–res qq and 2–res qg final states, and a four times larger rate for the gg final state.

The differences predicted by the recent parametrization of ref.[31] are even larger, as shown in fig. 5a,b; here we show results for the least extreme[§] of the three LAC parametrizations, normalized to the prediction of the DG parametrization. At very small x and small Q^2, its gluon density is about 7 times larger than that of the DG parametrization. This far over–compensates the reduction of α_s which is induced by the reduction of $\Lambda_{\rm QCD}$ from 0.4 GeV (DG) to 0.2 GeV (LAC); this reduction amounts to a factor of 0.7 (0.5) for the 1–res (2–res) processes at $p_T = 2$ GeV. This manifests itself in the 1–res qq final state, which comes from a $g\gamma$ initial state; the hard photon spectrum leads to a very small average x for the gluon inside the other photon. If both initial state particles are gluons, their average Bjorken x has to be increased; therefore the enhancement factor for the gg final state at the smallest transverse momentum shown is not 25, but "only" 15. The enhancement factors for both the 1–res qq and the gg final states eventually flatten out when one goes to even smaller x, which can be achieved by using a harder photon spectrum; e.g., at the $\gamma\gamma$ collider they reach 4.8 and 25, respectively, at $p_T = 2$ GeV.

In sharp contrast, the effective quark density at small x and small Q^2 predicted by the LAC2 parametrization appears to be only 70% of that of the DG parametrization. This is because the authors of ref.[31] treat the charm quark as a massless parton already at

[‡]Except for the region $x < .1$; however, here gluon initiated processes overwhelm quark initiated ones anyway.

[§]The LAC1 parametrization uses an even steeper gluon distribution function. We have already seen in sec. 2 that the LAC3 parametrization, whose gluon density peaks at large x, is strongly disfavoured by the AMY data [8] on jet production.

$Q^2 = 4\ GeV^2$; however, we do not include the contribution from c quarks if $Q^2 < 50\ GeV^2$. Without the c quark contribution, the LAC parametrization cannot reproduce data on F_2^γ at small Q^2; one might therefore argue that for $Q^2 < 50\ GeV^2$, we should have defined the effective LAC u quark density as the sum of the original u and c quark densities, rather than as the arithmetic mean as shown in eq.(11a).¶ In that case the predictions of the LAC2 parametrization for purely quark initiated processes would have been quite similar to that of the DG parametrization. Notice that the 2–res qq final state in fig. 5b shows no depletion at small p_T; the reason is that the LAC2 parametrization predicts a sizeable contribution to this final state from gg fusion, inspite of the smallness of the hard $gg \to q\overline{q}$ matrix element compared to the one for $qq \to qq$ [16]. In any case, we have already seen that even the DG parametrization with its small gluon density predicts quark–initiated processes to be sub–dominant for $p_T < 5$ GeV, so that this discussion is somewhat academic.

In view of these very large differences in the region of small p_T, and correspondingly small x and small Q^2, it is reassuring to note that the two parametrizations make quite similar predictions both for quark initiated and for gluon initiated processes once $p_T > 20$ GeV, which corresponds to $Q^2 > 400\ GeV^2$ and average Bjorken x for the parton in the photon larger than 0.15. Due to the increase of $\vec{q}^\gamma(x, Q^2) \propto \log Q^2$, the ansatz one assumes for $\vec{q}^\gamma$ at $Q^2 = Q_0^2 = 1$ to $4\ GeV^2$ has only little effect in this kinematical region, although deviations by 20–30% are still possible, e.g. due to the different values for $\Lambda_{\rm QCD}$ that have been assumed. This result also holds for the $\gamma\gamma$ collider, as far as the twice resolved contributions are concerned. However, due to the hardness of the photon spectrum and resulting small average Bjorken x in the 1–res qq final state, the prediction from the LAC2 parametrization still exceeds the one from DG by a factor of 2 at $p_T = 20$ GeV; the two parametrizations make approximately equal predictions only for $p_T > 40$ GeV. Finally, we remark that the use of the DO+VMD parametrization at such large p_T and correspondingly large Q^2 can be dangerous, since it assumes a Q^2 independent (scaling) VMD contribution, in contradiction to expectations from QCD that the assumed hard gluon component should "shrink" down to small values of x.

More detailed information about two–photon contributions to di–jet production can be gained from the triple differential cross section $d\sigma/dp_T dy_1 dy_2$. In fig. 6 we display predictions for this quantity as derived from the DG parametrization, at fixed $p_T = 30$ GeV for the case $y_1 = y_2 \equiv y$; fig. 6a is for the TESLA collider, while fig. 6b shows results for the $\gamma\gamma$ collider. Only the region $y \geq 0$ is shown; the distributions are symmetric in y, of course.

The shape of the curves can be understood from the observation that increasing y increases the Bjorken–x of one parton inside the electron, x_1, while decreasing the other, x_2; see eqs.(17). The requirement $x_1 \leq x_{\rm max}$ immediately gives

$$y \leq y_{\rm max} \equiv \log \frac{x_{\rm max}}{x_T}. \tag{21}$$

For an e^+e^- collider, $x_{\rm max} = 1$, while for the $\gamma\gamma$ collider, $x_{\rm max} = 0.828$, see eq.(8); therefore the curves in fig. 6b end at a somewhat smaller value of y than those in fig. 6a. In the

¶However, in this case it is not clear how the sizeable contribution from $\gamma\gamma^* \to c\bar{c}$ to F_2^γ should have been treated.

limit $y \to y_{\max}$, we have $x_1 \to x_{\max}$ and $x_2 \to x_T^2/x_{\max} \simeq 0.014$ (0.017) at e^+e^- ($\gamma\gamma$) colliders, for the given values of p_T and $\sqrt{s}$. The region of large y is therefore sensitive to both the photon density "in" the electron and the parton densities inside the photon at quite small Bjorken–x, even at this large value of p_T (which correpsonds to annihilation events at the TRISTAN collider).

We saw already in fig. 1a that, due to the beamstrahlung contribution, the TESLA photon spectrum increases rapidly in the region of small x; fig. 6a shows that this leads to an increase of the direct contribution at large y. This shows that one cannot ignore the beamstrahlung contribution even though it increases the di–jet cross section integrated over rapidities $|y_{1,2}| \leq 2$ by only approximately 15%; without this contribution, the rapdity distribution would have the bell shape familiar [7] from lower energy colliders. Of course, at most one of the two initial state photons at large y will come from beamstrahlung, since x_1 is large here; notice that the bremsstrahlung spectrum (2) remains finite as $x \to 1$. The direct contribution at the $\gamma\gamma$ collider also increases as y approaches its kinematical maximum. In this case, however, this is due to the increase of $f_{\gamma|e}$ at large x; fig. 1a shows that it remains essentially constant as $x \to 0$.

The once resolved contribution also remains finite as $y \to y_{\max}$. It is important to realize that in this case only the product of the photon energy and the Bjorken–x of the parton inside the photon is fixed, as shown by eq.(9). The once resolved contribution at TESLA remains large at large y mostly due to the contribution of hard quarks in soft photons. In contrast, the enormous spike[‖] at large y predicted for the $\gamma\gamma$ collider is entirely dominated by soft gluons and sea–quarks in hard photons. This difference also manifests itself in the energy of the spectator jet, which for $y \to y_{\max}$ always emerges at negative rapidities, well separated from the high–p_T jets. At $y = 2$, the DG parametrization predicts the average energy of this jet at TESLA to be 57 GeV, while at $y = 1.8$ at the $\gamma\gamma$ collider it should be as large as 135 GeV. The difference in spectator jet energy between the two 1–res final states at the TESLA collider is even larger: The qg final state only has an average spectator jet energy of 31 GeV, while the qq final state, which originates from a (soft) gluon in the initial state, is accompagnied by a spectator jet with average energy around 100 GeV. This large difference should be observable even in a detector with relatively poor angular coverage.

Finally, the rapidity distribution of the twice resolved contribution always has a maximum at $y = 0$. Since $\vec{q}^{\gamma}(x \to 1) \to 0$, this contribution always vanishes as y approaches its kinematical maximum. In principle, this does not exclude the possibility of having a maximum at intermediate values of y. Indeed, such a maximum does occur at the $\gamma\gamma$ collider for the 2–res qg final state; here the asymmetric initial state favours configurations where a hard quark scatters off a soft gluon. This maximum, which occurs at $y \simeq 1.4$, explains why the total twice resolved contribution shows a very flat rapidity distribution almost all the way out to the kinematical maximum.

We close this subsection with a comparison of two–photon and annihilation contributions to di–jet production at e^+e^- colliders with $\sqrt{s} = 500$ GeV. In fig. 7 we present the di–jet invariant mass distributions for the two most extreme examples of table 1, the Palmer G (fig. 7a) and TESLA (fig. 7b) designs. The contributions from the three

[‖]This is an example where the once resolved contribution dominates, at least in a limited region of phase space.

subclasses of two–photon contributions are shown separately, and compared to the annihilation contribution (dotted curves); both beamstrahlung and initial state radiation have been included for the latter, using eqs. (13) – (15).

The most prominent feature of the annihilation contribution is the peak at $M_{jj} = m_Z$. The annihilation spectrum is quite flat between about 130 and 300 GeV, since the reduction from the s–channel propagators is largely cancelled by the increase of the e^+e^- flux with increasing invariant mass. Of course, at both machines one finds a second, pronounced maximum at large M_{jj}, close to the nominal $\sqrt{s}$ of the machine. The shoulder in the TESLA annihilation contribution at $M_{jj} \simeq 70$ GeV occurs since requiring rapidities $|y_{1,2}| \leq 2$ and $M_{jj} < \sqrt{s} \cdot e^{-2}$ is inconsistent with $x_1 = 1$ or $x_2 = 1$, see eqs.(17); such events can thus only occur if *both* the electron and the positron emit a hard photon before annihilating each other. Fig. 2 shows that the e^+e^- flux at TESLA is little affected by beamstrahlung in the region $M_{jj} < 400$ GeV or so; hard bremsstrahlung only occurs with probability $\alpha_{\rm em}/\pi \log E/m_e \simeq 0.03$, so that double bremsstrahlung is much less likely than single bremsstrahlung. This shoulder is not visible for the Palmer G design, since here beamstrahlung affects the e^+e^- flux at all invariant masses.

It is obvious, however, that beamstrahlung affects the two–photon contribution *much* more than the annihilation contribution. While the rate of two–photon events shoots up by about a factor of 35 when going from TESLA to Palmer G, the annihilation cross section at $M_{jj} \simeq m_Z$ only increases by a factor of 3. In fig. 7a we have chosen the cut $p_T > 20$ GeV, which reduces the two–photon contribution at $M_{jj} \simeq m_Z$ by about 25%, while leaving the Z signal essentially unaltered. An optimal signal–to–background ratio can probably be achieved by choosing a cut around 30 GeV; applying an even stronger cut might not help much, since one then starts to loose significant numbers of annihilation events, partly due to mismeasurement of the true p_T. With the requirement $p_T > 30$ GeV, the di–jet annihilation cross section integrated over 87 GeV $\leq M_{jj} \leq$ 95 GeV at Palmer G becomes 6.0 pb, compared to a two–photon background of 2.9 pb. With the same cuts, the annihilation and two–photon cross sections at the first stage of the JLC are 2.6 pb and 0.38 pb; the corresponding numbers for TESLA are 2.0 and 0.09 pb. Predictions for the DESY–Darmstadt and Palmer F designs fall in between those for TESLA and JLC1.

In fig. 7b we have chosen a very loose explicit p_T cut; note, however, that the rapidity cuts imply $p_T > M_{jj}/8$. Nevertheless this figure nicely demonstrates the effect of the dynamical enhancement factor $\hat{s}/\hat{t} \propto M_{jj}^2/p_T^2$ of the twice resolved contribution, which we already discussed in connection with fig. 3b. We have seen that in the p_T spectrum of di–jet events at TESLA, resolved photon contributions dominate only for $p_T < 5$ GeV, and that the cross–over between the once and twice resolved contributions occurs at $p_T =$ 28 GeV. From naive kinematical considerations one would therefore expect the resolved photon contributions to dominate only for $M_{jj} < 10$ GeV, while fig. 7b shows that they actually are dominant up to $M_{jj} = 60$ GeV; similarly, the cross–over between 1–res and 2–res contributions occurs at $M_{jj} \simeq 200$ GeV, which is three times the value one would expect from kinematics alone, given the p_T spectrum. This enhancement factor also implies that twice resolved contributions will be even more strongly suppressed by a tight cut on p_T than the other two–photon contributions; this can be seen from fig. 7a, where the twice resolved contribution is always below the once resolved one.

This figure also shows that in the region $M_{jj} \geq m_Z$, the total two–photon background

is dominated by the direct contribution, once a modest cut on p_T has been applied; it is therefore almost independent of the parton densities $\vec{q}^{\gamma}$. We can thus conclude with some confidence that at a machine like TESLA or the narrow band beam version of the DESY–Darmstadt design one can study the process $e^+e^- \to q\overline{q}$ down to an invariant mass of about 85 GeV, with little backgound from two–photon reactions. This might offer the possibility to directly measure [45] the running of α_s in a single experiment, by comparing annihilation events at $M_{jj} \simeq m_Z$ with those at $M_{jj} \simeq \sqrt{s}$. At the intermediate machines (DESY–Darmstadt (wbb), Palmer F and JLC1) this should still be possible, but at Palmer G a substantial irreducible two–photon background will remain.

3b. Mono–jet production

So far we have only considered the case where both high–p_T jets are produced centrally. In this section we discuss the case where only one jet is produced centrally, while the other is produced at small angles and thus cannot be reconstructed. To be specific, we require

$$|y_1| \leq 1.5; \tag{22a}$$

$$|y_2| \geq 2, \tag{22b}$$

i.e. we demand a finite rapidity gap between the two jets. Since most of the forward jet will not be seen,* the p_T of the central jet will be approximately equal in magnitude to the total missing p_T in the event. Missing p_T is (part of) the signature for many interesting annihilation events. Within the standard model, these include events with semi–leptonically decaying heavy quarks; W^+W^- events where one gauge boson decays leptonically; and ZZ events where one Z decays into $\nu\overline{\nu}$. Mono–jets might be a particularly important background to one–sided or "Zen" events [46] that could signal the associate production of a heavy and a light supersymmetric neutralino.

Here we consider mono–jets from two–photon events, as well as from $e^+e^- \to q\overline{q}$ annihilation events with hard photon emission from the initial state, where again both beamstrahlung and bremsstrahlung are included. In fig. 8a we show results for the two most extreme designs of e^+e^- colliders with $\sqrt{s} = 500$ GeV listed in table 1, Palmer G and TESLA. We see again that an increase of Υ increases the $\gamma\gamma$ flux much more rapidly than the e^+e^- flux at invariant mass well below $\sqrt{s}$. At TESLA, two–photon events only dominate for $p_T \leq 24$ GeV, while at Palmer G they continue to dominate total mono–jet production up to $p_T \simeq 32$ GeV, and make important contributions also in the region 45 GeV $\leq p_T \leq$ 55 GeV.

The spectrum of the annihilation contribution is, as usual, largely determined by kinematic considerations. The cuts on the rapidities of the two jets imply

$$p_T \leq \frac{\sqrt{s}}{e^{-|y_{1,\max}|} + e^{|y_{2,\min}|}} \simeq 0.131\sqrt{s}, \tag{23}$$

where in the second step eqs.(22) have been used; this bound also applies for two–photon events, of course. Fig. 8a shows that the annihilation contribution stays at the level of

*Part of this jet should still be visible in most cases; the arguments for the detectability of the spectator jets in resolved two–photon events also apply here.

1 fb/GeV almost all the way to the kinematical limit; recall that 1 fb corresponds to at least 10 events per year (up to 60 at Palmer G). Obviously the annihilation contribution increases quite rapidly if the two jets can originate from the decay of a real Z boson. For given transverse momentum, the invariant mass of the $q\overline{q}$ pair is minimized when $y_1 = y_{1,\max}$ and $y_2 = y_{2,\min}$; on–shell Z bosons can therefore only contribute if

$$p_T \leq \frac{m_Z}{\sqrt{2\left[1+\cosh(y_{2,\min}-y_{1,\max})\right]}} \simeq 44.2 \text{ GeV}. \tag{24}$$

However, the contribution of real Z bosons will be suppressed if this final state can only be produced by radiation off both electron legs. On–shell Z bosons produced via single beam– or bremsstrahlung only contribute if

$$p_T = \frac{\sqrt{s}}{\frac{s}{m_Z^2}e^{-y_1}+e^{y_1}}. \tag{25}$$

The r.h.s. reaches its absolute maximum of $m_Z/2$ at $y_1 = \log\sqrt{s}/m_Z$; this, however, is in conflict with the constraint (22a) for the machines we are considering. The maximal achievable p_T is therefore bounded by the r.h.s. of eq.(25) with $y_1 = y_{1,\max}$. Of course, we also have to require that $y_2 \geq y_{2,\min}$. On–shell Z bosons produced via the emission of a single photon from the initial state can therefore only contribute if

$$p_T \leq \frac{\sqrt{s}}{\max\left(\frac{s}{m_Z^2}e^{-y_{1,\max}}, e^{y_{2,\min}}\right)+e^{y_{1,\max}}}. \tag{26}$$

For $\sqrt{s} = 500$ GeV, the r.h.s. amounts to 42.1 GeV; this is so close to the value of eq.(24) that no extra structure at this point is visible in fig. 8a. However, the near–equality of (24) and (26) explains why around the Jacobian peak of the Z boson, the cross section is actually smaller at Palmer G; in most real Z events that pass the cuts (22) at $\sqrt{s}=$ 500 GeV, the energy of one of the electrons is very close to the nominal beam energy, where the flux at Palmer G is depleted due to strong beamstrahlung, while the energy of one emitted photon is so large that it is in most case produced by bremsstrahlung even at Palmer G. Finally, we note that real Z bosons can only be produced in accordance with the cuts (22) if y_1 and y_2 have the same sign. In contrast, the absolute upper bound (23) is saturated if $|y_1 - y_2|$ is maximal, i.e. y_1 and y_2 have opposite signs. In the region $p_T \geq 50$ GeV the cross section at Palmer G therefore exceeds the one at TESLA again, since we are now in a region where the fractional momenta of both the electron and the positron are sizeable.

In fig. 8b we display the three classes of two–photon contributions separately, for the case of the $\gamma\gamma$ collider. Of course, there is no e^+e^- annihilation contribution here. Moreover, when computing the kinematic limit (23), $\sqrt{s}$ has to be replaced by $x_{\max}\sqrt{s} \simeq 0.828\sqrt{s}$; the curves in fig. 8b therefore terminate at a somewhat smaller value of p_T than those in fig. 8a. We see that the spectrum shows an even steeper threshold at the kinematical limit than do the annihilation contributions in fig. 8a; just 2 GeV below the maximum, the direct contribution still amounts to 10 fb/GeV. This is partly due to the slower decrease of the $\gamma\gamma \to q\overline{q}$ cross section with increasing energy, compared to the

$e^+e^- \to q\overline{q}$ cross section. Furthermore, since the photon spectrum at the $\gamma\gamma$ collider is quite flat (see fig. 1a), configurations close to the edge of the phase space region defined by the cuts (22) are not particularly suppressed, unlike the situation at the TESLA collider.

Fig. 8b also shows that the once resolved contribution plays an important role; we already saw in fig. 6b that asymmetric cuts, like in (22), favour this contribution. However, the most asymmetric initial state configuration ($y_1 = y_{1,\max}, y_2 = y_{2,\min}$) only contributes to part of the p_T spectrum; for the given case, it disappears for $p_T > 32$ GeV, which explains the small kink that occurs in the 1–res spectrum at this point. In contrast, the twice resolved contribution dominantly comes from rather symmetric initial state configurations, which imply that y_1 and y_2 have opposite signs, as can be seen from eqs.(17). Of course, $p_T \to p_{T,\max}$ implies that $x \to 1$ for the parton densities inside the photon; the 2–res spectrum in the threshold region is therefore not as steep as the direct or 1–res spectrum. Finally, we remark that the dependence of the relative importance of the three classes of two–photon contributions at the e^+e^- colliders is quite similar to the case of di–jet production, discussed in some detail in the previous subsection.

Fig. 8a shows that at $\sqrt{s} = 500$ GeV, at least the hard part of the mono–jet spectrum will be dominated by annihilation events, largely due to the contribution from real Z bosons. In Fig. 9 we compare the annihilation and two–photon contributions for the three stages of the JLC. We see that already at $\sqrt{s} = 1$ TeV, the two–photon contribution dominates over almost the whole kinematically accessible region. In particular, the contribution of real Z bosons now amounts to at most 5% of the two–photon contribution. The reason is that now the limit (26) gives $p_T \leq 32$ GeV. Most of the true Jacobian peak of the Z (which occurs at $p_T = m_Z/2$, of course) is therefore only accessible after emission of two hard photons, and is therefore strongly suppressed. Notice that we did not change the cuts (22) when increasing the beam energy. In reality it might be necessary to allow for smaller values of $y_{2,\min}$ at higher energy, since the coherent production of e^+e^- pairs rapidly increases [23, 43] with increasing Υ. Even with these fixed cuts, we find a rate of about 1,000 mono–jet events with $p_T \geq 100$ GeV per year at JLC2, and 300 events per year with $p_T \geq 150$ GeV at JLC3. At lower values of p_T, the rate shoots up very rapidly, due to the two–photon contribution; for instance, at JLC2 we expect about 35,000 mono–jet events with $p_T \geq 50$ GeV per year. We are therefore lead to the conclusion that missing p_T by itself will only be useful as a signal for "new physics" if it amounts to at least 20% of $\sqrt{s}$.

There is yet another source of mono–jet events in the standard model: Three jet annihilation events where two jets go in forward and backward direction, respectively, while the third jet emerges at a large angle. The cross section, integrated over $p_T \geq p_{T,min}$, for the dominant configuration where the central jet stems from the gluon can be estimated as

$$\sigma(e^+e^- \to q\overline{q}g) \simeq \sigma(e^+e^- \to q\overline{q}) \cdot \frac{1}{20} \cdot \frac{\alpha_s}{\pi} \cdot f(p_{T,min}), \tag{27}$$

where we have ignored both beam– and bremsstrahlung. f describes the relative weight for configurations where the q and $\bar{q}$ have an opening angle of at least 150°, while the angle between the g and the q or $\bar{q}$ has to be at least 10°, as dictated by the cuts (22); numerically, $f \simeq 10$ (1) for $p_{T,min} = 0.1$ (0.35) $\sqrt{s}$. The additional factor 1/20 comes from the requirement that the q and $\bar{q}$ be approximately aligned with the beam pipes. Notice

that this contribution extends to larger values of p_T than the contribution with only two hard partons in the final state:

$$p_{T,\max}(q\overline{q}g) = \sqrt{s}\frac{\sin\theta_{\max}}{1+\sin\theta_{\max}} \simeq 0.21\sqrt{s}, \tag{28}$$

where $\theta_{\max}$ is the maximal angle of the forward and backward jets; the cut (22b) corresponds to $\theta_{\max} = 15.4°$. Numerically, eq.(27) gives approximately 30 (3) fb for $p_{T,min} = 25$ (90) GeV at $\sqrt{s} = 500$ GeV. Comparison with figs. 8a and 9 shows that this contribution will only be important at very large p_T. It does therefore not change our previous conclusion about the relative importance of contributions from annihilation and two–photon processes to mono–jet events.

3c. Heavy quark production

In this subsection we study the production of c, b, and t quarks at future e^+e^- colliders. It should be clear from the results of the previous two subsections that the total cross sections for the production of $c\overline{c}$ and $b\overline{b}$ pairs at these colliders will be dominated by two–photon contributions. The direct process as well as the single resolved photon–gluon fusion process contribute with essentially the same strength as in case of jet production from light quarks. On the other hand, the twice resolved contribution is strongly suppressed here, since none of the processes that proceed via gluon exchange in the t or u channel can contribute. We therefore expect this latter class of contributions to be relatively less important here.

This is born out by the results of tables 2 and 3, where we list predictions for total $c\overline{c}$ and $b\overline{b}$ cross sections; all contributing processes are shown separately. As described in more detail in ref.[7], we have assumed different values for the "dynamical" quark mass entering the matrix elements, and the "kinematical" mass which determines the phase space. For charm and beauty production we have used [47] dynamical masses of 1.35 GeV and 4.5 GeV, respectively; the kinematical mass is always taken to be the mass of the lightest meson carrying the corresponding heavy flavor. We do not list results for the nbb version of the DESY-Darmstadt design, since they differ by only 10% or less from those of the TESLA design; this difference is smaller than the theoretical error of our esitmates.

We see that the direct $c\overline{c}$ cross section varies much less between the different designs than the resolved photon contributions do. This is because, as shown in sec. 2, designs with smaller beamstrahlung parameter Υ tend to have more soft photons, which contribute strongly to direct $c\overline{c}$ production, but have little impact on the resolved photon contributions; for instance, the direct $c\overline{c}$ cross section at TESLA is about the same as at Palmer F, but the latter has an about 2.5 times larger resolved photon contribution. Notice also that the $\gamma\gamma$ collider with its hard photon spectrum actually has the *smallest* direct $c\overline{c}$ and $b\overline{b}$ cross sections; due to the huge 1–res contribution, it nevertheless has by far the largest total $c\overline{c}$ and $b\overline{b}$ cross sections.

The 1–res and direct contributions are of similar size at the 500 GeV e^+e^- colliders, with the exception of the Palmer G design. At higher energies, however, the resolved photon contributions clearly begin to dominate. Notice also that the DG parametrization,

which we used here, predicts the ratio of 1–res and direct contributions to be roughly the same for $c\overline{c}$ and $b\overline{b}$ production; the more rapid decrease of the resolved photon contribution with increasing mass is balanced by the charge suppression factor of 1/4 of the direct contribution. The exception is the TESLA (and DESY–Darmstadt (nbb)) design, where the increase in mass also suppresses the direct contribution strongly, due to the very soft beamstrahlung spectrum.

Tables 2 and 3 also contain an entry for the production of the 1s vector quarkonium state. In leading order in α_s, this state can only be produced [7] in resolved photon reactions; by far the dominant contribution comes from the single resolved process $\gamma + g \to J/\psi + g$, and correspondingly for the $\Upsilon(1s)$. We estimate these cross sections using the color singlet model [48]. The cross section for J/ψ production is so large that it should be easily detectable via its decay into muons or electrons even at the TESLA collider. The cross sections for $\Upsilon(1s)$ production are smaller by a factor of about 500; in addition, the branching ratios for the leptonic decays are almost 3 times smaller than for the J/ψ. Nevertheless, at least 15 $\Upsilon(1s) \to \mu^+\mu^-$ per year are expected to occur even at TESLA.

The results of tables 2 and 3 have been obtained using the conservative DG parametrization for $\vec{q}^{\gamma}$. The other parametrizations discussed in sec. 2 lead to larger predictions for the resolved photon contributions. We can conclude from fig. 5a that the LAC2 parametrization predicts almost 5 times larger 1–res $c\overline{c}$ cross sections than DG does; in that case resolved photon contributions would dominate $c\overline{c}$ production at all colliders we have studied here. However, even though the 2–res contribution would increase by a factor of 15 or so, it would still be subdominant. Since the LAC2 parametrization contains a very steeply falling $G^{\gamma}(x)$, it predicts the ratio of resolved to direct contributions to decrease by approximately a factor of two when going from $c\overline{c}$ to $b\overline{b}$ production. The predictions of the modified DO+VMD parametrization for the single resolved contribution also lie a factor 1.5 to 2 above those of the DG parametrization.

So far we have only discussed total cross sections. The results seem to indicate enormous event rates, especially for $c\overline{c}$ pairs. This might be somewhat misleading, however, since in many events the heavy quarks emerge at such small angles that they remain unobserved. In particular, we saw in fig. 6 that single resolved $q\overline{q}$ production is concentrated at large rapidities, due to the asymmetric initial state. A realistic estimate of the number of *observed* $c\overline{c}$ and $b\overline{b}$ events needs a full simulation of the detector, which is beyond the scope of this paper. One might be able to get an idea of the result of such a full simulation by looking at the p_T spectrum of centrally produced heavy quark pairs. In fig. 10a,b we therefore show the transverse momentum spectrum of charm quarks produced at $\theta = 90°$, i.e. $y_1 = y_2 = 0$, for the TESLA (9a) and Palmer G (9b) designs. We see that at TESLA the resolved photon contribution is now well below the direct one for all p_T, while at Palmer G it exceeds the direct contribution by at most a factor of two. Notice that the relative importance of the 2–res contributions is actually enhanced by going to small rapidities; due to the symmetric initial state configuration and the soft parton distributions inside the electron the twice resolved contribution is concentrated at small y. At TESLA, the effective quark density in the electron is even softer than the gluon distribution; we are again seeing the contribution from quarks with large Bjorken–x inside soft photons.

Finally, fig. 10 shows that one can neglect* all resolved photon contributions if one is only interested in central events with hard muons or electrons; such events might be [49] a background to top production.

It has recently been pointed out [49] that $t\bar{t}$ production at future e^+e^- colliders might itself be dominated by two–photon events. In fig. 11a,b we compare $t\bar{t}$ production via $\gamma\gamma$ fusion and e^+e^- annihilation at two designs of e^+e^- colliders operating at $\sqrt{s} = 500$ GeV (10a), as well as the third stage of the JLC (10b). Notice that the two–photon contributions in fig. 11a have been multiplied with 10 (for Palmer G) and 100 (for TESLA), respectively. We see that at $\sqrt{s} = 500$ GeV, $\gamma\gamma$ processes will contribute at most 7% of the total $t\bar{t}$ cross section; their contribution at TESLA is always well below 1%. In fact, it might be very difficult to even detect the two–photon contribution, since some annihilation events will also have a $t\bar{t}$ invariant mass well below $\sqrt{s}$, due to the combined effects of brems– and beamstrahlung (see fig. 2).

Fig. 11b shows that at $\sqrt{s} = 1.5$ TeV the two–photon contribution could indeed dominate, but not by a large factor; moreover, for $m_t \geq 125$ GeV the annihilation contribution is still the more important one. In this figure the direct and total $\gamma\gamma$ contributions are shown separately; even though we have used the modified DO+VMD parametrization with its hard (and Q^2 independent) intrinsic gluon component here, we still find that at this collider, at most 10% of the total two–photon contribution comes from resolved photons. They can make important contributions to $t\bar{t}$ production only at $\gamma\gamma$ colliders operating at $\sqrt{s} \geq 1$ TeV.

Figs. 11 also show that an estimate of the annihilation contribution to $t\bar{t}$ production should include the effects of beam– and bremsstrahlung. At $\sqrt{s} = 500$ GeV, they increase (decrease) the cross section for $m_t < (>)$ 155 GeV. For light top quarks, the increase of the photon and Z propagators $\propto 1/m^2_{t\bar{t}}$ is the dominant effect, while for large m_t, the reduction of the available phase space is more important. At $\sqrt{s} = 1.5$ TeV, the top quark is always "light", of course. Radiation therefore increases the cross section by a factor between 1.5 and 1.7; it also leads to a decrease of the annihilation cross section by about 10% when m_t is increased from 90 to 200 GeV. About 30 to 40% of these effects is due to bremsstrahlung; beamstrahlung by itself increases the annihilation cross section by about 30% at JLC3. While this is certainly not negligible, it pales compared to the 800% increase of the $\gamma\gamma$ contribution to $t\bar{t}$ production which is also caused by beamstrahlung at this collider. Nevertheless, the total $t\bar{t}$ cross section at $\sqrt{s} = 1.5$ TeV remains considerably smaller than at $\sqrt{s} = 500$ GeV.

3d. Single W and Z production

We now turn to our final example of a hard two–photon process, the production of a single W or Z boson. The corresponding processes at ep colliders like HERA have been studied

*The production of high–p_T charm quarks via resolved photon mechanisms is probably dominated by flavor excitation processes, rather than the pair production process we have studied here. If $p_T^2 \gg m_c^2$, one can again treat the charm quark as an essentially massless parton inside the photon. However, the flavour structure of the photon is not well understood [37]; no existing parametrization treats the quark mass effects properly. In any case, although this process might be interesting in itself, it will be subdominant at large transverse momentum.

in some detail in the literature [50]. In particular, it has been shown that the total cross section can to 20–30% accuracy be estimated from the simple resolved photon process [51] $q\overline{q} \to W, Z$. We will assume that this is also true for two–photon reactions, and will estimate the total cross sections from the twice resolved $q\overline{q}$ annihilation (Drell–Yan) process alone.

Our results are summarized in table 4, where we list the total cross sections for single W and Z production in two–photon collisions at various colliders. The W cross section includes W^+ as well as W^- production; since the initial state has even C parity, the W^+ and W^- cross sections are, of course, equal. The results of table 4 have been obtained using the DG parametrization with $Q^2 = m^2_{W,Z}$. Increasing Q^2 by a factor of two increases the cross sections by about 10–20%; note that in the given case the increase of $\vec{q}^{\gamma}(x, Q^2)$ and $f^{\rm brems}_{\gamma|e}$ is not compensated by a decrease of the hard cross section, in contrast to the reactions we studied in secs. 3a–c. The modified DO+VMD parametrization predicts 50–70% larger W cross sections, and 30–50% larger Z cross sections, where the larger number refers to the hardest photon spectra (JLC3 and the $\gamma\gamma$ colliders). However, at least part of this excess is certainly fake. As noted before, the VMD contribution is assumed to be Q^2–independent, which overestimates its importance at high Q^2. Furthermore, the DO parametrization only includes $N_f = 4$ active flavours, while for $Q^2 \simeq m^2_W$, $N_f = 5$ seems more appropriate. The b quark itself does not contribute to W production, but the increase of α_s when going from 4 to 5 flavours leads to somewhat softer quark distributions in the photon.

This is also one of the few processes where the flavour structure of the photon plays an important role. For example, for the LAC2 parametrization we find W cross sections larger by 70–150% and Z cross sections higher by about 30–100% at the JLC3 and $\gamma\gamma$ colliders. While it is true that this parametrization again uses only $N_f = 4$ and part of the increase may be ascribed to that, the real reason for this difference lies in the different flavour structure of the DG and LAC2 parametrization. The LAC2 parametrization does not satisfy the constraint $u^{\gamma}(x) = 4d^{\gamma}(x)$ even at large x and Q^2. As a result it requires a higher d quark content of the photon (as compared to the DG parametrization) to fit the data on F_2^{γ}. This leads to higher cross sections for both W and Z production.

By comparing the results of table 4 with the integrated di–jet cross sections shown in figs. 3 and 7 one can immediately convince oneself that it will be very difficult to observe the gauge bosons in their hadronic decay modes. (This is again similar to the case of HERA [52].) One will thus have to use leptonic decay modes. W production would therefore be signalled by a hard lepton, with a Jacobian peak at $m_W/2$ in its p_T spectrum, in association with large missing transverse momentum; the signal for Z production is simply a hard lepton pair whose invariant mass equals m_Z. In both cases the event should contain two spectator jets. We remind the reader that the branching ratio for the leptonic decays are only 11% and 3.3% per generation for the W and Z boson, respectively. Even after summing over e and μ channels, we therefore only expect about 10 (35) detectable W events per year at TESLA (JLC1). This should be compared to an $e^+e^- \to W^+W^-$ cross section of about 8 pb; the cross sections for the annihilation processes $e^+e^- \to We\nu$ and $e^+e^- \to e^+e^-Z$ also amount to about 5 pb at $\sqrt{s} = 500$ GeV [53] even if beamstrahlung can be ignored. Of course, these annihilation events lack the spectator jets of the resolved photon events; moreover, the gauge bosons are usually produced with sizeable transverse

momentum. Nevertheless, it is quite clear that extraction of the two–photon signal will be quite difficult, if not impossible, at a 500 GeV e^+e^- collider.

The situation might be different, however, at higher energies. At the second stage of the JLC, we expect as many as 1500 $W \to l\nu$ and 230 $Z \to l^+l^-$ events from two–photon processes per year ($l = e, \mu$). The cross sections for the single production of a gauge boson also increase when going from $\sqrt{s} = 500$ GeV to 1 TeV, but only by about a factor of two [53]*. The rates at $\gamma\gamma$ colliders are even larger; assuming an integrated luminosity of 20 fb^{-1} per year, one has about 1,750 $W \to l\nu$ events and 275 $Z \to l^+l^-$ events per year already at $\sqrt{s} = 500$ GeV. Notice, however, that the $\gamma\gamma \to W^+W^-$ cross section amounts to about 80 pb, giving as many as 500,000 events with one leptonically decaying W boson per year; extraction of the W signal will therefore still not be trivial. On the other hand, the background for the Z signal should be much smaller. Finally, we remark that at $\gamma\gamma$ colliders, our cross sections increase almost linearly with energy, while the $\gamma\gamma \to W^+W^-$ cross section stays constant.

4. Semi-hard and soft two-photon reactions

In this section we discuss semi–hard (minijet) and soft (VMD) two–photon reactions at future e^+e^- linacs [11]. "Semi–hard" here merely means that we are trying to push leading order perturbative QCD to its limit of applicability. We do not attempt to re–sum $\log 1/x$ terms, or to include shadowing effects. The main emphasis will be on the question whether these events give rise to an "underlying event", where one or several two–photon reaction occurs simultaneously (within the time resolution of the detector) with every annihilation event; if such an underlying event does occur, we try to characterize it at least qualitatively.

We have already seen in figs. 3 that the cross section for the production of a pair of jets in two–photon collisions increases very rapidly with decreasing transverse momentum of the jets; figs. 3b and 4 show that this is mostly due to the contribution from resolved photon processes. In figs. 12a,b we extend these calculations to even lower values of the minimal transverse momentum $p_{T,min}$ of the partons participating in the (semi–)hard $2 \to 2$ scattering process. We show results for the DG (12a) and modified DO+VMD (12b) parametrization; since we are now considering reactions that are characterized by a relatively low momentum or Q^2 scale, the effect of the Q^2 variation of the hadronic VMD contribution to $\vec{q}^{\gamma}$, which we ignored, is probably not very large here. Notice that we have not applied any rapidity cuts in figs. 12. Due to the nontrivial colour flow between spectator and "hard" jets, a resolved photon event should always include some detectable particles, even if the "hard" jets emerge at very small angles, and will thus always contribute to the underlying event. A direct event might remain invisible if both jets are produced in the very forward or very backward direction, due to a strong boost between the $\gamma\gamma$ centre–of–mass frame and the lab frame; however, less than 1% of all minijet events will come from the direct process at the colliders we are considering.

Unfortunately, figs. 12 show that the leading order prediction for the cross section

*However, the calculation of Hagiwara et al. [53] does not include the contribution from beamstrahlung photons, e.g. $\gamma e \to W\nu$, which should be quite large at this collider.

depends quite sensitively on $p_{T,min}$. This is not surprising, since most of the hard $2 \to 2$ cross sections diverge like $1/p^2_{T,\rm min}$ as $p_{T,min} \to 0$. An additional $p_{T,min}$ dependence is produced by the growth of the parton densities at low x. Eqs.(17) and (18) show that the average x decreases linearly with p_T, while the kinematical minimum of x even decreases quadratically with decreasing p_T. The results of figs. 12 can to good approximation be parametrized by a power law, $\sigma(p_T \geq p_{T,min}) = a p^{-b}_{T,\rm min}$, where the power b is approximately independent of the photon spectrum (i.e., of the collider), but does depend on the parametrization we used; one has $b \simeq 3.3$ (3.6) for the DG (modified DO+VMD) parametrization. The prediction for the cross section therefore changes by a factor of 2 when $p_{T,min}$ is changed by 23 (21) %! It is therefore very important to at least try to get an idea down to which value of $p_{T,min}$ our calculation might be reliable.

We see from eq.(19) that $p_{T,min}$ determines the minimal virtuality of the exchanged parton in the $2 \to 2$ scattering, and thus the "hardness" of the process. It should therfore be analogous to the momentum transfer Q^2 in deep inelastic scattering. Standard parametrizations of hadronic structure functions [54], which rely on the validity of perturbative QCD, are assumed to be reliable down to some value Q^2_0 in the range between 1 and 5 GeV^2. Further support for the applicability of perturbative QCD at momentum scales between 1 and 2 GeV comes from its success in describing at least the gross features of charmonium physics [55], as well as of open charm production from hadrons [56].

Moreover, minijet calculations are also able to reproduce quite well the observed rise of the total $p\overline{p}$ cross section with energy. The basic idea that semi–hard QCD interactions could affect such a seemingly "soft" quantity as the total cross section dates back to 1973 [57]. Of course, minijet calculations for $p\overline{p}$ reactions also depend on a cut–off $p_{T,min}$. Recent fits to existing data [58] indicate that $p_{T,min}$ has to be chosen in the range between 1.3 and 2 GeV, if the rise of hadronic cross sections is to be described by minijets. It is sometimes even claimed that minijets have been seen experimentally by the UA1 collaboration [59]. However, the UA1 analysis only inlcuded "clusters" with transverge energy of at least 5 GeV, which corresponds to a minimal partonic p_T of approximately 3.5 GeV. The cross section for the production of such clusters does indeed grow very rapidly with energy, in the region 200 GeV $\leq \sqrt{s} \leq$ 900 GeV, in accordance with leading order QCD predictions. However, we have seen above that changing $p_{T,min}$ from 3.5 to 1.5 GeV would change the leading order prediction of the cross section by more than an order of magnitude. In our opinion the UA1 results are therefore not a direct proof for the validity of the minijet ansatz, although they are certainly not in disagreement with it. In fact, it seems quite unlikely that "jets" with (partonic) p_T as small as 1.5 to 2 GeV can ever be identified at hadron colliders.

Fortunately the situation is quite different for two–photon collisions, where "jets" with p_T as small as 1 GeV are routinely reconstructed [42]. The relationship between the transverse momenta of the parton and the resulting jet is quite complicated, however. At such small values of p_T, contributions from the hadronization process, as well as from the intrinsic p_T of the partons, are not negligible. Moreover, the whole event is forced into a two–jet topology; parts of the spectator jets of resolved photon events will thus be included in the reconstructed jets. One therefore needs a careful Monte Carlo analysis to derive the partonic p_T from the transverse momentum of the jets even on a statistical basis. So far the only analysis of this kind has been performed by the AMY

collaboration [8], using their data taken at the TRISTAN collider at $\sqrt{s} \simeq 60$ GeV. Their Monte Carlo generator was able to describe the real data quite well, both in shape and normalization, once the resolved photon contributions had been taken into account; this is in sharp contrast to older analyses [42] where these contributions were not included, and consequently an excess of data over the Monte Carlo predictions was oberved. AMY determined the minimal partonic p_T using only events with $p_T(jet) \geq 3$ GeV, where the soft or VMD component, which is characterized by an exponential p_T spectrum, is already essentially negligible; they found $p_{T,min} = 1.6$ (2.4) GeV for the DG (DO+VMD) parametrization. These numbers depend only weakly on the chosen fragmentation and hadronization scheme. At least in case of the DG parametrization, the AMY value for $p_{T,min}$ falls within the range of values favoured by other minijet analyses. We will therefore from now on use their values as out best guess for $p_{T,min}$.

The resulting predictions for the total semi–hard two–photon induced cross section at a variety of hypothetical future colliders are listed in columns 2 and 3 of table 5. We see immediately that the modified DO+VMD parametrization predicts a 1.4 to 1.7 times smaller cross section than the DG parametrization; the increase of G^γ is over–compensated by the increase in $p_{T,min}$. However, we should caution the reader that this is partly due to our rather arbitrary regularization (10) of the original DO parametrization [15]. Without this regularization, this prediction would be approximately 20% higher at the 500 GeV e^+e^- colliders; the effect of the regularization is even larger for harder photon spectra and higher electron beam energies.

In order to translate the cross sections of table 5 into a meaningful number of events, we first define an "effective bunch crossing". If within the time resolution of the detector only one bunch crossing occurs, the luminosity per effective bunch crossing is identical to the luminosity per bunch crossing $\hat{\mathcal{L}}$ listed in table 1. If the temporal separation of consecutive bunches Δt is smaller than the time resolution δt, we sum over $\delta t / \Delta t$ bunch collisions, or over a complete bunch train collision, whatever gives the smaller number. (Consecutive bunch train collisions can trivially be distinguished.) In table 5 we have assumed a rather poor time resolution of 10^{-7} seconds, which should be quite easy to achieve. However, table 1 shows that only the DESY–Darmstadt design would benefit from an improved time resolution of $3 \cdot 10^{-8}$ sec. In any case it is trivial to compute the effects of better time resolution from the numbers in the last column of table 5.

These numbers indicate that most designs for e^+e^- colliders operating at $\sqrt{s} = 500$ GeV should have at most one event per average effective bunch collision. Since these events should obey a Poisson distribution, an average of one event per effective bunch crossing still means that more than 35% of all bunch collisions are free of any two–photon events, independent of whether they contain an annihilation event or not. This would be equivalent to a reduction of the luminosity by a factor of 3 for those measurements where not even a single two–photon event can be tolerated, *if* the presence of a two–photon event can be reliably detected when an annihilation event occurs at the same time. For instance, the measurement of the mass of the top quark to sub–GeV precision [60] is not limited by statistics; such a measurement could thus still be performed at the JLC1 collider, if $t\bar{t}$ events that also contain a two–photon event can be reliably distinguished from "pure" $t\bar{t}$ events.

On the other hand, performing such a measurement at the Palmer G collider will

be almost impossible, if our estimate for the number of two–photon events that occur at each effective bunch crossing is at least approximately correct. Assuming that the spectator jets deposit about 1 – 2 GeV transverse energy per unit of rapidity, and adding another 4 GeV if the "hard" jets are produced centrally, we estimate that each minijet event will deposit between 5 and 12 GeV transverse energy in the central part of the detector, defined by the rapidity window $|y| \leq 2$. At Palmer G one would therefore have to expect at least 100 GeV of transverse energy in soft particles to underly every annihilation event; a similar number has to be expected at the second stage of the JLC, and the third stage would be even worse. This would cause a host of problem familiar from hadron colliders. Examples are: a deterioration of the experimental resolution of jet energies, which would, e.g., make it difficult to distinguish between hadronically decaying W and Z bosons; a large number of tracks, which complicates b–tagging with microvertex detectors; fluctuations in the underlying event, which could produce missing tranverse momentum; and difficulties in defining isolation criteria for hard leptons, which figure prominently in searches for semi–leptonically decaying heavy particles.

The fourth column of table 5 shows the VMD prediction for the total hadronic cross section for events with $\gamma\gamma$ invariant mass $W_{\gamma\gamma} \geq 5$ GeV, assuming a constant $\gamma\gamma \to hadrons$ cross section of 250 nb [61]. We see that for the 500 GeV e^+e^- colliders the predicted minijet cross section always falls below this conservative estimate of the total $\gamma\gamma$ cross section. In principle, the contributions from both these sources should be included, if one wants to estimate the total number of events; for instance, the AMY Monte Carlo generator needs both soft and hard two–photon reactions to explain their data. However, it is not clear whether a soft interaction will always be observable at high energy e^+e^- or $\gamma\gamma$ colliders. At low $W_{\gamma\gamma}$, the multiplicity of soft events seems to be quite low, at least according to standard MC generators [62]. No experimental information exists about two–photon events with $W_{\gamma\gamma} > 25$ GeV or so, but it seems possible that (part of) the soft component becomes diffractive, so that (almost) all particles are concentrated in the forward and backward regions. In any case, it is quite certain that the average E_T in a soft event will be smaller than in a minijet event. We will also see below that it may no longer be appropriate to simply sum the soft and hard contributions to the total $\gamma\gamma$ cross section if the hard contribution is of the same order as or larger than the soft one. For these reasons we have ignored the soft contribution when estimating the number of events per effective bunch crossing.

The results for the second and third stage of the JLC show that building a "clean" e^+e^- collider with $\sqrt{s} \geq 1$ TeV might be quite difficult. The same conclusion also holds for simple extrapolations of the X–band design with the smallest minijet cross section, Palmer F. In principle it might be possible to improve the time resolution of the detecor to something like 2 nanoseconds; the drift velocity of electrons in gas seems to make it impossible to achieve better time resolution with present technology [63]. Even a time resolution of 2 nanoseconds seems quite difficult to achieve, given that an ultrarelativistic particle needs about 10 to 15 nanoseconds to traverse the detector; at the JLC bunch spacing of 1.4 nanoseconds fast particles produced in the current bunch crossing can therefore overtake slower particles produced in previous bunch crossings. The problem is further complicated by the probable occurence of "loopers", i.e. of particles describing spiral orbits in the magnetic field of the detector, which could stay in the detector for

several 10^{-8} seconds. The assignement of a given particle to a certain bunch crossing can therefore only occur on the software level, by combining information about arrival times and energy/momentum of the particle, or by reconstructing its track. In view of these problems it seems unlikely that a detector for an e^+e^- supercollider would be much easier to build than a detector for a *pp* supercollider, *if* such a time resolution turns out to be necessary. Note also that even with this excellent resolution, the leading order DG calculation still predicts 2.5 (5) two–photon events to occur per effective bunch collision at the JLC2 (JLC3).

Nevertheless it might be possible to build TeV linear colliders with $\ll 1$ events per effective bunch crossing. This is demonstrated by the last 4 rows of table 5, where we have tried to extend the TESLA design as described in table 1 to higher energies. As discussed in sec. 2, quite simple considerations show that the beamstrahlung parameter Υ should grow between linearly and quadratically with the beam energy; in the first case one assumes a constant luminosity per bunch crossing $\hat{\mathcal{L}}$, while in the second case $\hat{\mathcal{L}}$ grows like s. Our predictions for the first, more optimistic scenario are given in rows 9 and 10, while rows 11 and 12 show results for the less favorable extrapolation; in both cases the number given in the first column is $\sqrt{s}$ in GeV. We don't claim these to be realistic extrapolations; e.g., we have not varied the bunch length at all. Nevertheless, they should be sufficient to give us some indication of the true situation.

We see that in the optimistic scenario one can achieve a clean environment even at $\sqrt{s} = 2$ TeV, with a large safety margin. Of course, the total luminosity has to grow like s if the machine is to retain its full potential. Since we assumed constant luminosity per bunch crossing, one has to increase either the number of bunches per train, or the number of bunch train collisions per second. At worst, one would have to increase the number of bunches by a factor of 16 when going from $\sqrt{s} = 0.5$ TeV to 2 TeV; this would still leave a time gap between subsequent bunches of 60 nanoseconds, so that single bunch collisions could be resolved quite easily. Of course, the large safety margin shows that one might for technical reasons prefer to increase $\hat{\mathcal{L}}$ at least slightly, even at the cost of a somewhat more rapid increase of Υ. However, our results for the less favourable projection show that even at a TESLA–like design one would have to deal with an underlying event at $\sqrt{s} > 1.5$ TeV, if both Υ and $\hat{\mathcal{L}}$ grow quadratically with the beam energy. Recall that they grow between linearly and quadratically at the JLC collider as currently planned.

Finally, notice the very large minijet cross section at the $\gamma\gamma$ collider already at $\sqrt{s} =$ 500 GeV. If the $\gamma\gamma$ collider originates from an e^+e^- collider like TESLA, one still only expects one event every 2 bunch crossing or so; a similar rate can be achieved at the DESY–Darmstadt (nbb) design, if a time resolution of around 50 nanoseconds can be realized. At all the other designs one would have to expect (much) more than one event per effective bunch collision; the higher number given in table 5 corresponds to the Palmer G design. In all cases the minijet rate at a $\gamma\gamma$ collider would be far larger than at its e^+e^- progenitor. The $\gamma\gamma$ option, while interesting in its own right [64], would therefore not help to solve the problem of hadronic backgrounds.

Note that the leading order prediction for the minijet cross section at the $\gamma\gamma$ collider is far *above* the VMD prediction for the *total* hadronic cross section; clearly at least one of these predictions must be wrong. The problem is illustrated in fig. 13, where we show the DG prediction for the total minijet cross section (with $p_{T,min} = 1.6$ GeV) as

a function of the $\gamma\gamma$ centre–of–mass energy $W_{\gamma\gamma}$. It obviously rises very quickly with energy. The AMY analysis [8] provided experimental evidence for the rapid growth of the resolved photon cross section when going from PETRA to TRISTAN energies, but this only probes the region $W_{\gamma\gamma} \leq 25$ GeV. We find that the leading order prediction exceeds the VMD prediction of 250 nb for $W_{\gamma\gamma} \geq 50$ GeV. The true value of the soft two–photon cross section even at low energies is quite uncertain; even a number as large as 420 nb has been quoted [65]. Moreover, one might envision a slow (logarithmic) increase of the soft cross section with energy. On the other hand, the result of fig. 13 can be parametrized as

$$\sigma^{LO}(\mathrm{DG}) = 250 \text{ nb} \left(\frac{W_{\gamma\gamma}}{50 \text{ GeV}} \right)^{1.4} ; \tag{29}$$

this reproduces the numerical leading order prediction to better than 10% in the region 10 GeV $\leq W_{\gamma\gamma} \leq$ 500 GeV. This cross section will be substantially larger than any conceivable VMD estimate in the region $W_{\gamma\gamma} \geq 100$ GeV.

Fig. 1 shows that the photon luminosity at e^+e^- colliders decreases quite rapidly with $W_{\gamma\gamma}$; on the other hand the leading order cross section (29) clearly gives great weight to the region of large $W_{\gamma\gamma}$. It is therefore possible that the region of $W_{\gamma\gamma}$ where $\sigma^{\mathrm{LO}}(\mathrm{DG}) > \sigma(\mathrm{VMD})$ contributes significantly to the total minijet cross section at a given collider, even if that cross section is still below the total VMD cross section at the same collider. This is demonstrated by the dashed curves in fig. 13, which refer to the scale on the right side of the frame; they depict the fraction of the total minijet cross section at a given collider that comes from events with two–photon invariant mass smaller than the $W_{\gamma\gamma}$ shown as the x-axis. We see that for the Palmer F design, most minijet events still have values of $W_{\gamma\gamma}$ where the leading order prediction for the hard cross section is less than or roughly equal to the VMD prediction for the total cross section; the same is true for the TESLA, DESY–Darmstadt and JLC1 designs. On the other hand, at the Palmer G collider 30% of the minijet events have $W_{\gamma\gamma} > 100$ GeV, where the DG prediction (29) clearly exceeds the VMD estimate. This feature becomes even more prominent for harder photon spectra, as shown by the curves for the JLC2 and $\gamma\gamma(500)$ colliders.

The problem that the leading order prediction for the minijet cross section exceeds the total cross section if $p_{T,min}$ is chosen in the GeV range is well known in the case of hadronic collisions. The standard remedy [58] is to interpret the leading order calculation not as a prediction of the total cross section σ, but as a prediction of σ times the number of minijet pairs per event. In this way two events with one jet pair each can be combined into one event with two pairs of jets. Formally, this is achieved by eikonalizing the cross section. Essentially one writes [58]

$$\sigma_{p\bar{p}}^{\mathrm{inel}} = \int d^2b \left[1 - e^{-\left(\sigma_{p\bar{p}}^{\mathrm{hard}}(s) + \chi_{p\bar{p}}^{\mathrm{soft}} \right) A(b)} \right] . \tag{30}$$

$A(b)$ describes the transverse distribution of partons in the proton, normalized such that $\int d^2b A(b) = 1$. $\chi_{p\bar{p}}^{\mathrm{soft}}$ is assumed to be (almost) independent of s. It is related to the soft inelastic $p\overline{p}$ cross section, which is essentially equal to the total inelastic $p\overline{p}$ cross section at low energies, by

$$\sigma_{p\bar{p}}^{\mathrm{soft}} = \int d^2b \left[1 - e^{-\chi_{p\bar{p}}^{\mathrm{soft}} A(b)} \right] . \tag{31}$$

If $\chi^{\rm soft}_{p\bar{p}} \ll 1/A(0)^*$, we simply have $\chi^{\rm soft}_{p\bar{p}}$=$\sigma^{\rm soft}_{p\bar{p}}$; moreover, eq.(30) reduces to $\sigma^{\rm inel}_{p\bar{p}} = \sigma^{\rm hard}_{p\bar{p}} + \sigma^{\rm soft}_{p\bar{p}}$ if the hard cross section is also small in this sense. However, if either cross section is large, of order of the geometrical cross section, eq.(30) predicts a much slower increase of the total cross section with energy than predicted by the simple leading order calculation.

Unfortunately, it is not entirely straightforward to apply this formalism to reactions involving photons in the initial state. As first pointed out by Collins and Ladinsky [66] for the case of the γp cross section, the ansatz (30) has to be modified. The point is that once a photon has "transformed" itself into a (virtual, but long–lived) hadronic state, which it does with probability $P_{\rm had} \simeq \mathcal{O}(\alpha_{\rm em})$, the production of additional pairs of jets should *not* be suppressed [67] by additional powers of $\alpha_{\rm em}$, as would be predicted by eq.(30) since $\sigma^{\rm hard}_{\gamma\gamma}$ is itself $\mathcal{O}(\alpha^2_{\rm em})$. For the case of $\gamma\gamma$ collisions one has to write [68]

$$\sigma^{\rm inel}_{\gamma\gamma} = \int d^2b P^2_{\rm had} \left[1 - e^{-\left(\sigma^{\rm hard}_{\gamma\gamma}(s) + \sigma^{\rm soft}_{\gamma\gamma}\right) A(b)/P^2_{\rm had}} \right] . \tag{32}$$

The problem is that it is not at all obvious how $A(b)$ and $P_{\rm had}$ are to be determined. For instance, it is generally accepted that $P_{\rm had}$ should be of order $\alpha_{\rm em}$, but it is not clear just how large it is. From the VMD model, one estimates [66] $P_{\rm had} \simeq 1/300$; in this case eikonalization reduces the minijet cross section at the 1 TeV collider of ref.[12] by approximately a factor of 2 [68]. On the other hand, parton model considerations lead to the estimate [67] $P_{\rm had} \simeq 1/170$. Recently it has been suggested [69] that $P_{\rm had}$ might even grow logarithmically with energy, so that $P_{\rm had}(100\ {\rm GeV}) \simeq 1/55$. Obviously the asymptotic hadronic $\gamma\gamma$ cross section as predicted by eq.(32) is proportional to $P^2_{\rm had}$; even if $p_{T,min}$, the parton densities inside the photon and $A(b)$ were all known, estimates of $\sigma_{\gamma\gamma}$ would still differ by a factor of 30! In particular, it is very well possible that even after eikonalization the cross section exceeds the VMD estimate substantially. In fact, one can argue that the smallness of the VMD cross section (250 nb) is hard to understand from perturbative QCD. Once hard interactions start to dominate the exponents in eqs.(30) and (32), one would expect the $\gamma\gamma$ cross section to lie very roughly between $\alpha^2_{\rm em}\sigma_{p\bar{p}}$ and $(\alpha_{\rm em}/\alpha_s(p_{T,min}))^2\sigma_{p\bar{p}}$, i.e. between 2 and 25 μb for $W_{\gamma\gamma} = 500$ GeV; recall that $\vec{q}^{\gamma} \propto 1/\alpha_s$. If this simple estimate is at least halfway correct, the total hadronic $\gamma\gamma$ cross section could be described by eq.(29) for $W_{\gamma\gamma} \leq 200$ GeV, and possibly up to $W_{\gamma\gamma} \simeq 1$ TeV. Fortunately, in the near future measurements of the total γp cross section at centre–of–mass energies up to about 250 GeV will be performed at HERA. Different ansätze for $P_{\rm had}$ and $A(b)$ also lead to quite different predictions [66, 67, 69] for $\sigma_{\gamma p}$, so that one should be able to reduce the uncertainty of theoretical predictions for $\sigma_{\gamma\gamma}$ by fitting model parameters to those HERA data.

Finally, we would like to argue that, while the total $\gamma\gamma$ cross section is certainly of great theoretical interest, since it could teach us important lessons about semi–hard QCD, it is in many cases *not* a good measure for the severity of problems caused by soft and semi–hard two–photon backgrounds. We see from table 5 that, whenever the leading order estimate predicts $\gg 1$ minijet events per effective bunch crossing, so does the conservative VMD estimate. Since it would be implausible to assume that the total $\gamma\gamma$ cross section at high energies is even smaller than the VMD estimate, we can in such cases be sure

*Obviously, $A(b)$ is maximal at $b = 0$.

that ≥ 1 two–photon event will indeed occur at (almost) every effective bunch crossing. As explained above, eikonalization basically combines two (or more) events with one pair of minijets each into one event with two (or more) pairs of minijets. This has very little impact on the underlying event, since in both cases the number of minijets contained in it will be approximately the same, as will be the particle multiplicity, the total transverse energy, etc. All these quantities are approximately proportional to the product of the total cross section and the jet multiplicity per interaction, which should to good approximation be described by the leading order calculation.

This simple argument is at least to some extent borne out by a full Monte Carlo simulation [70] of minijet events at $p\overline{p}$ colliders[†]; multiple interactions lead to higher multiplicities, and thus to larger underlying events. Moreover, it is known experimentally [59] that not only the total inelastic $p\overline{p}$ cross section, but also the average charged particle multiplicity $\langle n_{ch} \rangle$ per unit of rapidity as well as the average transverse momentum $\langle p_{T,ch} \rangle$ of charged particles increase with energy. If one has $\gg 1$ event per effective bunch crossing, the total scalar p_T in the underlying event, which should be a good measure of the background problems caused by it, is approximately proportional to the product $\sigma \cdot \langle n_{ch} \rangle \cdot \langle p_{T,ch} \rangle$; this product grows much more rapidly with energy than the total cross section alone does. We therefore believe that eq.(29) provides in most cases a good figure of merit for the background problems caused by the underlying event, even for values of $W_{\gamma\gamma}$ where it no longer describes the true total $\gamma\gamma$ cross section.

This is not true, however, if one expects the average number $\langle n \rangle$ of two–photon events per effective bunch crossing to be close to 1. In that case it might be important to know what fraction of bunch crossings, and thus annihilation events, will be entirely free of two–photon events, as discussed above. This fraction is given by $e^{-\langle n \rangle}$, which varies rapidly with $\langle n \rangle$ if $\langle n \rangle \simeq 1$; e.g. $e^{-1/2} = 0.61$, while $e^{-2} = 0.14$. In such a situation it is probably advisable to plan for the worst, or else to postpone a final decision until the total $\gamma\gamma$ cross section can be predicted more reliably.

5. Summary and Conclusions

In this paper we have studied various two–photon reactions leading to hadronic final states at future linear e^+e^- and $\gamma\gamma$ colliders. The photon spectrum at these machines will be quite different from the Weizsäcker–Williams bremsstrahlung spectrum familiar from e^+e^- storage rings. In case of the e^+e^- linacs, an important new contribution to the photon flux comes from beamstrahlung. We saw in sec. 2 that the shape and normalization of the beamstrahlung spectrum depends quite sensitively on the size and shape of the electron and positron bunches. Already at $\sqrt{s} = 500$ GeV, the beamstrahlung contribution to the total photon flux can be anywhere between almost negligible (except for very small photon energies) and clearly dominant (except for very hard photons); as a result, photon fluxes at existing designs of 500 GeV colliders differ by as much as a factor of 30. It is therefore very difficult to make definite statements about two–photon reactions at such

[†]We mention in passing that in this analysis $p_{T,min}$ was fixed from the total charged particle multiplicity, not from the cross section; again values in the range from 1.5 to 2 GeV were found for $\sqrt{s} \leq 1$ TeV

colliders which are valid for all possible designs; instead, we tried to give an impression of the range of cross sections or event rates one may expect.

In sec. 3 we studied several hard two–photon reactions. We showed in sec. 3a that even without beamstrahlung a large majority of all hard events at future e^+e^- colliders will come from two–photon reactions (unless a new Z' gauge boson is found). In principle, one could discard many of these events already at the trigger level, by requiring a large transverse or total energy in the event. This might be dangerous, however, since "new physics" events containing heavy stable neutral particles would also have visible energy substantially below $\sqrt{s}$. Moreover, two–photon events are interesting in their own right. In particular, it is important to study the transition from "hard" to "soft" events; more about that later. The price one has to pay for a low trigger threshold is that one has to deal with a very large number of events; for instance, one expects at least 4 million events per year with transverse energy $E_T \geq 10$ GeV even at the TESLA collider, which has the smallest beamstrahlung of all designs we studied. Such event rates are not unusual for hadron colliders, and pose no great technological problems; such numbers might be unexpected for high energy e^+e^- colliders, however. We described these events in some detail, giving distributions of variables of interest (transverse momentum, rapidity and di–jet invariant mass). In particular, we showed that at machines with very strong beamstrahlung, exemplified here by the Palmer G design (see Table 1), it would be difficult to study annihilation events containing a hard photon collinear with the beam pipe and a real Z boson. Such events are interesting [45] since they would allow to study the QCD evolution of hadronic systems between two quite different energy scales (m_Z and 500 GeV, respectively) in one experiment; they might also allow to self–calibrate the detector [29].

At future e^+e^- linacs a large number of soft e^+e^- pairs will be produced at each bunch crossing. Most of these pairs will emerge at small angles. These very large electron and positron fluxes probably force one to leave substantial dead zones around the beam pipes when constructing detectors for such colliders. One then expects a substantial cross section for mono–jet events; these can be produced from di–jet final states where one jet emerges at a small angle while the second jet is produced centrally. Such one–sided events are a signature for some "new physics" processes, like the production of supersymmetric neutralinos; it is therefore important to know the standard model prediction for this final state accurately. We found in sec. 3b that at $\sqrt{s} = 500$ GeV, it is dominated by annihilation events if $p_T > 30$ GeV; at higher energies, however, two–photon events dominate over most of the kinematically accessible region. Their cross section is well above that of typical new physics processes. When looking for such processes one might therefore have to require the missing p_T to be larger than the value that can be produced by boosted two–jet events.

Two–photon events also produce a large majority of all $c\bar{c}$ and $b\bar{b}$ pairs at future e^+e^- colliders. The total cross sections, listed in tables 2 and 3, are very large; e.g., even at the TESLA collider at least 80 million $c\bar{c}$ and 400,000 $b\bar{b}$ events would be produced per year. However, without a full detector–specific Monte Carlo study it is impossible to say what fraction of these events will be identified or even detected. In contrast, $t\bar{t}$ production will be dominated by annihilation events at least up to $\sqrt{s} = 1$ TeV, at all the proposed accelerators we studied.

Finally, two–photon processes can also contribute to the single production of W and

Z bosons. At $\sqrt{s} = 500$ GeV, the rates are still marginal, except for the case of the $\gamma\gamma$ collider. Even at higher energy e^+e^- colliders these process will not be able to compete with the single production of W and Z bosons from $2 \to 3$ processes like $e^+e^- \to eW\nu$, as far as the total cross section is concerned. However, the two–photon events always contain some hadronic activity, and produce gauge bosons with small transverse momentum, in contrast to the $2 \to 3$ reactions. It is therefore important to include the two–photon processes in a complete simulation of W and Z production.

In all cases we included direct as well as resolved photon contributions when estimating two–photon cross sections. The relative importance of these two classes of contributions depends on the process under consideration, on the photon spectrum, as well as on the region of phase space one is studying. If a given final state can be produced via gluon exchange in the t or u channel (e.g., jet production), the resolved photon contributions are more important than for reactions that can only proceed via s channel and quark exchange diagrams (e.g., heavy quark production). Moreover, harder photon spectra favour resolved photon events over direct ones, since at higher photon energies one probes the parton densities inside the photon at smaller values of Bjorken–x, where they increase rapidly. This can also be achieved by going to particular regions of phase space, which favour asymmetric initial state configurations; in this case, once resolved contributions are very important. As a rule we find that at e^+e^- colliders operating at $\sqrt{s} = 500$ GeV, resolved photon contributions never dominate if the typical momentum scale of the process exceeds 40 to 50 GeV; in heavy quark production they become subdominant already for $p_T \geq 10$ GeV. Of course, there are also final states which in leading order cannot be produced in direct two–photon reactions, like the vector quarkonium states discussed in sec. 3c and the W and Z bosons of sec. 3d.

We also find that beamstrahlung can have a significant effect on the annihilation cross section for events with visible energy well below $\sqrt{s}$; it can also affect the total cross section for the production of a given final state, as shown in the case of $t\bar{t}$ production in sec. 3c. However, in all cases beamstrahlung increases the two–photon cross section much more than the annihilation cross section.

In sec. 4 we discussed semi–hard and soft two–photon reactions in some detail. We showed that at certain designs one has to expect several such events to occur simultaneously (within the time resolution of the detector) with any annihilation event, giving rise to an "underlying event". Going to the $\gamma\gamma$ collider option only makes this problem worse. These qualitative conclusions are independent of whether one estimates the total cross section using semi–hard QCD (minijets), or relies on the VMD estimate. We presented arguments showing that from the point of view of perturbative QCD, it is quite natural to expect the $\gamma\gamma$ cross section at high energies to substantially exceed the VMD prediction, perhaps by as much as a factor of 10 or more. Furthermore, once an underlying event occurs, quantities like the total particle multiplicity and the total (transverse) energy in the underlying event are of more immediate experimental interest than the number of separate two–photon reactions that contributed to it. These quantities are proportional to the product of the total $\gamma\gamma$ cross section and the particle multiplicity or (transverse) energy per interaction; in the case of $p\overline{p}$ collisions these quantities are known experimentally to increase much more rapidly with energy than the total cross section does. We therefore argued that the simple leading order estimate for the cross section provides a

good figure of merit for the severity of the problems caused by the underlying event, even if it does not reproduce the total $\gamma\gamma$ cross section accurately.

A hard beamstrahlung spectrum also has other adverse effects. As already mentioned, beamstrahlung depends sensitively on the size and shape of the bunches; it also depends on the bunch overlap during collisions. (The expressions of sec. 2 always assume perfect overlap, i.e. fully central bunch collisions.) The same factors also determine the luminosity. Therefore the number of two–photon events due to beamstrahlung grows much more rapidly than linearly with the luminosity per bunch crossing. One will then have to accurately keep track of fluctuations and systematic changes of the luminosity in order to estimate two–photon backgrounds and signal–to–noise ratios precisely; this poses new challenges to the construction of realistic event generators. Finally, as well known [2, 23], beamstrahlung is also responsible for the large number of soft e^+e^- pairs mentioned above, which cause a multitude of technological and physics problems; and by smearing out the electron beam energy, it makes it difficult to study new thresholds in detail.

Of course, it has to be admitted that at present our predictions for hard as well as semi–hard two–photon cross sections suffer from several uncertainties. In the case of hard resolved photon events the biggest unknown is the parton content of the photon; in case of the gluon, it is at present only known to at best a factor of 2. We saw in sec. 3a that present parametrizations for G^γ even differ by as much as a factor of 5 at low Bjorken–x and small momentum scale Q^2. This leads to large uncertainties in the predictions for total jet rates for $p_T \leq 20$ GeV (for $\sqrt{s} = 500$ GeV), as well as for total $c\overline{c}$ and $b\overline{b}$ production rates. Fortunately, this situation should improve soon. In the near future, studies of heavy quark and jet production at TRISTAN and LEP [7, 71] as well as HERA [72] will provide new information on the hadronic structure of the photon. In a few years valuable new information should also come from measurements of the photon structure function F_2^γ at LEP [73] in the region of small x; in this region the evolution equations lead to a strong coupling of quark and gluon densities, while F_2^γ at large x is not very sensitive to G^γ. Measurements of deep inelastic scattering have the advantage that higher order corections ("k-factors") are expected to be smaller than in case of real $\gamma\gamma$ scattering. The ultimate F_2^γ measurement might come [74] from $e\gamma$ colliders, which are a hybrid of the e^+e^- and $\gamma\gamma$ colliders discussed in this paper.

More experimental information about the details of the spectator jets from resolved photons, as well as of the behaviour of total cross sections for hadronic processes involving real photons in the initial state, is also needed. TRISTAN can make important contributions also in this area, since it can study semi–hard two–photon events at energies where eikonalization does certainly not play a major role. This should allow to determine the cut–off parameter $p_{T,min}$ with greater confidence; we saw that already the first AMY measurement [8], which is based on some 300 events, was very helpful in this respect. A more detailed study of resolved photon events might necessitate to upgrade the detectors towards a better angular coverage, so that the spectator jets can be reconstructed more completely. In this area HERA seems to have an advantage. According to first Monte Carlo studies [75], HERA detectors should be able to isolate resolved photon events efficiently and reliably; the large cross sections expected at HERA should then allow detailed investigations of these events. Furthermore, as already mentioned in sec. 4, at HERA the total γp cross section will be measured at energies up to about 250 GeV; this should

help to weed out some of the existing ansätze [66, 67, 69] for the eikonalization of photon cross sections. In some models [66, 68] a first hint of eikonalization might even be visible at LEP200.

We already argued in sec. 4 that the presence of an underlying event would introduce many problems familiar from hadron colliders. In addition, it would become impossible to distinguish between hard resolved and direct two–photon reactions on an event by event basis; the spectator jets which are the tell–tale signature for the former would be lost among the soft hadrons of the underlying event. We therefore see that soft and semi–hard two–photon reactions can commit some sort of fratricide by making the detailed study of hard two–photon events very difficult. Even the measurement of the total $\gamma\gamma$ cross section, which is of great theoretical interest, is much easier if the probability to have more than one event per bunch crossing is very small, since only in this case the total cross section is directly proportional to the number of bunch crossings that contain some hadronic acitivity. Moreover, the ability to trigger against the presence of a spectator jet would (greatly) reduce many two–photon induced hard backgrounds; essentially one would only have to deal with the direct contributions, the cross sections of which can be calculated almost unambiguously. In fact, one can probably remove almost the whole mono–jet background to true one–sided events if the presence of a forward jet is detectable; we argued in sec. 3b that in principle such a jet should be visible, *if* it is not totally obscured by an underlying event.

In view of the undesirable consequences of having ≥ 1 event per bunch crossing, the most conservative attidute seems to be to design colliders such that there is a large safety margin, i.e. *not* to rely on the "conservative" VMD model prediction, nor on calculations of eikonalized cross sections that make use of it. Fortunately, we saw in sec. 4 that it seems to be possible at least in principle to extend the superconducting TESLA design to $\sqrt{s} = 2$ TeV and beyond without risking the occurence of an underlying event. This should not be misunderstood as our endorsement of a particular design; rather it is an (at least theoretical) existence proof for designs that maintain the traditional "clean" environment of e^+e^- colliders up to TeV energies, as far as hadronic backgrounds are concerned. Moreover, even an e^+e^- collider where $\mathcal{O}(1)$ two–photon event underlies every annihilation event has many advantages over hadron colliders, because at e^+e^- colliders the cross section for the production of almost any heavy new particle will be at least roughly comparable to the cross section for typical standard model annihilation processes; at hadron colliders this is only true if the new particle carries colour. Therefore we do *not* believe that soft and semi–hard two–photon events will be the demise of the e^+e^- collider; they do, however, provide a strong additional argument in favour of designs with low beamstrahlung.

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Table 1: Parameters of the machine designs we use. P–G, P–F, D–D and T stand for Palmer G, Palmer F, DESY–Darmstadt and TESLA, respectively; JLC1,2,3 stands for the three phases of the Japan Linear Collider. Notice that there are two versions of the DESY–Darmstadt design, the original wide band beam (wbb) design [29] and its narrow band beam (nbb) variant [28]. Υ is the beamstrahlung parameter, σ_z the bunch length, $\hat{\mathcal{L}}$ the luminosity per bunch crossing, N the number of bunches in each train, Δt the temporal separation between two conecutive bunches in one train, and $\mathcal{L}$ the luminosity of the collider.

	P–G	P–F	D–D(wbb)	D–D(nbb)	T	JLC1	JLC2	JLC3
Υ	0.42	0.11	0.065	0.015	0.0083	0.118	0.404	0.613
σ_z $[mm]$	0.11	0.11	0.4	1.0	2.0	0.152	0.113	0.095
$\hat{\mathcal{L}}$ $[\mu b^{-1}]$	5	1.1	0.29	0.12	0.26	0.8	2.9	4.2
N	10	10	172	172	800	20	20	20
Δt $[10^{-9} sec]$	1	1	11	11	1000	1.4	1.4	1.4
$\mathcal{L}[10^{33}/cm^2 sec]$	5.9	1.4	2.6	1.0	2.1	2.4	8.8	12.7

Table 2: Total $c\overline{c}$ cross sections from two–photon processes at 7 e^+e^- colliders of table 1, as well as for a "$\gamma\gamma$" collider made from an e^+e^- collider with $\sqrt{s} = 500$ GeV; results for the DESY–Darmstadt (nbb) design are very close to those for the TESLA design. We have used the DG parametrization to estimate the resolved photon contributions. $\sigma(q\overline{q})$ and $\sigma(gg)$ stand for the 2–res $q\overline{q}$ annihilation and gluon fusion cross sections, $\sigma(\gamma g)$ for the 1–res photon gluon fusion cross section, and $\sigma(\gamma\gamma)$ for the direct cross section; $\sigma(J/\psi)$ is the 1–res $\gamma + g \to J/\psi + g$ cross section in the color singlet model. All cross sections are in nb.

Collider	$\sigma(q\overline{q})$	$\sigma(gg)$	$\sigma(\gamma g)$	$\sigma(\gamma\gamma)$	σ(tot)	$\sigma(J/\psi)$
T	0.010	0.038	1.8	2.2	4.0	0.014
D–D(wbb)	0.041	0.11	7.0	6.4	13.5	0.053
P–F	0.017	0.08	4.0	2.4	6.4	0.030
P–G	0.14	1.1	38	9.9	49	0.28
JLC1	0.029	0.12	6.3	3.7	10.1	0.047
JLC2	0.064	1.3	31	3.9	36	0.22
JLC3	0.054	2.2	41	3.1	46	0.28
$\gamma\gamma(500)$	0.13	7.6	130	0.14	140	0.89

Table 3: Total cross sections for $b\overline{b}$ production from two–photon processes. The notation is as in table 2, except that now all cross sections are in pb.

Collider	$\sigma(q\overline{q})$	$\sigma(gg)$	$\sigma(\gamma g)$	$\sigma(\gamma\gamma)$	$\sigma(\mathrm{tot})$	$\sigma(\Upsilon(1\mathrm{s}))$
T	0.39	0.46	10.4	7.6	19	0.026
D–D(wbb)	2.0	1.2	38	30	71	0.097
P–F	1.1	1.1	26	12	41	0.066
P–G	10	13	260	65	350	0.66
JLC1	1.8	1.5	39	20	62	0.10
JLC2	6.4	24	280	26	330	0.66
JLC3	7.3	51	430	20	510	1.0
$\gamma\gamma(500)$	21	150	1,300	4.2	1,500	3.5

Table 4: Total cross sections for single production of W and Z bosons in $\gamma\gamma$ collisions at various colliders, estimated from the twice resolved $q\overline{q} \to W, Z$ contribution. The W cross section inlcudes both W^+ and W^- production. We have used the DG parametrization with $Q^2 = m^2_{W,Z}$. All cross sections are in fb.

Collider	$\sigma(W)$	$\sigma(Z)$
T	2.0	1.0
D–D (wbb)	5.0	2.3
P–F	4.7	2.2
P–G	58	28
JLC1	6.5	3.0
JLC2	77	39
JLC3	115	55
$\gamma\gamma(500)$	400	205
$\gamma\gamma(1000)$	800	340
$\gamma\gamma(2000)$	1,750	615

Table 5: Total semi–hard two–photon cross section at various colliders. The notation for the first 9 rows is like in the previous tables; rows 10 and 11 show results for an upgrade of the TESLA design where the beamstrahlung parameter Υ grows like $\sqrt{s}$, while the last two rows are for an upgraded TESLA if Υ grows like s. We have chosen $p_{T,min} = 1.6$ (2.4) GeV for the DG (DO+VMD) parametrization, as described in the text. For comparison, col. 4 shows the soft contribution for $W_{\gamma\gamma} \geq 5$ GeV, assuming a constant $\gamma\gamma$ cross section of 250 nb as predicted by the VMD model. Col. 5 shows the number of semi-hard events per bunch collision or per 10^{-7} sec, whatever is bigger.

Collider	σ^{hard}(DG) $[\mu b]$	σ^{hard}(DO + VMD) $[\mu b]$	σ^{soft} $[\mu b]$	no. of events (DG)
T	0.016	0.0090	0.041	0.004
D–D (nbb)	0.020	0.014	0.051	0.021
D–D (wbb)	0.075	0.041	0.20	0.20
P–F	0.042	0.024	0.072	0.46
P–G	0.48	0.29	0.51	24
JLC1	0.069	0.04	0.12	1.1
JLC2	0.41	0.28	0.19	24
JLC3	0.59	0.43	0.15	50
$\gamma\gamma$(500)	1.9	1.4	0.25	0.49 – 95
T(1000)	0.057	0.036	0.099	0.0036
T(2000)	0.21	0.15	0.13	0.013
T'(1000)	0.17	0.099	0.27	0.043
T'(2000)	3.4	2.4	1.2	3.4

Figure Captions

Fig. 1 Photon spectra at $\sqrt{s} = 500$ GeV (a) and evolution of the photon spectrum with $\sqrt{s}$ at the JLC design (b). T, D–D, P–F and P–G stand for the TESLA, DESY–Darmstadt, Palmer F and Palmer G designs, respectively; note that the DESY–Darmstadt design exists in wide band beam (wbb) and narrow band beam (nbb) versions. WW is the Weizsäcker Williams or bremsstrahlung spectrum. The curve labelled 'laser' shows the spectrum (8) that emerges when laser photons are backscattered off incident electrons.

Fig. 2 Electron spectra at $\sqrt{s} = 500$ GeV. The dotted curve shows the electron spectrum without beamstrahlung, but with initial state radiation included. The notation for the other curves is as in fig. 1. Notice that we have chosen to present the spectra as a function of $1 - x$, in order to better resolve the region of large x.

Fig. 3 Cross sections for the two–photon production of two central jets at $\sqrt{s} = 500$ GeV (a) and the three stages of the JLC (b), as a function of the minimal transverse momentum $p_{T,min}$ of the jets. The notation in (a) is like in Fig. 1a; notice that the results for the DESY–Darmstadt (nbb) design are almost identical to those for the TESLA collider. In (b), the dashed curves show the prediction from the direct process alone, while the solid curves show the prediction after inclusion of the resolved photon contributions. The DG parametrization has been used with $Q^2 = \hat{s}/4$ and a floating number of flavours, as described in the text.

Fig. 4 Various contributions to the two–photon production of two central jets at the first stage of the JLC. In (a) only the once resolved contributions are shown, while (b) depicts the twice resolved contributions. The curves are labelled according to the composition of the final state; here 'q' stands for any quark or anti–quark. We have used the same parameters as in fig. 3. In particular, the use of a Q^2 dependent number of flavors explains the kinks at $p_T \simeq 7$ GeV, where charm starts to contribute.

Fig. 5 Ratios of predictions of the LAC2 and DG parametrizations for the production of two central jets at the first stage of the JLC. Contributions with different final states have been shown separately, using the same notation as in fig. 4.

Fig. 6 Rapidity distribution of di–jet events at $p_T = 30$ GeV for the case $y_1 = y_2$, at the TESLA collider (a) and a 500 GeV $\gamma\gamma$ collider (b). The contributions from the direct, once resolved and twice resolved processes are shown separately. We have

used the same parameters as in fig. 3. Notice that (b) includes a very small, but nonzero direct contribution (short dashed curve).

Fig. 7 Invariant mass distribution of centrally produced di–jet events at the Palmer G (a) and TESLA (b) colliders. The solid and dashed curves show the two–photon contributions in the notation of fig. 6, while the dotted curves show the contribution from annihilation events. Notice that a stronger p_T cut has been applied in (a). We have used the same parameters as in fig. 3.

Fig. 8 The transverse momentum spectrum of mono–jets, at two different e^+e^- colliders operating at $\sqrt{s} = 500$ GeV (a), as well as a $\gamma\gamma$ collider (b). The solid and dotted curves in (a) show the total two–photon and annihilation contributions, respectively, while the various curves in (b) correspond to different classes of two–photon contributions; in (b) there is no annihilation contribution, of course. We have used the same parameters as in fig. 3.

Fig. 9 The transverse momentum spectrum of mono–jets at the three stages of the JLC. Notations and parameters are as in fig. 8a.

Fig. 10 The transverse mometum spectrum of central charm pairs produced from two–photon processes at the TESLA collider (a) and at Palmer G (b), respectively. Contributions from different classes of processes are shown separately; notice that in this figure, the two twice resolved contributions are labelled according to the initial state. We have used the DG parametrization with $N_f = 3$ active flavours.

Fig. 11 Total $t\bar{t}$ production cross sections at two different e^+e^- colliders operating at $\sqrt{s} =$ 500 GeV (a) and at the third stage of the JLC (b). The dotted and solid curves in (a) show contributions from the annihilation process and from two–photon reactions, respectively; notice that the latter have been multiplied with 100 (10) for the TESLA (Palmer G) collider. In (b) we show in addition the annihilation contribution if both initial state radiation and beamstrahlung could be switched off (long dashed curve), as well as the contribution from the direct two–photon process (short dashed curve).

Fig. 12 Total integrated semi–hard (minijet) two–photon cross section as a function of the transverse momentum cut–off parameter $p_{T,min}$, for the DG (a) and modified DO+VMD (b) parametrizations. The notation is the same as in fig. 1a.

Fig. 13 The total semi–hard $\gamma\gamma$ cross section as predicted by the DG parametrization with $p_{T,min} = 1.6$ GeV, as a function of the $\gamma\gamma$ centre–of–mass energy $W_{\gamma\gamma}$ (solid). The

dashed curves show which fraction of all semi–hard two–photon events at a given collider have a $\gamma\gamma$ energy less than $W_{\gamma\gamma}$; they refer to the scale at the right.