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Non-Critical Strings in Robertson-Walker Space Time

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Abstract

I consider a D+1 dimensional nonlinear σ model based on a possible interpretation of the Liouville field as a physical time. The Weyl invariance of this theory gives us restrictions for the background fields and the parameters of the theory, e.g. for trivial background one obtains the known regions for the dimension of the space-time (≤ 1 or ≥ 25). For a Robertson-Walker space time a special solution of these equations is discussed.

1 Introduction

The nonlinear σ model describing strings in non non-trivial background fields was extensively studied in the last years [1, 2]. By the interpretation of the Liouville field as physical time it is possible to regard the non-critical string theory as a special σ model [3, 4, 5]. Some interesting results for non-trivial target space were already obtained. First of all this is the black hole solution for $D = 1$ [3, 6, 7]. Secondly it is possible to quantize the theory in arbitrary dimension if one incorporates convenient spatial background charges [8]. And finally the new cosmological results are remarkable [4, 9, 10].

In this paper I consider a string living in a Robertson-Walker space time where the time correspond to the Liouville field and I want discuss a special solution of the Weyl invariance conditions. The following questions found my particular interest: 1) How is the timelike evolution of the metric? 2) What is the region of the allowed dimension of the space time? and 3) Is it possible to find a solution for the tachyon equation? Similar questions are already discussed for flat (spatial independent) Robertson-Walker metrics [5].

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2 The Model

Before I want consider the string in a Robertson-Walker space time let me make some introductory remarks for the theory in Minkowskean (or Euclidean) space time. For genus zero world sheets the corresponding partition function is given by [11, 12] :

$$Z = \int D_{\hat{g}}\sigma \, D_{\hat{g}}\mathbf{X} \; e^{-S} \tag{1}$$

with:

$$\begin{aligned} S(\mathbf{X},\sigma,\hat{g}) &= S_M(\mathbf{X},\hat{g}) + \tfrac{1}{4\pi}\int d^2z\sqrt{\hat{g}}\left(\hat{g}^{ab}\partial_a\sigma\partial_b\sigma + Q\hat{R}\sigma + \mu e^{2\alpha\sigma}\right) \\ S_M(\mathbf{X},\hat{g}) &= \tfrac{1}{4\pi\alpha'}\int d^2z\sqrt{\hat{g}}\,\hat{g}^{ab}\partial_a\mathbf{X}\cdot\partial_b\mathbf{X} \; . \end{aligned} \tag{2}$$

Here $\mathbf{X}$ is the position of the string in the space time and σ is the Liouville field connected with the 2d metric via the conformal gauge: $g_{ab} = e^{2\sigma}\hat{g}_{ab}$; $\hat{g}_{ab}$ is a reference metric and $\hat{R}$ denotes the 2d scalar curvature with respect to the reference metric $\hat{g}_{ab}$. The coefficients Q and α in S are not arbitrary [12]: Q is determinated by the vanishing of the total central charge and α by the requirement that the conformal dimension of $e^{2\alpha\phi}$ is equal to one.

If one interprets the Liouville field as a time variable it is possible to consider this model as a special σ model [3, 4, 5]. With

$$D \to D+1 \quad , \quad \sigma \to \tfrac{1}{\sqrt{\alpha'}}X^0$$

one obtains

$$\begin{aligned} Z &= \textstyle\int D_{\hat{g}}X \; e^{-S} \quad , \\ S &= \tfrac{1}{4\pi\alpha'}\textstyle\int_M d^2z\sqrt{\hat{g}}\left(\hat{g}^{ab}\partial_a X^\mu\partial_b X^\nu G_{\mu\nu} + \alpha'\hat{R}\phi + \alpha' T\right) \; . \end{aligned} \tag{3}$$

Here $G_{\mu\nu}$ is the metric in the target space (space time), ϕ is the dilaton field and T the tachyon field. For the special background fields: $G_{\mu\nu} = \delta_{\mu\nu}$, $\phi(X^0) = \frac{1}{\sqrt{\alpha'}}QX^0$ and $T(X^0) = \mu e^{2\frac{\alpha}{\sqrt{\alpha'}}X^0}$ one obtains again the original model (1), (2).

Since the functional integration includes the integration over the Liouville field this σ model must be Weyl invariant. This statement is equivalent to the vanishing of the total central charge [12]. As the original theory depends on g_{ab} only (not on $\hat{g}_{ab}$), this statement seems to be trivial. But if we regularize this theory we have to introduce a cutoff corresponding to $\hat{g}_{ab}$ and therefore the condition for Weyl invariance give us non-trivial restrictions for the background fields [4]. The vanishing of the Weyl anomaly correspond to the vanishing of all $\bar{\beta}$ functions of the theory [1]:

$$\begin{aligned} & \bar{\beta}^i \equiv 0 \quad \forall i && (\text{ i numerates all background fields }), \\ \text{with :} \quad & \bar{\beta}^G_{\mu\nu} = \beta^G_{\mu\nu} + D_{(\mu}M_{\nu)} \quad , && M_\nu = 2\alpha'\partial_\nu\phi + W_\nu \; , \\ & \bar{\beta}^\phi = \beta^\phi + \tfrac{1}{2}M^\mu\partial_\mu\phi \; , \\ & \bar{\beta}^T = \beta^T - 2T + \tfrac{1}{2}M^\mu\partial_\mu T \; . \end{aligned} \tag{4}$$

Up to the second order in α' one gets for the β functions [1, 13][a]:

$$\begin{aligned}
\beta^T &= -\tfrac{1}{2}\alpha' D^2 T \ , \\
\beta^G_{\mu\nu} &= \alpha' R_{\mu\nu} + \tfrac{1}{2}\alpha'^2 R_{\mu\alpha\beta\lambda} R_\nu{}^{\alpha\beta\lambda} \ , \\
\beta^\phi &= \tfrac{1}{6}(D-25) - \tfrac{1}{2}\alpha' D^2\phi + \tfrac{1}{16}\alpha'^2 (R_{\mu\nu\alpha\lambda})^2 \ .
\end{aligned} \tag{5}$$

In the following I want to investigate these equations and look for non-trivial solutions. As a first example I want rederive the results for Q and α obtained by David, Distler and Kawai [12]. Therefore I consider the background field configuration:

$$G_{\mu\nu} = \delta_{\mu\nu} \quad , \quad \phi = \frac{1}{\sqrt{\alpha'}} Q X^0 \quad \text{and} \quad T = \mu e^{2\frac{\alpha}{\sqrt{\alpha'}} X^0} \ . \tag{6}$$

As a result of the covariance W_ν in (4) is a function of the covariant derivatives of the curvature tensor only (torsion terms are absent in this model) and it would vanish if these curvature terms vanished, especially W_ν is independent from the dilaton and tachyon field [1]. For the equations (4) one obtains:

$$\begin{aligned}
\bar\beta^G_{\mu\nu} &\equiv 0 \ , \\
\bar\beta^\phi &= \tfrac{1}{6}(D-25) + Q^2 = 0 \ , \\
\bar\beta^T &= -2(\alpha^2 - Q\alpha + 1) T = 0 \ .
\end{aligned} \tag{7}$$

Thus we have the known results:

$$\begin{aligned}
Q &= \sqrt{\tfrac{25-D}{6}} \\
\alpha_\pm &= \tfrac{Q}{2} \pm \sqrt{\tfrac{Q^2}{4} - 1} = \tfrac{1}{\sqrt{24}} \left(\sqrt{25-D} \pm \sqrt{1-D} \right) \ .
\end{aligned} \tag{8}$$

Before I turn to a non-trivial space time metric I look for the most general dilaton and tachyon field in *flat* space time ($G_{\mu\nu} = \delta_{\mu\nu}$) solving the equations (4). In this case one has:

$$\bar\beta^G_{\mu\nu} = 2\alpha' \partial_\mu \partial_\nu \phi = 0 \tag{9}$$

and hence

$$\phi = \phi_0 + \frac{1}{\sqrt{\alpha'}} q_\mu X^\mu \ , \tag{10}$$

which was already discussed as one example for noncritical strings in arbitrary dimensions [8]. The tachyon field is given by:

$$\bar\beta^T = -\frac{1}{2}\alpha' \partial^2 T - 2T + \sqrt{\alpha'} q_\mu \partial^\mu T = 0 \ , \tag{11}$$

[a] I neglect in my consideration all " non-perturbative " contributions [4, 13].

with the solution:

$$T \sim e^{\frac{1}{\sqrt{\alpha'}} p_\mu X^\mu} \qquad \text{and:} \qquad -\frac{1}{2} p_\mu p^\mu - 2 + q_\mu p^\mu = 0 \ . \tag{12}$$

For the dilaton $\bar{\beta}$ function one gets:

$$\bar{\beta}^\phi = \frac{1}{6}(D-25) + q_\mu q^\mu = 0 \ . \tag{13}$$

Combining (12) and (13) one obtains finally [14, 15]:

$$\begin{aligned} &q_0 = \sqrt{\tfrac{25-D}{6} - \vec{q}^{\,2}} \ , \\ &p_{0_\pm} = q_0 \pm \sqrt{q_0^2 - \vec{p}^{\,2} + 2\vec{q}\vec{p} - 4} = \tfrac{1}{\sqrt{6}} \left(\sqrt{25 - D - 6\vec{q}^{\,2}} \pm \sqrt{1 - D - 6(\vec{q} - \vec{p})^2} \right) \ . \end{aligned} \tag{14}$$

3 A Solution for Robertson-Walker Space Time

Now I want to investigate the equations (4) in a Robertson-Walker space time. This metric describing a (spatial) homogeneous and isotropic universe is given by [16]:

$$(ds)^2 = a^2 (dX^0)^2 + \frac{K^2(X^0)}{(1 + \frac{1}{4}\epsilon r^2)^2} \left[(dX^1)^2 + (dX^2)^2 + ... + (dX^D)^2 \right] \ , \tag{15}$$

where $r^2 = (X^1)^2 + (X^2)^2 + ... + (X^D)^2$, $K(X^0)$ is the so-called world radius and the parameter ϵ determinates the spatial geometry: flat ($\epsilon = 0$), spherical ($\epsilon = +1$) or hyperbolical ($\epsilon = -1$)[b]. This metric has Euclidean signature if $a^2 > 0$ and Minkowskean signature if $a^2 < 0$. The only non-vanishing curvature terms are given by ($k, l, ... = 1, ..., D$):

$$R^m{}_{nkl} = \left[\frac{\epsilon}{K^2} - \frac{1}{a^2} \left(\frac{\dot{K}}{K} \right)^2 \right] (G^m_k G_{nl} - G^m_l G_{nk}) \quad , \quad R^0{}_{n0l} = -\frac{1}{a^2} \frac{\ddot{K}}{K} G_{nl} \ . \tag{16}$$

If one computes the β functions in terms of this curvature tensor one obtains very complicated non-linear differential equations for K, ϕ and T. In order to avoid these difficulties I will not consider $G_{\mu\nu}$ and ϕ as "physical" metric and dilaton [1, 9], but (for $D \neq 1$)[c]:

$$\tilde{G}_{\mu\nu} = e^{-\frac{4}{D-1}\phi} G_{\mu\nu} \quad , \quad \tilde{\phi} = \frac{2}{\sqrt{|D-1|}} \phi \ . \tag{17}$$

Therefore it is reasonable that the time evolution of the metric is controlled by the dilaton field, i.e.:

$$K = \text{const.} \ , \ \phi = \phi(X^0) \ . \tag{18}$$

[b] Strictly speaking ϵ has to carry space time dimensions and therefore $\epsilon = \pm \frac{1}{\alpha'}, 0$.

[c] By this reparametrization $S_{eff} = \int d^{D+1}X \sqrt{G} e^{-2\phi} \left[\frac{2}{3}(D-25) - \alpha' R - 4\alpha' (\partial\phi)^2 + ... \right]$ is for $D \neq 1$ just transformed in $S_{eff} = \int d^{D+1}X \sqrt{\tilde{G}} \left[\alpha' \tilde{R} + 2\alpha' (\partial\tilde{\phi})^2 + \frac{2}{3}(D-25) e^{\frac{2}{\sqrt{|D-1|}}\tilde{\phi}} + ... \right]$, i.e. an action with a standard kinetic part for the dilaton and the usual volume measure. In addition the theory in $\tilde{G}_{\mu\nu}$ and $\tilde{\phi}$ yield the correct vertex operators for the S-matrix [1].

Then one gets for (16):

$$R_{mnkl} = \frac{\epsilon}{K^2}\left(G_{mk}G_{nl} - G_{ml}G_{nk}\right) \quad , \quad R_{0n0l} = 0 \ . \tag{19}$$

This configuration corresponds for $\epsilon = 0$ or $D = 1$ to a flat space time and in addition all covariant derivatives and timelike components of the curvature tensor vanish. Hence it is impossible to construct a vector field W_μ or a timelike tensor β_{00} (they have to be independent of the dilaton and tachyon field [1]) and thus: $W_\mu = \beta^G_{00} = 0$, $M_\mu \equiv 2\alpha'\partial_\mu\phi$. So one finds for the metric and dilaton $\bar{\beta}$ functions (4) in arbitrary orders in α':

$$\begin{aligned}
&\bar{\beta}^G_{00} = 2\alpha' D_0\partial_0\phi(X^0) = 2\alpha'\ddot{\phi} \ , \\
&\bar{\beta}^G_{mn} = \beta^G_{mn} \qquad\qquad (D_m\partial_n\phi = 0 \text{ if } K = \text{const. and } \partial_m\phi = 0) \ , \\
&\bar{\beta}^\phi = \beta^\phi + \tfrac{\alpha'}{a^2}\left(\dot{\phi}(X^0)\right)^2 \ .
\end{aligned} \tag{20}$$

Note that from (19) follows: $\beta^G_{mn} = \Lambda G_{mn}$ and the vanishing of $\bar{\beta}^G_{mn}$ is equivalent to the vanishing of Λ which restrict the possible value of K and ϵ (presumed it exist real solution) and furthermore the vanishing of the metric β function ensure that one does not need renormalize the metric. The vanishing of the first equation gives us:

$$\phi(X^0) = \tfrac{a}{\sqrt{\alpha'}}QX^0 + \phi_0 \quad ; \quad \text{and with the last equation :} \quad Q = \pm\sqrt{-\beta^\phi} \ . \tag{21}$$

Here Q is a generalized background charge which depend from the ϵ and the dimension of the space time and ϕ_0 is the zero mode ($\sqrt{\alpha'}$ ensure that Q is dimensionless and a that Q is a scalar under time reparametrization). As for the dilaton field I want investigate at this point a tachyon field depending of the time only. Than β^T can not contain curvature terms ($R_{0n0l} = 0$) and one finds for β^T the same equation as in the flat case. In terms of (4) T is defined by the equation:

$$-\frac{1}{2}\frac{\alpha'}{a^2}\ddot{T}(X^0) - 2T(X^0) + \frac{\alpha'}{a^2}\dot{\phi}\dot{T}(X^0) = 0 \ , \tag{22}$$

with the solution:

$$T(X^0) \sim e^{2\frac{\alpha_\pm}{\sqrt{\alpha'}}aX^0} \qquad \text{wherein:} \ \ \alpha_\pm = \frac{1}{2}\left(Q \pm \sqrt{Q^2 - 4}\right) \tag{23}$$

or with (21):

$$\alpha_\pm = \frac{1}{2}\left(\sqrt{-\beta^\phi} \pm \sqrt{-\beta^\phi - 4}\right) \ . \tag{24}$$

Via β^ϕ in this equation one has again (cp. (5)) a restriction for the dimension of the space time. The tachyon field is real if: $\beta^\phi \leq -4$ (Euclidean space time) or $\beta^\phi \geq 0$ (Minkowskean space time). For $D = 1$ or $\epsilon = 0$ all curvature terms vanish and thus for $\epsilon = 0$: $\beta^\phi = \frac{1}{6}(D - 25)$; or for $D = 1$: $\beta^\phi = -4$. In addition the metric β function vanish and one does not have any restriction for K.

All expressions I have obtained up to this point are valid in arbitrary order in α'. Now I compute Q up to the second order and want discuss the modification for a spatial dependent tachyon field. In terms of (5) one finds:

$$\begin{aligned} \beta^G_{mn} &= \left[(D-1)\tfrac{\alpha'\epsilon}{K^2} + (D-1)\left(\tfrac{\alpha'\epsilon}{K^2}\right)^2\right] G_{mn} \ , \\ \beta^\phi &= \tfrac{1}{6}(D-25) + \tfrac{D(D-1)}{8}\left(\tfrac{\alpha'\epsilon}{K^2}\right)^2 \ . \end{aligned} \tag{25}$$

Therefore one gets for the vanishing of the metric β function the restrictions:

$$\begin{aligned} & K^2 = -\alpha'\epsilon \qquad \text{or} \qquad \epsilon = 0 \qquad \text{or} \qquad D = 1 \\ & \text{and thus if } D \neq 1 \text{ and } \epsilon \neq 0 : \\ & Q = \sqrt{-\beta^\phi} = \sqrt{\tfrac{25-D}{6} - \tfrac{D(D-1)}{8}} \ . \end{aligned} \tag{26}$$

In order to obtain a *real* tachyon field (23), (24) we get the restriction for Euclidean signature ($a^2 > 0$): $Q^2 \geq 4$ or $-\frac{4}{3} \leq D \leq 1$ and for Minkowskean signature ($a^2 < 0$): $Q^2 \leq 0$ which correspond to $D \geq \frac{1}{6}(\sqrt{1201} - 1) \approx 5.61$ or $D \leq -\frac{1}{6}(\sqrt{1201} + 1)$. Unfortunately the dependence on α' in Q drops out and hence one has not a perturbative result. Therefore one can not neglect the next terms and one has to deal with this result carefully.

A spatial dependent tachyon field is up to the second order defined by:

$$-\frac{1}{2}\alpha' D^2 T(X^0, X^m) - 2T(X^0, X^m) + \frac{\alpha'}{a^2}\dot{\phi}\dot{T}(X^0, X^m) = 0 \ . \tag{27}$$

With the ansatz: $T(X^0, X^m) = H(X^m)U(X^0)$ and the assumption that H should depend on $L = \frac{1}{1+\frac{1}{4}\epsilon r^2}$ only (motivated by the spatial dependence of $G_{\mu\nu}$), one can for $\epsilon \neq 0$ separate this equation in two ones (λ is the separation constant):

$$\begin{aligned} & -\tfrac{1}{2}\tfrac{\alpha'}{a^2}\ddot{U} + \tfrac{\sqrt{\alpha'}}{a}Q\dot{U} - \lambda U = 0 \ , \\ & L(1-L)H''(L) + (\tfrac{D}{2} - DL)H'(L) + 2(2-\lambda)H(L) = 0 \quad \left(\ H' = \tfrac{d}{dL}H \ \right) . \end{aligned} \tag{28}$$

The solution for U is given by (23) if one replaces $\alpha_\pm$ by $\alpha_\pm = \frac{1}{2}(Q \pm \sqrt{Q^2 - 2\lambda})$. The second equation is just the hypergeometrical differential equation and solutions around $L = 1$ ($r^2 \simeq 0$), around $L = 0$ ($r^2 \simeq \infty$) and for $\alpha'\epsilon = -1$ around $L \simeq \infty$ ($\alpha' r^2 \simeq 4$) can one finds in [17]. Hypergeometrical functions as correlation functions for conformal field theories in restricted domains are already discussed in [18]. For $\epsilon = 0$ one gets for the tachyon by replacing from $p_0 \to 2ap_0$ and $p_m \to 2Kp_m$ and setting $q_\mu = 0$ just the flat solution (12), (14).

Finally I return to the results (21)-(24) and transform they in the " physical" one. In order to interpret $\tilde{G}_{\mu\nu}$ in (17) as "physical" metric from Robertson-Walker type a time reparametrization $X^0 \to t = t(X^0)$ transforming $\tilde{G}_{00}$ in ± 1 ($+1$ for Euclidean and -1 for

Minkowskean signature) is quite reasonable. Of course this procedure makes sence only for $D \neq 1$. The corresponding transformation is given by:

$$\begin{aligned} aX^0_{\pm} &= \tfrac{\sqrt{\alpha'}(D-1)}{2Q} \log \tfrac{(D-1)\sqrt{\alpha'}}{2\tilde{Q}\sqrt{\eta}(t_0 \pm t)} \\ (\tilde{Q} = Qe^{\frac{2\phi_0}{D-1}} &\ , \ t_0 = \text{const.} \ , \ \eta = \pm 1 \text{ for Minkowskean or Euclidean signature }) \ . \end{aligned} \tag{29}$$

If one prefers for t the same direction as for X^0 one has to take the minus sign. After performing this transformation one finds finally for the "physical" metric, dilaton field and tachyon field:

$$\begin{aligned} &(ds)^2 = \eta(dt)^2 + \tfrac{\tilde{K}^2(t)}{(1+\frac{1}{4}\epsilon r^2)^2}\left[(dX^1)^2 + (dX^2)^2 + ... + (dX^D)^2\right] \ , \\ &\tilde{\phi} = \tfrac{\sqrt{|D-1|}}{2} \log \tfrac{K^2}{\tilde{K}^2} \qquad \text{and} \qquad \tilde{T}(t) \sim \left(\tfrac{K^2}{\tilde{K}^2}\right)^{\delta_{\pm}} \ , \\ &\text{with :} \qquad \tilde{K}^2 = \tfrac{4\eta Q^2}{\alpha'(D-1)^2}(t \pm t_0)^2 K^2 \qquad \text{and} \qquad \delta_{\pm} = \tfrac{\alpha_{\pm}(D-1)}{2Q} \ . \end{aligned} \tag{30}$$

Here for $\epsilon = 0$ K^2 is arbitrary and $Q = \sqrt{\frac{D-25}{6}}$, for $\epsilon = \pm\alpha'$ K^2 is given by the equation: $\beta_{mn} = 0$, Q is given by (21) and $\alpha_{\pm}$ by (24). Up to the order α'^2 the corresponding expressions are given in equation (26).

4 Conclusion

I have discussed solutions of the Weyl invariance condition for a string σ model in which the time correspond to the Liouville field. The vanishing of the Weyl anomaly coefficients ($\bar{\beta}$ function) restricts the possible background fields and the parameters of the theory. First I have rederived the known results for Q and $\alpha_{\pm}$ (David, Distler,Kawai). Secondly I investigated the general solution in a *flat* space time. The corresponding dilaton field must be linear and the tachyon field is just a exponential function. The zero components of the momentums are restricted, but the dimension is for special regions of spatial momentums (or spatial background charge resp.) arbitrary.

As an example for non-trivial metric I have studied in the third section a solution for a Robertson-Walker space time. If one transforms the metric and dilaton in the "physical" ones ($\tilde{G}_{\mu\nu}, \tilde{\phi}$) and furthermore assumes that the time evolution is controlled by the dilaton field it is possible to find solutions for the Wyel invariance condition (also in higher orders in α'). The value for the background charge and for $\alpha_{\pm}$ is in principle now given by the dilaton β function. The demand that the tachyon should be a real field restricts the dilaton β function and thus the dimension: for Euclidean space time $\beta^{\phi} \leq -4$ and for Minkowskean $\beta^{\phi} \geq 0$. For $\epsilon = 0$ or $D = 1$: $\beta^{\phi} = -4$ and else one has perturbative results only. The obtained results are valid for arbitrary order in α'. A perturbative computation of β^{ϕ} or the generalized background charge resp. Q yields up to the second order in α' the restriction: $-\frac{4}{3} \leq D \leq 1$ (Euclidean signature) or $D \geq \frac{1}{6}(\sqrt{1201} - 1) \approx 5.61$ or $D \leq -\frac{1}{6}(\sqrt{1201} + 1)$. Unfortunately the parameter of the perturbation theory α' drops out and one has to deal with this result carefully.

The final result for the "physical" background fields solving the Weyl invariance condition is given by (30). The world radius depends quadratic, the dilaton logarithmic and the

tachyon field potential on the time. The direction of the evolution (increasing or decreasing) is determined by ϵ and the signature of the space time (Euclidean or Minkowskean).

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