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BILOCAL FIELD APPROACH AND SEMILEPTONIC HEAVY MESON DECAYS

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Abstract

In this paper we consider the bilocal field approach for QCD. We obtain a bilocal effective meson action with a potential kernel given in relativistic covariant form. The corresponding Schwinger–Dyson and Bethe–Salpeter equations are investigated in detail. By introducing weak interactions into the theory we study heavy meson properties as decay constants and semileptonic decay amplitudes. Thereby, the transition from the bilocal field description to the heavy quark effective theory is discussed. Considering as example the semileptonic decay of a pseudoscalar B–meson into a pseudoscalar D–meson we obtain an integral expression for the corresponding Isgur–Wise function in terms of meson wave functions.

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1. Introduction

The investigation of heavy meson decays has become one of the important problems in heavy flavour physics. Especially, B–decays [1] play an important role in determining the Kobayashi–Maskawa matrix elements, including the CP violating phase. Furthermore, rare decays of heavy mesons may indicate deviations from the standard model. For the description of physical (hadronic) decay processes one needs to know the wave functions and form factors. There exist several approaches to attack these problems. Examples are phenomenological quark models, QCD sum rules, heavy quark effective theory, potential models.

Here we shall consider one more possibility – potential models in the bilocal field approach for QCD [2, 3]. Thereby, we start from an approximate QCD action with massive quark fields and hadronize it. Substitution of the interaction kernel containing the free gluon propagator by a instantaneous Lorentz–vector potential kernel yields a class of bilocal potential models for meson fields [4, 5]. Potential models do not possess full relativistic invariance because the Fock space is restricted to $(q\bar{q})$ pairs and an instantaneous interaction is assumed. Nevertheless, in our model the kernel is written in relativistic covariant form by introducing a special vector being proportional to the bound–state total momentum. This allows us to investigate equations for a bound state moving together with the potential kernel. This fact is important especially for the calculation of formfactors where one needs to know meson wave functions for moving particles. One further advantage of our relativistic model consists in the possibility of describing in a uniform manner both light and heavy mesons in dependence on the choice of the potential. So, in the case of a short–range potential one can use the separable approximation (for small orbital momenta) to obtain a new regularized version of the Nambu–Jona–Lasinio model [6]. We should also mention that we are able to extend some of the results obtained by A. Le Yaouanc et al. [7], who investigated a local quark model for massless quark fields within the Hamilton formalism with an instantaneous fourth–component Lorentz–vector colour confining potential (e.g. the harmonic oscillator potential).

In the first part of this paper the Schwinger–Dyson and Bethe–Salpeter equations for our model will be derived, and the meson wave functions fulfilling Schrödinger–like equations will be introduced. Thereby, the equations for the quark mass spectrum will be considered in application to oscillator, Coulomb and linear potentials. Concerning the equations on bound state masses special attention will be paid to the derivation of equations for wave functions of scalar, pseudoscalar and vector particles.

Then, in the second part of the paper we will apply the bilocal field method to heavy meson physics. Therefore we will introduce semileptonic weak interactions into the underlying potential theory by shifting the bilocal field correspondingly. Furthermore, now we will have to consider in general moving bound states. We will derive formulas for pseudoscalar meson decay constants as well as for semileptonic decays of heavy quarkonia. Moreover, we will establish a relation between bilocal field approach and heavy quark effective theory. This will allow us to consider the semileptonic decays in the limit to the heavy quark effective theory [8 – 10]. Thereby, as example the Isgur–Wise function [8] corresponding to decays $B \to D(l\nu_l)$ will be defined within the

bilocal field method.

In this paper general formulas containing the so far undetermined meson wave functions will be obtained. Nevertheless, as will be shown, these wave functions fulfil systems of integral equations which shall be solved in the near future for concrete potentials by different methods.

The paper is organized as follows. In sect. 2 the formulation of the model is given. The corresponding equations for the quark spectrum within a meson are derived in sect. 3. Sect. 4 contains the investigation of equations for bound state vertex and wave functions. In sect. 5 formulas for the pseudoscalar meson decay constants are derived. In sect. 6 we discuss the relation between bilocal field approach and heavy quark effective theory. Semileptonic decays of heavy quarkonia in the limit of heavy quark effective theory are investigated in sect. 7. The summary and conclusions are given in sect. 8.

2. Formulation of the model

2.1. Hadronization of QCD

Let us start with the approximate QCD action for quarks q in the form

$$W = \int d^4x \bar{q}(x)[G_0^{-1}(x)]q(x) - \frac{g^2}{2}\int\int d^4x d^4y j_\mu^a(x) D_{\mu\nu}^{ab}(x-y) j_\nu^b(y) \ . \qquad (1)$$

Here G_0 is the Green's function for free quarks,

$$G_0^{-1} = i\partial\!\!\!/ - \hat{m}_0 \ ,$$

where $\hat{m}_0$ is the bare quark mass matrix, $\hat{m}_0 = \mathrm{diag}(m_1^0, m_2^0, \ldots, m_{N_f}^0)$, N_f being the flavour number. The quark current $j_\mu^a(x)$ is defined by the relation

$$j_\mu^a(x) = \bar{q}(x)\Big(\frac{\lambda^a}{2}\Big)\gamma_\mu q(x) \ ,$$

where λ^a are the Gell–Mann matrices in colour space $\mathrm{SU}(3)_c$, μ is the Lorentz index and γ_μ denotes the Dirac matrix. The quark–gluon interaction with coupling constant g is mediated via the free gluon propagator $D_{\mu\nu}^{ab}(x-y) \equiv \delta^{ab} g_{\mu\nu} D(x-y)$.

For hadronization of action (1) let us first rewrite the bilocal interaction term

$$-\int\int d^4x d^4y \frac{1}{2}\Big(\bar{q}(x)\frac{\lambda^a}{2}\gamma_\mu q(x)\Big)\frac{g^2}{2}D(x-y)\Big(\bar{q}(y)\frac{\lambda^a}{2}\gamma^\mu q(y)\Big)$$

in the form

$$\int\int d^4x d^4y q_B(y)\bar{q}_A(x) K_{AB,EF}(x-y) q_F(x)\bar{q}_E(y) \qquad (2)$$

with the kernel

$$K_{AB,EF}(x-y) = \gamma_{ru}^\mu(\gamma_\mu)_{ts}\sum_{a=1}^{8}\frac{\lambda_{\alpha\delta}^a}{2}\frac{\lambda_{\gamma\beta}^a}{2}\delta_{il}\delta_{kj}\frac{g^2}{2}D(x-y) \ .$$

Here A, B, E, F are short–hand notations for the indices $A = \{r, \alpha, i\}$, $B = \{s, \beta, j\}$, $E = \{t, \gamma, k\}$ and $F = \{u, \delta, l\}$. The first index in the bracket refers to the Lorentz group , the second one to the colour group $SU(3)_c$ and the third one to the flavour group $SU(N_f)_f$.

Now, we make the colour Fierz rearrangement [11]

$$\sum_{a=1}^{8} \lambda^a_{\alpha\delta}\lambda^a_{\gamma\beta} = \frac{4}{3}\delta_{\alpha\beta}\delta_{\gamma\delta} + \frac{2}{3}\sum_{\rho=1}^{3} \epsilon_{\rho\alpha\gamma}\epsilon^{\rho\beta\delta} \tag{3}$$

with $\epsilon_{\alpha\beta\gamma}$ being the antisymmetric Levi- Civita tensor. This identity allows to decompose $K_{AB,EF}(x-y)$ completely into "attractive" colour singlet $(q\bar{q})$ and colour antitriplet (qq) channels. In this treatment "repulsive" colour octet $(q\bar{q})$ and sextet (qq) channels are absent in a natural way. In this paper we want to discuss $(q\bar{q})$ meson bound states. For this reason we will consider only the colour singlet part of (3) and therefore the kernel

$$K^{singlet}_{AB,EF}(x-y) = \gamma^\mu_{ru}(\gamma_\mu)_{ts}\frac{1}{N_c}\delta_{\alpha\beta}\delta_{\gamma\delta}\delta_{il}\delta_{kj}\frac{g^2}{2}D(x-y) \ , \tag{4}$$

where $N_c = 3$ denotes the colour number.

Inserting the kernel (4) into the bilocal interaction term (2) one obtains (omitting the group indices)

$$\frac{1}{N_c}\int\int d^4x d^4y q(y)\bar{q}(x)\gamma^\mu\frac{g^2}{2}D(x-y)q(x)\bar{q}(y)\gamma_\mu \ .$$

Then, action (1) can be represented in the form

$$\begin{aligned} W_M \ = \ & \int\int d^4x d^4y \Big\{ (q(y)\bar{q}(x))(-G_0^{-1})\delta(x-y) \\ & + \frac{1}{2N_c}\Big[(q(y)\bar{q}(x))\mathcal{K}(x-y)(q(x)\bar{q}(y))\Big]\Big\} \end{aligned}$$

with the Lorentz–vector kernel for QCD

$$\mathcal{K}(x-y) = g^2\gamma^\mu D(x-y)\otimes\gamma_\mu \ . \tag{5}$$

In symbolic notation it reads

$$W_M = (q\bar{q}, -G_0^{-1}) + \frac{1}{2N_c}(q\bar{q}, \mathcal{K}\, q\bar{q}) \ .$$

2.2. Schwinger–Dyson and Bethe–Salpeter equations

Let us now consider the functional integral

$$Z = \int \mathcal{D}q\mathcal{D}\bar{q}\exp\{iW_M[q,\bar{q}]\} \ .$$

After integrating over the quark fields with the help of the Legendre transform one gets

$$Z = \int \mathcal{D}\mathcal{M} \exp\Big\{ i W_{eff}[\mathcal{M}] \Big\} \ ,$$

with the effective action

$$W_{eff}[\mathcal{M}] = N_c \Big\{ -\frac{1}{2} (\mathcal{M}, \mathcal{K}^{-1}\mathcal{M}) - i \mathrm{Tr} \ln(-G_0^{-1} + \mathcal{M}) \Big\} \tag{6}$$

for meson fields $\mathcal{M}$. Here Tr means integration over the continuous variables and taking the traces over the discret ones (spinor and flavour indices). The condition of minimum for this effective action reads

$$\mathcal{K}^{-1}\mathcal{M} + i \frac{1}{-G_0^{-1} + \mathcal{M}} = 0 \ . \tag{7}$$

Let us denote the solution of this equation by $(\Sigma - \hat{m}^0)$. Then one obtains from (7) the Schwinger–Dyson equation

$$\Sigma = \hat{m}^0 + i \mathcal{K} G_\Sigma \ , \tag{8}$$

where

$$G_\Sigma^{-1} = i\partial\!\!\!/ - \Sigma \ .$$

Expanding action (6) around the minimum with $\mathcal{M} = (\Sigma - \hat{m}^0) + \mathcal{M}'$ one gets

$$\begin{aligned} W_{eff}[\mathcal{M}] &= W_{eff}[\Sigma] \\ &+ N_c \Big\{ -\frac{1}{2} (\mathcal{M}', \mathcal{K}^{-1}\mathcal{M}') - \frac{i}{2} \mathrm{Tr}(G_\Sigma \mathcal{M}')^2 - i \sum_{n=3}^{\infty} \frac{1}{n} \mathrm{Tr}(-G_\Sigma \mathcal{M}')^n \Big\} \ . \end{aligned} \tag{9}$$

The vanishing of the second variation of this effective action with respect to $\mathcal{M}'$,

$$\left. \frac{\delta^2 W_{eff}}{\delta \mathcal{M}' \delta \mathcal{M}'} \right|_{\mathcal{M}'=0} \cdot \Gamma = 0 \ ,$$

leads to the Bethe–Salpeter equation

$$\Gamma = -i \mathcal{K} (G_\Sigma \Gamma G_\Sigma) \tag{10}$$

for the vertex function $\Gamma(x, y)$ in the ladder approximation.

The Schwinger–Dyson equation (8) describes the quark spectrum in the meson, whereas the Bethe–Salpeter equation (10) yields the bound state spectrum. Solving together both equations one may obtain the wave functions of the bound states and calculate with their help not only static properties of mesons as the mass spectrum and the decay constants but also decay probabilities.

2.3. Relativistic covariant description of potential models

From the translation invariance of the two–particle bound state it follows that one can separate the c.m. motion of the $(\bar{q}q)$ system from the relative motion. But in a relativistic theory it is impossible to separate the c.m. coordinates. Nevertheless, one may separate the total momentum $\mathcal{P}_\mu$ being the momentum of the c.m. motion. After this one can more or less arbitrarily define some coordinate X_μ representing the absolute position in space–time and use the relative coordinate of the bound state only. For example, let X_μ be the position x or y of one of the particles (quarks) or a linear combination $X = \alpha x + (1-\alpha)y$ of them. For $\alpha = 1$ the relative coordinate is $z = x - y$. In this case one can look for the solution of the Bethe–Salpeter equation for the bound state wave function ψ in the form

$$\psi(x,y) = e^{i\mathcal{P}X}\psi(z) \ .$$

Let us now substitute the QCD kernel $\mathcal{K}$ of the integral equations (8) and (10) by a potential one. This can be done in a relativistic covariant way by choosing instead of (5)

$$\mathcal{K}^\eta(x,y) = \mathcal{K}^\eta(x-y|\frac{x+y}{2}) = \not{\eta}V(z^\perp)\delta(z\cdot\eta)\otimes\not{\eta} \tag{11}$$

with

$$z_\mu^\perp = z_\mu - z_\mu^{||} \ , \ \ z_\mu^{||} = \eta_\mu(z\cdot\eta) \ .$$

Here η_μ is a vector ($\not{\eta} = \eta_\mu\gamma_\mu; \eta^2 = 1$), being proportional to the momentum eigenvector $\mathcal{P}_\mu$ and describing the motion of the bound state as a whole

$$\eta_\mu = \frac{\mathcal{P}_\mu}{\sqrt{\mathcal{P}^2}} \ , \ \ \mathcal{P}_\mu\psi = -i\frac{\partial}{\partial X_\mu}\psi \ .$$

In (11) the transversality of the exchange interaction in the $(q\bar{q})$ system is ensured by $V(z^\perp)$ – some phenomenological potential for the description of quarkonia. Furthermore, the $\delta-$function $\delta(z\cdot\eta)$ guarantees the instantaneousness of the exchange interaction.

We should add that one can arrive at equation (11) by discussing moving bound states rigorously within the quantization theory for gauge theories [12]. For the bound state at rest one has $\vec{\mathcal{P}} = 0$, so that $\eta_\mu = (1,0,0,0)$ and the kernel takes the form

$$\mathcal{K}^\eta = \gamma_0 V(z^\perp)\delta(z_0)\otimes\gamma_0 \ . \tag{12}$$

Kernel (12) is known from the calculation of the (e^+e^-) positronium spectrum in electrodynamics with V as Coulomb potential, where an exchange of transversal photons takes place.

Phenomenologically we may choose the interaction potential in the form

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + V_0\delta(r) + ar + br^2 \ . \tag{13}$$

Here the first term is a Coulomb–type potential for one–gluon exchange, whereas the last two terms in (13) guarantee quark confinement. The second term leads to the Nambu–Jona–Lasinio model which has been considered within this approach in [6].

The potential (13) may be attributed to the sum of the Coulomb and oscillator potentials. In the applications we will consider only two potentials $V(r)$, the first one being the sum of Coulomb, constant and linear potentials and the second one – the sum of Coulomb, constant and oscillator potentials. In Table 1. all potentials under consideration are displayed in x space as well as in momentum space. (There the definition $\Delta_{\mathbf{p}} = \partial^2/\partial\mathbf{p}^2$ has been used.)

3. Equations for the quark mass spectrum within a meson

3.1. The case of an unspecified potential

The Schwinger–Dyson equation (8) takes for the potential kernel (11) in momentum space the form

$$\Sigma(\mathbf{p}) = \hat{m}^0 + i\int \frac{d^4q}{(2\pi)^4} V(\mathbf{p}-\mathbf{q}) \not{\eta} G_\Sigma(q) \not{\eta} \ , \tag{14}$$

where

$$\begin{aligned} V(\mathbf{p}-\mathbf{q}) &= \int d^4x e^{-i(p-q)x} V(x^\perp)\delta(x\cdot\eta) \ , \\ G_\Sigma(q) &= \int d^4x e^{-iqx} G_\Sigma(x) \ . \end{aligned}$$

Assuming flavour diagonality of $\Sigma(\mathbf{p})$, i.e. considering $\Sigma = \mathrm{diag}(\Sigma_1, \Sigma_2, ..., \Sigma_{N_f})$, equation (14) falls into identical equations for all Σ_n with bare mass term m_n^0 , $n = 1, 2, ..., N_f$. Omitting the flavour index n, these Schwinger–Dyson equations for the fields Σ of a given flavour read in the rest frame, $\eta_\mu = (1, 0, 0, 0)$:

$$\Sigma(\mathbf{p}) = m^0 + i\int \frac{d^4q}{(2\pi)^4} V(\mathbf{p}-\mathbf{q}) \gamma_0 G_\Sigma(q) \gamma_0 \ . \tag{15}$$

Now we make the ansatz

$$\Sigma(\mathbf{p}) = A(\mathbf{p})|\mathbf{p}| + B(\mathbf{p}) p_i \gamma_i \ , \qquad i = 1, 2, 3 \, . \tag{16}$$

Then, the propagator $G_\Sigma(q)$ for the free quark can be represented as follows

$$\begin{aligned} G_\Sigma(q) = \frac{1}{\not{q} - \Sigma(\mathbf{q}) + i\epsilon} &= \Big(\frac{\Lambda_+(\mathbf{q})}{q_0 - E(\mathbf{q}) + i\epsilon} + \frac{\Lambda_-(\mathbf{q})}{q_0 + E(\mathbf{q}) - i\epsilon} \Big) \gamma_0 \\ &= \gamma_0 \Big(\frac{\bar{\Lambda}_+(\mathbf{q})}{q_0 - E(\mathbf{q}) + i\epsilon} + \frac{\bar{\Lambda}_-(\mathbf{q})}{q_0 + E(\mathbf{q}) - i\epsilon} \Big) \ . \end{aligned} \tag{17}$$

Here the notations

$$\begin{aligned}\Lambda_{\pm}(\mathbf{q}) &= S^{-1}(\mathbf{q})\Lambda^0_{\pm}S(\mathbf{q}) = \frac{1}{2}(1 \pm S^{-2}(\mathbf{q})\gamma_0) = \frac{1}{2}(1 \pm \gamma_0 S^2(\mathbf{q})) \ , \\ \bar{\Lambda}_{\pm}(\mathbf{q}) &= S(\mathbf{q})\Lambda^0_{\pm}S^{-1}(\mathbf{q}) = \frac{1}{2}(1 \pm S^{2}(\mathbf{q})\gamma_0) = \frac{1}{2}(1 \pm \gamma_0 S^{-2}(\mathbf{q}))\end{aligned} \tag{18}$$

and

$$E(\mathbf{q}) = |\mathbf{q}|[A^2(\mathbf{q}) + (1 + B(\mathbf{q}))^2]^{1/2} \tag{19}$$

have been introduced, where

$$\left.\begin{aligned} S^{\pm 2}(\mathbf{q}) &= \sin\phi(\mathbf{q}) \pm \hat{\mathbf{q}}\cos\phi(\mathbf{q}) = \exp\{\pm 2\hat{\mathbf{q}}\nu(\mathbf{q})\} \ , \\ \sin\phi(\mathbf{q}) &= \frac{A(\mathbf{q})|\mathbf{q}|}{E(\mathbf{q})} \ , \qquad \cos\phi(\mathbf{q}) = \frac{(1 + B(\mathbf{q}))|\mathbf{q}|}{E(\mathbf{q})} \ , \\ S^{\pm 1}(\mathbf{q}) &= \cos\nu(\mathbf{q}) \pm \hat{\mathbf{q}}\sin\nu(\mathbf{q}) \ , \qquad \nu(\mathbf{q}) = \tfrac{1}{2}(-\phi(\mathbf{q}) + \tfrac{\pi}{2}) \ , \\ \hat{\mathbf{q}} &= \hat{q}_i\gamma_i \ , \quad \hat{q}_i = \frac{q_i}{|\mathbf{q}|} \ , \quad \hat{\mathbf{q}}^2 = -1 \ , \end{aligned}\right\} \tag{20}$$

and

$$\Lambda^0_{\pm} = \frac{1}{2}(1 \pm \gamma_0) \ . \tag{21}$$

With these definitions one can write the solution $\Sigma(\mathbf{p})$ of the Schwinger–Dyson equation (15) in the form

$$\Sigma(\mathbf{p}) = E(\mathbf{p})\sin\phi(\mathbf{p}) + \hat{\mathbf{p}}(E(\mathbf{p})\cos\phi(\mathbf{p}) - |\mathbf{p}|) = (\mathbf{p}\gamma) + E(\mathbf{p})S^{-2}(\mathbf{p}) \ . \tag{22}$$

By inserting (17) and (22) into (15) we can rewrite the Schwinger–Dyson equation as

$$\begin{aligned} E(\mathbf{p})\sin\phi(\mathbf{p}) &- \hat{\mathbf{p}}(E(\mathbf{p})\cos\phi(\mathbf{p}) - |\mathbf{p}|) \\ &= m^0 - \frac{1}{2}\int \frac{d\mathbf{q}}{(2\pi)^3} V(\mathbf{p} - \mathbf{q})\gamma_0(1 - \gamma_0(\sin\phi(\mathbf{q}) + \hat{\mathbf{q}}\cos\phi(\mathbf{q}))) \ . \end{aligned}$$

Taking the trace on both sides of this equation, one obtains a system of two equations,

$$\left\{\begin{aligned} E(\mathbf{p})\sin\phi(\mathbf{p}) &= m^0 + \frac{1}{2}\int \frac{d\mathbf{q}}{(2\pi)^3} V(\mathbf{p} - \mathbf{q})\sin\phi(\mathbf{q}) \\ E(\mathbf{p})\cos\phi(\mathbf{p}) &= |\mathbf{p}| + \frac{1}{2}\int \frac{d\mathbf{q}}{(2\pi)^3} V(\mathbf{p} - \mathbf{q})\chi(\mathbf{p},\mathbf{q})\cos\phi(\mathbf{q}) \ , \end{aligned}\right. \tag{23}$$

which defines the mass spectrum of two quarks forming a bound state. Here the notation

$$\chi(\mathbf{p},\mathbf{q}) = (\hat{\mathbf{q}} \cdot \hat{\mathbf{p}}) = \cos\theta(\mathbf{p},\mathbf{q}) \tag{24}$$

has been introduced. For solving the system of equations (23) it is necessary to fix the interaction potential.

3.2. Application to concrete potentials

Let us now investigate the system of equations (23) obtained from the Schwinger–Dyson equation (15) for different potentials. First of all we rewrite the potential (13) in momentum space:

$$V(\mathbf{p}-\mathbf{q}) = -(\frac{4}{3}\alpha_s)\frac{4\pi}{(\mathbf{p}-\mathbf{q})^2} - a\frac{8\pi}{(\mathbf{p}-\mathbf{q})^4} + V_0 - b(2\pi)^3\Delta_{\mathbf{q}}\delta(\mathbf{p}-\mathbf{q}) \ . \tag{25}$$

The last term in this potential – the oscillator term – should be considered separately. Inserting it into the system (23) one gets

$$\begin{cases} E(\mathbf{p})\sin\phi(\mathbf{p}) = m_0 - \dfrac{b}{2}\displaystyle\int d\mathbf{q}(\Delta_{\mathbf{q}}\delta(\mathbf{p}-\mathbf{q}))\sin\phi(\mathbf{q}) \\ \\ E(\mathbf{p})\cos\phi(\mathbf{p}) = |\mathbf{p}| - \dfrac{b}{2}\displaystyle\int d\mathbf{q}(\Delta_{\mathbf{q}}\delta(\mathbf{p}-\mathbf{q}))\chi(\mathbf{p},\mathbf{q})\cos\phi(\mathbf{q}) \ . \end{cases} \tag{26}$$

Now we make use of the relations

$$\begin{aligned} \Delta_{\mathbf{q}}(\sin\phi(\mathbf{q}))|_{\mathbf{p}=\mathbf{q}} &= \frac{2}{p}\cos\phi\cdot\phi' - \sin\phi\cdot(\phi')^2 + \cos\phi\cdot(\phi'') \ , \\ \Delta_{\mathbf{q}}(\cos\theta(\mathbf{p},\mathbf{q})\cdot\sin\phi(\mathbf{q}))|_{\mathbf{p}=\mathbf{q}} &= \cos\theta\cdot(-\frac{2}{p^2}\cos\phi - \frac{2}{p}\sin\phi\cdot(\phi') \\ &\quad -\cos\phi\cdot(\phi')^2 - \sin\phi(\phi'')) \ , \end{aligned}$$

where the notation $p = |\mathbf{p}|$ has been used. Then, the system (26) corresponding to the Schwinger–Dyson equation for the oscillator potential leads to the differential equation

$$-\frac{b}{2}(p^2\phi')' = p^3\sin\phi - m_0p^2\cos\phi + \frac{b}{2}\sin(2\phi)$$

on the function $\phi(p)$.[1] Knowing it's solution one may in principle calculate the constituent quark mass m in dependence on two parameters – the current mass m^0 and the potential parameter b. Indeed, let us consider relations (16), (19), (20) for the case $A(\mathbf{p}) = m/|\mathbf{p}|, B(\mathbf{p}) = 0$, so that one has

$$\begin{aligned} \Sigma(\mathbf{p}) &\equiv m \ , \qquad E(\mathbf{p}) = \sqrt{\mathbf{p}^2+m^2} \ , \\ \cos\phi(\mathbf{p}) &= \frac{|\mathbf{p}|}{E(\mathbf{p})} \ , \qquad \sin\phi(\mathbf{p}) = \frac{m}{E(\mathbf{p})} \ . \end{aligned} \tag{27}$$

Here the last relation is the equation on the constituent mass m.

[1] For $m_0 = 0$ this equation has already been given in [7].

Now we turn to the linear and Coulomb potentials in (25). In this case one has to investigate the renormalized Schwinger–Dyson equation because the second equation of system (23) contains an ultraviolet divergence. We introduce into the latter a renormalization constant Z,

$$E(\mathbf{p})\cos\phi(\mathbf{p}) = Z|\mathbf{p}| + \frac{1}{2}\int \frac{d\mathbf{q}}{(2\pi)^3} V(\mathbf{p}-\mathbf{q})\,\chi(\mathbf{p},\mathbf{q})\cos\phi(\mathbf{q})\ .$$

and choose it as

$$Z = 1 - \frac{1}{2|\mathbf{p}|}\int \frac{d\mathbf{q}}{(2\pi)^3} V(\mathbf{p}-\mathbf{q})\,\chi(\mathbf{p},\mathbf{q})\ .$$

Then the system (23) can be represented in the following manner

$$\left\{\begin{array}{l} E(\mathbf{p})\sin\phi(\mathbf{p}) = m_0 + \dfrac{1}{2}\displaystyle\int \frac{d\mathbf{q}}{(2\pi)^3} V(\mathbf{p}-\mathbf{q})\sin\phi(\mathbf{q}) \\ E(\mathbf{p})\cos\phi(\mathbf{p}) = |\mathbf{p}| + \dfrac{1}{2}\displaystyle\int \frac{d\mathbf{q}}{(2\pi)^3} V(\mathbf{p}-\mathbf{q})\chi(\mathbf{p},\mathbf{q})(\cos\phi(q)-1)\ . \end{array}\right. \tag{28}$$

From here the energy $E(\mathbf{p})$ may also be expressed via the function $\phi(\mathbf{p})$:

$$\begin{aligned} E(\mathbf{p}) &= m_0\sin\phi(\mathbf{p}) + |\mathbf{p}|\cos\phi(\mathbf{p}) \\ &+ \frac{1}{2}\int \frac{d\mathbf{q}}{(2\pi)^3} V(\mathbf{p}-\mathbf{q})\{\sin\phi(\mathbf{p})\sin\phi(\mathbf{q}) - \chi(\mathbf{p},\mathbf{q})\cos\phi(\mathbf{p})(\cos\phi(\mathbf{q})-1)\}\ . \end{aligned} \tag{29}$$

The infrared singularity appearing in (28) and (29) for the Coulomb potential may be removed if one changes the physical observable – the excitation energy $\Delta E(\mathbf{p}) = E(\mathbf{p}) - E(\mathbf{0})$.

Furthermore, from (28) one obtains the renormalized integral equation

$$\begin{aligned} |\mathbf{p}|\sin\phi(\mathbf{p}) = m_0 \ &+ \ \frac{1}{2}\int \frac{d\mathbf{q}}{(2\pi)^3} V(\mathbf{p}-\mathbf{q})\{\cos\phi(\mathbf{p})\sin\phi(\mathbf{q}) \\ &-\chi(\mathbf{p},\mathbf{q})\sin\phi(\mathbf{p})(\cos\phi(\mathbf{q})-1)\}\ . \end{aligned} \tag{30}$$

Now, we integrate in (30) over the angular variables. Then, for the Coulomb potential one gets

$$\begin{aligned} |\mathbf{p}|\sin\phi(\mathbf{p}) &= m_0\cos\phi(\mathbf{p}) + \frac{1}{2\pi}\frac{4\alpha_s}{3}\int d|\mathbf{q}|\Big\{\Big(\frac{|\mathbf{q}|}{|\mathbf{p}|}\ln\Big|\frac{|\mathbf{q}|-|\mathbf{p}|}{|\mathbf{q}|+|\mathbf{p}|}\Big|\cos\phi(\mathbf{p})\sin\phi(\mathbf{q}) \\ &-\Big[\frac{|\mathbf{q}|}{|\mathbf{p}|} + \frac{1}{2}\frac{\mathbf{q}^2+\mathbf{p}^2}{\mathbf{p}^2}\ln\Big|\frac{|\mathbf{q}|-|\mathbf{p}|}{|\mathbf{q}|+|\mathbf{p}|}\Big|\Big]\sin\phi(\mathbf{p})(\cos\phi(\mathbf{q})-1)\Big\}\ . \end{aligned}$$

In the case of the linear potential equation (30) becomes

$$\begin{aligned} |\mathbf{p}|\sin\phi(\mathbf{p}) &= m_0\cos\phi(\mathbf{p}) - \int d|\mathbf{q}|\Big\{\Big(\frac{\mathbf{q}^2}{(\mathbf{q}^2-\mathbf{p}^2)^2}\cos\phi(\mathbf{p})\sin\phi(\mathbf{q}) \\ &+\Big[\frac{|\mathbf{q}|}{|\mathbf{p}|}\frac{\mathbf{q}^2+\mathbf{p}^2}{(\mathbf{q}^2-\mathbf{p}^2)^2} + \frac{1}{2\mathbf{p}^2}\ln\Big|\frac{|\mathbf{q}|-|\mathbf{p}|}{|\mathbf{q}|+|\mathbf{p}|}\Big|\Big]\sin\phi(\mathbf{p})\cos\phi(\mathbf{q})\Big\}\ . \end{aligned}$$

The singularity in this equation can be regularized by introducing a small cut-off λ around the singular point $\mathbf{p} = \mathbf{q}$ and taking the limit $\lambda \to 0$ afterwards.

4. Equations on bound state masses

4.1. Equations for bound state vertex functions

The Bethe–Salpeter equation (10) for the vertex functions Γ reads in the case of the potential kernel (11) in momentum space

$$\Gamma(\mathbf{p}|\mathcal{P}) = -i\int \frac{d^4q}{(2\pi)^4} V(\mathbf{p}-\mathbf{q}) \not{\eta} G_1\Big(q+\frac{\mathcal{P}}{2}\Big)\Gamma(\mathbf{q}|\mathcal{P})G_2\Big(q-\frac{\mathcal{P}}{2}\Big)\not{\eta}\ , \tag{31}$$

where $G_n \equiv G_{\Sigma_n}$ is given by (17). Here the index $n = 1, 2$ is used to distinguish between the two quarks forming the bound state. Therefore the quantities $\Sigma, A, B, \Lambda, S, E, \phi, \nu$ defined by (15)–(20) will now carry this index. The quantity $\mathcal{P}$ denotes as before the total momentum of the bound state.

For what follows we will investigate equation (31) in the rest frame, $\eta_\mu = (1, 0, 0, 0)$:

$$\Gamma(\mathbf{p}) = -i\int \frac{d^4q}{(2\pi)^4} V(\mathbf{p}-\mathbf{q})\gamma_0 G_1\Big(q+\frac{M}{2}\Big)\Gamma(\mathbf{q})G_2\Big(q-\frac{M}{2}\Big)\gamma_0\ . \tag{32}$$

Here M means the bound state mass. Then, according to (17) the Green's functions G_n may be expressed as

$$\begin{aligned} G_n(q \pm \frac{M}{2}) &= \left(\frac{\Lambda_+^{(n)}(\mathbf{q})}{q_0 \pm M/2 - E_n(\mathbf{q}) + i\epsilon} + \frac{\Lambda_-^{(n)}(\mathbf{q})}{q_0 \pm M/2 + E_n(\mathbf{q}) - i\epsilon}\right)\gamma_0 \\ &= \gamma_0\left(\frac{\bar\Lambda_+^{(n)}(\mathbf{q})}{q_0 \pm M/2 - E_n(\mathbf{q}) + i\epsilon} + \frac{\bar\Lambda_-^{(n)}(\mathbf{q})}{q_0 \pm M/2 + E_n(\mathbf{q}) - i\epsilon}\right)\ , \end{aligned} \tag{33}$$

whereby $E_n(\mathbf{q}), n = 1, 2$ are the solutions of the system of equations (23). Integrating the Bethe–Salpeter equation (32) over q_0 we obtain the Salpeter equation

$$\Gamma(\mathbf{p}) = \int \frac{d\mathbf{q}}{(2\pi)^3} V(\mathbf{p}-\mathbf{q})\gamma_0\Big(\frac{\Pi_{+-}(\mathbf{q})}{E(\mathbf{q})-M} + \frac{\Pi_{-+}(\mathbf{q})}{E(\mathbf{q})+M}\Big)\gamma_0 \tag{34}$$

with $E(\mathbf{q}) = E_1(\mathbf{q}) + E_2(\mathbf{q})$ and

$$\Pi_{\pm\mp}(\mathbf{q}) = \Lambda_\pm^{(1)}(\mathbf{q})\gamma_0\Gamma\gamma_0\bar\Lambda_\mp^{(2)}(\mathbf{q})\ . \tag{35}$$

Let us now decompose $\Gamma(\mathbf{p})$ with respect to it's Dirac structure,

$$\Gamma = \Gamma_1 + \gamma_0\Gamma_2,$$

where

$$\Gamma_l = \gamma^S\cdot\Gamma_l^S + \gamma^P\cdot\Gamma_l^P + \gamma_i^V\cdot\Gamma_l^{Vi} + \gamma_i^A\cdot\Gamma_l^{Ai}\ ,\quad l = 1,2,\ \ i = 1,2,3\ .$$

Here we have defined $\gamma^S = 1$, $\gamma^P = \gamma_5$, $\gamma_i^V = \gamma_i$, $\gamma_i^A = \gamma_i\gamma_5$ for scalar, pseudoscalar, vector and axial–vector bound states Γ^S, Γ^P, Γ^V, Γ^A, respectively. It is favourable to write Γ in the form

$$\Gamma = \sum_{I=S,P,V,A} (\Gamma_1^I + \gamma_0\Gamma_2^I)\gamma^I \ , \tag{36}$$

so that the expression $(\gamma_0\Gamma\gamma_0)$ in (34) may be rewritten as

$$(\gamma_0\Gamma\gamma_0) \ = \ \sum_{I=S,P,V,A} \alpha_I(\Gamma_1^I + \gamma_0\Gamma_2^I)\gamma^I \tag{37}$$

with

$$\{\alpha_I, \ I = S,P,V,A\} = \{1,-1,-1,1\} \ .$$

Then, inserting (37) into (35) one can derive from (34) the Bethe–Salpeter equations for the vertex functions Γ_l^I. Here we shall give only the result of the calculation for the simple case, in which the relations (27) hold for both quarks. Then the Green's functions (17) are given by $G_n(q) = (\not{q} - m_n + i\epsilon)^{-1}$, $n = 1,2$ with m_n as constituent quark mass. One obtains

$$\begin{aligned}
\Gamma_1^I(\mathbf{p}) \ &= \ \int \frac{d\mathbf{q}}{(2\pi)^3} V(\mathbf{p}-\mathbf{q}) \frac{1}{E^2 - M^2} \Big\{ \Big[(m_1 - \alpha_I m_2)\Big(\frac{m_1}{E_1} - \frac{m_2}{E_2}\Big) \\
&\quad + (1 - \alpha_I\beta_I)\mathbf{q}^2\Big(\frac{1}{E_1} + \frac{1}{E_2}\Big)\Big]\Gamma_1^I(\mathbf{q}) + M\Big(\frac{m_1}{E_1} - \alpha_I\frac{m_2}{E_2}\Big)\Gamma_2^I(\mathbf{q})\Big\} \ , \\
\Gamma_2^I(\mathbf{p}) \ &= \ \int \frac{d\mathbf{q}}{(2\pi)^3} V(\mathbf{p}-\mathbf{q}) \frac{1}{E^2 - M^2} \Big\{ M\Big(\frac{m_1}{E_1} - \alpha_I\frac{m_2}{E_2}\Big)\Gamma_1^I(\mathbf{q}) \\
&\quad + \Big[(m_1 - \alpha_I m_2)\Big(\frac{m_1}{E_1} - \frac{m_2}{E_2}\Big) + (1 + \alpha_I\beta_I)\mathbf{q}^2\Big(\frac{1}{E_1} + \frac{1}{E_2}\Big)\Big]\Gamma_2^I(\mathbf{q})\Big\}
\end{aligned}$$

with $E_n \equiv E_n(\mathbf{q}) = \sqrt{\mathbf{q}^2 + m_n^2}$, $n = 1,2$, $E = E_1 + E_2$ and

$$\{\beta_I, \ I = S,P,V,A\} = \{-1, 1, \frac{1}{3}, -\frac{1}{3}\} \ .$$

4.2. Equations for bound state wave functions

Sometimes it is favourable to work not with the vertex function Γ but with the Bethe–Salpeter wave function Ψ. For our potential theory (9), (11) both quantities are connected with each other in the rest frame by the relation

$$\Gamma(\mathbf{p}) = \int \frac{d\mathbf{q}}{(2\pi)^3} V(\mathbf{p}-\mathbf{q})\gamma_0\Psi(\mathbf{q})\gamma_0 \ , \tag{38}$$

such that according to (34) Ψ is defined as

$$\Psi(\mathbf{q}) = \frac{\Pi_{+-}(\mathbf{q})}{E(\mathbf{q}) - M} + \frac{\Pi_{-+}(\mathbf{q})}{E(\mathbf{q}) + M} \tag{39}$$

with $\Pi_{\pm\mp}(\mathbf{q})$ given by (35). Furthermore, it is more suitable to introduce instaed of (39) a new wave function

$$\overset{0}{\Psi}(\mathbf{q}) = S_1(\mathbf{q})\Psi(\mathbf{q})S_2(\mathbf{q}) \ . \tag{40}$$

Then one may express the quantities $\Pi_{\pm\mp}(\mathbf{q})$ appearing in the definition (39) of the wave function $\Psi(\mathbf{q})$ as follows:

$$\Pi_{\pm\mp}(\mathbf{q}) = S_1^{-1}(\mathbf{q})\ \overset{0}{\Pi}_{\pm\mp}(\mathbf{q})S_2^{-1}(\mathbf{q}) \tag{41}$$

with

$$\overset{0}{\Pi}_{\pm\mp}(\mathbf{q}) = \overset{0}{\Lambda}_{\pm}\overset{0}{\Gamma}(\mathbf{q})\ \overset{0}{\Lambda}_{\mp} \ , \tag{42}$$

where $\overset{0}{\Lambda}_{\pm}$ is defined by (21) and

$$\overset{0}{\Gamma}(\mathbf{q}) = S_1^{-1}(\mathbf{q})\Gamma(\mathbf{q})S_2^{-1}(\mathbf{q}) \ . \tag{43}$$

With the help of these relations one obtains from (38) a Salpeter equation for the new quantities. It reads

$$\overset{0}{\Gamma}(\mathbf{p}) = -\int \frac{d\mathbf{q}}{(2\pi)^3} V(\mathbf{p}-\mathbf{q})S_1'(\mathbf{p},\mathbf{q})\ \overset{0}{\Psi}(\mathbf{q})S_2'(\mathbf{q},\mathbf{p}) \ , \tag{44}$$

where

$$S_n'(\mathbf{p},\mathbf{q}) = S_n^{-1}(\mathbf{p})\gamma_0 S_n^{-1}(\mathbf{q}) \ , \quad n = 1,2 \ .$$

Notice, that according to (40) and (41) the wave function $\overset{0}{\Psi}(\mathbf{q})$ has a representation in analogy to relation (39) for $\Psi(\mathbf{q})$:

$$\overset{0}{\Psi}(\mathbf{q}) = \frac{\overset{0}{\Pi}_{+-}(\mathbf{q})}{E(\mathbf{q})-M} + \frac{\overset{0}{\Pi}_{-+}(\mathbf{q})}{E(\mathbf{q})+M} \ .$$

From here one gets the two relations

$$\overset{0}{\Lambda}_{\pm}\overset{0}{\Psi}(\mathbf{q})\ \overset{0}{\Lambda}_{\mp} = \frac{1}{E(\mathbf{q})\mp M}\ \overset{0}{\Pi}_{\pm\mp}(\mathbf{q}) \ . \tag{45}$$

Let us rewrite them in a form similar to Schrödinger equations

$$[E(\mathbf{q})\mp M]\ \overset{0}{\Lambda}_{\pm}\overset{0}{\Psi}(\mathbf{q})\ \overset{0}{\Lambda}_{\mp} = \overset{0}{\Pi}_{\pm\mp}(\mathbf{q}) \ . \tag{46}$$

Next, we decompose the wave function $\overset{0}{\Psi}(\mathbf{q})$ of two–particle bound states,

$$\overset{0}{\Psi} = \overset{0}{\Psi}_1 + \gamma_0 \cdot \overset{0}{\Psi}_2 \ ,$$

and expand $\overset{0}{\Psi}_l$, $l=1,2$ in analogy to (36) over the full system $\{\gamma^J,\, J=1,2,3\} = \{\gamma_5, \hat{e}^a(p), \hat{p}\}$, so that

$$\overset{0}{\Psi} = \sum_{J=1}^{3} [\overset{0}{\Psi}{}_1^J + \gamma_0 \cdot \overset{0}{\Psi}{}_2^J]\gamma^J \ . \tag{47}$$

After inserting decomposition (47) into the Schrödinger–like equations (46) one may derive the following systems of equations on $\overset{0}{\Psi}{}_1^J$ and $\overset{0}{\Psi}{}_2^J$ for $J=1,2,3$:

$$\begin{aligned} M\ \overset{0}{\Psi}{}_2^J\ (\mathbf{p}) \cdot \frac{1}{4}\mathrm{tr}(\gamma^K\gamma^J) &= E(\mathbf{p})\ \overset{0}{\Psi}{}_1^J\ (\mathbf{p}) \cdot \frac{1}{4}\mathrm{tr}(\gamma^K\gamma^J) \\ &- \int \frac{d\mathbf{q}}{(2\pi)^3} V(\mathbf{p}-\mathbf{q}) T_{12}^{KL}(\mathbf{p},\mathbf{q})\ \overset{0}{\Psi}{}_1^L\ (\mathbf{q}) \ , \\ M\ \overset{0}{\Psi}{}_1^J\ (\mathbf{p}) \cdot \frac{1}{4}\mathrm{tr}(\gamma^K\gamma^J) &= E(\mathbf{p})\ \overset{0}{\Psi}{}_2^J\ (\mathbf{p}) \cdot \frac{1}{4}\mathrm{tr}(\gamma^K\gamma^J) \\ &- \int \frac{d\mathbf{q}}{(2\pi)^3} V(\mathbf{p}-\mathbf{q}) T_{12}^{KL}(\mathbf{p},\mathbf{q})\ \overset{0}{\Psi}{}_2^L\ (\mathbf{q}) \ , \end{aligned} \tag{48}$$

where

$$T_{12}^{KL}(\mathbf{p},\mathbf{q}) = \frac{1}{4}\mathrm{tr}(\gamma^K S_1'(\mathbf{p},\mathbf{q})\gamma^L S_2'(\mathbf{q},\mathbf{p})) \ .$$

The calculation of the traces yields

$$T_{12}^{KL}(\mathbf{p},\mathbf{q}) = c_{\mathbf{p}}^{\rho_K} c_{\mathbf{q}}^{\rho_L} \mathrm{tr}(\gamma^L\gamma^K) - s_{\mathbf{p}}^{\rho_K} s_{\mathbf{q}}^{\rho_L} \mathrm{tr}(\gamma^L\gamma^K\hat{\mathbf{p}}\hat{\mathbf{q}})$$

with

$$\{\rho_K; K=1,2,3\} = \{-1,-1,1\}$$

and

$$\begin{aligned} c_{\mathbf{q}}^{\pm\rho_K} &= c_2(\mathbf{q})c_1(\mathbf{q}) \mp \rho_K s_2(\mathbf{q})s_1(\mathbf{q}) \ , \\ s_{\mathbf{q}}^{\pm\rho_K} &= s_2(\mathbf{q})c_1(\mathbf{q}) \mp \rho_K s_2(\mathbf{q})c_1(\mathbf{q}) \ , \\ s_n(\mathbf{q}) &= \sin\phi_n(\mathbf{q}) \ , \quad c_n(\mathbf{q}) = \cos\phi_n(\mathbf{q}) \ , \quad n=1,2 \ . \end{aligned}$$

The systems (48) of integral equations for the bound state wave functions $\overset{0}{\Psi}{}_1^J$ and $\overset{0}{\Psi}{}_2^J$ need to be specified for every particle type. To do this let us introduce the notations

$$\overset{0}{\Psi}{}_l^1 = \overset{0}{L}_l \ , \quad \overset{0}{\Psi}{}_l^2 = \overset{0}{N}{}_l^a \ , \quad \overset{0}{\Psi}{}_l^3 = \overset{0}{\Sigma}_l \ , \quad l=1,2 \ , \tag{49}$$

for pseudoscalar, vector and scalar particles, respectively. Then, for pseudoscalar particles (48) reads

$$\begin{aligned} M\ \overset{0}{L}_2\ (\mathbf{p}) &= E(\mathbf{p})\ \overset{0}{L}_1\ (\mathbf{p}) - \int \frac{d\mathbf{q}}{(2\pi)^3} V(\mathbf{p}-\mathbf{q})(c_{\mathbf{p}}^- c_{\mathbf{q}}^- - \chi s_{\mathbf{p}}^- s_{\mathbf{q}}^-)\ \overset{0}{L}_1\ (\mathbf{q}) \ , \\ M\ \overset{0}{L}_1\ (\mathbf{p}) &= E(\mathbf{p})\ \overset{0}{L}_2\ (\mathbf{p}) - \int \frac{d\mathbf{q}}{(2\pi)^3} V(\mathbf{p}-\mathbf{q})(c_{\mathbf{p}}^+ c_{\mathbf{q}}^+ - \chi s_{\mathbf{p}}^+ s_{\mathbf{q}}^+)\ \overset{0}{L}_2\ (\mathbf{q}) \ . \end{aligned} \tag{50}$$

Here we have introduced as short–hand notation $s^{\pm} \equiv s^{\pm 1}$, $c^{\pm} \equiv c^{\pm 1}$. The quantity $\chi \equiv \chi(\mathbf{p},\mathbf{q})$ is given by (24). For vector particles system (48) takes the form

$$\begin{aligned} M \overset{0}{N}{}_2^a(\mathbf{p}) &= E(\mathbf{p}) \overset{0}{N}{}_1^a(\mathbf{p}) + \int \frac{d\mathbf{q}}{(2\pi)^3} V(\mathbf{p}-\mathbf{q}) \\ &\quad \cdot \Big\{ (c_{\mathbf{p}}^- c_{\mathbf{q}}^- \underline{\delta}^{ab} - s_{\mathbf{p}}^- s_{\mathbf{q}}^- (\underline{\delta}^{ab}\chi - \eta^a \underline{\eta}^b)) \overset{0}{N}{}_1^b(\mathbf{q}) + \eta^a c_{\mathbf{p}}^- c_{\mathbf{q}}^+ \overset{0}{\Sigma}_1(\mathbf{q}) \Big\} , \\ M \overset{0}{N}{}_1^a(\mathbf{p}) &= E(\mathbf{p}) \overset{0}{N}{}_2^a(\mathbf{p}) + \int \frac{d\mathbf{q}}{(2\pi)^3} V(\mathbf{p}-\mathbf{q}) \\ &\quad \cdot \Big\{ (c_{\mathbf{p}}^+ c_{\mathbf{q}}^+ \underline{\delta}^{ab} - s_{\mathbf{p}}^+ s_{\mathbf{q}}^+ (\underline{\delta}^{ab}\chi - \eta^a \underline{\eta}^b)) \overset{0}{N}{}_2^b(\mathbf{q}) + \eta^a c_{\mathbf{p}}^+ c_{\mathbf{q}}^- \overset{0}{\Sigma}_2(\mathbf{q}) \Big\} , \end{aligned} \tag{51}$$

where we have used the definitions

$$\eta^a \equiv \eta^a(\mathbf{p},\mathbf{q}) = \hat{q}_i \hat{e}_i^a(\mathbf{p}) , \quad \underline{\eta}^a \equiv \underline{\eta}^a(\mathbf{p},\mathbf{q}) = \hat{p}_i \hat{e}_i^a(\mathbf{q}) , \quad \underline{\delta}^{ab} \equiv \underline{\delta}^{ab}(\mathbf{p},\mathbf{q}) = \hat{e}_i^a(\mathbf{q}) \hat{e}_i^a(\mathbf{p}) .$$

And for scalar particles one has

$$\begin{aligned} M \overset{0}{\Sigma}_2(\mathbf{p}) &= E(\mathbf{p}) \overset{0}{\Sigma}_1(\mathbf{p}) \\ &\quad + \int \frac{d\mathbf{q}}{(2\pi)^3} V(\mathbf{p}-\mathbf{q}) \Big\{ (\chi c_{\mathbf{p}}^+ c_{\mathbf{q}}^+ - s_{\mathbf{p}}^+ s_{\mathbf{q}}^+) \overset{0}{\Sigma}_1(\mathbf{q}) + \underline{\eta}^b c_{\mathbf{p}}^- c_{\mathbf{q}}^+ \overset{0}{N}{}_1^b(\mathbf{q}) \Big\} , \\ M \overset{0}{\Sigma}_1(\mathbf{p}) &= E(\mathbf{p}) \overset{0}{\Sigma}_2(\mathbf{p}) \\ &\quad + \int \frac{d\mathbf{q}}{(2\pi)^3} V(\mathbf{p}-\mathbf{q}) \Big\{ (\chi c_{\mathbf{p}}^- c_{\mathbf{q}}^- - s_{\mathbf{p}}^- s_{\mathbf{q}}^-) \overset{0}{\Sigma}_2(\mathbf{q}) + \underline{\eta}^b c_{\mathbf{p}}^+ c_{\mathbf{q}}^- \overset{0}{N}{}_2^b(\mathbf{q}) \Big\} . \end{aligned} \tag{52}$$

The relations (50)–(52) for bound state wave functions have a very compact form. In principle, they may be solved in dependence on the concrete form of the underlying potential. Thereby, in general an exact solution would be possible only by numerical calculations. But one can investigate also approximate solutions for different limiting procedures as, for example, nonrelativistic limits and the heavy quark mass limit. Let us add, that in [7] similar equations have been obtained for the oscillator potential, and the light meson mass spectrum has been calculated numerically.

5. Meson decay constants

In the remaining sections of this paper we want to apply the bilocal meson model (9) with the relativistic covariant written kernel (11) to the investigation of heavy meson properties. Therefore we have to include into our QCD–motivated model the weak interaction. We will restrict ourselves here to the discussion of semileptonic weak processes. This allows us first of all to determine meson decay constants. Let us consider the quadratic part

$$W_{eff}^{(2)} = -i\frac{N_c}{2}\mathrm{Tr}(G_\Sigma \mathcal{M})^2 \tag{53}$$

of the effective action (9). First of all we expand the bilocal field $\mathcal{M}$ over creation (a_H^+) and annihilation (a_H) operators

$$\begin{aligned}\mathcal{M}(x,y) &= \mathcal{M}\Big(x-y|\frac{x+y}{2}\Big) \\ &= \sum_H \int \frac{d\overrightarrow{\mathcal{P}}}{(2\pi)^{3/2}\sqrt{2\omega_H}} \int \frac{d^4q}{(2\pi)^4} e^{iq(x-y)} \Big\{ e^{i\mathcal{P}\frac{x+y}{2}} a_H^+(\mathbf{q}|\mathcal{P})\Gamma_H(\mathbf{q}|\mathcal{P}) \\ &\qquad + e^{-i\mathcal{P}\frac{x+y}{2}} a_H(\mathbf{q}|\mathcal{P})\bar{\Gamma}_H(\mathbf{q}|\mathcal{P})\Big\} \ .\end{aligned} \tag{54}$$

Here the sum runs over the set of quantum numbers H of hadrons contributing in the bilocal fields $\mathcal{M}(x,y)$. The bound state has the total momentum $\mathcal{P} = \{\omega_H, \overrightarrow{\mathcal{P}}\}$, the energy $\omega_H(\overrightarrow{\mathcal{P}}) = \sqrt{\overrightarrow{\mathcal{P}}^2 + M_H^2}$ and the mass M_H. The bound state vertex functions Γ_H and $\bar{\Gamma}_H$ satisfy the Bethe–Salpeter equation (10) with kernel (11).

Furthermore, we have to include the weak interactions into the effective action (53). The effective Lagrangian of semileptonic weak interaction has the form

$$\mathcal{L}_{semi} = \frac{G_F}{\sqrt{2}}\{V_{ij}(\bar{Q}_i O_\mu q_j) l_\mu + h.c.\} \tag{55}$$

with the leptonic current

$$l_\mu \equiv \bar{l} O_\mu \nu_l \ , \quad l = e, \mu, \tau \ , \qquad O_\mu = \gamma_\mu(1+\gamma_5) \ ,$$

the elements V_{ij} of the Kobayashi–Maskawa matrix and the Fermi constant $G_F = 10^{-5}/m_p^2$. Q denotes the column of (u,c,t) quarks and q – the column of (d,s,b) quarks. The lagrangian (55) can be incorporated into the bilocal action (53) by substituting

$$\mathcal{M}(x,y) \to \mathcal{M}(x,y) + \hat{L}(x,y) \ , \tag{56}$$

i.e., by adding to the bilocal field $\mathcal{M}(x,y)$ the local weak leptonic current

$$\hat{L}_{ij}(x,y) = \frac{G_F}{\sqrt{2}}\delta^4(x-y) V_{ij} \hat{l} e^{i\mathcal{P}_L \frac{x+y}{2}} \ , \tag{57}$$

where $\hat{l} \equiv O_\mu l_\mu$ and $\mathcal{P}_L$ being the momentum of the leptonic pair. Then, the terms of interest standing in (53) after the substitution (56) and corresponding to semileptonic

weak interaction are

$$\begin{aligned} W^{(2)}_{semi} &= -iN_c \mathrm{Tr}(G\mathcal{M}G\hat{L}) \\ &\equiv -iN_c \int dxdydzdt\, \mathrm{tr}\Big[G_{i'}(t-x)\mathcal{M}_{i'k}(x,y)G_{j'}(y-z)\hat{L}_{j'k}(z,t)\Big] \ . \end{aligned}$$

Here the trace runs over Dirac and flavour indices.

Now we are able to derive a formula for pseudoscalar meson decay constants. The matrix element for a decay of a meson $H_{ij} \sim (q_i \bar{q}_j)$ into a leptonic pair reads

$$\begin{aligned} < l\nu|W^{(2)}_{semi}|H_{ij}> &= -iN_c(2\pi)^4 i\delta^{(4)}(\mathcal{P}_H - \mathcal{P}_L)\frac{G_F}{\sqrt{2}} < l\nu|l_\mu|0> \\ &\cdot \int \frac{d^4q}{(2\pi)^4}\mathrm{tr}_\gamma\Big\{O_\mu G_i\Big(q-\frac{\mathcal{P}_H}{2}\Big)\bar{\Gamma}(\mathbf{q}|\mathcal{P}_H)G_j\Big(q+\frac{\mathcal{P}_H}{2}\Big)\Big\} \ . \quad (58) \end{aligned}$$

Using the relations (33) for Green's functions and the definition of a bound state wave function (from (32) and (38))

$$i\int \frac{dq_0}{2\pi}G_i(q-\frac{\mathcal{P}}{2})\bar{\Gamma}(\mathbf{q}|\mathcal{P})G_j(q+\frac{\mathcal{P}}{2}) \equiv \bar{\Psi}_{H_{ij}}(\mathbf{q}|\mathcal{P}) \ ,$$

we can write

$$\begin{aligned} &\int \frac{d^4q}{(2\pi)^4}\mathrm{tr}_\gamma\Big\{O_\mu G_i\Big(q-\frac{\mathcal{P}_H}{2}\Big)\bar{\Gamma}(\mathbf{q}|\mathcal{P}_H)G_j\Big(q+\frac{\mathcal{P}_H}{2}\Big)\Big\} \\ &= -i\int \frac{d\mathbf{q}}{(2\pi)^3}\mathrm{tr}_\gamma[O_\mu \bar{\Psi}_{H_{ij}}(\mathbf{q}|\mathcal{P}_H)] \ , \quad (59) \end{aligned}$$

whereby $\bar{\Psi}(\mathbf{q}|\mathcal{P}_H) \equiv \Psi(\mathbf{q}|-\mathcal{P}_H)$. In analogy to (40) we introduce now the "dressed" wave function $\overset{0}{\Psi}$ and expand it in correspondence with (47) over the Lorentz matrices. Because we are interested in pseudoscalar mesons only, we take $J=1$ and have then in the moving frame (using notation (49))

$$\overset{0}{\Psi}\ (\mathbf{q}|-\mathcal{P}_H) = \Big(\overset{0}{L}_1\ (\mathbf{q}|-\mathcal{P}_H) - \not{\eta}\cdot\ \overset{0}{L}_2\ (\mathbf{q}|-\mathcal{P}_H)\Big)\gamma^5 \ . \quad (60)$$

Remember, that $\eta^\mu = \mathcal{P}^\mu_H/\sqrt{\mathcal{P}^2_H}$ with $\mathcal{P}^0_H = M_H$. Inserting (60) into (59) one gets after the calculation of the trace

$$i4\eta_\mu \int \frac{d\mathbf{q}}{(2\pi)^3}\Big(\cos\nu_i(\mathbf{q})\cdot\cos\nu_j(\mathbf{q}) - \sin\nu_i(\mathbf{q})\cdot\sin\nu_j(\mathbf{q})\Big)(\overset{0}{L}_2)_{H_{ij}}(\mathbf{q}|\mathcal{P}_H) \ ,$$

where $\nu(\mathbf{q})$ is defined in (20). Then, this expression is inserted into (57). Comparing the result with the general formula

$$< l\nu|W^{(2)}_{semi}|H_{ij}> = (2\pi)^4 i\delta^4(\mathcal{P}_H - \mathcal{P}_L)\frac{G_F}{\sqrt{2}} iF_{H_{ij}}\mathcal{P}^\mu_H < \omega|l_\mu|0>$$

one obtains for the decay constant of a pseudoscalar meson H_{ij} at rest

$$F_{H_{ij}} = \frac{4N_c}{M_H}\int \frac{d\mathbf{q}}{(2\pi)^3}(\overset{0}{L}_2)_{H_{ij}}(\mathbf{q})\cos(\nu_i(\mathbf{q})+\nu_j(\mathbf{q})) \ . \quad (61)$$

Notice, that in the rest frame the functions $\overset{0}{L}_1$ and $\overset{0}{L}_2$ satisfy the system of equations (50). So we have for B and D mesons the relations

$$\begin{aligned}
F_D &= \frac{4N_c}{M_D}\int\frac{d\mathbf{q}}{(2\pi)^3}(\overset{0}{L}_2)_D\cos(\nu_c+\nu_u)\ ;\\
F_{D_s} &= \frac{4N_c}{M_{D_s}}\int\frac{d\mathbf{q}}{(2\pi)^3}(\overset{0}{L}_2)_{D_s}\cos(\nu_c+\nu_d)\ ;\\
F_{B_u} &= \frac{4N_c}{M_{B_u}}\int\frac{d\mathbf{q}}{(2\pi)^3}(\overset{0}{L}_2)_{B_u}\cos(\nu_b+\nu_u)\ ;\\
F_{B_c} &= \frac{4N_c}{M_{B_c}}\int\frac{d\mathbf{q}}{(2\pi)^3}(\overset{0}{L}_2)_{B_c}\cos(\nu_b+\nu_c)\ .
\end{aligned}$$

6. The relation between bilocal field approach and heavy quark effective theory

For the description of the properties of heavy quarkonia one can employ the heavy quark effective theory, which has been developed recently [8 – 10]. Let us here consider mesons $(Q\bar{q})$ consisting of a heavy quark Q and a light antiquark $\bar{q}$. Now we make use of the fact that the quarkonium velocity v^μ is determined more or less by the velocity v_Q^μ of its heavy quark constituent Q. Then, the momentum of a bound state of velocity v^μ is given by

$$\mathcal{P}^\mu = Mv^\mu\ ,$$

where M is the bound state mass. The latter is approximately equal to the heavy quark mass m_Q, i.e.

$$M \approx m_Q\ .$$

Then one can write for the heavy quark momentum

$$p_Q^\mu = m_Q v_Q^\mu = \mathcal{P}^\mu - p_q^\mu = m_Q v^\mu + k^\mu \tag{62}$$

with

$$k^\mu = (M - m_Q)v^\mu - p_q^\mu\ , \tag{63}$$

where p_q is the light antiquark momentum. From (61) it follows that the heavy quark velocity is given by

$$v_Q^\mu = v^\mu + \frac{1}{m_Q}k^\mu\ . \tag{64}$$

Therefore, in the limit $m_Q \to \infty$ one has $v_Q^\mu \to v^\mu$. In (62) and (63) the light antiquark momentum p_q^μ is small as compared with $\mathcal{P}^\mu$, and it has only little influence on the direction of the heavy quark motion. As result one obtains that a meson bound state

containing a heavy quark can be considered by a theory describing heavy quark motion, for which the QCD corrections are small. Therefore the bound state is formed by a colour Coulomb field.

Let us now assume that the bilocal field $\mathcal{M}(x,y)$ contains a heavy quark (b or c) and a light antiquark ($\bar{u}$, $\bar{d}$ or $\bar{s}$). Now, the integral kernel $\mathcal{K}^\eta(x-y)$, eq. (11), is defined with the help of the vector η_μ, which has been introduced for two reasons. Firstly, it defines the transversality of the interaction given effectively by the phenomenological potential $V(z^\perp)$. And secondly, the vector η_μ determines the motion of the interaction potential together with the motion of the $(Q\bar{q})$ bound state. In the limit of the heavy quark effective theory one has

$$\eta_\mu = \frac{\mathcal{P}_\mu}{\sqrt{\mathcal{P}^2}} \to v_\mu \ ,$$

where $v^2 = 1$. Therefore the interaction kernel (11) takes in this limit the form

$$\mathcal{K}^v(x-y) = \not{v} V(z^\perp)\delta^{(4)}(v\cdot z) \otimes \not{v} \ ,$$

where $z_\mu^\parallel = v_\mu(v\cdot z)$. We conclude that for considering the heavy quark effective theory one has to investigate the case of moving bound states in our bilocal field theory for $m_Q \to \infty$. Concering the problems that have been discussed in this paper so far it was sufficient to consider the bound states at rest. Let us note the main modifications that appear if one works in an arbitrary reference frame. Instead of the 3–momentum $\mathbf{q}$ one has now $q^\perp$, and q^0 is substituted by $q^\parallel$. The product $q_i\gamma_i$ changes to $-\not{q}^\perp = -q_\mu^\perp\gamma^\mu$. Instead of $|\mathbf{q}|$ one has $q^\perp \equiv \sqrt{(q^\perp)^2}$, so that $\hat{q}^\perp = \hat{q}_\mu^\perp\gamma^\mu, \hat{q}_\mu^\perp = q_\mu^\perp/q^\perp$. Then, for instance, eqs. (16) and (22) for Σ read

$$\Sigma(q^\perp) = A(q^\perp)q^\perp + B(q^\perp)\not{q} = \not{q}^\perp + E(q^\perp)S^{-2}(q^\perp) \ .$$

For illustration let us now consider the Schwinger–Dyson equation in the heavy mass limit. Thereby, we will restrict ourselves to the investigation of the case (27), in which $\Sigma(p_Q^\perp) \equiv m_Q$. Then, for $S^{\pm 2}(q^\perp)$ one obtains according to (20)

$$S^{\pm 2}(q^\perp) = \frac{m_Q}{E(q^\perp)} \pm \hat{q}^\perp \frac{|q^\perp|}{E(q^\perp)} \ .$$

The Schwinger–Dyson equation (15) for a heavy quark reads

$$m_Q = m^0 - i\mathrm{tr}\int \frac{d^4q}{(2\pi)^4} V(q^\perp)\not{v} G_{m_Q}(p_Q - q)\not{v} \ , \tag{65}$$

where the trace runs over colour and Dirac indices. In the heavy mass limit we take inside the loop integral of (65) for the quark propagator the expression

$$G_{m_Q}(p_Q - q) = \frac{m_Q(1+\not{v}) - \not{q}}{2m_Q v(k-q) + q^2 + i\epsilon} \ . \tag{66}$$

Here we have used equation (62) and from (64) the relation $k^\mu/m_Q \ll v_Q^\mu \approx v^\mu$. Inserting (66) into (65) one gets

$$\begin{aligned} m_Q &= m^0 - i4N_c \int \frac{d^4q}{(2\pi)^4} V(q^\perp) \frac{m_Q}{2m_Q v(k-q) + q^2 + i\epsilon} \\ &= m^0 + 2N_c \int \frac{d^3q^\perp}{(2\pi)^3} V(q^\perp) \frac{m_Q}{\sqrt{m_Q^2 v^2 - 2m_Q vk + (q^\perp)^2}} \ . \end{aligned}$$

7. Semileptonic decays of heavy quarkonia in the limit of heavy quark effective theory

Let us now consider semileptonic decays of heavy mesons. Therefore we have to investigate the cubic part of the effective action (9), thereby substituting one of the bilocal fields $\mathcal{M}$ by the local leptonic current $\hat{L}$ from (57):

$$\begin{aligned} W_{semi}^{(3)} &= i\frac{N_c}{3} \mathrm{Tr}[(G_\Sigma M)^2 (G_\Sigma \hat{L})] \\ &\equiv \frac{iN_c}{3} \int \frac{d^4x_1}{(2\pi)^4} \cdots \int \frac{d^4x_6}{(2\pi)^4} \, \mathrm{tr}[G_\Sigma(x_1 - x_2) \mathcal{M}(x_2, x_3) G_\Sigma(x_3 - x_4) \\ &\qquad \hat{L}(x_4, x_5) G_\Sigma(x_5 - x_6) \mathcal{M}(x_6, x_1)] \ . \end{aligned} \tag{67}$$

Here the arguments in the integrand have been introduced according to fig.1. After rewriting (67) in momentum space and using the decomposition (54) for $\mathcal{M}(x,y)$ one obtains for the matrix element describing a semileptonic decay of a B–meson H_{ib} with a meson H'_{ji} in the final state

$$\begin{aligned} < (l\nu_l) H'_{ji} | W_{semi}^{(3)} | H_{ib} > &= -i\frac{N_c}{3} (2\pi)^4 \delta(\mathcal{P} - \mathcal{P}' - \mathcal{P}_L) \frac{1}{2(2\pi)^3 \sqrt{\omega\omega'}} \\ &\quad \cdot < l\nu | l_\mu | 0 > \cdot \frac{G}{\sqrt{2}} V_{jb} \, I^\mu_{bji}(\mathcal{P}, \mathcal{P}') \end{aligned} \tag{68}$$

with

$$\begin{aligned} I^\mu_{bji}(\mathcal{P}, \mathcal{P}') &= \mathrm{tr} \int \frac{d^4k}{(2\pi)^4} G_b(k - \mathcal{P}) O^\mu G_j(k - \mathcal{P}') \\ &\quad \cdot \Gamma_{H'}(\mathbf{k} - \frac{\mathcal{P}'}{2} | \mathcal{P}) G_i(k) \bar{\Gamma}_H(\mathbf{k} - \frac{\mathcal{P}}{2} | \mathcal{P}) \ , \end{aligned} \tag{69}$$

$O^\mu = \gamma^\mu(1 + \gamma_5)$ and the latain indices b, $j = c, u$ and $i = u, d, s, c$ indicating the quark content of the quantities. The momenta in the integral I^μ_{bji} have been introduced according to fig.2. In (69) G_b, G_j and G_i are the Green's functions of the quarks. For example, for $j = c$ and $i = u, d, s, c$ the matrix element (68) describes the decays $B_u^- \to D^0(l\nu_l)$, $B_d^0 \to D^+(l\nu_l)$, $B_s^0 \to D_s^+(l\nu_l)$, and $B_c^- \to (c\bar{c})(l\nu_l)$, respectively.

For definiteness let us investigate the matrix element (68) for a semileptonic B–decay into a D–meson ($j = c$). The calculation will be done as follows. Knowing from

sect.6 the relation between bilocal field approach and heavy quark effective theory we will work at first in the rest frame. Only at the end we shall turn to the moving frame by substituting γ_0 by $\not{\eta}$ what corresponds for $\not{\eta} = \not{v}$ to the heavy quark mass limit.

Proceeding in this way let us start with rewriting the integral (69). At first we insert the expressions (33) for the Green's functions into the latter:

$$\begin{aligned} I^{\mu}_{bci}(M,M') &= \mathrm{tr}\int \frac{d^4k}{(2\pi)^4}\left(\frac{\Lambda^{(b)}_+(\mathbf{k})}{k_0-k^{(b)}_1+i\epsilon}+\frac{\Lambda^{(b)}_-(\mathbf{k})}{k_0-k^{(b)}_2-i\epsilon}\right)\gamma_0 O^{\mu} \\ &\quad\cdot\left(\frac{\Lambda^{(c)}_+(\mathbf{k})}{k_0-k^{(c)}_1+i\epsilon}+\frac{\Lambda^{(c)}_-(\mathbf{k})}{k_0-k^{(c)}_2-i\epsilon}\right)\gamma_0\Gamma_H(\mathbf{k})\gamma_0 \\ &\quad\cdot\left(\frac{\bar{\Lambda}^{(i)}_+(\mathbf{k})}{k_0-k^{(i)}_1+i\epsilon}\frac{\bar{\Lambda}^{(i)}_-(\mathbf{k})}{k_0-k^{(i)}_2-i\epsilon}\right)\bar{\Gamma}_{H'}(\mathbf{k}) \ , \end{aligned} \tag{70}$$

with

$$\begin{aligned} k^{(b)}_{1/2} &= M \pm E_b(\mathbf{k}) \ , \\ k^{(c)}_{1/2} &= M' \pm E_c(\mathbf{k}) \ , \\ k^{(i)}_{1/2} &= \pm E_i(\mathbf{k}) \ . \end{aligned}$$

Now we represent the integrand of I_{bci} as a sum of eight terms the numerators of which have the form

$$\begin{aligned} &\mathrm{tr}[\Lambda^{(b)}_{\pm}(\mathbf{k})\gamma_0 O^{\mu}\Lambda^{(c)}_{\pm}(\mathbf{k})\gamma_0\Gamma_H(\mathbf{k})\gamma_0\bar{\Lambda}^{(i)}_{\pm}(\mathbf{k})\bar{\Gamma}_{H'}(\mathbf{k})] \\ &= \mathrm{tr}[S_b^{-1}(\mathbf{k})O^{\mu}S_c^{-1}(\mathbf{k})(\pm\overset{0}{\Lambda}_{\pm})\,\overset{0}{\Gamma}_H\,(\mathbf{k})(\pm\overset{0}{\Lambda}_{\pm})\,\overset{0}{\bar{\Gamma}}_{H'}\,(\mathbf{k})(\pm\overset{0}{\Lambda}_{\pm})] \ . \end{aligned}$$

Here the last line is obtained by using (18) and (43), whereby $\overset{0}{\Gamma}$ fulfils the Bethe–Salpeter equation (44). Because of the relations

$$\overset{0}{\Lambda}_{\pm}\overset{0}{\Gamma}_H\overset{0}{\Lambda}_{\pm}= 0 \ , \quad \overset{0}{\Lambda}_{\pm}\overset{0}{\Gamma}_H\overset{0}{\Lambda}_{\mp}=\overset{0}{\Pi}{}^{H}_{\pm\mp}$$

(cf. (42)) only two numerators remain. Then (70) is given by

$$\begin{aligned} I^{\mu}_{bci}(M,M') &= \mathrm{tr}\int\frac{d^4k}{(2\pi)^4}S_b^{-1}(\mathbf{k})O^{\mu}S_c^{-1}(\mathbf{k}) \\ &\quad\cdot\left(\frac{\overset{0}{\Pi}{}^{H'}_{-+}\,(\mathbf{k})\,\overset{0}{\Gamma}_H\,(\mathbf{k})\,\overset{0}{\Lambda}_-}{(k_0-k^{(b)}_2-i\epsilon)(k_0-k^{(c)}_1+i\epsilon)(k_0-k^{(i)}_2-i\epsilon)}\right. \\ &\quad\left.-\frac{\overset{0}{\Pi}{}^{H'}_{+-}\,(\mathbf{k})\,\overset{0}{\bar{\Gamma}}_H\,(\mathbf{k})\,\overset{0}{\Lambda}_+}{(k_0-k^{(b)}_1+i\epsilon)(k_0-k^{(c)}_2-i\epsilon)(k_0-k^{(i)}_1+i\epsilon)}\right) \ . \end{aligned}$$

Now we can perform the k_0 integration. The result reads

$$I^{\mu}_{bci}(M,M') = i\mathrm{tr}\int\frac{d\mathbf{k}}{(2\pi)^3}S_b^{-1}(\mathbf{k})O^{\mu}S_c^{-1}(\mathbf{k})$$

$$\cdot\Bigg(\frac{\overset{0}{\Pi}{}^{H'}_{-+}(\mathbf{k})\,\overset{0}{\bar{\Gamma}}_H(\mathbf{k})\,\overset{0}{\Lambda}_-}{[E_c+E_b-(M-M')](E_c+E_i+M')}$$
$$-\frac{\overset{0}{\Pi}{}^{H'}_{+-}(\mathbf{k})\,\overset{0}{\bar{\Gamma}}_H(\mathbf{k})\,\overset{0}{\Lambda}_+}{[E_c+E_b+(M-M')](E_c+E_i-M')}\Bigg)\ . \qquad (71)$$

Here the expression in the brackets can be rewritten according to (45) as follows

$$\frac{\overset{0}{\Lambda}_-\overset{0}{\Psi}_{H'}(\mathbf{k})\,\overset{0}{\Lambda}_+\overset{0}{\bar{\Gamma}}_H(\mathbf{k})\,\overset{0}{\Lambda}_-}{E_c+E_i+M'}-\frac{\overset{0}{\Lambda}_+\overset{0}{\Psi}_{H'}(\mathbf{k})\,\overset{0}{\Lambda}_-\overset{0}{\bar{\Gamma}}_H(\mathbf{k})\,\overset{0}{\Lambda}_+}{E_c+E_i-M'}$$
$$=\frac{\overset{0}{\Lambda}_-\overset{0}{\Psi}_{H'}(\mathbf{k})\,\overset{0}{\Pi}{}^H_{+-}(\mathbf{k})}{E_c+E_i+M'}-\frac{\overset{0}{\Lambda}_+\overset{0}{\Psi}_{H'}(\mathbf{k})\,\overset{0}{\Pi}{}^H_{-+}(\mathbf{k})}{E_c+E_i-M'}\ .$$

Using (45) again and inserting the result in (71) one obtains

$$I^\mu_{bci}(M,M') = i\mathrm{tr}\int\frac{d\mathbf{k}}{(2\pi)^3}S_b^{-1}(\mathbf{k})O_\mu S_c^{-1}(\mathbf{k})\Big(\overset{0}{\Lambda}_-\overset{0}{\Psi}_{H'}(\mathbf{k})\,\overset{0}{\Lambda}_+\overset{0}{\bar{\Psi}}_H(\mathbf{k})\,\overset{0}{\Lambda}_-$$
$$-\overset{0}{\Lambda}_+\overset{0}{\Psi}_{H'}(\mathbf{k})\,\overset{0}{\Lambda}_-\overset{0}{\bar{\Psi}}_H(\mathbf{k})\,\overset{0}{\Lambda}_+\Big)\ . \qquad (72)$$

In the following we will further rewrite the integral (72) in the heavy quark limit. To do this we have first of all to turn to the moving frame. We introduce the momenta as shown in fig.2. In the moving frame we consider the decomposition $k_\mu = k^{\|}_\mu + k^\perp_\mu$ of the momentum $k_\mu = (k_0,\mathbf{k})$ with respect to the momentum $\mathcal{P}$ of the initial boson. Furthermore, γ_0 has to be replaced by $\not\eta = \not v$ for the incoming b quark and by $\not\eta' = \not v'$ for the outgoing c quark. Therefore in the moving frame the integral (72) takes the form

$$I^\mu_{bci}(v,v') = -\frac{i}{2}\mathrm{tr}\int\frac{d^3k^\perp}{(2\pi)^3}S_b^{-1}(k^\perp)O^\mu S_c^{-1}(k^\perp)$$
$$\cdot(1+\not v')\,\overset{0}{\Psi}_{H'}(k^\perp)\,\overset{0}{\bar{\Psi}}_H(k^\perp)(1+\not v)\ . \qquad (73)$$

In this relation we have also made use of the fact that for heavy quarks the change to the moving frame leads immediately to the heavy quark effective theory. So for heavy quark fields Q with momentum $\mathcal{P}^\mu = mv^\mu + k^\mu, k^\mu \ll M$ one has

$$(1-\not v)Q(\mathcal{P}) \approx \frac{\not k}{M}Q(\mathcal{P}) \approx 0\ .$$

Therefore the projector on the antipartilce vanishes: $\overset{0}{\Lambda}{}^{(Q)}_- \to 0$. This means that for heavy quarks the only relevant contribution comes from $\overset{0}{\Lambda}{}^{(Q)}_+ \to \Lambda^{(Q)}_+ = (1+\not v)/2$. Furthermore, we have used in (73) for the light quark q in the heavy quark limit $\overset{0}{\Lambda}{}^{(q)}_\pm \to 1/2$.

For definiteness let us now consider the semileptonic decay of a pseudoscalar B meson into a heavy pseudoscalar meson of the type $(c\bar{i})$, $i = u, d, s, c$. In this case we can use the decomposition (47) of the meson wave function which for moving fields takes the form

$$\begin{aligned} \overset{0}{\Psi}_{H'}(k^\perp|\mathcal{P}') &= (L_1^{H'} + \not{v}' L_2^{H'})(k^\perp|\mathcal{P}')\gamma_5 \ , \\ \overset{0}{\bar{\Psi}}_{H}(k^\perp|\mathcal{P}) &= \gamma_5(\bar{L}_1^{H} + \not{v}\bar{L}_2^{H})(k^\perp|\mathcal{P}) \ . \end{aligned}$$

Inserting these decompositions into (73) one gets

$$\begin{aligned} I_{bci}^{(PS)\mu}(v,v') &= -\frac{i}{2}\mathrm{tr}\int\frac{d^3k^\perp}{(2\pi)^3} S_b^{-1}(k^\perp) O^\mu S_c^{-1}(k^\perp) \\ &\qquad \cdot(1 + \not{v}' + \not{v} + \not{v}'\not{v}) W(k^\perp|v,v') \ , \end{aligned} \tag{74}$$

$$\begin{aligned} W(k^\perp|v,v') &= L_1^{H'}(k^\perp|\mathcal{P}')\bar{L}_1^{H}(k^\perp|\mathcal{P}) + L_1^{H'}(k^\perp|\mathcal{P}')\bar{L}_2^{H}(k^\perp|\mathcal{P}) \\ &\quad + L_2^{H'}(k^\perp|\mathcal{P}')\bar{L}_1^{H}(k^\perp|\mathcal{P}) + L_2^{H'}(k^\perp|\mathcal{P}')\bar{L}_2^{H}(k^\perp|\mathcal{P}) \ . \end{aligned} \tag{75}$$

According to (20) we have

$$S_i^{-1}(k^\perp) = c_i^\nu - \hat{k}^\perp s_i^\nu$$

with

$$c_i^\nu \equiv cos\nu_i(k^\perp) \ , \quad s_i^\nu \equiv sin\nu_i(k^\perp) \ .$$

After calculating the trace in (74) one obtains

$$I_{bci}^{(PS)\mu}(v,v') = i4\pi(v+v')^\mu \int\frac{d^3k^\perp}{(2\pi)^3}(c_b^\nu c_c^\nu - s_b^\nu s_c^\nu) W(k^\perp|v,v') \ . \tag{76}$$

Inserting expression (76) for $I_{bci}^{(PS)\mu}$ into (68) one obtains for the matrix element in the case of a semileptonic decay of a pseudoscalar B meson into a pseudoscalar meson of the type $(c\bar{i})$, $i = u, d, s, c$ within the heavy quark effective theory the result

$$\begin{aligned} <(l\nu)H'_{ji}|W_{semi}^{(3)}|H_{ib}> &= \frac{N_c}{3}(2\pi)^4\delta(\mathcal{P} - \mathcal{P}' - \mathcal{P}_L) <l\nu|l_\mu|0> \\ &\quad \sqrt{MM'}[\xi_+(v\cdot v')(v+v')^\mu + \xi_-(v\cdot v')(v-v')^\mu] \end{aligned} \tag{77}$$

with the Isgur–Wise functions [8] of the form

$$\begin{aligned} \xi_+(v\cdot v') &= \frac{1}{\sqrt{MM'}}\frac{1}{(2\pi)^2\sqrt{\omega\omega'}}\frac{G}{\sqrt{2}}V_{jb} \\ &\quad \cdot\int\frac{d^3k^\perp}{(2\pi)^3}(c_b^\nu c_c^\nu - s_b^\nu s_c^\nu) W(k^\perp|v,v') \ , \end{aligned} \tag{78}$$

$$\xi_-(v\cdot v') = 0 \ ,$$

where $W(k^\perp|v,v')$ is defined by (75).

8. Summary and conclusions

In this paper we have presented the bilocal field approach for relativistic covariant potential models in QCD. The main issue consists in the derivation of general integral expressions for meson properties as decay constants and semileptonic decay amplitudes. Therefore, it has been necessary to handle moving bound states. Doing this a special feature of our model has been important, namely, that the potential kernel (11) moves together with the bound state because of the presence of the vector η_μ. Furthermore, to obtain semileptonic decay amplitudes we have established the relation between the bilocal field approach for our model and heavy quark effective theory. In this way we have been able to obtain an integral expression for the Isgur–Wise function.

The formulas for the meson decay constants (61) and the Isgur–Wise function (78) appearing in the semileptonic decay amplitude (77) depend on the concrete form of the potential and contain trigonometric functions and meson wave functions. Thereby, the trigonometric functions fulfil together with the energy function the system of equations (23) which has been derived from the Schwinger–Dyson equation. For constituent quark masses one has the relations (27) and the system (23) simplifies significantly. The meson wave functions satisfy systems of equations (50)-(52) resulting from the Bethe–Salpeter equation. To obtain numerical results for the physical quantities one has to solve these systems of equations for concrete potentials. Work in this direction by using different approximations is in progress.

We hope that the hadronization scheme with the use of the Foldy–Wouthuysen transform can be applied to the hadronization of quarkonia of the type $(Q\bar{Q})$ as well. This would allow one to describe also nonleptonic decays of heavy mesons in the same fashion.

References

[1] For a recent review see: A. Ali, *B-Decays–Introduction*, DESY-Preprint 91-137 (to be published in *B Decays*, ed. S. Stone (World Scientific Publishers, Singapore).

[2] H. Kleinert, *Phys. Lett.* **B62** (1976) 429 and in: *Understanding the Fundamental Constituents of Matter* (Erice Lectures 1976), ed. A. Zichichi (Plenum, New York, 1978).

[3] D. Ebert and V.N. Pervushin, *Theor. Math. Phys.* **36** (1978) 759;
D. Ebert, H. Reinhardt and V.N. Pervushin, *Sov. J. Part. Nucl.* **10** (1979) 444.

[4] Yu.L. Kalinovsky et al., *Sov. J. Nucl. Phys.* **49** (1989) 1059;
V.N. Pervushin, *Nucl. Phys.* **B15** (Proc. Suppl.) (1990) 197.

[5] V.N. Pervushin et al., *Fortschr. Phys.* **38** (1990) 333;
Yu.L. Kalinovsky, L. Kaschluhn and V.N. Pervushin, *Fortschr. Phys.* **38** (1990) 353;
Yu.L. Kalinovsky et al., *Few–Body Systems* **10** (1991) 87.

[6] Yu.L. Kalinovsky, L. Kaschluhn and V.N. Pervushin, *Phys. Lett.* **231** (1989) 288.

[7] A. Le Yaouanc et al., *Phys. Rev.* **31** (1985) 137

[8] N. Isgur and M.B. Wise, *Phys. Lett.* **B232** (1989) 113; *ibid* **237** (1990) 527.

[9] E. Eichten and B. Hill, *Phys. Lett* **B234** (1990) 511;
H. Georgi, *Phys. Lett* **B240** (1990) 447;
B. Grinstein, *Nucl. Phys.* **B339** (1990) 253.

[10] For recent reviews see:
M.B. Wise, *New Symmetries of the Strong Interaction*, Lake Louise Winter Inst., Lake Louise, Canada, Feb. 1991, CALT-68-1721;
H. Georgi, *Heavy Quark Effective Field Theory*, HUPT-91-a039;
B. Grinstein, *Lectures on Heavy Quark Effective Theory*, Workshop on High Energy Phenomenology, Mexico City, Mexico, July 1991, HUPT-91-A040 and SSCL-Preprint 17.

[11] R.T. Cahill, J. Praschifka and C.J. Burden, *Aust. J. Phys.* **42** (1989) 161.

[12] V.N. Pervushin, *Riv. Nuovo Cim.* **8** (1985) 1;
Nguyen Suan Han and V.N. Pervushin, *Fortschr. Phys.* **37** (1986) 611 and *Mod. Phys. Lett.* **A2** (1987) 367.

Tables

Potential	V(r)	V(**p**)
Coulomb	$\frac{4}{3}\frac{\alpha_s}{r}$	$-(\frac{4}{3}\alpha_s)\frac{4\pi}{\mathbf{p}^2}$
Linear	ar	$-a\frac{8\pi}{\mathbf{p}^4}$
Oscillator	br^2	$-b(2\pi)^3\Delta_{\mathbf{p}}\delta(\mathbf{p})$
Yukawa	$\frac{\alpha}{r}e^{-\eta r}$	$-\alpha\frac{4\pi}{\mathbf{p}^2+\eta^2}$
NJL	$V_0\delta(r)$	V_0
Constant	V_0	$V_0(2\pi)^3\delta(\mathbf{p})$

Tab.1: Interaction potentials in x space and momentum space.

Figure captions

Fig.1: Diagram for the semileptonic decay of a heavy meson $\mathcal{M}(x_2, x_3)$ into a heavy meson $\mathcal{M}(x_6, x_1)$ and a leptonic current $\hat{L}(x_4, x_5)$ in bilocal field theory.

Fig.2: Diagram corresponding to the integral I^{μ}_{bji}, eq.(69), figurating in the semileptonic decay amplitude (68).

Figures

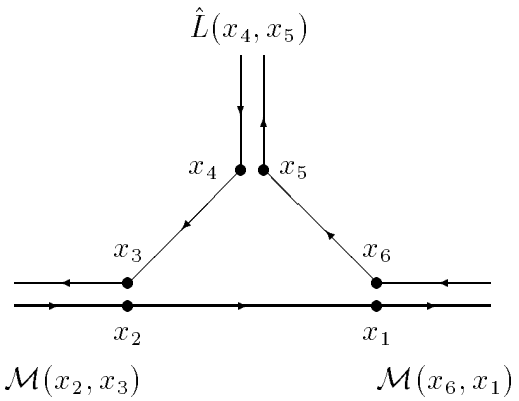

Figure 1.

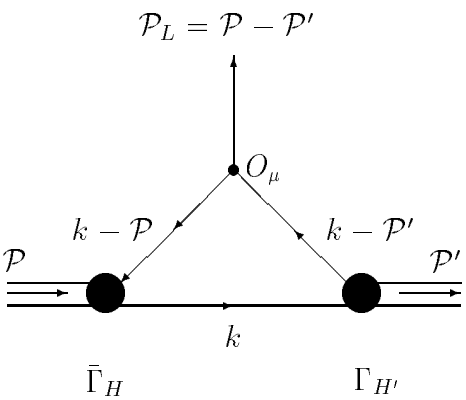

Figure 2.