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Abstract

We present an approach to the evolution of the valence structure functions based on the Colour Dipole Cascade Model for deep inelastic lepto-production. We show that this approach leads to an evolution equation similar to the DGLAP equation. In our approach the dependence on Q^2 is however much weaker and the evolution levels out at high Q^2.

1 Introduction

1.1 The Model for Ocean Partons

In an earlier note [1] we presented (some parts) of a scenario for the structure functions of a hadron, i.e. the number of partons available per unit energy momentum fraction $x_{Bj} \equiv x$, at a given resolution power Q^2. This paper contains an extension of the model to large values of x together with a more thorough comparison of our model to the conventional perturbative QCD approach.

We will use as the framework of all our considerations the Colour Dipole Model (CDM) [2] and the Soft Radiation Model (SRM) [3], which for convenience are briefly described below in sections 3.1 and 3.2.

The basic properties of these models are that a partonic cascade is described in terms of repeated dipole emissions between adjacent (colour connected) partons (CDM). Further, due to the spatial extension of the hadronic state, there is in accordance with a simple colour coherence argument a particular softening of the radiation (SRM) in connection with colour separation between the partons of such a state.

This softening occurs mostly in the target region and it is governed by an inverse length parameter μ_0, expected from present day experiments to be of the order of 1 GeV. We are thereby providing a "new" (non-perturbative) scale, which may be very noticeable in the coming HERA experiments if it does exist.

In our model we consider the interaction of a lepto-production probe and the hadron (which is assumed to have a large negative longitudinal momentum, i.e. a large $E - P_l = P_-$) as *a measuring process*. The field pulse $q = (Q_+, -Q_-, \vec{0})$ resolves all virtual (string) states $(P_+, P_-, \vec{0})$ with $P_+ \leq Q_+$ and interacts with a parton with $xP_- = Q_-$.

In the conventional approach (c.f. e.g. reference [4], where a particularly lucid physics description is given) it is assumed that the partonic quanta of a fast-moving hadron will (independently) have long (virtual) lifetimes. Each one can give rise to a cascade chain, which later may reassemble in accordance with the coherence properties of the field. In this way there are in general many chains available and the interaction probe picks out one parton with x, thereby breaking the coherence in that particular chain and realizing the corresponding radiation state.

In a more colourful language; in this picture the virtual partons exist "long before" the collision while in our picture they are suddenly realized by the measurement. There is, however, no real physics difference between these pictures because in a quantum state it is impossible to discuss notions like "before" or "suddenly" unless one prescribes a measurement.

In connection with a gauge field theory the notion of quanta is gauge dependent and even frame dependent. A general gauge-invariant matrix element is usually the sum of contributions from several different Feynman graphs. A particular emission from one part of the current in a given Lorentz frame and a given gauge may very well stem from a different part of the current in another frame or gauge. Any measurement of these properties in general destroys the coherence of the state.

The question of coherence and the corresponding interference effects has, however, a physics content. In our approach the radiation states are totally coherent. There is no part which independently of the others may emit and we are consequently defining the virtuality as *the virtuality of the total state* and not in terms of the individual partons. There is further no difference between "Initial State" and "Final State Bremsstrahlung" in our approach, although as we will show below we do reproduce the cross sections in the presently available energy regime.

For the ocean parton region (small x, but still above the scale of the vacuum fluctuations) there may in the approach of reference [4] sometimes occur such a large density of virtual quanta that they will start to interact. In that way the number of available chains will diminish. Consequently the number of available partons will be smaller.

In our approach there is a different mechanism which will lead to a saturation to an even larger degree. We have shown in reference [1] that there will be essentially fewer low x-partons in our approach as compared to the ordinary perturbative QCD approaches [5, 6, 7, 4]. We obtain e.g. for the ocean parton distributions f_o

$$f_o \propto \frac{g(Q^2)}{x} \quad \text{if } x < \left(\frac{\mu_0}{Q}\right)^4 \tag{1}$$

as compared to small x-behaviours like $x^{-(1+\epsilon)}$ with $\epsilon \sim 0.5$ in other approaches.

The reason is that the radiation emitted in the CDM is of a local character in angle and consequently in rapidity. For a given rapidity the amount of radiation is basically given by the largest possible transverse momentum that can occur for this rapidity [8]. For sufficiently small x-values we are considering rapidity regions which are so far away from "the front" of the hadron that the maximal transverse momentum of the radiation is no longer influenced by the (initial) hadron energy momentum P_-.

It is of some interest to note that the approach of reference [4] to take binary interactions between the chains into account is only expected to be valid up to a certain region in the $(\ln\frac{1}{x}, \ln Q^2)$-plane. Outside this region it is expected that also more complex interactions must be treated.

A finite energy description of this boundary region, expected to be valid over the HERA energy region [9], coincides very closely with the region where we expect xf_o to become constant in x. Whether the more complex interactions will work as we suggest i.e. as such a strong coherence condition that xf_o becomes a constant in x is a question which may be experimentally answered at HERA.

The evolution equations of a DGLAP-type [5] corresponds to a search for the possible gluon emissions, which can produce a final state parton with x at the given virtuality Q^2. We will term this approach Initial State Bremsstrahlung (ISB) models and the natural kinematical variables are in that case the pair (x, Q^2). In our approach it is rather the value of the total state variable $\nu = 2Pq$ that play the main role in the investigation.

Thus in a Monte Carlo simulation (where we use Ariadne [10], which contains the CDM and the SRM) states are produced with given values of ν. The structure functions of the ocean partons are then sampled by summing over those partons which have the right values of (x, Q^2) (c.f. reference [1]).

1.2 The Valence Constituents

Our model does not, however, provide the distributions of the valence constituents, i.e. the large x-distributions of the hadrons. They correspond to the large energy momentum fluctuations in the wave functions. It is possible inside a semi-classical scenario to obtain such distributions for an undisturbed string state and we show the results of such considerations in section 2 below. They should not be taken for more than tentative "toy models", however.

Whatever the "undisturbed" distributions are it is evident that the virtual emissions in our model demand that some energy should be transferred to the radiation. There is actually inside the SRM a particular such "transport" mechanism which is very similar to the Initial State Bremsstrahlung. We will in this paper provide a detailed study of the relationship.

The basic dynamics in the CDM (c.f. section 3.1) is the production and decay of the dipoles. The interaction in an inelastic lepto-production event produces an original dipole between the kicked-out parton (a colour-3 or -$\bar{3}$) and the (extended) remainder state (then in a $\bar{3}$ or 3 colour state).

The SRM (c.f. section 3.2) is based upon the idea that a large transverse momentum $(k_\perp)$ gluon has a small wavelength $\lambda \sim 1/k_\perp$. Therefore it will only resolve a fraction $\lambda/l \equiv \mu/k_\perp$ of the

extended remainder state, where l is the effective transverse size of this state and the mass μ its inverse.

In that way the effective emission stems from the dipole formed between the kicked-out parton and this fraction, which is treated as a gluonic excitation in the model. Consequently *there will in the emission, according to the SRM, occur both the "really" emitted gluon and the recoiling emitter fraction*, which we will term "the recoil gluon".

This implies that there is a complete dipole - "the recoil dipole" - produced in the process, between the really emitted gluon and the recoil gluon. This dipole will take energy and momentum both from the kicked-out parton and the remainder state. It is necessary for the emission to use up both a negative light-cone energy momentum fraction x_r of the remainder as well as to produce a (virtual) mass corresponding to the dipole mass.

Thus this collective phenomena implies that in order to kick out a parton and allow for the necessary ensuing radiation, energy-momentum is brought out from the remainder state. After the emission mentioned above this state only contains the fraction $x_u = 1 - x_r$.

In our picture there is, however, no way to trace this loss back to a particular (valence) constituent because the radiation is not provided for in terms of single parton emitters. Instead the energy momentum necessary for the radiation is given away in "chunks" like the (collective) recoil dipole emission mentioned above.

We will in this paper assume *that the energy momentum necessary for the radiation process is taken from the whole remainder state*. Thus we will assume that there is a collective loss which is proportional to the available energy in each part of the original hadron. If some parton existing on a certain "level of virtuality", Q_0^2, has the energy x it will at a larger virtuality $Q^2 > Q_0^2$, in case there has been a recoil gluon emitted be left with $x' = x_u x$.

Such an "equipartition of losses" may be too extreme but it is the simplest and most symmetrical approach we have found.

In this way there is also in our case evolution equations. As we will see later on, the harder the hadronic state is hit the more (collective) energy momentum will be dragged out. The rate of loss is, however, essentially smaller than the rate obtained in e.g. the DGLAP-equations. In our case there is a large change between low and medium Q^2 but it tends to level out for large values of Q^2.

1.3 The Further Structure of the Paper

Besides the toy models for the undisturbed string structure functions in section 2 and the survey of the CDM and the SRM in section 3 we consider in section 4 some properties of a (simple but realistic) Initial State Bremsstrahlung Model, and provide a detailed kinematical and dynamical comparison between the model predictions in the two cases.

We show in particular that *the cross sections are the same for the emission of the recoil dipole in the SRM and the initial state bremsstrahlung splitting in an ISB model.* We note, however, that by insisting upon the emission of the whole dipole we are forcing a particular phase space density (characteristic for the CDM as well as all cascade models with strong angular ordering) upon the ISB model.

In section 5 we exhibit the resulting evolution equation in our model. We show that it is has several similarities to the DGLAP-equation. It contains, however, a particular Sudakov form factor, which acts in a way so that there will be a lower phase space density of the radiation in our approach than in the DGLAP one.

Finally in section 6 we present our conclusions.

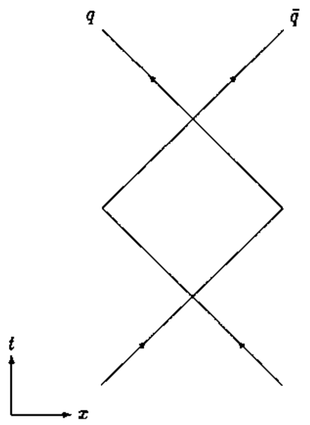


Figure 1: *The movement of the quarks in a meson according to the MRS [11].*

2 The Structure functions of an Undisturbed String

2.1 The Mesonic String

The states of a meson used in The Lund String Fragmentation Model are the so-called yoyo-mode of the MRS [11]. In case one would take them seriously as a description of *an interaction state of such a hadron* then it is possible to calculate the corresponding structure functions from semi-classical considerations.

In the yoyo-mode there is a (mass-less) q- and $\overline{q}$-particle joined by a flat string piece. They move to and through with the velocity of light (c.f. figure 1) under the influence of the tension of the string (denoted $\kappa_s \sim 1GeV/fm$).

It is easily seen that the period of motion is $2E/\kappa_s$ with E the total energy of the state. They "start out" e.g. at the central meeting point, each with the energy $E/2$ in the c.m.s. They move away in opposite directions until they have lost the energy (constant loss per time unit), then they turn around and are dragged together (constant gain per time unit), go out again in the opposite directions and return.

The properties of such a state is in the c.m.s. completely determined by an internal time (or correspondingly an energy momentum variable) together with the direction of the string and the particle motion. Given the values of these variables both the q, $\overline{q}$ and the string field energy momentum variables are determined. Due to the Lorentz covariance of the model the corresponding values are easily constructed in any Lorentz frame.

Now suppose that one would measure (instantaneously) the light-cone energy-momentum ($e + p\cos\theta) \equiv zE$ of anyone of these three degrees of freedom along an axis with the angle θ with respect to the direction of motion.

The variable z is evidently an invariant for all Lorentz boosts along the measuring axis. Therefore the calculation can be done in the c.m.s. of the yoyo state. Then the variables will have the ranges $0 \leq e = |p| \leq E/2$ for the q and $\overline{q}$ and $0 \leq e \leq E, p = 0$ for the string field. Finally the angle θ acts like the polar angle in a spherically symmetric system.

It is possible to construct the distribution in z if we prescribe some weighting procedure for the different internal times and solid angles. In case *all possible times and all possible directions are given the same weight* then we obtain for the q ($\overline{q}$) distributions

$$f_{q(\overline{q})}(z) = \int \frac{ded(\cos\theta)}{E}\delta(z - \frac{e(1-\cos\theta)}{E}) = \ln\frac{1}{z} \qquad (2)$$

and similarly for the total field energy momentum fraction distribution

$$f_{field}(z) = 1 \qquad (3)$$

From these we immediately find that

$$< z^j >_{q(\overline{q})} = \begin{cases} 1 & \text{if } j = 0 \\ 1/4 & \text{if } j = 1 \end{cases} \qquad (4)$$

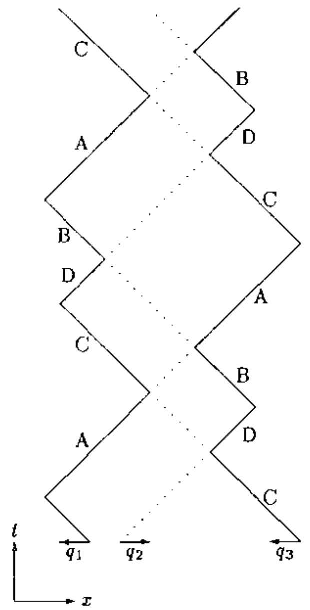


Figure 2: *The movement of the quarks in a one-dimensional baryon.*

This means that there is one endpoint q ($\overline{q}$) with a mean energy momentum fraction 1/4 each. Similarly we find that the mean field energy fraction is 1/2.

2.2 The Baryonic String

The model up to now only accounts for meson states. It is possible to construct a baryonic model of a similar kind. To that end we consider again a flat string piece but this time containing three q-particles. There is one q at each endpoint and one in between along the string. We assume that the state is a colour singlet all in all, which means that each neighboring pair makes up a colour-$\overline{3}$ compensating for the third colour-3.

One notes that while there are attractive forces working on each one of the endpoint q's (of the same size as for the corresponding meson-yoyo state, i.e. κ_s) there is no force acting upon the middle q. Consequently the middle one will move along the string without any changes in its energy momentum. When it passes an endpoint it will, however, take over its role and become the "new" endpoint acted upon by κ_s.

This model of a one-dimensional baryon is unconventional but has a similar relationship to the baryon-anti-baryon production mechanism ("the popcorn model" [12]) as the yoyo states bear to the mesons in the Lund Fragmentation Model. We will adopt it as a toy model for calculating baryon structure functions.

A closer analysis reveals that there are four (internal) degrees of freedom in the model. In figure 2 we show a space-time (or equivalently after some reinterpretation an energy-momentum space) description of the state (together with the corresponding simpler "ordinary" yoyo mode of the string). One may chose e.g. the four "side-lengths", denoted A, B, C and D in the figure as independent variables.

It is evident that the total energy E of the state is (in suitable units)

$$E = A + B + C + D \qquad (5)$$

and that in the c.m.s.

$$A + D = B + C \qquad (6)$$

Therefore there are in reality only two degrees of freedom. In case we again make an instantaneous measurement of the same variables as before and again give all independent degrees of freedom the same weighting then for anyone of the q's we obtain the distribution

$$f_{q,B}(z) = 2(\ln\left(\frac{1}{z}\right) - (1 - z)) \qquad (7)$$

(with the index B for baryon) There are in this case two field pieces, one to the right and one to the left of the central q. For each one we obtain the distribution

$$f_{field,B} = 3(1 - z)^2 \qquad (8)$$

The combined field distribution becomes $6z(1 - z)$.

Both the distributions $f_{q,B}$ and $f_{field,B}$ are softer than the corresponding yoyo meson distributions,i.e.

$$f_{q,B} \sim (1-z)^2 \tag{9}$$

just as the field distribution when $z \to 1$. The mean energy carried by each of the endpoint "constituents" is

$$< z >_{q,B} = 1/6 \tag{10}$$

and the field carries the remaining 1/2 just as for the meson case.

3 The Colour Dipole and Soft Radiation Models

3.1 The Dipole Approximation

When there is colour separation between a colour charge and an anti-charge then there will be gluonic (colour8) radiation in accordance with the well-known dipole radiation formula

$$d\sigma \propto \alpha_S \frac{x_1^u + x_3^w}{(1-x_1)(1-x_3)} dx_1 dx_3 \tag{11}$$

with the $x_j, j = 1,3$ the (final state) center of mass energy fractions of the emitters and $x_2 = 2 - x_1 - x_3$ the corresponding energy fraction of the emitted gluon. They are related to the mass squares of the parton pairs through

$$s_{12} = s_{di}(1-x_3) \tag{12}$$

etc. The exponents u and w are 2 and 3 depending upon the nature of the emitters [2] and α_S is the ordinary running coupling constant

$$\frac{3\alpha_S}{2\pi} \equiv \frac{\alpha_0}{\ln(k_t^2/\Lambda^2)} \tag{13}$$

The transverse momentum k_t and the (pseudo-) rapidity y of the gluon may be defined in terms of the dipole mass square s_{di} and the x_j as

$$\begin{aligned} k_t^2 &= s_{di}(1-x_1)(1-x_3) \\ y &= \frac{1}{2}\ln\left(\frac{1-x_1}{1-x_3}\right) \end{aligned} \tag{14}$$

In terms of these variables the cross section is approximately

$$\begin{aligned} d\sigma &\simeq \frac{\alpha_0}{\kappa} d\kappa dy \\ \kappa &\equiv \ln(k_t^2/\Lambda^2) \end{aligned} \tag{15}$$

and the available emission region from energy momentum conservation is

$$k_t \cosh(y) \le \sqrt{s_{di}}/2 \tag{16}$$

Emitted gluons in the Lund Model play the role of kinks along the string-like force field and the variables k_t and y describe "the bends" of the forcefield around the kinks.

Both the variables y and k_t, and in particular k_t, are "theoretical" variables and the "true" rapidity and transverse momentum of the gluon (e.g. defined with respect to an axis related the $q\bar{q}$-directions) are in general better approximated by e.g.

$$\begin{aligned} k_\perp^2 &= s_{di}\frac{(1-x_1)(1-x_3)}{(1-x_2)} \\ y_\perp &= \frac{1}{2}\ln\left(\frac{x_1(1-x_1)}{x_3(1-x_3)}\right) \end{aligned} \tag{17}$$

The cross-section is the same in terms of $k_\perp$ and $y_\perp$ as in k_t and y.

In an e^+e^--annihilation event the original $q\bar{q}$-pair starts to emit a gluon in accordance with the formulas above. The resulting state, containing a colour-3, colour-$\bar{3}$ and a colour-8, can (for a second emission) to a good approximation be considered as *two independent dipoles, one between the q and the g (gluon) and one between the g and the $\bar{q}$* [2].

The process is repeated and one obtains an ever increasing number of independent dipoles with decreasing masses and consequently quickly decreasing k_t of the "last" emitted gluon. The phase space limitations above is equivalent to "the strong angular ordering" condition in the partonic cascades [13]. This phase space can be well approximated by

$$|y| \le \ln\left(\frac{\sqrt{s_{di}}}{k_t}\right) \tag{18}$$

which corresponds to the inside of a triangular region in the (κ, y)-plane.

In reference [3] an extension to the CDM is introduced, also containing the gluon splitting process $g \to q\bar{q}$. This is generally a small correction to the gluon emission process, of the order of 10%.

In the e^+e^--annihilation reactions the emitters can be considered as essentially pointlike objects but this may no longer be the case in connection with lepto-production. The probe, i.e. the field pulse is well defined in size but the target hadron is extended.

3.2 The Soft Radiation Model

The Soft Radiation Model suggests one possible way to treat this extension. The starting point is to note that the colour radiation can be described in two complementary ways. One can consider the emission in terms of an initial state (spacelike) cascade and a final state (timelike) cascade. But it can also be argued that, as the hadron is in a bound state, there is no radiation before the interaction and that all radiation stems from the dipole formed between the struck parton and the hadron remnant.

In case e.g. a q-parton is "kicked out" then the whole of the remainder contains the corresponding energy and $\bar{3}$ colour. Thus the initial dipole contains one pointlike object and one extended, in particular with respect to the carrying of energy-momentum.

Then a gluon emitted in the phase space element $(k_\perp, y_\perp)$ will need both a positive and a negative energy-momentum light-cone component

$$k_\pm = k_\perp \exp(\pm y_\perp) \tag{19}$$

While the positive light-cone component is easily available from the large energy concentration in the struck parton, the negative one is spread over some region. It is a well-known property that a coherent wave, built in an emitter of size l with a wavelength $\lambda << l$ only stems from a fraction of the emitter comparable to λ.

Then for a large $k_\perp$ gluon (which has a small $\lambda \sim 1/k_\perp$) the total negative light-cone component will stem from a fraction of the emitter P_{-r} (the index r stands for the remainder after the struck parton is kicked out). This means that there will be a strong damping of the radiation in the backward direction.

In The Soft Radiation Model it is assumed that the phase space limits are changed into

$$\begin{aligned} k_- &< \left(\frac{\mu}{k_\perp}\right)^\alpha P_{-r} \\ k_+ &< Q_+ \end{aligned} \tag{20}$$

with the parameter μ corresponding to the inverse size and with α a number describing the dimensionality of the source-remnant.

The data from the EMC collaboration prefer a value of $\mu \approx 0.6GeV/c$ and $\alpha \approx 1$. This corresponds to a mean transverse extension $\sim \pi/\mu \approx 1fm$ and an essentially one-dimensional energy density like in a string. The data covers a rather small (Q^2, x) range but we will nevertheless assume in the model that the result is valid for arbitrary energies .

We will make one adjustment, however. In the case when there is a large x-interaction the remnant only contains $P_{-r} = (1-x)P_-$, i.e. essentially less than the incoming hadronic state P_-. Assuming that it is *the energy density in the rest system that is a constant* we are lead to expect that the effective l is correspondently smaller and thus that μ behaves as

$$\mu \equiv \mu(x) = \frac{\mu_0}{(1-x)} \tag{21}$$

This will mean that *the radiation in the target fragmentation region is the same, independent of the interaction*, i.e. the damping is governed by $(\mu_0/k_\perp)P_-$, independent of x and Q^2. The available energy will, however, only permit an emission in case the energy momentum component of the emitted gluon, k_-, is smaller than $P_{-r} = (1-x)P_-$.

The allowed emission region in an e^+e^--annihilation event is in this way changed (some-

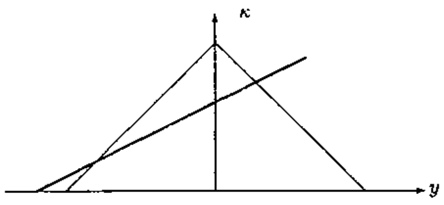


Figure 3: *The available phase space for gluon emission in DIS. Positive rapidity corresponds to the direction of the struck quark.*

times very much) in the target fragmentation region in connection with lepto-production (c.f. figure 3). From equations (20, 21) we conclude that the new phase space boundaries are

$$y > \ln\left(\frac{k_\perp^2}{\mu_0 P_-}\right)$$
$$y < \ln\left(\frac{Q_+}{k_\perp}\right) \qquad (22)$$

i.e. again two straight lines in the (κ, y)−plane.

The damping is in our case of a step function character. In case there would be a power suppression in the transverse momentum this would in the κ-variable correspond to an exponential damping. The results presented here are however insensitive to such an exponential tail.

3.3 The Recoil Structure

There are two kinds of recoil phenomena in connection with the emission of radiation, i.e. the energy loss of the emitters and the necessary momentum compensation. The energy loss corresponds to the obvious requirements that the variables $x_j < 1$. The x_j-variables also determines the relative angles in the emission, e.g.

$$\sin^2(\theta_{23}/2) = \frac{(1 - x_1)}{x_2 x_3} \qquad (23)$$

The relative orientation between the original dipole direction and the final state partons is, however, not fixed by the x_j variables but instead according to the transition matrix elements by the directions of the currents "before" and "after" the emission.

If a (hard) gluon emission occurs then the resulting colour current for the partons is rather complex. There is, however, an alignment between the original current direction and a direction for the final state, i.e. the axis along which the emitters in the final state has a minimum for the transverse momentum combination $(k_{\perp 1}^2 + k_{\perp 3}^2)$.

A prescription given by Kleiss [14] allows a simple implementation of this alignment in Monte Carlo simulation programs. There is, however, no prescription known at present for handling the emission of two or more gluons.

In Ariadne the emission from a dipole composed of two gluons is handled by aligning the original dipole direction with this "minimal $k_\perp$-axis". In a dipole composed of a gluon and a q or $\bar{q}$ the total recoil is in the default version given to the q or $\bar{q}$, however. A general investigation of the possible recoil strategies and their properties is given in reference [15].

For the particular case when the softening mechanism in The Soft Radiation Model is used, i.e. when the radiation occurs between the remnant and an earlier emitted gluon, it is necessary to make further assumptions. We have treated the remnant fraction

$$(\mu_0/k_\perp)P_- \equiv x_r P_- \qquad (24)$$

as an already existing gluon for this k_t-emission. Provided with a particular recoil strategy in the emission these "fraction-gluons" (this notation will be used henceforth) will become recoil gluons. Together with the "really" emitted gluon they form a "recoil dipole" which plays a role very similar to the splitting process gluons which in standard QCD occur in the lepto-production cross section as initial state corrections.

4 Comparison with an Initial State Bremsstrahlung Scenario

In this section we will describe an ISB scenario and consider the relationship between our SRM recoil gluons and the ISB gluons in detail both from a kinematical and a dynamical point of view.

4.1 An Initial State Bremsstrahlung Scenario

We will assume that $x_{Bj} \equiv x$ is large, although not close to 1. Then in an ISB model a parton will come in with a fraction $x_- \equiv x'$, emit a gluon with $x_r \equiv (1-\zeta)x'$, thereby turning into a virtual parton with a light-cone fraction $x = \zeta x'$. Its (virtual) mass square $-Q_I^2$ is related to the transverse momentum of the emission $k_{\perp,I}$ by

$$Q_I^2 = \frac{k_{\perp I}^2}{(1-\zeta)} \qquad (25)$$

We are then assuming that we may neglect the corresponding virtual mass and transverse momentum of the incoming x'-parton.

The transverse momentum is in the general ISB scenario compensated by the two emitted partons, i.e. they will take $\vec{k}_\perp$ and $-\vec{k}_\perp$ respectively.

The probability for the emission is (for fixed x)

$$dP(\zeta, Q_I^2) \propto \alpha_s \frac{dQ_I^2}{Q_I^2} \frac{d\zeta}{\zeta(1-\zeta)} \qquad (26)$$

which is identical to the cross section given in equations (11, 15) (note that the ζ-dependent part is equal to $dx_r/x_r = dy$. We will from now on neglect the numerator in equation (11), which stem from a polarization sum).

In case one identifies the virtual parton mass square with the (square) of the inelastic field pulse q (which in a light-cone frame will have components $(Q_+, -Q_-, \vec{0})$) then one obtains the

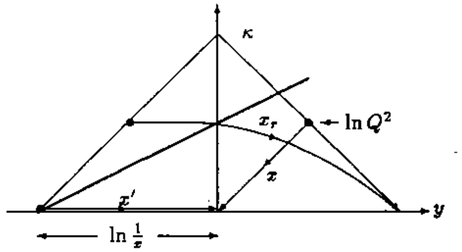


Figure 4: *The possible gluon emissions contributing to the structure function at x for a given Q^2. The arrows indicates the orbits of the x, x' and x_r partons when varying ζ in equation (27)*

following kinematical relations for the emitted gluon with light-cone components $k_\pm = k_{\perp,I}\exp(\pm y_\perp)$

$$Q_- = x'\zeta P_- = \frac{\zeta}{1-\zeta}k_-$$
$$Q_+ = \frac{k_+}{\zeta} \qquad (27)$$

In figure 4 we exhibit the possible gluon emissions which will contribute to the structure function at x for a given Q^2, i.e. the results of varying ζ in the equations above. We have shown "the orbits" of the emitted x_r-gluon, the x-parton and the x'-parton.

For large values of x' the emitted x_r-gluon may be outside the phase space boundary of the SRM, i.e. the boundary curve in equation (22). It is, however, easy to convince oneself that if

$$x' \leq x_0 \equiv \frac{\mu_0}{\sqrt{Q^2}} \qquad (28)$$

then the emission regions of the two models will coincide.

We note that the fraction x_0 is a "natural" scale in the SRM. It corresponds to a (one-dimensional, i.e. along the string) resolution of a constant energy density by means of the field pulse q.

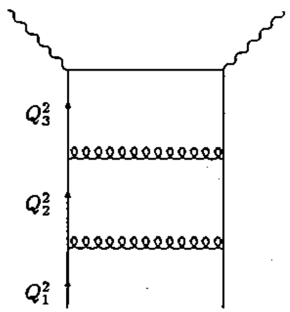


Figure 5: *A Feynman diagram indicating successive gluon emissions in an ISB scenario.*

In reference [4] the region $x \geq (\mu_0/Q)^2$ is considered as the place of the large structure function fluctuations in that two-dimensional scenario. The scale x_0 has the corresponding meaning in our one-dimensional string model.

If $x' > x_0$, however, then the SRM conditions will not allow emission of gluons with a negative light-cone fraction x_r larger than $x_{r,max}$:

$$x_{r,max} \sim (x' x_0^2)^{1/3} \tag{29}$$

although such emissions are allowed in the ISB model.

In case the virtual mass-square $Q_I^2 < Q^2$ (which would be the case if there is an initial virtuality or/and an initial transverse momentum of the x'-parton) then the quantity x_0 in equation (29) is exchanged for the larger quantity $x_{0I} = \mu_0/Q_I$. Consequently the commonly allowed regions between the models increase.

In an ISB scenario the above-described emission may be iterated repeatedly. From an analysis of the Feynman graphs in figure 5 one concludes [16] that there must be an ordering in the virtualities in this iteration so that the occurring Q_I^2- and $k_{\perp I}^2$-values are strongly increasing. (In general this implies that the x_r-values are decreasing).

In case we concentrate on the x-parton one may ask about the number, $f(x, Q^2)dx$, of such partons available in the state at different Q^2-values. This problem is solved by the DGLAP-equation(s):

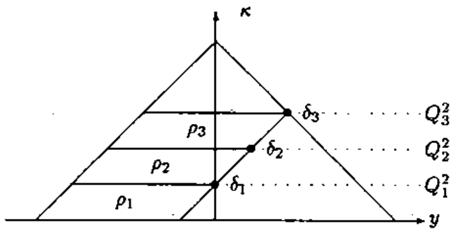


Figure 6: *An interpretation of the DGLAP-equations with notations described in the text*

$$\frac{df_a}{d\ln(Q^2)}(x, Q^2) = \sum_b \int_x^1 \alpha_s \frac{d\zeta}{\zeta} P_{ab}(\zeta) f_b(x/\zeta, Q^2) \tag{30}$$

Here the indices (a, b) correspond to different parton species and P_{ab} describes the splitting probability of b into a.

The interpretation of the equations is obvious from figure 6, where we show a set of different Q^2-values. The left hand side of equation (30), i.e. the increase in the number distribution corresponds to the possible emission at x "just above" the corresponding Q_j^2 level (denoted δ_j). This increase is then related in the right hand side to the available number of (x')-partons inside the corresponding regions $R_j = R_{j-1} + \rho_j$.

4.2 The Properties of the Recoil Gluons

We will now study the kinematics in connection with the recoil gluon emissions in the SRM. We assume that there is a large amount of energy momentum continuing along the negative z-axis even after the emission, $x_1 P_-$ with $x_1 > x_r$. The recoil gluon will be indexed 2 and the "really" emitted one according to the SRM will be indexed 3. We will represent their energy momentum vectors in the light-cone basis as

$$\begin{aligned} k_2 &= A_2\left(\frac{k_r^2}{x_r P_-}, x_r P_-, -k_r \vec{e}_2\right) \\ k_3 &= A_3\left(\frac{t k_r^2}{x_r P_-}, \frac{x_r P_-}{t}, k_r \vec{e}_3\right) \end{aligned} \tag{31}$$

This is the most general possible representation.

We finally assume that the remaining part of the state has a large positive light-cone component P_+ and we neglect the remaining components. Thus we assume that there is a colour connection in the emission chain

$$x_1 P_- \to k_2 \to k_3 \to P_+ \tag{32}$$

In this framework it is possible to define the rapidity and transverse momentum variables for 2 and 3 by the prescriptions used in connection with the equations (14) and/or (17).

In this way the gluon variables $(k_{\perp 2}, y_{\perp 2})$ and $(k_{\perp 3}, y_{\perp 3})$ corresponds to the bending properties of the string in the regions $x_1 P_- \to k_2 \to k_3$ and $k_2 \to k_3 \to P_+$, respectively.

We obtain

$$\begin{aligned} k_{\perp 2} &= k_r A_2 R/t \\ y_{\perp 2} &= -\ln(P_-) + \ln(1/x_r) + \ln(k_r) \\ k_{\perp 3} &= k_r A_3 R/t \\ y_{\perp 3} &= -\ln(P_-) + \ln(1/x_r) + \ln(k_r) + \ln(t) \end{aligned} \tag{33}$$

with

$$R = \sqrt{(\vec{e}_2 + t\vec{e}_3)^2} \tag{34}$$

The requirement that the two final state vectors should have a combined negative light-cone fraction x_r together with the definition (24) of x_r in the SRM gives

$$\begin{aligned} A_2 + A_3/t &= 1 \\ x_r &= \mu_0/k_{\perp 3} \end{aligned} \tag{35}$$

Now all gluon variables are fixed besides one, which can be taken as the scale k_r. The requirement that

$$\frac{d^2 k_{\perp 2}}{k_{\perp 2}^2} dy_2 = \frac{d^2 k_{\perp 3}}{k_{\perp 3}^2} dy_3 \tag{36}$$

constitutes a (first order partial) differential equation for k_r. A little algebra shows that this equation and all the other conditions are fulfilled in case

$$k_{\perp 2}^2 \exp(-y_{\perp 2}) = k_{\perp 3}^2 \exp(-y_{\perp 3}) \tag{37}$$

This condition, which also can be written as

$$t = \frac{k_{\perp 3}^2}{k_{\perp 2}^2}, \tag{38}$$

corresponds to a balancing requirement for the two bending kinks along the colour force field or in other words to *a particular recoil strategy.*

The part of the $\vec{k}_{\perp 3}$ that is not compensated by the gluon 2 will be given to the other end of the emitting dipole.

We note that the scale k_r will always be close to $k_{\perp 2}$ (within 20%) although $k_{\perp 3}$ may be much larger.

4.3 The Implications of The Recoil Strategy

We will now consider the meaning and the implications of the recoil strategy and the equality of the cross sections. Firstly, we note that the recoil strategy described above in equations (37) and (38) is a very reasonable one. It means that when the rapidity difference between 2 and 3 is large then 2 will be less influenced by 3. Instead the momentum recoil will be taken up in the other end of the dipole.

The equality of the two cross sections in equation (36) means that it does not matter whether the gluon 3 or the gluon 2 is considered to be the primarily emitted one.

We note that the ordering in the emissions in general is related to the assumptions that are made on the coherence properties in the partonic cascade. Such an ordering is regulated through a Sudakov formfactor

$$dP = \frac{d\sigma(A)}{\sigma_0} \exp\left(-\int_\Omega \frac{d\sigma}{\sigma_0}\right) \tag{39}$$

The form-factor describes the probability of no emission inside the region Ω together with one emission in the boundary point A.

In the CDM one orders the emissions according to the size of the transverse momenta, always emitting the larger ones "before" the smaller ones. In that case the cascade comes from "above" in the phase space triangle and the region Ω is defined as the transverse momenta above $k_{\perp 3}$ (and below the one of the last emission).

If we compare to the ISB scenarium as described in connection with the equation (30) above we note that there is a very interesting difference in the two approaches. *The region with necessary emission knowledge in the ISB model is complementary to the region needed in the SRM.* For a given (x, Q_j^2) the emitted gluon at δ_j is influenced by the emissions in the R_j region for the ISB. It is, however, the emissions in the region "above" R_j (and inside the kinematical limits) which is essential for the CDM and SRM.

We note that for the SRM it is a basic assumption that the emission of 3 implies the (recoil) emission of 2. *We will now show that the combined emission of both the gluons 2 and 3 as a splitting process in an ISB model is equal to the emission of 3 (thereby obtaining 2 as a recoil) in the SRM.*

We will use the same notations for ζ and x_r as before for the combined emission. The mass of the $(2,3)$-dipole (which is identical to a virtual mass for one of them in case the emitter and the other one are mass-less) will be

$$Q'^2 \simeq x_r P_-(k_{+2} + k_{+3}) \propto k_{\perp 3}^4 \quad (40)$$

The process would then in an ISB scenarium have the probability $dP(\zeta, Q'^2)$ according to equation (26).

In order to see that

$$\frac{3}{2\pi}\alpha_s(k_{\perp 3}^2)\frac{dk_{\perp 3}^2}{k_{\perp 3}^2}dy_{\perp 3} \simeq dP(\zeta, Q'^2) \quad (41)$$

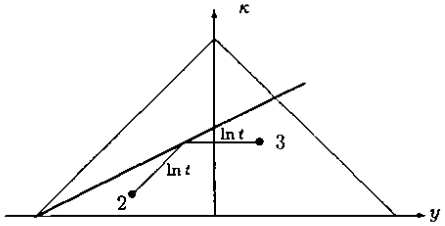


Figure 7: *The placement in phase space of the recoil gluon (2) and the "really emitted" gluon (3).*

we use that

$$\frac{dx_r}{x_r} = \frac{dk_{\perp 3}}{k_{\perp 3}}$$
$$y_{\perp 3} + \ln(k_{\perp 3} x P_-) \simeq \ln(Q'^2) \quad (42)$$

together with equation (40).

(Actually the factor in front of α_s in equation (41) should have been $4/3\pi$ for a quark emission - but the relative factor 8/9 would be 1 for infinitely many colours).

The combined emission of 2 and 3 in this way implies that a full dipole is created which can continue to radiate. In the CDM such radiation will, however, only occur for transverse momenta below $k_{\perp 2}$.

We are therefore by the combined emission "forcing" a density of gluons (which is characteristic for the CDM) upon the ISB model. In particular there is a characteristic distance between the gluons in the (κ, y) phase space. The recoil strategy means that 2 and 3 are placed at the corners of a four-sided area (with equal height and baseline size, i.e. $\ln(t)$, c.f. figure 7).

This distance is a typical one in the CDM. To see that in detail we note that the (rapidity) distance between the production point for the gluon 3 and the phase space boundary of the SRM is λ_i (i for initial)

$$\lambda_i = 1/2\ln(t) \quad (43)$$

This quantity was termed the dipole size at the virtuality $\kappa_3 = \ln(k_{\perp 3}^2)$ in reference [17]. It corresponds to the rapidity region inside which the dipole can emit gluons with that transverse momentum. The corresponding "final" dipole size at the virtuality κ_2 is λ_f

$$\lambda_f = 2\ln(t) \quad (44)$$

One of the results in [17] is that in a typical dipole cascade the mean value of the dipole size should equal the mean "step" in virtuality,i.e.

$$\delta\kappa = \kappa_3 - \kappa_2 = \ln(t) \quad (45)$$

"before" there is a new decay. This is evidently what happens in case we require no gluons between 2 and 3.

In case $\kappa_3 \gg \kappa_2$ this "hole"' is appreciable, however (although this situation occurs rather seldom). The typical "hole size area", A should evidently be governed by

$$\alpha_s A \sim 1 \quad (46)$$

in a "leading log cascade". This means that $\ln(t) \sim 1/\sqrt{\alpha_s}$ for the typical gluon densities in the CDM. Actually this is also the typical one in all (angular ordered) leading-log cascade models [16].

In order that our prescriptions should be self-contained with respect to the gluon density we have introduced a test procedure. We have at every stage when there is an emission containing a recoil gluon investigated whether it is possible to emit from the dipole between the gluon 3 and the remainder a gluon with a larger $k_\perp$ than $k_{\perp 2}$.

If this is the case then this gluon will be emitted together with its corresponding recoil gluon and the earlier recoil gluon 2 is neglected. Therefore the introduction of the recoil gluons with our particular recoil strategy will not inhibit possible radiation.

In conclusion, we have shown that inside the kinematical scenario in which the gluon energy momenta are related to the bending around the kinks of the force field and their momenta and rapidities are constrained by the recoil strategy in equation (37);

- The probabilities are equal to emit the gluon indexed 3 in the SRM (and obtain 2 as a recoil gluon) or to emit both in an ISB scenario as a virtual dipole.
- In an ISB scenarium the emission would be a splitting of an earlier emitted parton. There is a correspondence in the SRM, i.e. the collective fraction gluon.
- The regions of phase space needed for the emission are "opposite", i.e. in the SRM it is the region of all transverse momenta above the emitted one while in the ISB it is the region of all transverse momenta below.

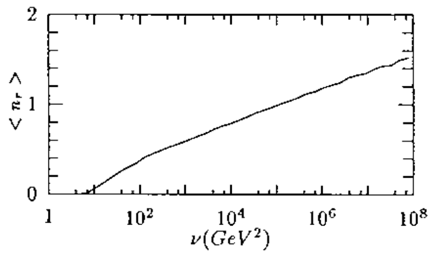
5 The Recoil Gluons and a DGLAP-like Equation

We have introduced two different views of the emission of radiation in the target region of an inelastic lepto-production event in sections 3 and 4.1. There is the scenario stemming from The Soft Radiation Model on the one hand and a scenario from an Initial State Bremsstrahlung Model and we have shown that certain cross sections agree in the two models. While there is an obvious relationship between the gluon emission and the changes in the structure functions in the ISB model there is, however, not necessarily a similar one in the SRM.

In the SRM all the radiation is of a final state type, i.e. "as if" it occurred between a (point-like) kicked-out q or $\overline{q}$ and an (extended) remnant.

The large x distributions, typically x of the order $(\mu_0/\sqrt{Q^2})$ in our approach, corresponds to the fluctuations in the valence parton wave functions. For these distributions the radiation from the SRM does not give predictions besides the obvious fact that the radiation takes energy away and "somebody must pay".

In order that such a valence parton should be kicked out and then be able to radiate, the recoil gluons are necessary in our picture. We will



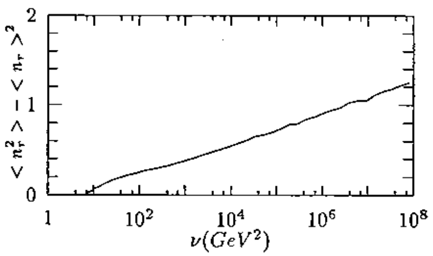


Figure 8: *The mean number of recoil gluons and the variation around this mean as a function of ν.*

now *assume that the negative light-cone fraction x_r of the recoil gluons, needed for the radiation is taken in a proportional way from all parts of the (undisturbed) hadronic wave function*, i.e. an "equipartitioning of losses". Thus when Q^2 increases and more and more radiation is emitted (and correspondingly more recoil gluons are necessary) in particular the valence partons will be correspondingly softer. This is the basis for an evolution equation.

We will start to investigate the number of recoil gluons, n_r, emitted as a function of the variable ν. In figure 8 we exhibit the mean number $< n_r >$ and also the variation around the mean $< n_r^2 > - < n_r >^2$ as a function ν. We note an almost linear rise in the number of recoil gluons with $\ln \nu$. The corresponding softening of the valence partons seems, however, to flatten out at

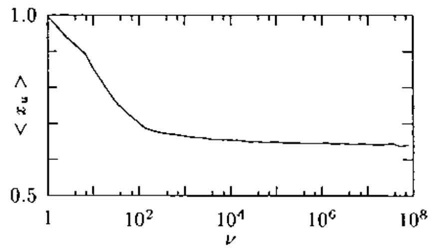


Figure 9: *The mean energy-momentum fraction x_u left to the remnant as a function of ν.*

high ν as can be seen in figure 9 where we show the mean energy-momentum fraction x_u left in the remnant.

The reason for this can be found in figure 7, where it is evident that the extra phase space obtained when increasing ν, corresponds to emissions with larger t or larger $k_\perp$. An increase in t will result in lower $k_{\perp 2}$ making it more likely that the recoil gluon is neglected due to the $k_\perp$ ordering procedure described in section 4.3. A larger $k_\perp$ on the other hand will give recoil gluons with smaller x_r. Hence although the number of recoil gluons increase, the average energy they carry away from the remnant decrease.

In order to discuss the evolution in the valence distributions we will calculate the total amount of recoil gluon x_r that is emitted for different values of the initial state energy momentum P and the field pulse q. We will show that this leads to a DGLAP-like evolution equation (30), using the kinematical scenario in section 4.2.

We firstly notice that there is a minimal value of Q_+ for given x and x_r. As Q_+ always must be larger than $(k_{+2} + k_{+3})$ in order to emit the two gluons at all, we obtain as a minimal requirement on Q_+:

$$Q_+ \geq \frac{k_{\perp 3}^2}{x_r P_-} \tag{47}$$

(corresponding to $t = 1$ and $k_{\perp 2} = k_{\perp 3}$). In

terms of (x, Q^2) and the splitting variable ζ this is

$$x \geq \frac{\mu_0}{\sqrt{Q^2}} \left(\frac{\zeta}{1-\zeta} \right)^{3/2} \tag{48}$$

We next assume that we know the distribution f of the valence partons at a given (x, Q^2). Then we increase $Q^2 \to (Q^2 + dQ^2)$. From equation (26) we deduce that the change in the structure function is

$$Q^2 \frac{\partial f}{\partial Q^2} = \int_x^1 \frac{3\alpha_s(Q^2)}{2\pi} f(\frac{x}{\zeta}) \times \tag{49}$$
$$(\frac{d\zeta}{\zeta(1-\zeta)_P} P_{Sud}(\zeta) - C d\zeta \delta(\zeta - 1))$$

The occurrence of the Sudakov form factor P_{Sud} corresponds to the requirement that there is no other emission inside a certain (ζ-dependent) region, i.e. that there is no gluon emission with transverse momentum $k_\perp > k_{\perp 3}$ and rapidities smaller than $y_{\perp 3}$. This requirement transforms itself into

$$k_\perp > \frac{\mu_0 \zeta}{x(1-\zeta)} \tag{50}$$

In case such an emission would occur then there will be no recoil gluon corresponding to the increase in Q^2. The $k_{\perp 3}$-emission occurs in a dipole "forward to" and isolated from the target remnant. The emission would, however, occur already for smaller values of Q^2 and is therefore already taken into account in the evolution equation.

The occurrence of this Sudakov factor corresponds to the different coherence requirements in the SRM and the ISB scenarios (c.f. the discussion after equation (39). In the SRM the emissions "above" $k_{\perp 3}$ do influence the $k_{\perp 3}$-emission and thereby also the changes in the structure functions.

The notation $1/(1-\zeta)_P$ means that the principal value is used at the integration endpoint $\zeta = 1$ and the constant C is equal to

$$\int_0^1 \frac{d\zeta}{(1-\zeta)_P} P_{Sud}(\zeta) \tag{51}$$

In this way the singularity at $\zeta = 1$ (corresponding to the possibility that x is not changed) is

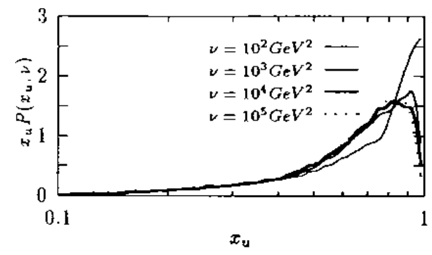


Figure 10: *The distribution in x_u (the fraction of energy-momentum left in the remnant after allowing for dipole emission) for different values of ν.*

defined in such a way that the total probability to find a (valence) parton is a constant, i.e.

$$\frac{\partial}{\partial Q^2} \int_0^1 dx f(x) = 0 \tag{52}$$

Although the integro-differential equation (49) can be solved (at least numerically) we have preferred to directly calculate from Ariadne the distribution in $x_u \approx 1 - \sum x_r$. The reason is that there are several kinematical corrections not included in the equation which are directly incorporated in the Monte Carlo simulation. The (normalized) distributions $P(x_u, \nu)$ are shown in figure 10 for a set of different values of the variable ν.

In order to obtain the valence distribution at a given Q^2, $f(x, Q^2)$, given an "original" distribution $g(x)$ for the undisturbed state, one calculates

$$f(x, Q^2) = \int_x^1 \frac{dx_u}{x_u} \int_{Q_0^2/x}^{Q^2/x} \frac{d\nu}{\nu} P(x_u, \nu) g(\frac{x}{x_u}) \tag{53}$$

In figure 11 we show the result of using Morfin-Tung (set 1) [18] valence structure functions (defined as $xd(x) - x\bar{d}(x)$ for the proton) at small values of Q^2 to obtain the function g in equation (53) and then a comparison to the results

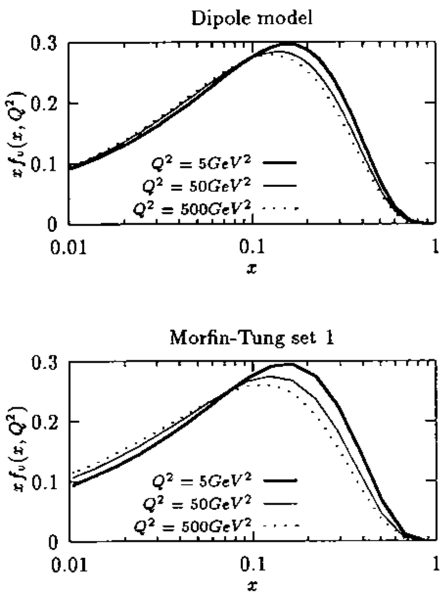


Figure 11: *The valence structure function for different values of Q^2 for the Dipole model and for Morfin-Tung set 1 [18]. For the Dipole model, the function $g(z)$ in equation 53 is chosen so that the two functions coincide for $Q^2 = 4GeV^2$.*

of the same structure function at different Q^2-values. From this figure it is evident that our approach gives a much slower evolution than the DGLAP-type used in the Morfin-Tung structure functions.

In figure 12 we show the same functions as figure 11 as a function of Q^2 fore some fixed x-values.

6 Conclusions

We feel that the Dipole Model approach to the hadronic structure functions presented in this paper and in reference [1] provides a possible dynamical alternative to the conventional QCD evolution models á la DGLAP. There are however still some questions to be answered before we are able to obtain firm quantitative predictions from the approach.

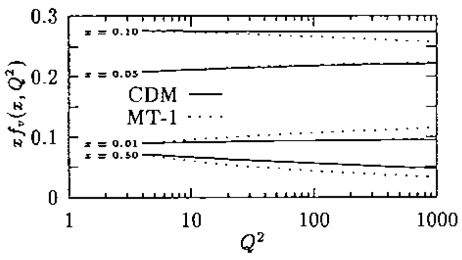


Figure 12: *The valence structure function for different values of x for the Dipole model and for Morfin-Tung set 1 [18].*

One question concerns the shape of the x-distribution in the undisturbed hadronic state. The approach presented in section 2 should only be regarded as toy models. We have in that case only taken the into account the degrees of freedom corresponding to orientation and different energy momentum configurations and totally neglected the possible internal degrees of freedom.

Thus in a string model of a vortex line kind the field configuration will (via a Weizsäcker-Williams approach) give further structure and in particular make the wave functions softer due to the extensions of the fields. At the present state of the model it is necessary to use experimentally measured structure functions as input in the same manner as was done in figure 11.

It is interesting to note that the change in the structure functions with Q^2 is in our model basically the same as in the ordinary DGLAP approach inside the presently available Q^2 region, i.e. up to $Q^2 \sim 100\ GeV^2$. The results from HERA will evidently be able to differentiate between the two approaches.

The question of parameter dependence is also an important one. In principle the four parameters of the model ($k_{\perp cut}$, Λ_{QCD}, μ and α) can be determined by fits to hadronic final state distributions from present experiments. However, these fits necessarily depend on the particular hadronization model used, which especially gives a large uncertainty for the value of $k_{\perp cut}$. This parameter is also the one to which, according to preliminary investigations, our evolution is the most sensitive.

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