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Parton Density at Small x_B

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Review talk at " QCD - 20 Years Later" Aachen, June 9 - 13 , 1992.

Abstract

This is a status report on new phenomena that are anticipated in deeply inelastic scattering in the low x region. I am viewing on this talk as summary of the theoretical situation in the region of small x, as is just before this new area of physics will be studied experimentally at HERA.

1 Introduction.

I think that the main goal of this conference is to sum up the ideas, problems and hopes of the current theory of strong interaction - QCD, which are circulating now among experts. In my talk I would like to address you the problem of low x_B behaviour of the parton density and hope to convince you that this problem is one of the most difficult and important problems of QCD. At least it should be solved before QCD will be a real theory of the strong interactions at high energy.

In my talk I am going to answer the following the questions:

- What kind of new phenomena we anticipate at low $x \to 0$?
- Why the region of low x is so important for our understanding of high energy interaction in QCD?
- What do we know about low x behaviour of deep inelastic structure functions?
- What kind of phenomenology we are able to develop at $x \to 0$?
- What questions and how HERA will be able to answer?

2 New physical phenomena at low x_B:

Here I would like to discuss the new phenomena that we anticipate in the region of low x_B in the perturbative QCD for deeply inelastic scattering. It is three of them that are the most important and determine the physical picture of the parton cascade in the region of low x_B, namely:

1. The increase of the parton density [1] at $x_B \to 0$.

2. The growth of the mean transverse momentum of the parton inside of the parton cascade at low x_B [1] [2].

3. So called saturation of the parton density [2].

Definitions:

First of all let me remind you several definitions. If N is the number of gluons, the gluon structure function says us what is the number of gluons with definite value of $y = \ln \frac{1}{x_B}$, so

$$x_B G(x_B, Q^2) \;=\; \frac{dN}{d \ln \frac{1}{x_B}} .$$

However it is more convenient to introduce the new function ϕ which is defined as:

$$\phi = \frac{dx_B G(x_B,Q^2)}{dQ^2}\cdot\frac{1}{\pi R^2} = \frac{1}{Q^2}\cdot\frac{x_B G(x_B,Q^2)}{\pi R^2}. \qquad (1)$$

The second factor in eq.(1) is the density of partons in the transverse plane at fixed $y = \ln\frac{1}{x_B}$, while the first one is the area that feels our probe (virtual photon). Thus

$$\phi = number\ of\ gluons\ in\ the\ area \propto \frac{1}{Q^2}.$$

In other words ϕ is the number of gluons that could interact with the target (photon with virtuality Q^2).

Perturbative QCD.

To understand the new phenomena at low x_B let me remind you that in the perturbative QCD the deep inelastic structure function or the basic branching process (see Fig.1) can be presented as a seria of the following type:

$$x_B G(x_B,Q^2) = \Sigma_n C_n(\alpha_s L)^n + O(\alpha_s(\alpha_s L)^n) , \qquad (2)$$

where L is the large log in our problem.

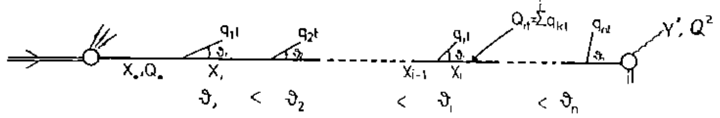

Fig. 1 : The basic branching process in the deeply inelastic scattering.

What kind of large logs could be in the parton cascade one can see just from the explicit expression for the probability of emission of i-th parton in the cascade that can be written in very simple form at least at $x \to 0$

$$P_i = \frac{N_c\alpha_s}{\pi}\cdot\frac{dx_i}{x_i}\cdot\frac{dq_{it}^2}{q_{it}^2}$$

1. In the region of large virtualities of the photon in the deep inelastic scattering but not at very small values of x_B L is equal to $\log Q^2$ and eq.(2) leads to the ordinary GLAP [3] evolution equation.

2. At Q^2 fixed ($Q^2 = Q_0^2 \gg m^2$) but at small value of x_B ($x_B \to 0$ we have the new scale for large log in eq.(2), namely $L = \ln\frac{1}{x_B}$. In this kinematical region the

seria of eq.(2) can be summed using the new evolution equation (so called BFKL - equation [1]). Namely this evolution equation gives the new phenomena at low x_B which I am going to discuss a little bit later.

3. When both $\ln Q^2$ and $\ln\frac{1}{x_B}$ are large ($Q^2 \gg Q_0^2$ $x_B \to 0$) our log scale has a more complicated form, namely $L = \ln Q^2 \ln\frac{1}{x_B}$. In this case we have so called double log approximation (DLA) of the perturbative QCD. It is very important to note that both evolution equation either GLAP or BFKL give the same DLA answer for the seria of eq.(2) providing the smooth matching between two extreme kinematical regions at small values of x_B.

2.1 Increase of the gluon (quark)density at $x_B \to 0$.

After this fast sketch of different approaches in the perturbative QCD let me discuss the new phenomena at low x_B.

Gribov - Lipatov - Altarelli - Parisi evolution equation.

In GLAP equation we sum such contribution in the parton cascade which compensates the smallness of the coupling constant of QCD (α_s) by large $\ln Q^2$. It is also easy to see that we need strong ordering in transverse momenta of emitted partons since only under such a condition we are able to get $\log Q^2$ contribution for each integration over q_{it} in our parton cascade(see Fig.1).

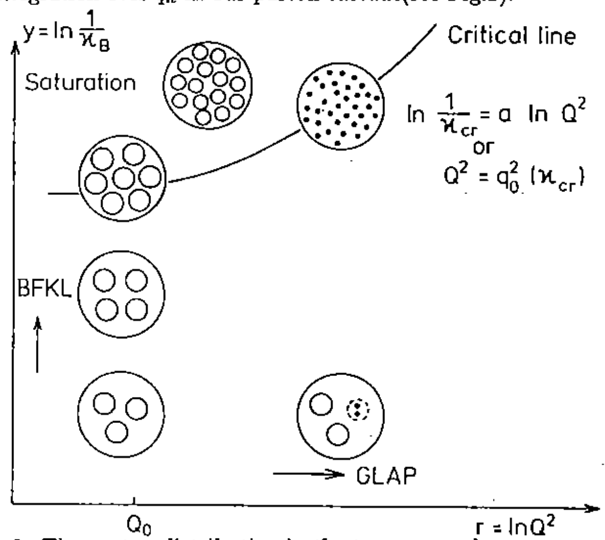

Fig.2: The parton distribution in the transverse plane.

Indeed, directly from the expression for P_i you can get $\log Q^2$ only if

$$Q^2 \gg q_{nt}^2 \gg .... \gg q_{it}^2 \gg q_{(i-1)t}^2 \gg ..... \gg \frac{1}{R_p^2} = Q_0^2$$

This strong ordering means that GLAP evolution equation allows us to to calculate the probability to find the parton with the transverse size $r_t \approx \frac{1}{Q}$ inside of the initial parton (quark or gluon) in the hadron with fixed x_B (see Fig. 2). And we can see immediately from the explicit solution of the GLAP equation that ϕ falls down at large Q^2. It seems very naturally from our intuition, since the asymptotic freedom of QCD means that hadron is almost empty at small distances.

Balitski- Fadin -Kuraev - Lipatov evolution equation.

However the situation changes crucially if we consider the behaviour of the parton density ϕ in another extreme case: at fixed Q^2 but in the region of small x_B. Here we have the new scale for large log in eq.(2),namely $L = \log \frac{1}{x_B}$. Even the slight glance at the parton cascade shows that the number of partons drastically increases in the region of small x_B, since each parton in the basic branching process (see Fig.1) is able to decay in its own chain of daughter partons (see Fig.3). We can even estimate the total multiplicity of gluons N_G associated with the above complicated process. Indeed, our target can interact only with one of produced parton while the others gather back and give the contribution into the renormalization of gluon mass as shown in Fig.3.

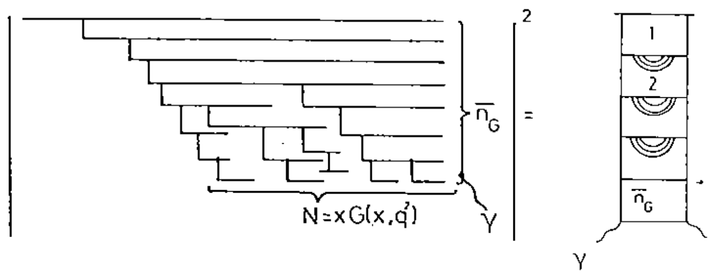

Fig.3: Structure of the parton cascade at low x_B and the coherence in the "ladder" diagram.

Thus the gluon structure function can be described by "ladder" diagram of Fig.3 and we can estimate the number of cells in this diagram or in other words the typical number of parton (gluon) emission in one subprocess in our parton cascade (n_G). Indeed, the characteristic value of $\Delta y_{i,i+1}$ in this diagram is equal to

$$\Delta y_{i,i+1} \approx \frac{1}{\alpha_s} .$$

The above statement is obvious just from the expression for the probability for gluon emission P_i. So

$$n_G = \frac{\ln \frac{1}{x_B}}{\Delta y_{i,i+1}} \approx \alpha_s \cdot \ln \frac{1}{x_B} . \quad (3)$$

The total number of the parton that can interact with the target N_G can be calculate as follows (see Fiog.3):

$$N_G \propto e^{n_G} = e^{c\alpha_s \ln 1/x_B} = (\frac{1}{x_B})^{c\alpha_s} , \quad (4)$$

where constant c should be calculated using the precise form of the BFKL equation [1] that sums all $(\alpha_s \ln(1/x_B))^n$ contribution.

2.2 The new scale for the deeply inelastic process at $x_B \to 0$.

Thus the density of the partons increases at the region of small x_B, but the careful study of the behaviour of the parton cascade shows that we have also the increase of the mean transverse momentum of the parton. Indeed,in the case when we neglect the running coupling constant of QCD our theory is dimensionless so each emission leads to the value of the transverse momenta of the daughter gluons of the same order as the transverse momentum of the parent gluon. This fact could be seen just from the explicit expression of P_i and we can introduce

$$|\ln \frac{q_{i,t}^2}{q_{i+1,t}^2}| = \Delta ,$$

which characterizes the emission.

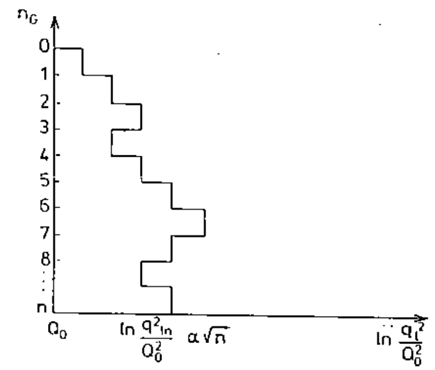

Fig. 4: The random walk picture in $\ln q_t^2$.

After n - emission the mean

$$|\ln(q_{n,t}^2/Q_0^2)| \propto \sqrt{n} \approx \sqrt{\alpha_s \ln\frac{1}{x_B}}\,, \qquad (5)$$

since we have something like random walk in $\ln(q_{n,t}^2/Q_0^2)$ (see Fig.4).

I think that it will be very instructive to adduce here the analytical solution of the BFKL equation with the initial condition $x_B G(x_B = x_0) = \delta(ln(Q^2/Q_0^2)$ which contains in explicit form all discussed properties:

$$G_y(y-y_0, r-r_0) = \frac{\sqrt{Q_0^2}}{\sqrt{q^2}} \cdot \frac{1}{\sqrt{\pi\Delta\omega_0(y-y_0)}} \cdot exp\{\omega_0(y-y_0) \; - \; \frac{(r-r_0)^2}{8\Delta\omega_0(y-y_0)}\}.$$

where

$$\omega_0 \;=\; \frac{\alpha_s(Q_0^2)N_c}{\pi}\cdot 4\ln 2\,,\;\; \Delta \;=\; \frac{14\xi(3)}{4\ln 2}\;,\; y - y_0 \;=\; \ln\frac{x_0}{x_B},\;\; r - r_0 \;=\; \ln\frac{Q^2}{Q_0^2}\,.$$

2.3 Saturation of the gluon density.

The increase of the parton density leads us to the new problem in the deeply inelastic scattering, namely to the violation of the s - channel unitarity. Indeed, unitarity tells us that the total cross section for virtual photon absorption should be smaller than the size of a hadron.

$$\sigma(\gamma^* N) \ll \pi R_h^2 \qquad (6)$$

We can express $\sigma(\gamma^* N)$ through the deep inelastic structure function $xD(x, Q^2$ $(F_2 = xD(x.Q^2)$) :

$$\sigma(\gamma^* N) \;=\; \frac{\alpha_{em}}{Q^2}\cdot xD(x,Q^2) \qquad (7)$$

However, we have shown previously that the value of the structure function is expected to increase very rapidly when $x \to 0$. Using the DLA result (see eq. (4)), we can rewrite the unitarity constraint in the following way:

$$\frac{\alpha_s(Q^2)}{Q^2}\cdot(\frac{1}{x_B})^{c\alpha_s(Q^2)} \ll \pi R_N^2. \qquad (8)$$

Here we put α_{em} has been replaced by α_s since the probe can be also a virtual gluon not only a photon.

From this expression alone one can conclude that **unitarity will be violated** [2] at

$$x < x_{cr} \;\; where \;\; \log\frac{1}{x_{cr}} \;=\; \frac{1}{c}\cdot\frac{1}{\alpha_s(Q^2)}\log Q^2 \qquad (9)$$

Therefore unitarity is violated even for large (or even for very large) values of Q^2 when $x < x_{cr}$ and clearly the miraculous confinement force can not prevent this phenomena. Thus we have to look for the origin of such a violation inside perturbative QCD.

Let us try to understand what happens in the region of small x_B looking at the parton distribution in the transverse plane (see Fig.2) Our probe (photon) feels those partons whose size is of the order of $\frac{1}{Q^2}$.

1. At $x \sim 1$ we have only several partons that are distributed in the hadronic disc. Let's take Q^2 such that

$$r_p^2 \approx \frac{1}{Q^2} \;\; \ll \;\; R_h^2. \qquad (10)$$

The distance between partons in the transverse plane is much larger than their size. We can neglect the interaction between partons. The only process which is essential here is the emission of partons that is taken into account in the usual evolution equation.

2. For smaller x the number of partons increases and at some value of $x = x_{cr}$ partons start to populate densely the whole disc of the hadron.

3. For $x < x_{cr}$ partons must overlap spatially and begin to interact in a whole disc that they occupy. For such small x- values the processes of recombination and annihilation of parton should be as essential as well as their emission. However these processes were completely omitted both in the GLAP and in the BFKL evolution equations.

The kinematical region $x < x_{cr}$ is free for any phenomenological hypothesis, but there is enough experience with some models [4] to suggest the so called **saturation of the parton density** [3]. It implies that the parton density ϕ , which we discussed before is equal to unity in this domain, so the parton distribution for $x_B < x_{cr}$ looks as shown in Fig.2.

3 Why region of small x_B is important?

After this brief review of the theoretical problems at $x_B \to 0$ I hope that you will able to accept my firm belief that the small- x physics is the most interesting and difficult problem in QCD. Only the periphery of this problem can be touched in perturbative QCD, while its kernel is a nonperturbative one.

• At $x \to 0$ we deal with a dense system of partons in the weak coupling limit in which the interactions between partons become large due to the high density of partons in spite of the fact that the coupling constant α_s is small here. The theoretical understanding of such a system of partons which will be probed experimentally at the new generation of accelerators (HERA, LHC and SSC), is relevant not only to the low -x_B behaviour of deep inelastic structure functions but also to the structure of typical inelastic event produced at high energy hadron- hadron interactions, to the understanding of nuclear shadowing as well as to the problem of baryonic number nonconservation in the electroweak theory at high energies.

• We cannot even start to discuss the structure of the typical inelastic event at high energy without understanding the behaviour of the deep inelastic structure function at $x_B \to 0$.

• Therefore HERA is a calibration experiment for new generation of the accelerator such as LHC and SSC.

4 Microreview of the theoretical situation with deeply inelastic scattering.

At present we understand that the whole kinematical region of deeply inelastic scattering can be divided in three separate parts as shown in Fig.5.

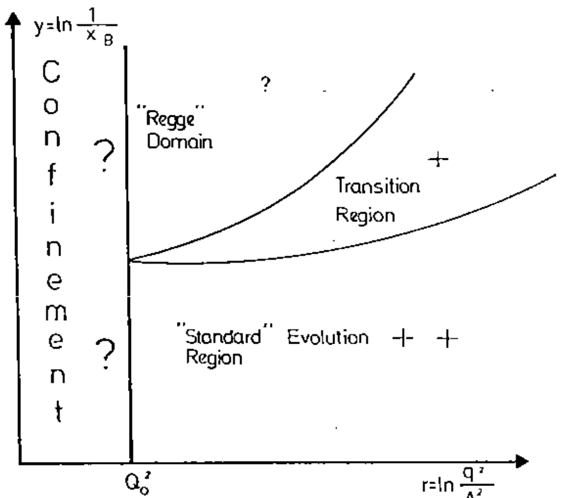

Fig. 5: Kinematical subregions for the deeply inelastic scattering.

Each of these subregions has a different underlying physics and a different level of theoretical understanding. I shall discuss the situation in each subregion separately.

4.1 "Standard" evolution region.

This is a region with very large value of transferred momentum Q^2 and moderately small values x. Deeply inelastic processes reveal here properties typical for hard processes. Namely:

1. The total cross section for virtual photon absorption is very small, $\sigma(\gamma^* N) \ll \alpha_{e.m.} \cdot \pi R_h^2$ where πR_h^2, is a typical area of the hadron. It falls down as inverse power of Q^2 at large values of Q^2 ($\sigma(\gamma^* N) \propto \frac{1}{Q^2}$).

2. We dispose of a transparent physical language to discuss the deeply inelastic process, namely, the parton language, specially concieved for this process.

3.In this region we can apply the leading log approximation (LLA) of perturbative QCD, which leads to a linear evolution equation for deep inelastic structure function. This equation is not the same as the Gribov-Lipatov- Altarelli-Parisi one, but it is known quite well.

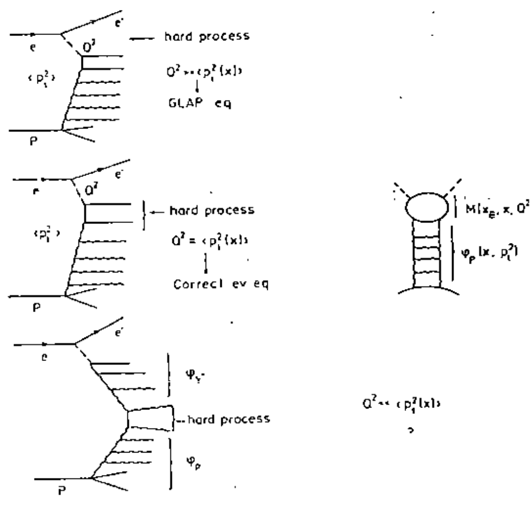

Fig.6: The deep inelastic e p scattering in different kinematical regions. The difference comes from the fact that we have **two scales of the hardness**

in deeply inelastic scattering , namely the value of Q^2 and the value of the mean transverse momenta of the emitted gluons q_t^2. So we cannot use the renormalization group approach in a such direct way as was done in the GLAP evolution equation and should first of all to study what scale works in the region of small x_B.

In Fig.6 we show the ep scattering in three kinematical regions where the different scales of hardness work.

1. If Q^2 is larger than the mean transverse momentum of gluon $\langle q_t^2 \rangle$ the hard cell of our process is the exchange of virtual photon. In this case we have typical deeply inelastic process and we can apply here the GLAP evolution equation.

2. If Q^2 is of the order of $\langle q_t^2 \rangle$ we have to calculate the interaction of the gluon with virtual photon more accurately since in this situation namely this interaction says us what scale works and what cell of our process is the hardest one. It could be not the exchange of virtual photon but the quark - antiquark production as shown in Fig.6 . Fortunately we know how to calculate the deep inelastic process in this region and correct amplitude ($M(x_B, x, Q^2, q_t^2)$ in Fig.6) together with integration over all variables was written in refs. [5]. Namely in this kinematical region we need to improve the GLAP equation.

Fortunately this work has been done by Italian group and Webber [6]. They managed to rewrite the emission of the parton in our basic branching process(see Fig.1) that it depends only on previous stage of emission. Such a form is very well suited for Monte - Carlo simulation and allows to get a numerical solution for the evolution of the parton cascade. Even more G.Marchesini [7] wrote an evolution equation for the unintegrated gluon structure function

$$xG(x, q^2) \;=\; \int d^2 q_t \phi(x, Q, q_t), \tag{11}$$

which looks as follows

$$\frac{\partial \phi(x, Q, q_t)}{\partial \log Q^2} \;=$$

$$= \int_x^1 dz\, [\, P(z) - \frac{N_c \alpha_s}{\pi z)} \cdot \frac{1}{z}] \cdot \phi(\frac{x}{z}, \frac{Q}{z}, q_t - \frac{1-z}{z} Q) \;+$$
$$+ \int_x^1 dz (\frac{N_c \alpha_s}{\pi z} \Delta_{ns}(z, \frac{Q}{z}, q_t)) \cdot \frac{1}{z} \cdot \phi(\frac{x}{z}, \frac{Q}{z}, q_t - \frac{1-z}{z} Q) \tag{12}$$

where $P(z)$ is the usual GLAP kernel and $\Delta_{ns}(z, \frac{Q}{z}, q_t))$ is equal to

$$\Delta_{ns} \;=\; \exp(-\frac{N_c \alpha_s}{\pi} \int_z^1 \frac{dz}{z} \int_{z q_t^2}^{Q^2} \frac{dk^2}{k^2}) \,.$$

It is very difficult to solve this evolution equation analytically. It is quite not a Volterra type of equation and in some sense not even evolution equation since

it depends on two scales. However this equation turns into the GLAP equation for structure function for x of the order of 1 and into the FKL equation (and the GLR generalization) for the function ϕ in the region of small x. What is most important is that this correct equation provides a smooth matching of all equations that have been discussed and describes the evolution of the parton cascade in the whole "standard" evolution region in a unique way.

It is worthwhile to mention that this correct equation provides energy conservation for deeply inelastic processes, which otherwise is violated by order of α_s both in the LL(x)A and in the DLA. This is so because the correct equation describes the region of $x \approx 1$ as well as the low-x region.

3. For the case $\langle q_t^2 \rangle \gg Q^2$ the hard cell is somewhere in the middle of our parton chain (see Fig.6). It means that in this kinematical region the deeply inelastic process looks rather as the high q_t jet production in hadron- hadron collisions than the probe of the parton contens of the target nucleon. We have all ingredients to write the correct formulas for this region such as the structure functions of the partons inside the virtual photon . However this job has not been done yet.

Thus concluding this section I can claim that by now we able to include two scales of hardness in deep inelastic scattering both in the amplitude of the interaction of the virtual photon with the partons [5] and in the correct evolution equation [7]. So I am glad to say that this region now is in the best theoretical shape.

4.2 Transition region.

In the transition region (see Fig.5) the situation changes crucially:

1. The total cross section $\sigma(\gamma^* N)$ becomes large and, near the border with "Regge" domain, even compatible with the geometrical size of the hadron at small x . It means that $\sigma(\gamma^* N) \to \alpha_{e.m.} \cdot \pi R_h^2$. In this kinematical region it depends only smoothly on $\log Q^2$, that is $\sigma(\gamma^* N) \propto F(\log Q^2)$.

2. The parton language can be used to discuss the main properties of our process but the interactions between parton become important. This interaction induces substantial screening (shadowing) corrections.

3. Fortunately, in this particular kinematical region the screening corrections are under theoretical control. We may go beyond the LLA and write the correct evolution equation, which becomes nonlinear. As we have discussed the number of partons in the parton cascade becomes extremely large at $x \to 0$, since each parton can decay in many daughter partons. For so small x the structure of the parton cascade should be the result of competition of two main processes: the emission of gluons that is proportional to the density of partons (ϕ) with fixed transverse momentum and the annihilation of gluons with probability proportional to ϕ^2 (see eq.(1) for the relation between ϕ and $xG(x, Q^2)$).

The simple parton picture allows to write the equation that takes properly into account these processes. Indeed, the number of partons in a cell of the phase space $(\Delta y.\Delta \log Q^2)$ increases due to emission and decreases as result of annihilation. As an outcome the balance of particles for this cell looks as follows:

$$\frac{\partial^2 \phi}{\partial y \partial \log Q^2} \;=\; \alpha_s \phi \;-\; \mathrm{Const} \cdot \alpha_s^2 \phi^2. \qquad (13)$$

Here the total cross section for gluon - gluon annihilation was written as $\sigma(GG \rightarrow G) = \mathrm{Const}\, \alpha_s^2$.

We can rewrite this equation in terms of $xG(x,Q^2)$:

$$\frac{\partial^2 xG(x,Q^2)}{\partial \log \frac{1}{x} \partial \log Q^2} \;=\; \alpha_s xG(x,Q^2) \;-\; \alpha_s^2 \cdot \frac{\mathrm{Const}}{Q^2 R_h^2} \cdot (xG(x,Q^2))^2. \qquad (14)$$

The equation is exactly the same as one we has suggested many years ago from analysis of Feynman diagrams in the DLA limit (see ref.[2]) . As it happens, even today the same kind of analysis is needed to calculate the value of Const (see ref. [8] where it was done) and to determine the kinematical region where such simple equation holds.

Two years ago we knew only a semiclassical solution of the nonlinear GLR equation.

By now we have accumulated enough experience in solution of the GLR equation and know the main properties of such a solution in the whole deeply inelastic scattering region. This was made possible, mostly due to the hard work of Bartels, Shuler and Blumlein[9], Kwiecinski, Martin and Sutton [10], Kim and Ryskin [11], Collins and Kwiecinski [12], Bartels and Levin [13], Collins and Landshoff [14]. Let me summarize their results.

1. The existence of the critical line that appeared in semiclassical solution was confirmed and the exact form of the critical line was calculated. This form in the subasymptotical kinematical region turnes out to be very different from the asymptotical one but coincides with it at very large values of Q^2.

2. They found the numerical and even semiclassical analytical solution to the left of the critical line . In this solution the parton density (ϕ) goes to the unitarity limit at very small x.

3. The solution to the right of the critical line does not depend on the solution to the left of it. In particular, it does not depend on our hypothesis on the confinement of quarks and gluons, which is still an open question in QCD.

New development.

Bartels [15] and Levin,Ryskin and Shuvaev [16] made the next step in the improvement of the GLR equation calculated the anomaluos dimension of the twist four gluon operator, using comletely different technique. . It turns out that the value of the anomalous dimension is equal to

$$\gamma_4 \;=\; 2\gamma_2(\frac{\omega}{2})[1 \,+\, \delta^2] \qquad (15)$$

where $\delta^2 \sim (\frac{1}{N_c^2-1})^2 \approx 10^{-2}$ is very small. I would like to discuss first several lessons that we have learned from this calculation:

1. Eq. (15) confirms the main hypothesis of ref. [2] that the small x_B behaviour of the deep inelastic structure function is the determined by the exchange of many Pomeron in t-channel and their interaction.

2. The smallness of δ mentioned above reflects the smallness of pomeron - pomeron interaction which is nonplanar and proportional to $\frac{1}{N_c^2-1}$.

3. Strictly speaking the pomeron-pomeron interaction was not taken into account in the GLR - equation.However the good news is the fact that the correction to the GLR equation is so small that they could give the noticeable contribution only at ultra high energies.

4. However the main theoretical conclusion from this exercise looks rather pessimistic because it was shown that QCD cannot cure the old problem of the reggeon approach that was pointed out in refs. [17], namely, the fact that pomeron cannot be correct first approximation at high energy interaction at least in the perturbative QCD. In other words the pomeron - pomeron interactions turns out to be attractive and the system of many pomerons cannot be stable. Of course we made only the fist step to study selfconsistently the above problem and the next one will be to consider the value of anomalous dimension of the higher twist operators,but there is no reason to think that the specific coherence effects in QCD will be able to help us for the twist $n > 4$ operators.

I think it is very instructive to understand the physical meaning of this result in terms of the evolution equation for the parton cascade (see eqs. (13) and (14)). We can suggest the new evolution equation taking into account the interaction of the pomerons for the twist four operator. It looks as the system of two equations which are simpler to write down in the integral- differential form form in DLA of the perturbative QCD.

$$\frac{\partial^2 xG(x,Q^2)}{\partial \log \frac{1}{x} \partial \log Q^2} \;=\; \alpha_s xG(x,Q^2) \;-\; \alpha_s^2 \cdot \frac{4\pi N_c^2}{(N_c^2-1)Q^2} \cdot (x^2 G^{(2)}(x,Q^2)). \qquad (16)$$

and

$$x^2 G^{(2)}(x,Q^2) \;=\; \frac{9}{8\pi R^2} \cdot (xG(x,Q^2))^2 \;+$$

$$+\frac{2N_c\delta\alpha_s}{\pi}\int_x^1\frac{dx'}{x'}\int^{Q^2}\frac{dk^2}{k^2}\int_{x'}^1\frac{dx"}{x"}\frac{x"}{x}G(\frac{x}{x"},\frac{Q^2}{k^2})\frac{x'}{x}G(\frac{x}{x'},\frac{Q^2}{k^2})x'^2G^{(2)}(x',k^2)-$$

$$-\frac{\alpha_s^2 g_{2\to 3}}{R^2}\int_x^1\frac{dx'}{x'}\int^{Q^2}\frac{dk^2}{k^4}(\frac{x'}{x})^2G^{(2)}(\frac{x}{x'},\frac{Q^2}{k^2})x'^2G^{(2)}(x',k^2)x'G(x',k^2). \qquad (17)$$

The first equation is the equation for the parton cascade which is written with better accuracy than eq.(14),since the probability for two parton to have the same x and Q^2 ($G^{(2)}$) was introduced (see ref. [8] where it was done first for details). The second one is new. From this equation you can see that in eq.(14) we assumed that there was no correlation between two gluons exept the fact that they are distributed in the hadron disc of the radius R. However the second equation shows that it is not true and the correlation increases untill the screening correction enters to the game and this growth will be stopped due to them. We can express the same physics saying that the correlation radius between two gluons increases with x_B and they create more compact system then the hadron.

4.3 "Regge" domain.

The most interesting, of all, is the "Regge" domain (see Fig.5) where new physics is anticipated.

1. The total cross section becomes huge, namely, $\sigma \propto \alpha_s \cdot \pi R_h^2 \cdot \frac{1}{\alpha_s}$.

2. Since the coupling constant of QCD remains small ($\alpha_s << 1$), we hope to be able to probe this region theoretically.

3. Unfortunately, because the parton density is so large in this region that the standard methods of perturbative QCD cannot be used.

This kinematical region escapes any theoretical control. There are plenty of ideas, but no well established theoretical results. The only good indication that we might have a chance to understand this domain in the future is the fact that the variety of ideas increases [18][19] [20][21] .

The working hypothesis in this domain is so called **saturation of the parton density** . It implies that the density of partons ϕ is equal to unity in the Regge domain. Saturation means that partons with an intrinsic size $r_G \sim \frac{1}{q_0(x)}$ are densely distributed in the hadron disc.

Perhaps the most important thing is that this hypothesis can be checked experimentally as the resulting behaviour of the deep inelastic structure function looks as shown in Fig.7. Saturation leads to $xG(x,Q^2) \propto Q^2$ in the Regge domain which induces a large scaling violation at low x.

The behaviour of $x_BG(x_B,Q^2)$ we can translate into the behaviour of the total cross section of virtual photon absorbtion as shown in Fig. 8. The saturation hypothesis says that the total cross section should have sufficiently broad transition

region where this cross section is moreless constant.For small virtuality of the photon we have developed phenomenology based on so called "soft" pomeron which leads to $\sigma_t \propto (\frac{1}{x_B})^\epsilon$ with the small value of $\epsilon \sim 0.008$. In the framework of the saturation hypothesis we are able to interprete this sufficiently smooth behaviour of the total cross section at small value of Q^2 as a result of large screening (shadowing) corrections on the gluon level. In this case the natural way to match the deep inelastic scattering with "soft" one is to assume the constituent quarks interact as two black discs. This assumption can describe the experimental data on hadron - hadron collisions [22] [23]. The most attractive features of this scenario is the unique description of the diffractive and inclusive process based on the properties of the shadowing correction in the perturbative QCD.

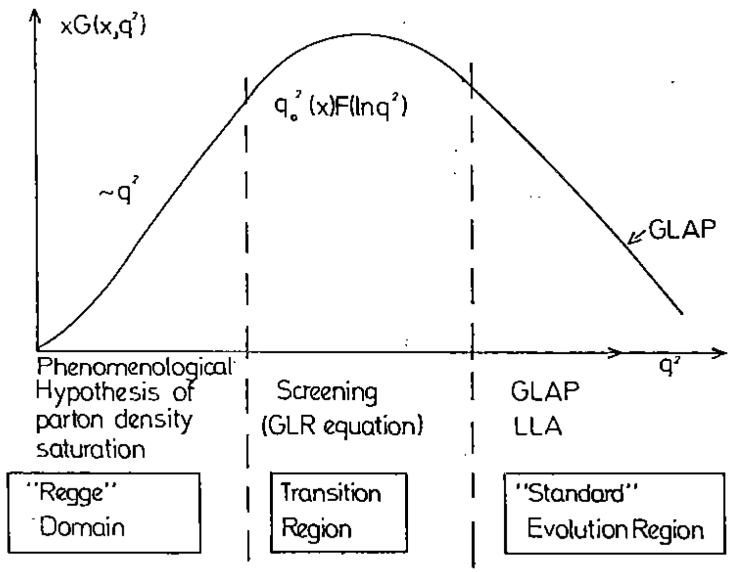

Fig.7: The behaviour of $x_BG(x_B,Q^2)$ versus Q^2 at fixed x_B.

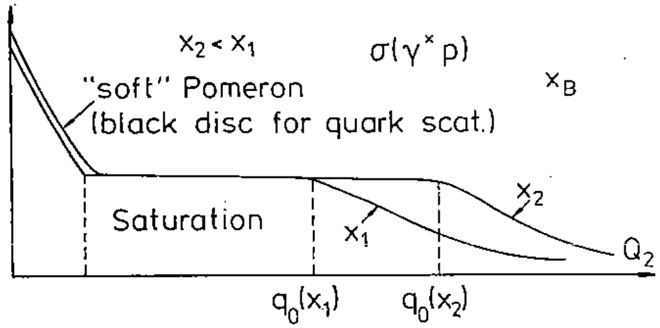

Fig.8: The behaviour of the total cross section of virtual photon absorbtion versus Q^2 at fixed x_B in the saturation hypothesis.

However I must admit that this scenario is in contradiction with elegant and very simple Landshoff picture for soft interaction [24] in which all experimental data has been described assuming the exchange one or two the "soft" Pomerons with intercept $\alpha_P = 1 + \epsilon$. This picture is rather in favour of the behaviour of the total cross section shown in Fig.9 with different physics for diffractive and inclusive processes at high energy.

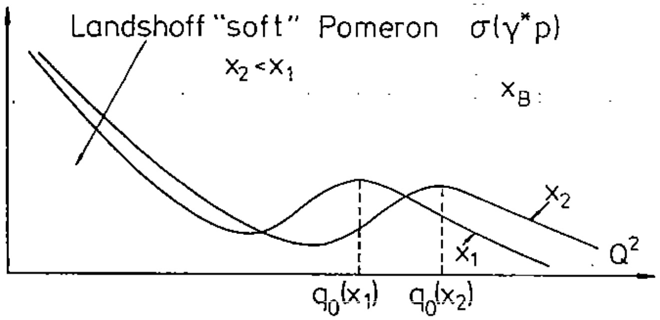

Fig.9: The behaviour of the total cross section of virtual photon absorbtion in the Landshoff picture for the" soft" pomeron.

Fortunately HERA will be able to clarify this question during the first several runs. The current experimemtal information tells us only that there is some transition region but cannot differentiate between two approaches (see ref.[25] for details).

5 Phenomenology.

I would like to restrict myself by only discussion of the estimates of the value of screening corrections in deeply inelastic scattering and the kinematical borders between different regions (see Fig.5). As you could see from GLR equation the value of screening corrections depends crucially on the value of radius R_h in the nonlinear term of the equation. So the main question arises, what we can say about this value. Unfortunately, we are not able to estimate R_h from theory and we can only address to our experiance with the models for structure of hadron,to common sense and to available experimental data concerning deeply inelastic scattering or hadron interaction.Now we have really two estimates for value of R_h, one pessimistic and one optimistic.

Pessimistic estimates.

This way of thinking looks even very natural. Let us suppose that R_h is equal to the radius of proton ($R_N \sim 1\,Fm$). It means that gluons are distributed uniformly in the whole hadron disc, as shown in Fig.10a. In this case the screening correction turnes out to be very small at least at $x > 10^{-4}$. I think the best presentation of this approach one can find in the Mueller's talk at DESY low-x workshop [26].

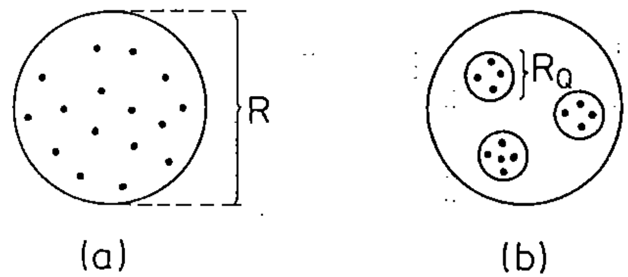

Fig.10: The gluon distribution in the transverse plane.

Optimistic approach .

In this approach we assume that gluons in a hadron are confined in the disc with smaller radius than the size of the hadron (see Fig.10 b). This picture of hadron of course remind us the old days of constituent quark model, which nicely described main property of hadrons and their interaction. Such a picture leads us to sufficiently large value of screening correction, which depends crucially on the value of radius for these "hot spots" (constituent quark) inside of hadron. In our estimates we extract from hadron production experiments that R_Q is of the order of 0.2 Fm [2] while Kwiecinski, Matrin and Sutton [10] used to traditional value of

R_Q for constituent quark, namely, $R_Q = 0.4$ Fm .

The difference between two approaches you can see in Fig.11 which I took from KMS paper [10]. So in the optimistic picture we have a good chance to see the screening correction at HERA experiments.

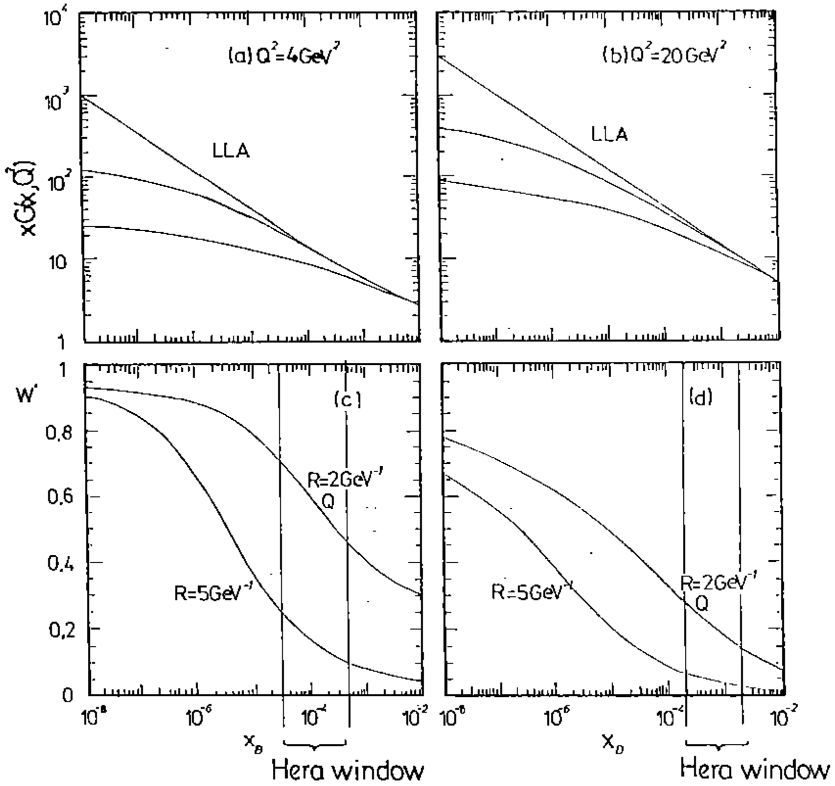

Fig.11: The KMS estimates [10] for the ratio of the nonlinear term to the linear one in the GLR equation (see eq.(14)).

6 HERA physics.

In this section I am going to discuss what new phenomena could be seen at HERA. The above discussion we can summarize saying that at HERA we hope to see:

1. The increase of the parton density at $x_B \to 0$

$$F_2(x_B, Q^2) \propto (\frac{1}{x_B})^{\omega_0}$$

with $\omega_0 \propto \alpha_s$.

2. The new scale of hardness in the deep inelastic process,namely $\langle p_t^2(x_B)\rangle$ instead of Q^2.

3. The large value of the shadowing (screening) correction.

The problem is how to measure these new phenomena, since HERA has very restricted kinematical region. The present estimates (see " Physics at HERA" proceedings of the workshop,Hamburg,Oct. 29 -30, in particular ref. [27]) of the behaviour of the deep inelastic structure functions show us that it is very difficult to differentiate the contribution of the screening correction from the different initial parton distribution at $Q^2 = q_0^2 \sim 4GeV^2$. It means that the classic deep inelastic measurement will be certainly not very decisive at HERA. So we need to suggest new experiments in which new phenomena give more pronounced signatûre. I would like to discuss here the ideas of four such experiments to illustrate the four different ways how the new physics could be seen in the deep inelastic experiments.

6.1 Energy correlation in the deep inelastic scattering.

The first idea is to look on the structure of the typical inelastic event in the deep inelastic scattering with the hope to find some correlations between produced particle that crucially and qualitatively depend on the main properties of the new physical phenomena in the region of small x_B.

First of all I would like to draw your attention to the fact that in inclusive processes there is a lot of the secondary partons that are emitted in the final stage of the deep inelastic process. Of course such partons do not contribute to the total cross section or in other words to the deep inelastic structure function. However, namely emission of these parton is responsible for the value of the inclusive cross section and leads to infrared unstable answer for it. It means that we have to look for correlations that are infrared stable and do not depend on the hadronization stage of the process that is out of the theoretical control. We are only in the beginning of this study. Here I would like to mention that the basic theoretical paper on the inclusive production is ref. [28]. By now we can discuss only one correlation in the deep inelastic process,namely the energy correlation between two produced hadrons (see ref.[29]).

The main idea of the experiment looks as follows. Let us measure the correlation function for two hadron

$$R(y_1, y_2) \;=\; \frac{\frac{1}{\sigma_{tot}} \cdot \frac{d\sigma}{dy_1 dy_2}}{\{\frac{1}{\sigma_{tot}} \cdot \frac{d\sigma}{dy_1}\}\{\frac{1}{\sigma_{tot}} \cdot \frac{d\sigma}{dy_2}\}} \;-\;,\; 1,$$

where

$$\frac{d\sigma}{dy} \;=\; \int_{p_0^2}^{Q^2} \frac{d^2\sigma^{jet}}{dy d^2 p_t} d^2 p_t$$

and $y_1 = \ln(1/x_1)$, $y_2 = \ln(1/x_2)$. Function R is infrared stable with respect to cuttof p_0. Let me list the main property of function R:

1. R$<$ 0 due to ordering in the transverse momenta of emitted partons in usual GLAP evolution equation without screening corrections.

2. R peaks at small value of $\Delta y = |y_1 - y_2|$due to possibility that both hadrons are produced from one parton jet.

3. Screening correction gives large contribution to R, much larger than to the deep inelastic structure function.

The first numerical estimates show that the above features of R can be measured at HERA [29].

6.2 "Hot spot" hunting.

Mueller suggested the beautiful idea of the experiment in which we can measure the main properties of low x_B deep inelastic processes in the situation when they give a big effect [26] Let us measure the inclusive production of gluon jet with transverse momentum p_{jt} which is very close to Q and with fraction of energy as closer to one as possible to provide the small value of x_B/x_j .In this case

1. the cross section of this process depends on x_B only due to low x gluon emission while in GLAP approach this cross section is constant with respect to x_B.

2, the scale of the absorption correction is determinated by the size of the "hot spot",namely $R \propto \frac{1}{p_{jt}}$ and so should be large.

More close investigation of the problem [30] shows that HERA has a chance to see this process and distinguish the smooth GLAP behaviour from the steep BFKL one.

6.3 Diffraction dissociation of virtual photon.

The next idea is to measure at HERA the diffraction dissociation of photon or the reaction

$$\gamma^* p \;\rightarrow\; X \;+\; p(x_F, p_t).$$

The most important advantage of this reaction is the fact that the cross section is proportional to $(xG(x,))^2$ where x = 1 - x_F . So the theoretical prediction for diffraction dissociation is very sensitive to the value of the gluon structure function and its behaviour at small x[31].It means that HERA will be able to measure the scale of the screening correction as well as the increase the gluon density at low x_B in the above reaction.

Ingelman and Janson - Prytz [32] proposed to measure the value of the screening correction in this process using the conception of the Pomeron structure function, suggested in ref.[33]. The physical idea is very clear. Indeed the radius of the soft pomeron turnes out to be very small ($R_P \propto 0.1 Fm$.It means that the GLR equation induces the large screening correction for the Pomeron structure function which could be measured at HERA. Of course, at the moment we do not understand how reliable theoretically the pomeron structure function approach but Ingelman-Jansen-Prytz"s idea illustrates the big difference between diffraction dissociation process and the standart deep inelastic one. So I firmly believe that the the measurement of the diffraction dissociation at HERA will provide deeper insight in the structure of the pomeron and interrelation between " soft" and "hard" processes.

7 Conclusions.

I hope I have convinced you that

1. between "soft" and "hard" processes there is no "holy water" but very specific processes (so called "semihard") for which the typical transverse momentum of produced parton increases with energy. It gives the possibility to study these process in the framework of the perturbative QCD. Not all properties of " semihard" processes have been understood but we are certainly on the way to deeper theoretical inspection of them.

2. the statement of Wilczek that the deep inelastic scattering is 1/2 solved means that we can provide the reliable theoretical prediction only in the kinematical region to the right of the border between "Regge"domain and the transition region (see Fig.5).

3. at the moment we have reached such understanding of the basical problems in the deeply inelastic scattering at low x that we can formulate the nearest steps to solve them.

Let me list here the most important problems that are needed to be be solved.

1. We need need transparent physical language to discuss all phenomena in the saturarion region where we cannot apply the parton picture. I hope, that we can guess this language studing the kinetic equation for partons inside the parton cascade in the transition region.

2. Systematical theoretical study of multipartical production in the deeply inelastic process.

At the moment there are only several theoretical papers devoted to this problem [28] while the coming HERA experiment will provide a lot of information on inclusive production in the deeply inelastic scattering.

3. Consistent calculation of the next order correction to correct evolution equation in the deeply inelastic scattering as well as to inclusive and to exclusive processes.

Only such a calculation might provide the predictions which are independent of the renormalization scale and answer the questions what is the scale for the running coupling constant of QCD or what is the value of the cross section for jet or heavy quark production (so called K-factor).

4. Systematic study of the higher twists contribution to the deep inelastic structure function.

This project should contain the solution of the evolution equation for the higher twist structure function and also the study of different models of the hadron structure to specify the form of the matrix element of the operator of the higher twists. The first result shows that we can expect a lot of unexpectable results in this project that will be able to move us to much deeper understanding of deep inelastic processes as well as the matching them with "soft" ones.

For the smooth transition to the next talk and as my comment to all theoretical talks at this conference I would like to conclude my talk with the words of the most famous Englishmen at least in my country:

"The temptation to form premature theories upon insufficient data is the bane of our proffession " (Sherlock Holmes).

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