a) for the singlet state of the nucleon pair

$$(3.4) \qquad <0\ \ 0;\ 0\ \ 0\ |T|\ 0\ 0;\ 1\nu> \ = \delta_{\nu,o}\ i\ a_g\ k\ [\psi(\vec{p}_5)-\psi(-\vec{p}_4)]$$

$$(3.5) \qquad <1\ T_z;\ 0\ 0\ |T|\ 0\ 0;\ 1\ \nu> \ = \delta_{\nu,o}\ i\ b_g <N\ N\ |\ \bar{\tau}.\bar{\tau}_4|\ D> \times$$
$$\times\ K\ [\psi(-\vec{p}_5)+\ \psi(-\vec{p}_4)]$$

b) for the triplet

$$(3.6) \qquad <0\ 0;\ 1\ \nu'\ |T|\ 0\ 0;\ 1\ \nu> \ = <\nu'|a_f+i\ \sigma_{4z}\ a_g|\nu> \times$$
$$\times\ K\ [\psi(-\vec{p}_5)+\psi\ (-\vec{p}_4)]$$

$$(3.7) \qquad <1\ T_z;\ 1\ \nu'\ |T|\ 0\ 0;\ 1\ \nu> = <NN|\vec{\tau}.\vec{\tau}_4|D> \times$$
$$\times <\nu'|b_f+\ i\ \sigma_{4z}\ b_g|\nu'> \times$$
$$\times\ K[\psi(-\vec{p}_5)-\psi(-\vec{p}_4)]$$

where $K = (16\ \pi^2\ m_1)^{1/2}$ and the isospin reduced matrix element on the right hand side $<NN|\bar{\tau}.\bar{\tau}_4|D> = <1\ T_z;\ \ \tau\ \tau_z'|\ \bar{\tau}.\bar{\tau}_4|0,0;\tau\ \tau_z>$; in the latter τ_z, τ_z' represent the isospin third component in the initial and final state of the meson. Here we have neglected the D-state of the deuteron; but the above formula can easily be generalized to include it.

The differential cross section for the three body process as function of the angle of particle 3 in the center of mass of the particles 3 and 4, or 3 and 5 is (*)

$$(3.8) \qquad \frac{d\sigma}{d\Omega^*} = \frac{1}{\Phi}\ \sum_f \bar{\sum_i}\ |T_{fi}|^2\ \frac{dV^{(3)}}{d\Omega^*}$$

or if we assume

$$(3.9) \qquad T_{fi} = \tilde{T}_{fi} \times [\psi(-\vec{p}_5)\ \pm\ \psi(-\vec{p}_4)]$$

we obtain, integrating on all variables except Ω^*

$$(3.10) \qquad \frac{d\sigma}{d\Omega^*} = \sum_f \bar{\sum_i}\ |\tilde{T}_{fi}|^2 \omega^{\pm}$$

(*) We call this angle Ω^*, from now onwards where

$$\omega^{\pm} = \frac{K^2}{\Phi} \int |\psi(-p_5) \pm \psi(-p_4)|^2 \frac{dV^{(3)}}{d\Omega^*}$$

The ambiguity in the definition of the system where Ω_3^* is defined is easily found to be only apparent: the weight factor $\omega^{\pm}$ can be expressed in the following way, using the symmetry of the invariant phase space volume for the exchange of p_4 with p_5

$$(3.11) \qquad \omega^{\pm} = \frac{2K^2}{\Phi} \left[\int |\psi(p_5)|^2 \frac{dV^{(3)}}{d\Omega^*} \pm \int \psi(p_4)\psi(p_5) \frac{dV^{(3)}}{d\Omega^*} \right]$$

The second term has an integrand function, still symmetric for the interchange of p_4 with p_5: therefore the ambiguity is completely eliminated. We can choose for instance the C.M.S. of the particles 3 and 4 to calculate Ω^*. To complete our task, to express the differential cross-section (3.10) in terms of the C.M. amplitudes, we perform the Lorentz transformation (2.17) of the amplitudes. For this purpose we calculate separately the charge exchange and the quasi-elastic scattering.

$$(3.12) \qquad \left.\frac{d\sigma}{d\Omega^*}\right|_{CEX} = |\langle NN|\bar{\tau}.\bar{\tau}_4|D\rangle|^2 \left[\frac{|b_g|^2}{3} \omega^+ + (|b_f|^2 + \frac{2}{3}|b_g|^2)\, \omega^- \right]$$

$$(3.13) \qquad \left.\frac{d\sigma}{d\Omega^*}\right|_{QE} = \frac{|a_g|^2}{3} \omega^- + (|a_f|^2 + \frac{2}{3}|a_g|^2)\omega^+ + |\langle NN|\bar{\tau}.\bar{\tau}_4|D\rangle|^2 \times \left[\frac{|b_g|^2}{3} \omega^+ + (|b_f|^2 + \frac{2}{3}|b_g|^2)\, \omega^- \right]$$

We will go through all steps, only for the charge exchange cross section, while for the quasi elastic scattering we give later on, the final result, in order to avoid useless repetitions. We substitute now, to the laboratory amplitudes, the expression in terms of the C.M. amplitudes, taking in account the invariance property (2.20)

$$(3.14) \qquad \left.\frac{d\sigma}{d\Omega^*}\right|_{CEX} = |\langle NN|\bar{\tau}.\bar{\tau}_4|D\rangle|^2 \left[\frac{|\beta_g|^2}{3} \omega^+ + (|\beta_f|^2 + \frac{2}{3}|\beta_g|^2)\, \omega^- + C \right]$$

where

$$(3.15) \qquad C = \frac{|\beta_f|^2 - |b_f|^2}{3} (\omega^+ - \omega^-)$$

C is the relativistic correction to the traditional formula ([4]). This correction seems to be important only near the forward direction, because for high momentum transfer is negligible since $\omega^+ \sim \omega^- \sim 2\frac{K^2}{\Phi}$. But around the forward direction as from (2.. $\beta_f \sim b_f$, therefore the correction is very small and will be neglected for the mom...

The last step is to change the normalization of the C.M. amplitudes, such as to connect the usual formalism of the phase shift analysis

(3.16)
$$\frac{d\sigma}{d\Omega^*}\bigg|_{CEX} = \frac{|\langle NN|\vec{\tau}.\vec{\tau}_4|D\rangle|^2}{|\langle N|\vec{\tau}.\vec{\tau}_4|N\rangle|^2} \times \left[\frac{|g^{CEX}|^2}{3} W^+ + \left(|f^{CEX}|^2 + \frac{2}{3}|g^{CEX}|^2\right) W^-\right]$$

$$\frac{d\sigma}{d\Omega^*}\bigg|_{CEX} = \frac{1}{2}\left[\frac{|g^{CEX}|^2}{3} W^+ + \left(|f^{CEX}|^2 + \frac{2}{3}|g^{CEX}|^2\right) W^-\right]$$

where

$$f^{CEX} = \frac{1}{2}[f_1 - f_0] \qquad \text{for i-spin } 1/2$$

and

$$f^{CEX} = \frac{\sqrt{2}}{3}[f_3 - f_1] \qquad \text{for i-spin } 1$$

and analogously for the spin flip amplitude.

(3.17)
$$W^{\pm} = \frac{d\Omega^*}{dV^{(2)}} \Phi^{(2)} \omega^{\pm}$$

The factor 1/2 is the squared ratio between the isospin matrix element for the deuterium and for the nucleon; that is

$$\langle PP|\vec{\tau}.\vec{\tau}_4|D\rangle = \frac{1}{\sqrt{2}} \langle P|\vec{\tau}\,\vec{\tau}_4|N\rangle$$

where

$$\langle P|\vec{\tau}.\vec{\tau}_4|N\rangle = \langle \tau\,\tau'_z; 1/2\ 1/2\ |\vec{\tau}.\vec{\tau}_4|\tau\,\tau_z;\ 1/2\ 1/2\rangle$$

the above relation in general and holds for K, $\bar{K}$ and π.

The relation between the conventional amplitudes f^{CEX}, g^{CEX} and the amplitudes β_f, β_g

$$\beta_f \langle P|\vec{\tau}_1\vec{\tau}_4|N\rangle \sqrt{\frac{dV^{(2)}}{d\Omega^*}} \frac{1}{\Phi^{(2)}} = f^{CEX}$$

Going through the same steps, we find for the charge preserving scattering

(3.18)
$$\frac{d\sigma}{d\Omega^*} = \frac{|g^+|^2}{3} W^- + \left(|f^+|^2 + \frac{2}{3}|g^+|^2\right) W^+ + $$
$$+ \frac{|g^-|^2}{3} W^+ + \left(|f^+|^2 + \frac{2}{3}|g^-|^2\right) W^-$$

where for isospin 1/2

$$f^{+} = \frac{1}{4}(3 f_1 + f_0)$$

$$f^{-} = \frac{1}{4}(f_1 - f_0)$$

and for isospin 1

$$f^{+} = \frac{1}{3}(2f_3 + f_1)$$

$$f^{-} = \frac{1}{3}(f_3 - f_1)$$

The connection with the α and β amplitudes is

$$f^{+} = \sqrt{\frac{dV^{(2)}}{d\Omega^*}} \frac{1}{\Phi(2)} \alpha_f$$

$$f^{-} = \sqrt{\frac{dV^{(2)}}{d\Omega^*}} \frac{1}{\Phi(2)} \beta_f \langle N|\bar{\tau}_1\bar{\tau}_4|N\rangle$$

In the charge preserving differential cross section, there are no factors coming from the isospin matrix elements because the matrix elements for the bound nucleon are equal to the ones for free nucleon

$$\langle NP|\bar{\tau}.\bar{\tau}_4|D\rangle = \frac{1}{2}(\langle P|\bar{\tau}.\bar{\tau}_4|P\rangle - \langle N|\bar{\tau}.\bar{\tau}_4|N\rangle) = \langle P|\bar{\tau}.\bar{\tau}_4|P\rangle$$

The $W^{\pm}$ are the weight factors calculated by Stenger et al. (5). He uses certain approximations to calculate the kinematics and the factor $\frac{dV^{(3)}}{d\Omega^*}$, which will be examined in detail later on. The result of Glauber and Franco (4) is found by noticing that the three body invariant phase space volume element contains the two body phase space volume element

$$dV^{(3)} = dV^{(2)} \frac{d^3 p_5}{(2\pi)^3 2E_5} \tag{3.21}$$

This volume element is calculated with the exact kinematics with $p_5 \neq 0$: however it is easy to convince oneself that it is a slowly varying function of p_5, compared with the wave function, and can be calculated for $p_5 = 0$, taken out of the integration symbol and cancelled (*): then

$$W^{\pm} = \frac{\Phi^{(2)}}{\Phi} \frac{2K^2}{2(2\pi)^3} \left[\int \psi^2(p_5) \frac{d^3p_5}{E_5} \pm \int \psi(p_4)\psi(p_5) \frac{d^3p_5}{E_5}\right] \tag{3.22}$$

(*) This implies that the definition of Ω^* is given in the situation $p_5 = 0$: as shown by Stenger (5) this makes a very small difference.

As

$$\Phi^{(2)} = 4\ p_2\ m_5$$
$$\Phi = 4\ p_2\ m_1$$
$$E_5 \sim m_5$$

$$(3.23) \qquad W^{\pm} \sim 2\left[\int \psi^2(p_5)d^3p_5 \pm \int \psi(p_4)\psi(p_5)d^3p_5\right] = 2\ [1 \pm S(\Delta)] \qquad \text{(closure)}$$

if $\qquad \vec{\Delta} = \vec{p}_2 - \vec{p}_3$

where S is the deuteron form factor

$$S = \int \psi\ (\vec{p}_5)\psi(\vec{p}_5-\vec{\Delta})d^3p_5 = \int e^{i\vec{\Delta}\cdot\vec{r}}\ \psi(\vec{r})d^3r$$

The result of Jew (6) is obtained taking into account the Fermi motion in the factor $\frac{d\Omega^*}{dV(^2)}\ \Phi(^2)$, that is considering this kinematical factor for an inelastic scattering of the incident particle on a light nucleon ($m^{*2} \cong m^2 - 2\ p_5^2$).

In the interference term there are choice problems: we assume that the scattering occurs on the particle 4 like in the first diagram of Fig. 1; this is not a bad approximation, as the overlapping between the two diagrams is important when p_4 and p_5 have similar values. In this way the cancellation occurs inside the integral and we obtain

$$(3.24) \qquad W^{\pm} = 2\ m\left[\int \psi^2(p_5)\ \frac{\Phi^{(2)}}{\Phi}\ \frac{d^3p_5}{E_5} \pm \int \psi(p_4)\psi(p_5)\ \frac{\Phi^{(2)}}{\Phi}\ \frac{d^3p_5}{E_5}\right]$$

Here the first definition of the energy (3.2a) was implicitly chosen and the flux factor is

$$(3.25) \qquad \Phi^{(2)} = 4\sqrt{(p_2p^*)^2-(m_2m^*)^2} = 4\sqrt{p_2^2(m^{*2}+p_5^2)+\ p_2^2p_{5\,z}^2+2E_2\sqrt{m^{*2}+p_5^2}p_2p_{5\,z}}$$

As seen from the above formula the factor is strongly dependent on the angle of p_5 with the beam direction. This variation is actually seen in the experimental data (7), as it will be discussed in the next section. Since only this definition of the weight factors has this asymmetry built in, we believe that this is a good reason to prefer this last one respect to others.

The same procedure can be followed in the C.M. system of the particle 3 and 4: if we neglect the relativistic correction of the deuteron, or in other words we mantain the rule for the deuteron vertex, given in Sec. 2, we arrive to the same formula for the differential cross-section (3.14), with the difference that C is rigorously zero. However the main difference is that we don't need to make the approximation that nucleon 5 is at rest in the laboratory system, to simplify the spin structure of the amplitude: this is very simple and uniquely defined in the C.M. system.

4. - NUCLEON MOMENTUM DISTRIBUTIONS

Before actually doing the integration on the phase space and to discuss the behaviour of the weight factors with the scattering angle, we study here the features of the integrand function

$$E^{\pm} = |\psi(p_4) \pm \psi(p_5)|^2 \qquad (4.1)$$

that we call extraction factor. We will think for the moment that particles 4 and 5 are physically distinguishable, as it actually occurs for a charge preserving process, where we have a neutron and a proton in the final state. In this case we can keep the form (4.1) for the extraction factor and study its behaviour, as function of $\mathbf{p}_5 \equiv (p_5, \vartheta_5, \varphi_5)$: Fig. 3 and 4 represent the perspective of the function $E^{+} \cdot dV^{(3)}/d\Omega_3 \; dp_5 \; d\Omega_5$ for $\varphi_5 = \pi$, $\cos\vartheta_3^* = 0$ and $\cos\vartheta_3^* = .9$ in the process $K^+D - K^\circ pp$ at .6 GeV/c. In both drawings we see a moraine and a peak: the first is the region where the particle 5 has small momentum and therefore is called spectator, the second is characterized by large momenta of particle 5, which is recoiling after having interacted with the incident particle. While the first is characterized by a negligible depencence on ϑ_5, the second is highly dependent on the angle. The top of the peak (socalled "quasi-elastic peak") is moving with the scattering angle and its coordinates p_5, ϑ_5 represents the recoil momentum of the corresponding two body process: in other words, with a hydrogen target we would obtain a peak, much thinner, exactly in the same position.

It is seen from the drawing at large angles that there is no overlapping between the two bumps: in experimental terms at large angles, we know that the slower nucleon is the spectator. However at small angles there is overlapping or, in other words, the wave character of the phenomenon shows off: therefore it is not possible to make the choice of the spectator, in the same way as in the two slits experiment it is not possible to determine the slit crossed by the particle. The range of angles, where this difficulty occurs, is obviously energy dependent: it tends to become more and more limited with raising energy.

One can ask oneself: why the choice of the spectator is necessary? The answer is that we want to determine the energy and the C.M. angle of the two body process for any event. This can be done, only if we assume a spectator model and know the momentum of the spectator. In the forward region, it is therefore not possible to define the kinematics of the two body process, and the analysis has to be done in terms of the external variables, as for instance the invariant momentum transfer.

However, if we are interested in the whole angular distribution, we can forget about this problem and define the "spectator": the procedure is to give a definition for it and to check afterwards its momentum and angle distribution. We define it as the nucleon which has lower energy. If we consider the distribution of events on the momentum of the nucleon satisfying this condition, as shown of Fig. 5 for the case K^+D--$K^\circ pp$ at .98 GeV/c, we realize that up to 250 MeV/c, the experimental spectrum is reproduced by the simple function depending only on the deuteron wave function and the kinematics

(4.2)
$$\frac{d\sigma}{dp_5} \sim E^- \frac{\Phi^{(2)}}{\Phi^{(3)}} \frac{p_5^2}{E_5} \qquad (*)$$

The region of high momentum (so called "tail") is however showing the still unexplained disagreement.

In the same way we can study the distribution in the spectator angle with the same model, for various momentum region. We consider to gain statistic, the forward-backward asymmetry for different momentum regions, that is the difference between the number of events with $0<\cos\vartheta_5\leq 1$ and $-1\leq\cos\vartheta_5\leq 0$, normalized with the sum of all events in the considered momentum bin. As shown in Table I, for .98 GeV/c the agreement between theory and experiment is satisfactory between 0. and .2 GeV/c, but surprisingly the agreement is broken in the .2 ÷ .3 GeV/c bin. On the "tail" region the agreement is restored, but it is not very significant because of the disagreement in the momentum distribution. If we go to higher energies, the simple function (4.2) tends to be not sufficient to explain the angular behaviour. The reason is that at 1.13 and 1.5 GeV/c the total charge exchange cross section is rapidly decreasing with energy and the energy of the K^+n system is very sensitive to the spectator angle

(4.3)
$$s = m_2^2 + m_4^2 + 2E_2E_5 + 2p_2p_{5z}$$

This fact is an indication that the analysis of these data should take in account of the Fermi motion.

TABLE I

The forward backward asymmetry as function of the momentum (in percentage).

p_5 GeV/c	p_2 = .98 GeV/c		p_2 = 1.13		p_2 = 1.5	
	Exp	Th	Exp	Th	Exp	Th
0 ÷ .1	10.0± 4.3	9.1	3.2± 4.8	8.9	0.0± 7.3	8.9
.1 ÷ .2	16.7± 9.8	19.1	4.3± 9.5	13.9	8.4±13.0	13.7
.2 ÷ .3	2.1±13.5	24.7	23.4±14.2	22.3	0.0±20.4	12.8
.3 ÷ .6	44.7±10.7	60.0	28.6± 9.7	51.7	42.9±15.3	43.2

(*) We choose E^- because we assume that the process occurs through a charge exchange without spin-flip (see (3.4),(3.7)). It is reasonable to assume that the amplitudes contribute in this distribution only as a normalization factor.

5. - COMPARATIVE DISCUSSION AND RESULTS

The present section is devoted to the analysis of the Stenger (5) procedure and to the comparison of his and our results. Eventually, we consider the weight factors of Jew (6) and compare it with ours.

The procedure of Stenger is very similar to ours: the main difference is in the normalization of the scattering amplitude, which doesn't affect the result for the cross-section:

$$t^s_{fi} = \frac{T_{fi}}{\sqrt{8E_3E_4E_5}} \sqrt{E_1E_2} \tag{5.1}$$

Therefore the kinematical factor is in our notation

$$\frac{\omega(k')}{k'^2} \frac{dE'_f}{dk'} \frac{k^2 d_k}{\omega(k)} P^2 \frac{dP}{d E_f} d\Omega_p = \frac{d\,\Omega^*}{dV(^2)} \frac{\Phi(^2)}{\Phi} \frac{dV(^3)}{d\Omega^*} k^2 \tag{5.2}$$

where $\vec{k} = \vec{p}$, $E_f = E_3+E_4+E_5$ and k', E_f are the corresponding quantities for the two body process on the nucleon at rest in the laboratory system, and $\vec{P} = \frac{\vec{p}_4-\vec{p}_5}{2}$

His approximation is to take out of the integration symbol in k and Ω_p, the phase space volume element and the flux factor for the two body process, but mantaining the dependence of the integration variables in the three body phase space volume element. The value of P is calculated for any value of k and Ω_p using energy conservation

$$E_f = m_1+E_2= E_3+E_4+E_5 = E_3+\sqrt{m^2+Q^2+P^2+2\ \vec{P}\cdot\vec{Q}}+\sqrt{m^2+Q^2+P^2-2\ \vec{P}\cdot\vec{Q}} \tag{5.3}$$

where $\vec{Q} = \frac{\vec{p}_4+\vec{p}_5}{2}$.

Q is thought fixed in the integration and determined with the two body kinematics on the nucleon at rest, from the C.M. scattering angle.

Squaring twice the relation (5.3), one obtains

$$P^2 = \frac{A^2 - 4\ B^2}{4(1-4C^2/A^2)} \tag{5.4}$$

where
$A = m_1+E_2-E_3$
$B^2= m^2+Q^2$
$C = Q \cos P\hat{}Q$

The main approximation of Stenger consists in neglecting the term $4C^2/A^2$ with respect to 1 and obtaining therefore for P the simple relation

$$P^2 = \frac{A^2 - 4B^2}{4} \tag{5.5}$$

This approximation allows the immediate integration on the angle Ω_p. He claims to obtain this relation simply expanding the square roots in (5.3) in terms of $2PQ \cos P\hat{}Q/(m^2+ +P^2+Q^2)$; but this expansion is possible only if the scattering angle is sufficiently small: in the backward direction $\vec{P} \sim \vec{Q} \sim \vec{p}_4/2 \sim \frac{\omega}{E+\omega} \vec{p}_2$ (where E, ω are energies of the meson and the nucleon in their C.M. system) therefore $\vec{P}\cdot\vec{Q} \simeq P^2$; the above expansion parameter becomes $2P^2/(m^2+2P^2)$, which is very small only if $m^2 >> 2\ P^2$.

This condition is obviously violated for high energy and we expect this procedure to fail in this region. To point out the dependence on the angle $P\hat{}Q$ of the value of P^2 we have inserted in the Stenger calculation the exact determination of P^2 in terms of various fixed values of cos (PQ) (Fig. 4).

As shown on the Fig. 6 the Stenger calculation corresponding to cos $P\hat{}Q = 0$ (dot-dashed curve) at .98 GeV/c for $K^+D \rightarrow K^\circ pp$ is very low in the backward direction. If we give to cos $P\hat{}Q$ values .5 and 1, (dashed and full curve respectively), the behaviour in the backward direction changes considerably and it tends to the exactly calculated curve (see Fig. 7). This last calculation is done using the same method of Stenger, that is

1) taking out of the integration the term $\frac{d\Omega^*}{dV^{(2)}} \Phi^{(2)}$

2) determining the 3 body kinematics in terms of cos ϑ^*, using the approximation $\vec{p}_5 = 0$.

but no approximation is done to reduce the number of integrations and a numerical method is used to calculate the three-fold integral on the volume element $d^3p_5 = p_5^2 d\, p_5 d\, \varphi_5 d \cos \vartheta_5$.

On Fig. 7 we show the results for W^+ at the different momenta of the K^+ .65 (- - -) and 1.51 GeV/c (———). From that figure we see that the overlapping region which is characterized by $W^+ > 1$, is rapidly decreasing with energy.

On Fig. 8 we show the behaviour with energy of the W in the "closure" approximation (3.23). The main difference with the exact calculation is in the backward direction, where the closure result is, for p_2 = .65 GeV/c, 10% higher than the "exact" result. This is due either to the cut of the spectator momentum $p_5 \leq 250$ MeV/c either to the exclusion from the integration region of the kinematically impossible values of $\vec{p}_5$.

In Fig. 9 we show the behaviour of W^+ with the momentum cut at $p_2 = 1.51$ (- - - - ; $p_5 \leq 100$ MeV/c; -·-·-·-·- $p_5 \leq 150$ MeV/c; ——— $p_5 \leq 250$ MeV/c).

Eventually in Fig. 10 we compare the results of W^+ obtained by the method, described above, (dashed lines) with the results of the calculation without making any approximation (continuous line), that is

1) keeping inside the integration $\frac{d\Omega^*}{dV^{(2)}} \Phi^{2}$

2) determining the 3 body kinematics in terms of cos ϑ_3^* taking in account the Fermi motion of the nucleon.

The result for the weight factors is very simple (3.24): however, we have to say that in the second term of formula (3.24) we cannot in principle determine the three body kinematics because, as from appendix B, we have to know the energy of the two body scattering which in this case has to be chosen between s_{34} and s_{35}. However we make the choice s_{34} and calculate it, remembering that the overlapping term is large only around the forward direction and in this region $s_{34} \sim s_{35}$.

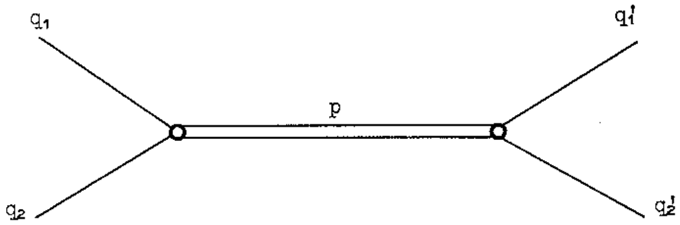
The results for the two different procedure look quite similar, a part from the backward region, which is influenced by the different approximations for the phase space volume Fig. 10, 11).

ACKNOWLEDGMENTS

It is a pleasure for us to thank Prof. L. Bertocchi for many illuminating discussions.

APPENDIX A

A practical method to determine the constant in the relation between the deuteron vertex and the non relativistic function is given in this appendix. The method consists in the calculation of the pole diagram and the comparison of its expression with the T matrix with one pole in potential scattering. The diagram is



$$\text{(A.1)} \qquad \langle q_1' q_2' | (S-1) | q_1 q_2) = i\ (2\pi)^4 \delta(q_1+q_2-p)\ i\ \frac{G^2}{p^2-M^2}\ \frac{d^4p}{(2\pi)^4} \cdot i(2\pi)^4 \frac{\delta(p-q_1'-q_2')}{N_1 N_2\ N_1' N_2'} =$$

$$= i\ (2\pi)^4 \delta(q_1+q_2-q_1'-q_2')(-)\ \frac{G^2}{p^2-M^2}\ \frac{1}{N_1 N_2\ N_1' N_2'}$$

where M is the mass of the deuteron.

Let us now consider this expression the C.M. system and the non relativistic limit: then

$$\text{(A.2)} \qquad \begin{aligned} p^2 &= (2\sqrt{q^2+m^2})^2 = 4\ m^2 + \frac{q^4}{m^2} + 4\ q^2 \sim 4\ m^2 + 4\ q^2 \\ 4\ m^2 &= (M + \frac{\chi^2}{m})^2 = M^2 + \chi^4 + \frac{2M}{m}\chi^2 \sim M^2 + 4\chi^2 \end{aligned}$$

where m is the mass of the nucleon, $\frac{\chi^2}{m}$ is the binding energy of the deuteron.

Extracting from the above expression the invariant T matrix, we obtain

$$\text{(A.3)} \qquad T = -\frac{G^2}{4(q^2+\chi^2)}$$

let us now calculate the scattering amplitude in potential scattering, assuming that it is simply a pole term in q^2 plane

$$\text{(A.4)} \qquad F = \frac{1}{2iq}\ \frac{f_0(q) - f_0(-q)}{f_0\ (-q)} = -\frac{1}{N^2}\ \frac{1}{q^2+\chi^2}$$

where

$$N^2 = \int_0^\infty f_0^2(-i\ \chi, r)\,dr$$

and $f_0(q)$, $f_0(q,r)$ are respectively the Jost function and the irregular solution of the radial Schroedinger equation.

The relation between the invariant T matrix and the scattering amplitude in the C.M. system is

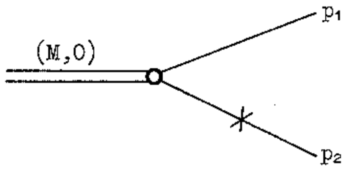
(A.5) $$T = 8\,\pi\,\sqrt{s}\;f$$

The comparison between the two results gives the relation between G and N

(A.6) $$G^2 = 64\,\pi\,E\;N^{-2}$$

where E is the energy of the nucleon.

Let us now consider the deuteron vertex,



and assume that p_2 in on the mass shell. The expression of this vertex, apart from the delta function in energy and momentum and factors, is

(A.7) $$\frac{G}{p_1^2 - m^2} \simeq \frac{G}{(-2)(q^2+\chi^2)}$$

The asymptotic wave function in momentum space is

(A.8) $$\psi(q) = -\frac{1}{\pi\,\sqrt{2}\,N}\;\frac{1}{q^2+\chi^2}$$

which compared with (A.7), gives the following relation

(A.9) $$\frac{G}{p_1^2 - m^2} = (32\,\pi^3\,E)^{1/2}\;\psi\,(q)$$

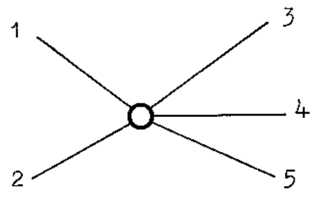
where E is the energy of the on-mass-shell particle.

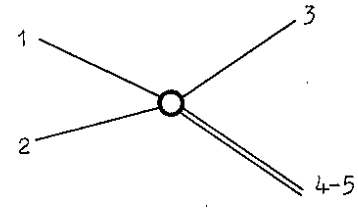
We want to stress here that although this relation seems to be valid only for an asymptotic wave function, it turns out to be true even for a general deuteron wave function, as shown more rigorously by Gross ([4]). In this case G is thought as a function of q.

APPENDIX B

In this appendix we consider in detail the procedure of calculating the full kinematics of the 3-body final state once 4 variables are fixed.

The method is based on the kinematical relation between the invariant quantities built with the energies and momenta of the 5 particles involved in the process





These relations are easily derived if we consider two of the 5 particles as a single one and reduce therefore the 3-body to a normal 2 body process; the relations are of the following type:

$$s_{12} + t_{23} + t_{13} = s_{45} + m_1^2 + m_2^2 + m_3^2 \tag{B.1}$$

This is the usual relation between the Mandelstan variables for the two body case. The definition of the invariants is the usual one:

$$\begin{aligned} s_{ij} &= (p_i + p_j)^2 \\ t_{ij} &= (p_i - p_j)^2 \end{aligned} \tag{B.2}$$

Obviously we can consider as one body any pair from the 5 different particles and obtain therefore 10 different relations, relative to the 10 possible pairs of "incident" particles.

In the analysis of the experimental data and the successive phase shift analysis, the best choice of the independent variables is given by the three-momentum of one of the outgoing nucleons and the C.M. angle of the incident particle-nucleon process.

The definition of $\vec{p}_5$ and the complete knowledge of the initial state fixes the invariants s_{12}, t_{15} and t_{25} ; the energy and momentum conservation fixes $s_{34}=(p_2+p_1-p_5)^2$.

The knowledge of ϑ_3^* gives the possibility of determining the squared four momentum transfer t_{23}, once the energy of the two body process is known. There are now two different ways of choosing this energy:

1) taking the internal line off-mass-shell and keeping energy conservation on the upper vertex of diagram

(B.3) $$s = s_{34}$$

2) taking the internal line on the mass shell and violating energy conservation

(B.4) $$s = \left(\sqrt{p_2^2+m_2^2} + \sqrt{p_5^2+m_5^2}\right)^2 - (\vec{p}_2-\vec{p}_5)^2$$

Once t_{23} is determined through one of these two methods, the procedure follows without further ambiguities.

The first step is the determination of t_{24} using equation (B.3) and the one of the same kind obtained considering as one body particles 1 and 3.

(B.5) $$t_{24}+s_{45} + t_{25} = t_{13}+m_2^2+m_4^2+m_5^2$$

Summing these two relations it follows.

(B.6) $$t_{24} = -t_{23}-t_{25}-s_{12}+m_1^2+2m_2^2+m_3^2+m_4^2+m_5^2$$

The second step is the derivation of linear equation for the invariants s_{35} and t_{13}; considering the two Mandelstan relations

$$\begin{aligned} t_{14}+t_{24}+s_{12} &= s_{35}+m_1^2 + m_2^2 + m_4^2 \\ t_{13}+t_{14}+s_{34} &= t_{25}+m_1^2 + m_3^2 + m_4^2 \end{aligned}$$

it follows

(B.7) $$s_{35}= t_{25}+t_{24}+s_{12}-s_{34}-t_{13}+m_3^2 -m_2^2$$

The invariant s_{35} can be also expressed using (B.2)

(B.8) $$s_{35} = m_3^2 + m_5^2 + 2E_3E_5- 2\ \vec{p}_3\vec{p}_5$$

where

(B.9) $$\begin{aligned} E_3 &= (m_1^2 + m_3^2 -t_{13})/2m_1 \\ p_{3z} &= (t_{23}-m_2^2-m_3^2+2E_2E_3)/2p_2 \\ p_{3x} &= (E_3^2+m_3^2-p_{3z}^2)^{1/2} \end{aligned}$$

By substitution of (B.9) in(B.8) we obtain another relation between s_{35}, t_{13} and t_{23}. Subtracting (B.8) from (B.7) we find a quadratic relation for t_{13}. The solution for t_{13} suffers for a sign ambiguity, which is solved comparing with the two body kinematics.

The knowledge of t_{13}allows the complete derivation of $\vec{p}_3$ and momentum conservation gives then $\vec{p}_4$.

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FIGURE CAPTIONS

Fig. 1 - Two-body scattering diagram. The dashed lines correspond to mesons and the full lines to nucleons.

Fig. 2 - Feynman graphs describing the deuteron break-up in the impulse approximation. 1 is the deuteron, 2 and 3 are the incoming and outgoing mesons, 4 and 5 are the two nucleons in the final state.

Fig. 3 - Perspective representation of the function $E^{+}dV^{(3)}/d\Omega_3 dp_5 d\Omega_5$ for $\varphi_5 = \pi$ and $\cos\vartheta_3^* = 0$. The quasi elastic peak and the spectator moraine are apparent.

Fig. 4 - As Fig. 3 with $\cos\vartheta_3^* = 0.9$. In this case the overlapping of the quasi elastic peak and the spectator moraine clearly shows off.

Fig. 5 - Momentum distribution of the spectator nucleon for the $K^+d \rightarrow K^o pp$ process at 0.98 GeV/c. Thick line refers to the experimental data and the thin line to the simplified model described in the text.

Fig. 6 - W^+ versus $\cos\vartheta^*$ for the $K^+d \rightarrow K^o pp$ process at 0.98 GeV/c, as calculated in ref.[5] for:

$\cos P\hat{}Q = 0.0$ dot-dashed
$\cos P\hat{}Q = 0.5$ dashed and
$\cos P\,Q = 1.0$ full line.

Fig. 7 - W^+ versus $\cos\vartheta^*$ for the $K^+d \rightarrow K^o pp$ process at 0.65 (dashed curve) and 1.51 GeV/c (full line), calculating the three-fold integral on the volume element d^3p_5 exactly.

Fig. 8 - As Fig. 7 but in closure approximation.

Fig. 9 - W^+ versus $\cos\vartheta^*$ for the $K^+d \rightarrow K^o pp$ process at 1.51 GeV/c for three different cuts on p_5 : < 0.100 GeV/c dashed, < 0.150 dot-dashed and < 0.250 full line.

Fig. 10 - Comparison of W^+ versus $\cos\vartheta^*$ for the $K^+d \rightarrow K^o pp$ process at 0.65 GeV/c with $p_5 < 0.100$ GeV/c. The two curves are computed with the two different procedures explained in the text.

Fig. 11 - As Fig. 10 with $p_5 < 0.250$ GeV/c.

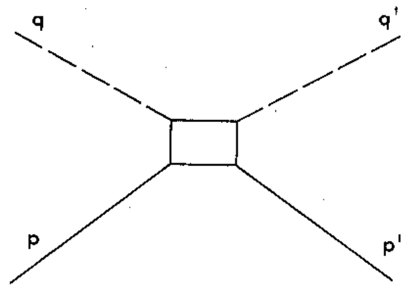
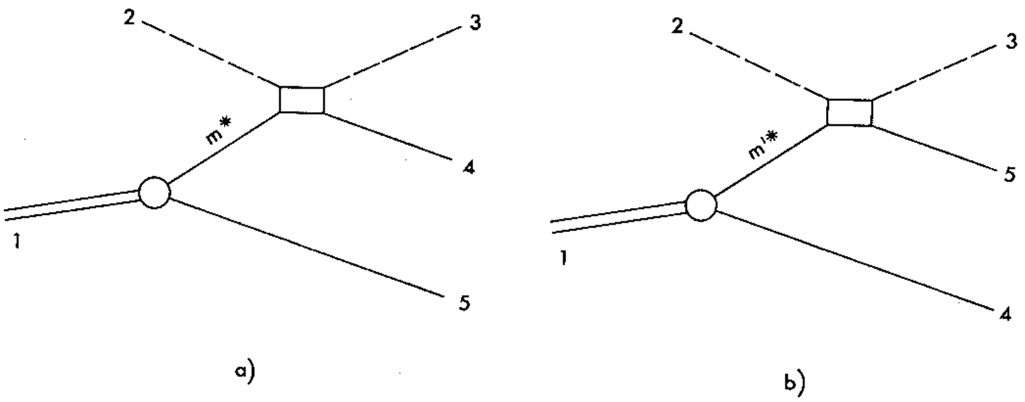


Fig. 1



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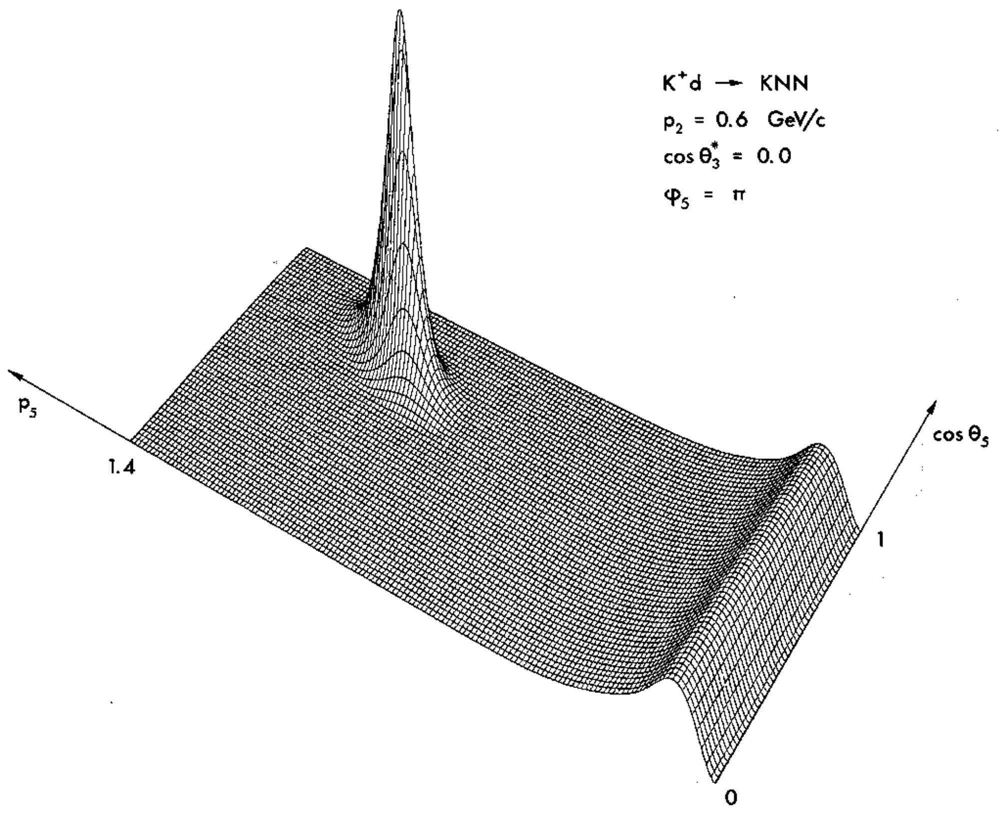
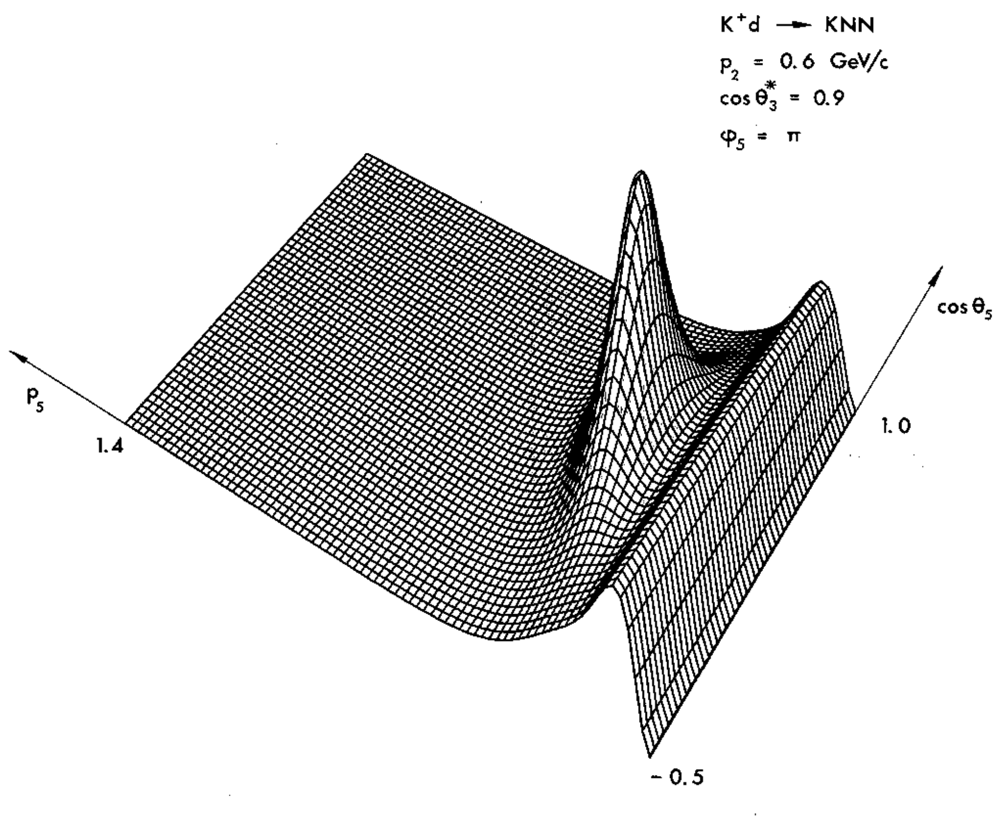


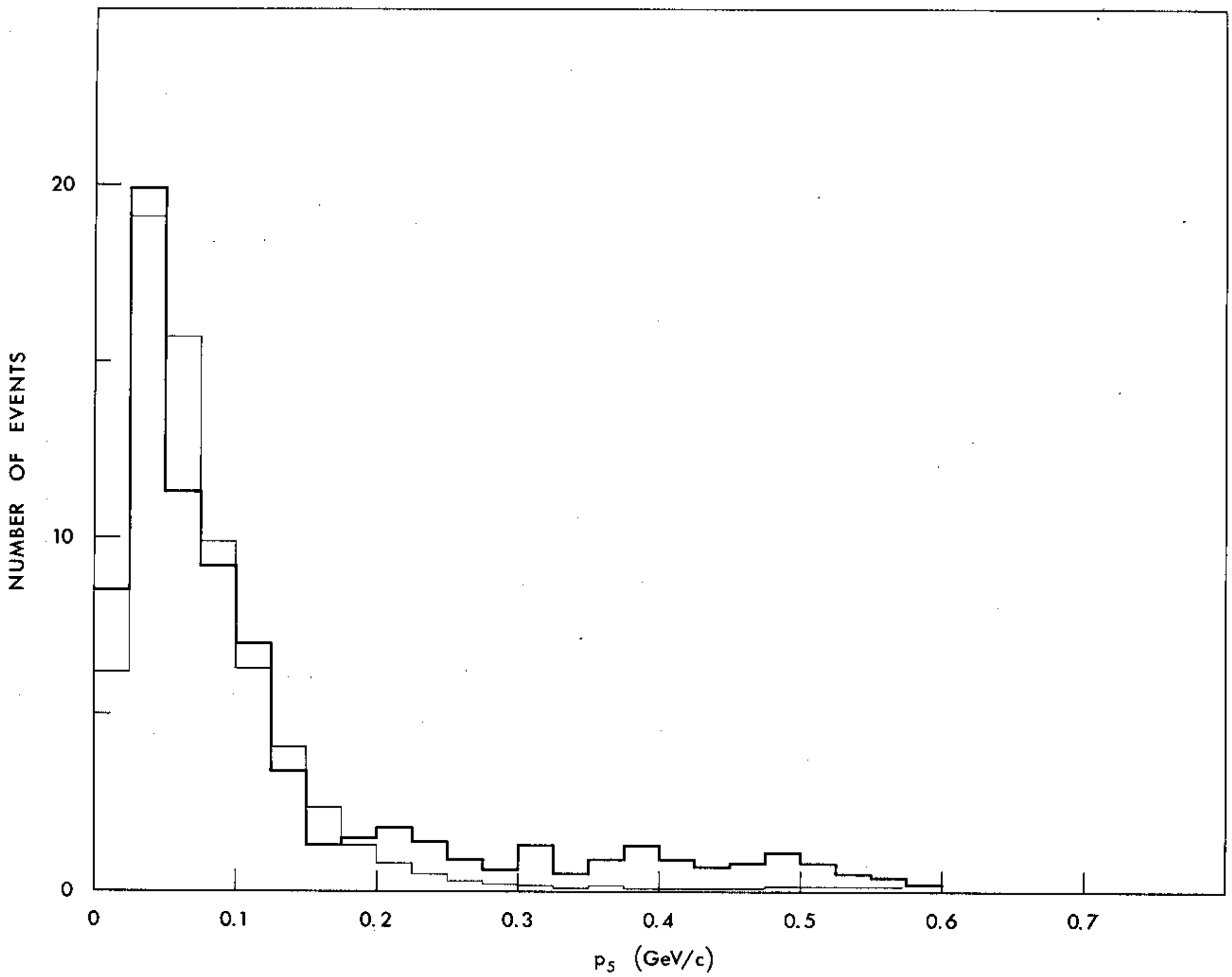
$K^+d \rightarrow KNN$
p_2 = 0.6 GeV/c
$\cos\theta_3^*$ = 0.0
φ_5 = π
p_5
1.4
$\cos\theta_5$
1
0

Fig. 3



$K^+d \rightarrow KNN$
p_2 = 0.6 GeV/c
$\cos\theta_3^*$ = 0.9
φ_5 = π
p_5
1.4
$\cos\theta_5$
1.0
− 0.5

Fig. 4



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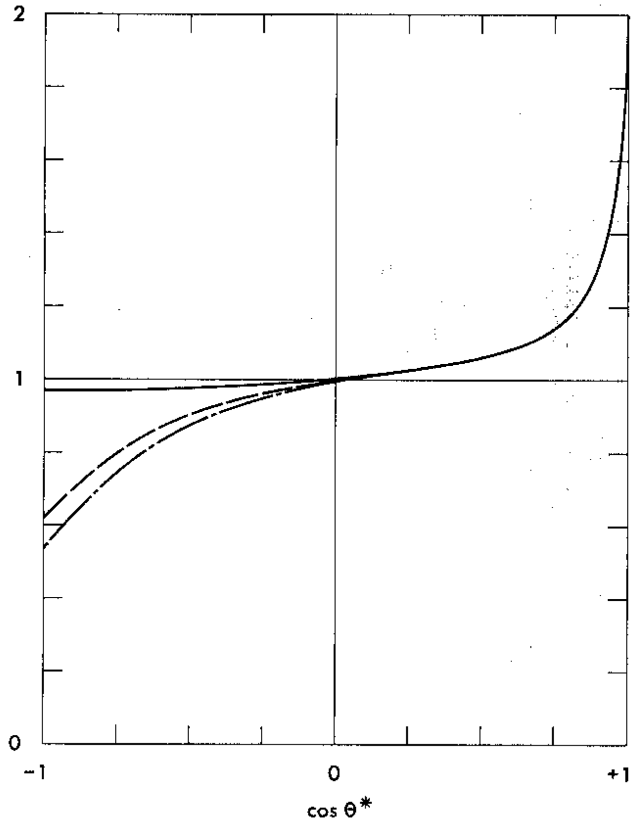

Fig. 6

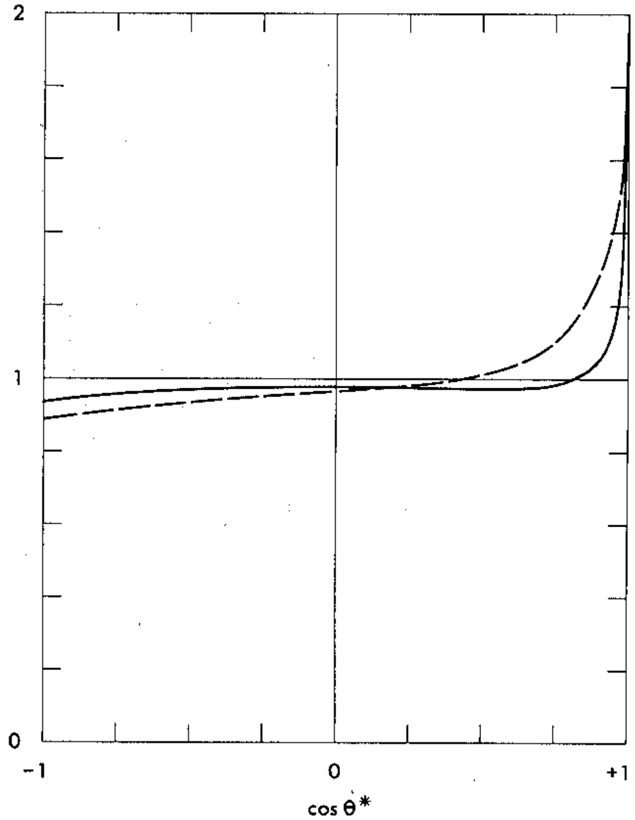

Fig. 7

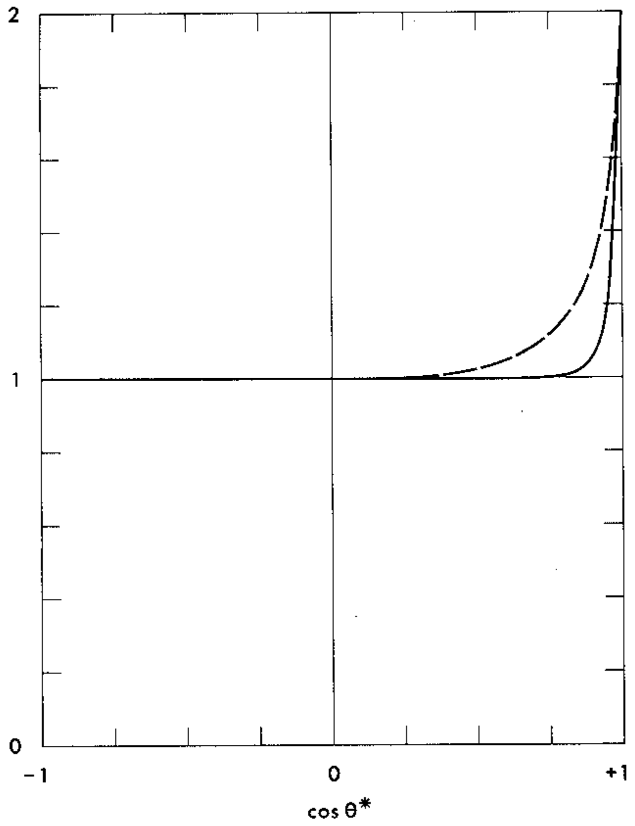

Fig. 8

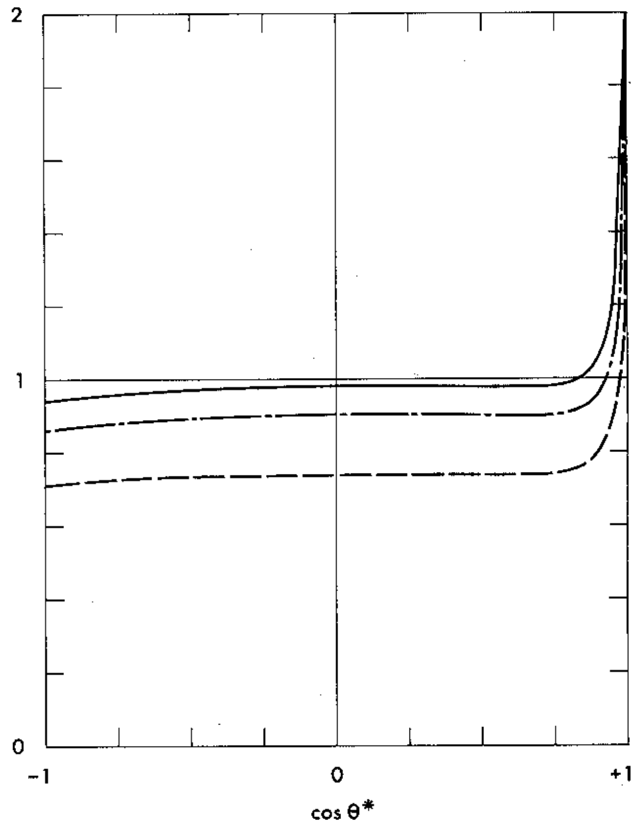

Fig. 9

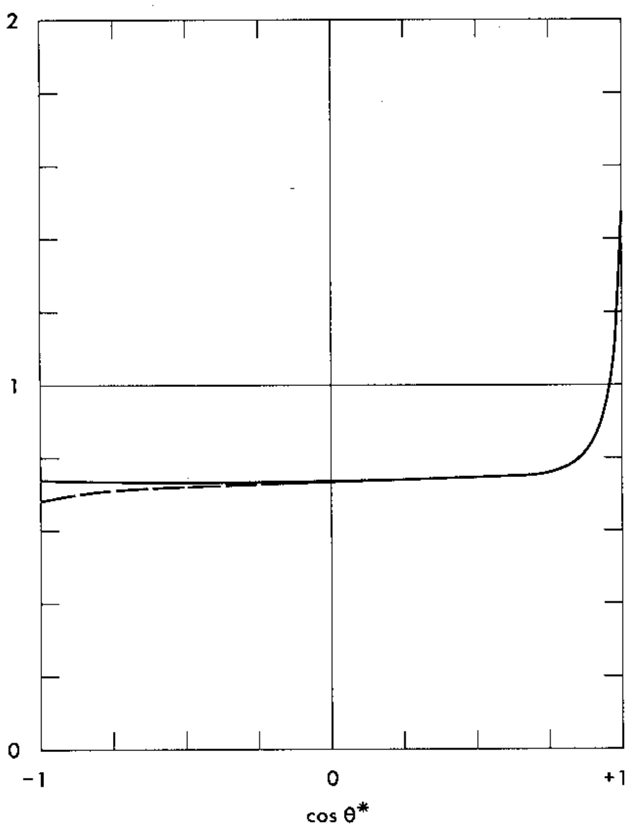

Fig. 10

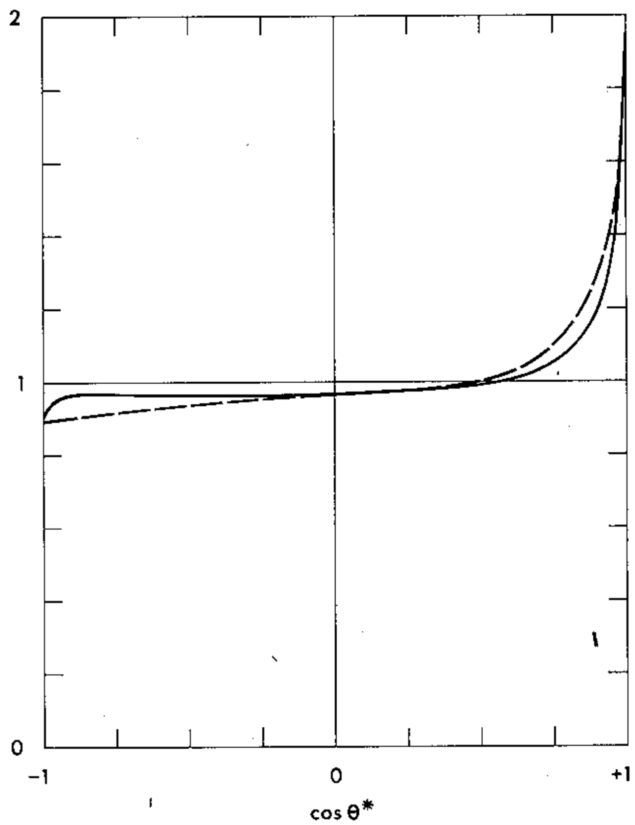

Fig. 11