ANNALS OF PHYSICS 236, 43–68 (1994)

Pressure Fluctuations and the Quantization of Electrodynamics with Boundary Conditions

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Received November 5, 1993

Canonical quantization of electrodynamics in the presence of parallel plates is studied in covariant as well as Coulomb gauges. In both cases the models of thick and thin plates are considered. The corresponding Wightman functions are given in a closed form. They depend strongly on the given geometry. Its structure is reflected by the fluctuations of the electromagnetic field strength as well as the components of the energy momentum tensor. In between the two-plate system the fluctuations in x-space depend critically on the considered space-time points. The reason is, that the correlation functions contain an infinite set of poles corresponding to events connected by n-times reflected light signals. Whereas in most cases the results are independent on the chosen models of the plates for special conditions of the measuring process the fluctuations of the Casimir pressure depend on them. © 1994 Academic Press, Inc.

1. INTRODUCTION

In the following we continue the investigation of the vacuum state in quantum field theory. In quantum mechanics the ground state is equally well investigated compared with other states. The wave function yields the necessary basic information. In quantum field theory the situation is quite different. The vacuum state is usually represented by the formal Fock space vector $|0\rangle$, only the Green functions of the field operators contain further information. Already in free field theory simple expectation values of the stress tensor or energy densities lead to divergent quantities. Because this seems to be unphysical in most applications these infinities are subtracted by the normal ordering procedure.

But this is not the right way. At least in part these infinities are direct consequences of the quantization procedure. For example, the infinities of the ground state

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0003-4916/94 $9.00

energy of QED could be understood as the added-up zero-point energies of the harmonic oscillators describing all the field modes. So possible infinities should be handled carefully and one should look for physically interesting finite parts.

A well-known nontrivial example is the Casimir pressure. Another interesting quantity is the fluctuation of the electromagnetic field strength [1].

Recently Barton [2] has raised the question of the fluctuations of the Casimir pressure. Similar investigations have been performed in [3, 4]. Fluctuations of observables are determined appropriately from correlation functions averaged with characteristic functions describing the measuring procedure. In [2] the correlation functions have been evaluated as matrix elements of operator products using a complete set of intermediate states.

Here we prefer another method. We write down the complete expression for the product of the two operators considered and then apply the standard methods of quantum field theory. These methods allow a simultaneous treatment of different interesting cases for which the Green functions are explicitly known.

As a case of physical importance we consider QED with one or two parallel conducting plates [5–7]. We start with the treatment of ideal conductors characterized by the vanishing of the tangential component of the electric field strength E_t and the normal component of the magnetic field strength B_n on the plates. It turns out that this characterization of the ideal conductor is not sufficient. The problem arises from the modes propagating parallel to the plates. These modes satisfy the boundary conditions automatically, so that formally all questions are solved. However, from a physical point of view we must decide whether these modes penetrate the plate or not. So we have again two idealized possibilities: the cases of thick or thin plates. For thick plates we assume that these modes cannot penetrate the boundary surface, whereas for thin plates these modes pass the boundary surface unchanged. Therefore in the first case we have an additional boundary condition and the photon field is quantized on one side of the plate only (or on both sides of the plate independently), whereas in the second case the standard boundary conditions are sufficient and both sides of the plate have to be taken into account.

For this reason we analyse the canonical quantization of the photon field A_μ in the presence of two plates to clarify this question. The problem is that historically the standard quantization procedures using the Coulomb gauge use automatically the model of thick plates [8], whereas quantizations using covariant gauge conditions have thin plates in mind [9,10]. We show that both gauges allow the treatment of thin or thick plates.

Of course most of the real plates are thick plates. However, the consideration of thin plates simplifies the calculations and in most cases it is unimportant which model of the plates will be used as it can be seen from different calculations of the Casimir force, mass shifts between parallel plates and other problems [5, 6, 8, 9, 11–15]. Different physical results can be expected in very special situations only. For example, for the space between two parallel plates it depends on the model of the plates whether the modes propagating parallel to them appear as a part of the continuous (thin plates—the modes are normalized with respect to the infinite space) or of the discrete (thick plates—in one direction the modes are normalized with respect to the distance between the two plates) spectrum. Such a different behaviour of the spectrum may have physical consequences. For a one-plate system we expect no physical consequences because of the infinite normalization volume; in both cases the spectrum remains unchanged. For a two-plate system, indeed, the result of special measurements of the fluctuations of the Casimir force depends on the model of the plates.

We study the fluctuations of the components of the energy momentum tensor and also the fluctuations of the Casimir pressure. It turns out that the fluctuations of the components of the energy-momentum tensor depend in an essential manner on the external conditions, i.e., whether there are one or two plates, or no plate at all. Consequently the vacuum fluctuations constitute an important indication of the physical situation. The Wightman functions for the considered case can be constructed by the help of the reflection principle. Accordingly we observe inside the two-plate systems a resonance structure of the correlation functions. Such resonances appear if the distances between the considered events correspond to a classical light signal that is n-times reflected at the plates. This structure concerns the fluctuations of the electromagnetic field strength, the pressure, and the Casimir force.

At the beginning we study the canonical quantization for thin and thick plates in covariant and Coulomb gauges. In the following section we derive the expression for the fluctuations of the components of the energy-momentum-tensor using straightforward methods of quantum field theory. The fluctuations of the pressure and the Casimir force are discussed for different geometrical situations in the fourth section. At this place we show that for a special measuring process of the Casimir force the result depends on the model of the plates.

2. Canonical Quantization and Wightman Functions in the Presence of Plates

Quantum electrodynamics with boundary conditions has been used in many calculations. There are many different approaches for the quantization of electrodynamics in the presence of conductors [6, 8–10, 16–18]. Here we discuss the canonical quantization in a covariant gauge or with the Coulomb gauge condition. In all cases we consider ideally conducting plates. The boundary conditions $E_t = B_n = 0$ can be written in terms of the electromagnetic potentials A_μ by

$$\varepsilon_{\mu\nu\rho\sigma} n^\rho \, \partial^\sigma A^\nu |_S = 0, \tag{2.1}$$

where n^ρ denotes the normal vector.

2.1. *Quantization in Covariant Gauge*

A straightforward formulation within the covariant gauges has been given by the help of the functional integration. In the perturbation theory it leads to the standard Feynman diagram technique with a modified photon propagator [9]. In the present case we are interested in the Wightman functions of the electromagnetic field. Because it is not a priori clear how to derive, especially in arbitrary gauges, the Wightman functions from the propagators (T-products), we present a straightforward construction based on canonical quantization.

As usual, we expand the free photon field A_μ in terms of four polarization vectors e^i_μ,

$$A_\mu(x) = \sum_{i=0}^{3} e^i_\mu f_i(x). \tag{2.2}$$

In order to find the boundary conditions to be fulfilled by the wave functions $f_i(x)$ one has to choose a suitable basis e^i_μ. It follows from Eq. (2.1) that the boundary conditions act actually in the space perpendicular to the vectors n_ρ and ∂_σ at the surface S. Because of the triviality of the surface considered here we are able to introduce globally the polarization vectors which satisfy the necessary conditions at the surface. For the case that the plates are perpendicular to the x_3-axis we choose

$$e^1_\mu = \frac{1}{\sqrt{\Delta_\perp}} \begin{pmatrix} 0 \\ -\partial_2 \\ \partial_1 \\ 0 \end{pmatrix}, \qquad e^2_\mu = \frac{1}{\sqrt{\Delta_\perp \tilde{\Delta}}} \begin{pmatrix} \Delta_\perp \\ \partial_0 \partial_1 \\ \partial_0 \partial_2 \\ 0 \end{pmatrix} \tag{2.3}$$

with $\tilde{\Delta} = \tilde{\partial}^2 = \partial_0^2 - \partial_1^2 - \partial_2^2$ and $\Delta_\perp = \partial_\perp^2 = \partial_1^2 + \partial_2^2$. The remaining orthogonal polarization vectors are

$$e^3_\mu = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \qquad e^0_\mu = \frac{1}{\sqrt{\tilde{\Delta}}} \begin{pmatrix} \partial_0 \\ \partial_1 \\ \partial_2 \\ 0 \end{pmatrix}. \tag{2.4}$$

As it should be the vectors e^i_μ satisfy the relations

$$e^i_\mu e^j_\nu g_{ij} = g_{\mu\nu}, \qquad g^{\mu\nu} e^i_\mu e^j_\nu = g^{ij}, \qquad a_\mu = (\tilde{a}_\mu, a_3), \qquad \tilde{a}_\mu = (a_0, a_1, a_2),$$

$$\sum_{i=1}^{2} e^i_\mu e^j_\nu g_{ij} = \tilde{g}_{\mu\nu} - \frac{\tilde{\partial}_\mu \tilde{\partial}_\nu}{\tilde{\Delta}}, \qquad \sum_{i=0,3} e^i_\mu e^j_\nu g_{ij} = \begin{pmatrix} \frac{\tilde{\partial}_\mu \tilde{\partial}_\nu}{\tilde{\partial}^2} & 0 \\ 0 & -1 \end{pmatrix}. \tag{2.5}$$

The boundary condition (2.1) leads to

$$\varepsilon_{\mu\nu\rho\sigma} n^\rho \partial^\sigma A^\nu |_S = \frac{1}{\sqrt{\Delta_\perp}} \begin{pmatrix} \Delta_\perp \\ -\partial_0 \partial_1 \\ -\partial_0 \partial_2 \\ 0 \end{pmatrix} f_1 |_S + \frac{1}{\sqrt{\Delta_\perp \tilde{\Delta}}} \begin{pmatrix} 0 \\ \partial_2 \tilde{\partial}^2 \\ -\partial_1 \tilde{\partial}^2 \\ 0 \end{pmatrix} f_2 |_S = 0 \tag{2.6}$$

on the plates. All the derivatives act in the (x_0, x_1, x_2)-subspace so that we have the Dirichlet conditions

$$f_i |_S = 0 \qquad (i = 1, 2), \tag{2.7}$$

whereas f_0 and f_3 are free of boundary conditions. To obtain the field equation for A_μ one has to start from the Lagrangian of the photon field. As is well known the quantization of a gauge field theory demands the choice of a gauge fixing condition or a gauge invariance breaking term in the Lagrangian. We choose the Feynman gauge with the Lagrangian

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{2}(\partial_\mu A^\mu)^2 = \tfrac{1}{2}A_\mu \partial^2 A^\mu. \tag{2.8}$$

As an interesting example we consider the quantization in the presence of two parallel plates located at $x_3 = 0$ and $x_3 = d$. The quantizations for thin and thick plates are treated separately.

Thin Plates

As a physical situation we assume that the plates are infinitely thin and characterized by the boundary conditions (2.1) only. The quantization will be performed on both sides of the plate [9, 10]. The wave equations $\partial^2 f_i = 0$, together with the boundary conditions (2.7), have the normalized solutions

$$f_i^\pm = \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{2k_0}} e^{\pm ikx}, \qquad k_0 = \sqrt{\mathbf{k}^2} \qquad (i = 3, 0), \tag{2.9}$$

$$f_{i,n}^\pm = \frac{1}{2\pi}\sqrt{2/d}\frac{1}{\sqrt{2k_0}} e^{\pm i\tilde{k}\tilde{x}} \sin n\frac{\pi}{d}x_3, \qquad 0 \leqslant x_3 \leqslant d, \tag{2.10}$$

$$k_0 = \sqrt{k_\perp^2 + (n\pi/d)^2}, \qquad n = 1, 2, 3, ... \quad (i = 1, 2),$$

$$f_{i,l}^\pm = \frac{1}{2^{1/2}(\pi)^{3/2}} \frac{1}{\sqrt{2k_0}} e^{\pm i\tilde{k}\tilde{x}} \sin k_3 x_3, \qquad x_3 < 0 \quad (i = 1, 2), \tag{2.11}$$

$$f_{i,r}^\pm = \frac{1}{2^{1/2}(\pi)^{3/2}} \frac{1}{\sqrt{2k_0}} e^{\pm i\tilde{k}\tilde{x}} \sin k_3(x_3 - d), \qquad x_3 > d \quad (i = 1, 2). \tag{2.12}$$

The quantization has to be performed in the full space, not only in the region $0 < x_3 < d$. According to standard procedure the quantized field A_μ reads

$$\begin{aligned} A_\mu(x) = {} & \sum_{i=0,3} e_\mu^i \int \frac{d^3k}{(2\pi)^{3/2}\sqrt{2k_0}} [e^{-ikx}a_i(\mathbf{k}) + e^{ikx}a_i^*(\mathbf{k})] \\ & + \Bigg\{ \sum_{i=1,2} e_\mu^i \Theta(x_3)\,\Theta(d - x_3)\frac{1}{2\pi}\int \frac{d^2k_\perp}{\sqrt{2k_0}} \\ & \times \sqrt{2/d}\sum_{n=1}^{\infty} \sin\frac{n\pi}{d}x_3[e^{-i\tilde{k}\tilde{x}}a_i(\mathbf{k}_\perp, n) + e^{+i\tilde{k}\tilde{x}}a_i^*(\mathbf{k}_\perp, n)] \\ & + \Theta(-x_3)\int \frac{dk_3}{2\pi^{3/2}k_0^{1/2}} \sin k_3 x_3[e^{-i\tilde{k}\tilde{x}}a_i(\mathbf{k}, l) + e^{+i\tilde{k}\tilde{x}}a_i^*(\mathbf{k}, l)] \\ & + \Theta(x_3 - d)\int \frac{dk_3}{2\pi^{3/2}k_0^{1/2}} \sin k_3(x_3 - d)[e^{-i\tilde{k}\tilde{x}}a_i(\mathbf{k}, r) + e^{+i\tilde{k}\tilde{x}}a_i^*(\mathbf{k}, r)]\Bigg\} \end{aligned} \tag{2.13}$$

with

$$
\begin{aligned}
[a_i(\mathbf{k}), a_j^*(\mathbf{q})] &= -g_{ij}\,\delta(\mathbf{q}-\mathbf{k}), \qquad (i=0,3),\\
[a_i(\mathbf{k}_\perp, n), a_j^*(\mathbf{q}_\perp, m)] &= -g_{ij}\,\delta_{nm}\delta(\mathbf{q}_\perp-\mathbf{k}_\perp) \qquad (i=1,2),\\
[a_i(\mathbf{k}, s), a_j^*(\mathbf{q}, t)] &= -g_{ij}\,\partial_{st}\delta(\mathbf{q}-\mathbf{k}) \qquad (i=1,2),\ (s=l,r).
\end{aligned} \tag{2.14}
$$

The difficulties of the commutation relation (2.14) with $i=0$ can be resolved in the standard way using the Gupta–Bleuler method of indefinite metric [19]. Defining the Hermitian conjugate operator of a_i as $a_i^\dagger = \eta a_i^* \eta$ (where η is the metric operator) and taking into account the special properties of this operator, it turns out that for vacuum expectation values a formal calculation using the commutation relations in a straightforward manner leads to the right result. With this procedure we directly define the Wightman function $\langle 0|\, A_\mu(x)\, A_\nu(y)\, |0\rangle$. Because the treatment of the region outside the slab is quite simple, we restrict our considerations to the space between the two plates. Taking into account Eqs. (2.13), (2.14), and (2.5) we obtain

$$
\begin{aligned}
&\langle 0|\, A_\mu(x)\, A_\nu(y)\, |0\rangle\\
&= -\sum_{i=1,2} e_\mu^i e_\nu^j g_{ij} \frac{2}{(2\pi)^2 d} \sum_{n=1}^{\infty} \int \frac{d^2k_\perp}{2k_0} e^{-i\tilde{k}(\tilde{x}-\tilde{y})} \sin\frac{n\pi}{d}x_3 \sin\frac{n\pi}{d}y_3\\
&\quad -\sum_{i=0,3} e_\mu^i e_\nu^j g_{ij} \frac{1}{(2\pi)^3} \int \frac{d^3k}{2k_0} e^{-ik(x-y)} \qquad (2.15)\\
&= i\left(g_{\tilde{\mu}\tilde{\nu}} - \frac{\partial_{\tilde{\mu}}\partial_{\tilde{\nu}}}{\tilde{\partial}^2}\right) {}^sD_2^-(\tilde{x}-\tilde{y}, x_3, y_3) + i\begin{pmatrix} \dfrac{\tilde{\partial}_\mu \tilde{\partial}_\nu}{\tilde{\partial}^2} & 0\\ 0 & -1 \end{pmatrix} D^-(x-y) \qquad (2.16)
\end{aligned}
$$

with

$$
D^-(x-y) = \frac{i}{(2\pi)^3} \int d^4k e^{ik(x-y)}\, \delta(k^2)\, \Theta(-k_0) \tag{2.17}
$$

and

$$
{}^sD_2^-(\tilde{x}-\tilde{y}, x_3, y_3) = \frac{2i}{(2\pi)^2 d} \sum_{n=1}^{\infty} \int \frac{d^2k_\perp}{2k_0} e^{-i\tilde{k}(\tilde{x}-\tilde{y})} \sin\frac{n\pi}{d}x_3 \sin\frac{n\pi}{d}y_3. \tag{2.18}
$$

Alternative expressions of the foregoing functions are

$$
\begin{aligned}
D^-(z) &= \frac{-i}{4\pi^2[(z_0-i\varepsilon)^2 - \mathbf{z}^2]},\\
&= \int \frac{d\tilde{k}}{(2\pi)^3} e^{i\tilde{k}\tilde{z}}\, \Theta(-k_0)\, \Theta(\tilde{k}^2) \frac{i}{2\Gamma}(e^{i\Gamma z_3} + e^{-i\Gamma z_3})
\end{aligned} \tag{2.19}
$$

and

$$
{}^{s}D_2^-(x, y) = +\int \frac{d\tilde{k}}{(2\pi)^3} e^{i\tilde{k}\tilde{z}} \frac{-i}{2\Gamma} \Theta(-k_0)\,\Theta(\tilde{k}^2) \left\{\frac{1}{2i\sin(\Gamma d)} - \frac{1}{2i\sin(\Gamma^* d)}\right\}
$$

$$
\times [(2\cos\Gamma(x_3 - y_3)\cos\Gamma d - 2\cos\Gamma(x_3 + y_3 - d)] \tag{2.20}
$$

$$
= -\frac{1}{8\pi\, d\zeta} \left\{\frac{1}{e^{(i\pi/d)(\zeta - x_3 - y_3)} - 1} + \frac{1}{e^{(i\pi/d)(\zeta + x_3 + y_3)} - 1} - \frac{1}{e^{(i\pi/d)(\zeta - x_3 + y_3)} - 1} - \frac{1}{e^{(i\pi/d)(\zeta + x_3 - y_3)} - 1}\right\} \tag{2.21}
$$

and

$$
{}^{s}D_2^-(x, y) = \sum_{l=-\infty}^{+\infty} [D^-(\tilde{z}, x_3 - y_3 + 2ld) - D^-(\tilde{z}, x_3 + y_3 + 2dl)], \tag{2.22}
$$

where we have used the notations with $\tilde{z} = \tilde{x} - \tilde{y}$, $\zeta = \sqrt{(z_0 - i\varepsilon)^2 - \mathbf{z}_\perp^2}$, and $\Gamma = \sqrt{\tilde{k}^2}$ (for the derivation compare Appendix A).

In the first representation of ${}^{s}D_2^-(x, y)$ the mode summation of Eq. (2.18) is converted into a Sommerfeld–Watson type integral using

$$
\frac{\cos\Gamma d}{\Gamma} \left\{\frac{1}{2i\sin(\Gamma d)} - \frac{1}{2i\sin(\Gamma^* d)}\right\}\Bigg|_{\Gamma d = n\pi} = -\frac{\pi}{dk_0}\,\delta\left(k_0 - \sqrt{k_\perp^2 + (n\pi/d)^2}\right).
$$

The representation (2.22) coincides with the elementary construction based on the reflection principle. It comes as a surprise that ${}^{s}D_2^-$ can be given by the closed summed-up expression (2.21). We remark that the explicitly constructed Wightman functions ${}^{s}D^-$ are in accordance with the expression obtained from the earlier given propagator [9] using the relation between the propagator ${}^{s}D^c$ and the Wightman function

$$
{}^{s}D^c(x, y) - {}^{s}D^{c*}(x, y) = {}^{s}D^-(x, y) - {}^{s}D^+(x, y). \tag{2.23}
$$

The Wightman function D^- is that part in $D^c - D^{c*}$ which allows an analytic continuation $z_0 = x_0 - y_0 \to z_0 - i\eta$ $(\eta > 0)$ and correspondingly contains $\theta(-k_0)$ in its Fourier representation.

As a typical result for thin plates in the representation (2.18) the sum over the eigenmodes starts with the term n = 1. The plan waves (2.9) belonging to e_μ^3 and e_μ^0 are non-transversal and in this sense unphysical and do not have to obey the boundary conditions. However, in the special case $k_3 = 0$ the solution

$$
A_\mu = e_\mu^3 \frac{1}{(2\pi)^{3/2}\sqrt{2k_0}} e^{\pm i\tilde{k}\tilde{x}} \tag{2.24}
$$

becomes transversal and fulfills automatically the boundary conditions. It describes a physical wave propagating parallel to the plates. This wave is not restricted to the space between the plates and because of its normalization on an infinite volume it is one point of the continuous spectrum. It has to be distinguished from the discrete solution

$$A_\mu = e^3_\mu \frac{1}{2\pi\sqrt{2k_0 d}} e^{i\tilde{k}\tilde{x}}, \qquad \tilde{k}^2 = 0, \tag{2.25}$$

normalized on the interval $0 < x_3 < d$ which does not appear here.

For completeness we write down the Wightman function in the presence of one plate. Here we have to substitute in the expression (2.16) the scalar Wightman function for the slab ${}^sD_2^-$ by the Wightman function for the case of one plate ${}^sD_1^-$,

$${}^sD_1^-(\tilde{x}-\tilde{y}, x_3, y_3) = D^-(\tilde{x}-\tilde{y}, x_3 - y_3) - D^-(\tilde{x}-\tilde{y}, x_3 + y_3). \tag{2.26}$$

Thick Plates

For ideal thick plates we assume that electromagnetic waves cannot penetrate the plates and that the quantization region consists of the considered side of the plate only. For the two-plate system this is the space between the two plates. Instead of restricting the space to this region we can adopt quantization methods used in solid state physics. We choose the second possibility and in addition to the boundary conditions (2.7) we impose a periodicity condition on all the other amplitudes (among the unphysical amplitudes this concerns the questionable mode too),

$$f_i(\tilde{x}, x_3) = f_i\left(\tilde{x}, x_3 + \frac{2\pi}{d}\right) \qquad \text{for} \quad i = 0, \tag{2.27}$$

This procedure guarantees the hermiticity of the Klein–Gordon operator. Then the solutions (2.9) have to be replaced by

$$f_{3,0} = \frac{1}{2\pi\sqrt{2k_0 d}} e^{\pm ikx} \qquad \text{with} \quad k_3 = \frac{2n\pi}{d}, n = 0, \pm 1, \pm 2, ... \tag{2.28}$$

To avoid singular gauge contributions we drop the mode with $n = 0$ in the solutions for f_0. In fact now all modes are discrete. The mode decomposition of the photon field reads now

$$\begin{aligned}
A_\mu(x) = {} & \sum_{i=0,3} e^i_\mu \sum_{n=-\infty}^{\infty} \int \frac{d^2k_\perp}{(2\pi)\sqrt{2k_0 d}} \\
& \times [e^{-i\tilde{k}\tilde{x} + ix_3(2n\pi)/d} a_i(\mathbf{k}) + e^{i\tilde{k}\tilde{x} - ix_3(2n\pi)/d} a_i^*(\mathbf{k})] \\
& - e^0_\mu \int \frac{d^2k_\perp}{(2\pi)\sqrt{2k_0 d}} [e^{-i\tilde{k}\tilde{x}} a_0(\mathbf{k}_\perp) + e^{i\tilde{k}\tilde{x}} a_0^*(\mathbf{k}_\perp)] \\
& + \sum_{i=1,2} e^i_\mu \sum_{n=1}^{\infty} \frac{1}{2\pi}\sqrt{\frac{2}{d}} \int \frac{d^2k_\perp}{\sqrt{2k_0}} \sin\frac{n\pi}{d} x_3 \\
& \times [e^{-i\tilde{k}\tilde{x}} a_i(\mathbf{k}_\perp, n) + e^{+i\tilde{k}\tilde{x}} a_i^*(\mathbf{k}_\perp, n)].
\end{aligned}$$

In this case the Wightman function takes the form

$$
{}^sD^-_{2,\mu\nu}(x, y) = \left(g_{\bar\mu\bar\nu} - \frac{\partial_{\bar\mu}\partial_{\bar\nu}}{\bar\partial^2}\right) {}^sD^-_2(\tilde x - \tilde y, x_3, y_3)
$$

$$
+ \begin{pmatrix} \dfrac{\tilde\partial_\mu \tilde\partial_\nu}{\tilde\partial^2} & 0 \\ 0 & 0 \end{pmatrix} (D^-(x-y) - \tilde D^-(x-y)) + \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} D^-(x-y). \tag{2.29}
$$

The representation for ${}^sD^-_2$ remain unchanged, only the representation (2.17) for D^- is replaced by

$$
D^-(x-y) = \frac{i}{(2\pi)^2 d} \int d\tilde k \sum_{n=-\infty}^{+\infty} e^{i(\tilde k(\tilde x - \tilde y) - (x_3 - y_3)(2n\pi)/d)} \delta(k^2)\, \Theta(-k_0).
$$

The physical mode propagating parallel to the plates is included in the function

$$
\tilde D^-(\tilde z) = \frac{i}{(2\pi)^2 d} \int d\tilde k\, e^{i\tilde k(\tilde x - \tilde y)}\, \delta(\tilde k^2)\, \Theta(-k_0). \tag{2.30}
$$

This quantization procedure can be extended in a straightforward manner to interacting QED by the inclusion of the electron field. In the last case we have to postulate periodicity conditions for the electron field too.

We underline that the quantization in the case of thin plates using all four components of the photon field and the basis vectors e^i is completely equivalent to our former procedure [9] which rests on the functional integral for QED in covariant gauges.

2.2. *Quantization in Coulomb Gauge*

In the classical treatment of the propagation of waves it is customary to start with the electric and the magnetic Hertz vectors (taking into account the $\mathbf{E}-\mathbf{B}$ symmetry of free electrodynamics). They are in principle defined globally and are adapted for giving acceptable boundary conditions in the case of conducting surfaces [1, 16, 18]. By using these polarization vectors as elements of the vierbein system, difficulties for the interacting QED are not excluded. The reason is that the boundary conditions act in the space orthogonal to the normal vector n_μ and to the four-dimensional gradient ∂_μ. Therefore in this case also the unphysical waves satisfy the boundary conditions following from an inappropriate choice of the polarization vectors. The problem is whether these additional boundary conditions have physical consequences [20] or not.

The Coulomb gauge is besides the covariant gauge, the most fundamental gauge in QED. Unlike to the covariant gauge one quantizes the physical degrees of freedom only. The representation of the photon field

$$
A_\mu(x) = \sum_{i=1,2} h^i_\mu g_i(x) \tag{2.31}
$$

is realized with the help of two transversal polarization vectors [8]

$$h^1_\mu = e^1_\mu, \qquad h^2_\mu = \frac{1}{\sqrt{\Delta\Delta_\perp}} \begin{pmatrix} 0 \\ \partial_1\partial_3 \\ \partial_2\partial_3 \\ -\Delta_\perp \end{pmatrix} \tag{2.32}$$

defined in the space-like subspace. The zero-component of the photon field does not have to be quantized; it is a Lagrange multiplier, defined by solving the field equations [21]. Using the representation of the photon field (2.31) then the functions $g^i(x)$ satisfy the wave equation. The boundary condition (2.1) leads to

$$\varepsilon_{\mu\nu\rho\sigma} n^\rho \partial^\sigma A^\nu|_S = \frac{1}{\sqrt{\Delta_\perp}} \begin{pmatrix} \Delta_\perp \\ -\partial_0\partial_1 \\ -\partial_0\partial_2 \\ 0 \end{pmatrix} g_1|_S + \frac{1}{\sqrt{\Delta\Delta_\perp}} \begin{pmatrix} 0 \\ -\partial_0\partial_2\partial_3 \\ \partial_0\partial_1\partial_3 \\ 0 \end{pmatrix} g_2|_S = 0. \tag{2.33}$$

This condition has to be exploited for the two cases of thick and thin plates.

Thick Plates

For thick plates (where we have to consider only one side of the plate) the conclusion is: the function g_2 satisfies the Neumann boundary condition

$$\partial_3 g_2(x)|_S = 0,$$

whereas the first function g_1 satisfies the Dirichlet condition as before. For the two-plate system the solutions for g_2 satisfying the Neumann condition are

$$g^\pm_{2,n} = \frac{1}{(2\pi)} \frac{1}{\sqrt{k_0 d}} e^{\pm i\tilde{k}\tilde{x}} \cos n\frac{\pi}{d} x_3, \qquad k_0 = \sqrt{k_\perp^2 + (n\pi/d)^2},\ n = 1, 2, 3, ...,$$

$$g^\pm_{2,0} = \sqrt{1/(2dk_0)}\,\frac{1}{2\pi} e^{i\tilde{k}\tilde{x}}, \qquad k_0 = \sqrt{k_\perp^2}. \tag{2.34}$$

Here the last solution is necessary for a completion of the set of solutions satisfying the Neumann condition. So in this case from a mathematical point of view the discrete mode with n = 0 (which is characteristic for thick plates in the case of the two-plate system) is included in a natural way. The solutions for g_1 (Dirichlet condition) are given by (2.10). If we start the quantization procedure with the mode expansion of the photon field we write:

$$A_\mu(x) = \Theta(x_3)\,\Theta(d-x_3)$$
$$\times\left[h^1_\mu \int \frac{d^2k_\perp}{2\pi\sqrt{dk_0}} \sum_{n=1}^{\infty} \sin\frac{n\pi}{d} x_3 [e^{-i\tilde{k}\tilde{x}} a_1(\mathbf{k}_\perp, n) + e^{+i\tilde{k}\tilde{x}} a_1^*(\mathbf{k}_\perp, n)]\right.$$
$$+ h^2_\mu \left\{ \int \frac{d^2k_\perp}{2\pi\sqrt{dk_0}} \sqrt{\frac{2}{d}} \sum_{n=1}^{\infty} \cos\frac{n\pi}{d} x_3 [e^{-i\tilde{k}\tilde{x}} a_2(\mathbf{k}_\perp, n) + e^{+i\tilde{k}\tilde{x}} a_2^*(\mathbf{k}_\perp, n)]\right.$$
$$\left.\left. + \frac{1}{2\pi} \int \frac{d^2k_\perp}{\sqrt{2k_0}} \sqrt{\frac{1}{d}} [e^{-i\tilde{k}\tilde{x}} a_i(\mathbf{k}_\perp, 0) + e^{+i\tilde{k}\tilde{x}} a_1^*(\mathbf{k}_\perp, 0)] \right\}\right]. \tag{2.35}$$

For the Wightman function we obtain for thick plates

$$\langle 0|\, A_\mu(x)\, A_\nu(y)\, |0\rangle = i(h^1_\mu(\partial_x)\, h^1_\nu(\partial_y)\, {}^sD^-_{2D}(x, y) + h^2_\mu(\partial_x)\, h^2_\nu(\partial_y)\, {}^sD^-_{2N}(x, y)), \tag{2.36}$$

where ${}^sD^-_{2D}(x, y)$ denotes here the Wightman function (2.18) satisfying the Dirichlet boundary condition and ${}^sD^-_{2N}(x, y)$ the Wightman function satisfying the Neumann boundary condition,

$${}^sD^-_{2N}((\tilde{x}-\tilde{y}), x_3, y_3)$$
$$= \frac{i}{(2\pi)^2 d} \sum_{n=-\infty}^{+\infty} \int \frac{d^2k_\perp}{2k_0} e^{-i\tilde{k}(\tilde{x}-\tilde{y})} \cos\frac{n\pi}{d} x_3 \cos\frac{n\pi}{d} y_3. \tag{2.37}$$

Thin Plates

The question remains: How do we treat thin plates? The answer is simple: The solutions of the field equations have to be extended over the full space, inside and outside the plates. In the case of the two-plate system for g_1 we can simply apply the solutions f_2 (2.10), (2.11), (2.12), for g_2 the solutions (2.34) have to be extended to the regions outside the plates. Thereby the mode propagating parallel to the plates is no longer restricted to the region $0 < x_3 < d$. Therefore the corresponding discrete mode (with $n = 0$ (2.34)) is replaced by an infinitely extended one. Moreover, the single infinite extended mode with vanishing k_3 gives a vanishing contribution (as one point with the measure zero) to the mode summation (integral), so that it can be dropped. Therefore up to this restriction we can assume $\partial_3 \neq 0$ and we are able to write

$$h^2_\mu g_2 = \frac{1}{\sqrt{\Delta\Delta_\perp}} \begin{pmatrix} 0 \\ \partial_1\partial_3 \\ \partial_2\partial_3 \\ -\Delta_\perp \end{pmatrix} g_2(x) \tag{2.38}$$

$$= \frac{1}{\sqrt{(1+\Delta_\perp/\partial_3^2)\,\Delta_\perp}} \begin{pmatrix} 0 \\ \partial_1 \\ \partial_2 \\ -\Delta_\perp/\partial_3 \end{pmatrix} \tilde{g}_2(x)|_{\partial_3^2 \neq 0} = \tilde{h}^2_\mu \tilde{g}_2. \tag{2.39}$$

Inserting this expression into the boundary condition (2.1) then it turns out that for $\partial_3^2 \neq 0$ the amplitude $\tilde{g}_2$ satisfies the Dirichlet boundary condition.

As a peculiarity of boundary conditions for more dimensional problems we see that changed conditions can transform the Neumann problem to a Dirichlet problem. In a very formal sense the solutions of the Neumann problem are here equivalent to the solutions of the Dirichlet problem completed by one exceptional mode. In cases where this additional mode does not contribute we reach the Dirichlet condition. The change of the boundary conditions from the Neumann problem to the Dirichlet problem, together with the change of the quantization region, is responsible for the completeness of both systems of solutions. (For the basic vector system (2.3), (2.4) the questionable mode is contained in the solutions multiplied by the "unphysical" polarization vector e_μ^3 which becomes transversal for the exceptional case $\partial_3^2 = 0$ only.)

The quantization procedure starts with the mode expansion of the photon field. For thin plates we have to treat the quantization on the full x_3-axis; however, we will hint at the differences occurring in the space between the two plates only. In the mode expansion (2.35) the last written line is not present. For that part of the Wightman function lying between $0 < x_3 < d$ we obtain for thin plates

$$\langle 0| A_\mu(x) A_\nu(y) |0\rangle = i(h_\mu^1(\partial_x) h_\nu^1(\partial_y) + \tilde{h}_\mu^2(\partial_x) \tilde{h}_\nu^2(\partial_y)) \, {}^sD_{2D}^-(x, y)), \qquad (2.40)$$

where ${}^sD_{2D}^-(x, y)$ denotes here the Wightman function (2.18) satisfying the Dirichlet boundary condition.

In all cases the polarization vectors are defined with the help of differential operators. In the case of the translation invariant subspace the differential operators act on Fourier representations and therefore $\tilde{\partial}$ corresponds to $\pm i\tilde{k}$ and the inverse differential operators are well defined. This is not the case for differentiations with respect to the third direction, where the translation invariance is broken. Near the boundary the wave equation is valid, so that $\partial_3^2 g = \tilde{\Delta} g$ can be expressed by derivatives with respect to the translation invariant subspace. But this is not the case for ∂_3 and consequently ∂_3^{-1} is indeed an integral operator. So it is more convenient to use the representation (2.36) with a modified expression (2.37) for the scalar Wightman function ${}^sD_{2N}^-(x, y)$. As is to be expected the mode with $n = 0$ in (2.37) has to be excluded in the sum over all the modes.

As an example we derive in Appendix B the special Wightman function for the field strength $\langle 0| F_{03}(x) F_{03}(x') |0\rangle$ using the model of thick and thin plates. Let us remark that for the determination of A_0 the Green function satisfying the Dirichlet condition has to be applied.

3. Field Theoretic Description of Fluctuations

In general the fluctuation of an observable T in the vacuum state is defined by

$$(\Delta T)^2 = \langle 0| (T - \bar{T})^2 |0\rangle = \langle 0| T^2 |0\rangle - \langle 0| \bar{T} |0\rangle^2, \qquad (3.1)$$

where

$$\bar{T} = \langle 0 | \, T \, | 0 \rangle, \qquad T = \int f(x) \, T(x) \, dx \tag{3.2}$$

is determined by a local field-theoretic observable $T(x)$ and a function $f(x)$ describing the measuring procedure. Therefore the essential information for the fluctuation is contained in the expectation values

$$\begin{aligned} W(x, x') &= \langle 0 | \, T(x) \, T(x') \, | 0 \rangle - \langle 0 | \, T(x) \, | 0 \rangle \langle 0 | \, T(x') \, | 0 \rangle \\ &= \langle | T(x) \, T(x') | \rangle'. \end{aligned} \tag{3.3}$$

For simplicity we do not symmetrize the Wightman-type function here, although only the symmetrized functions are necessary for these considerations.

In our case we consider the diagonal $T_{\mu\mu}$ components of the energy-momentum tensor. For a discussion of the Casimir pressure we need the 33-component. From this quantity the Casimir pressure on a plate located at $x_3 = a$ can be obtained as the difference of T_{33} across the plates

$$p(x) = T_{33}(x_3 = a + \varepsilon) - T_{33}(x_3 = a - \varepsilon). \tag{3.4}$$

For the energy-momentum tensor we use the symmetric tensor

$$-T_{\mu\nu} = F^{\rho}_{\mu} F_{\rho\nu} - 1/4 g_{\mu\nu} F_{\sigma\tau} F^{\sigma\tau} \tag{3.5}$$

with the field strength

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

As a regularization procedure we use the point-splitting technique. So we write for the diagonal elements of the energy-momentum tensor

$$\begin{aligned} T_{\mu\mu} &= \lim_{y \to x} -\tfrac{1}{2} g_{\mu\mu} [({}^{(\mu)}h^{\rho\lambda} \, {}^{(\mu)}h^{\sigma\tau} - {}^{(\mu)}h^{\rho\tau} \, {}^{(\mu)}h^{\sigma\lambda}) \, \partial^x_\rho \partial^y_\lambda] \, A_\sigma(x) \, A_\tau(y) \\ &= \lim_{y \to x} -\tfrac{1}{2} g_{\mu\mu} [\partial^{\overline{xy}} \, {}^{(\mu)}h^{\sigma\tau} - \partial^{x\underline{\tau}} \partial^{y\underline{\sigma}}] \, A_\sigma(x) \, A_\tau(y). \end{aligned} \tag{3.6}$$

In the last equation the indexes $\mu\mu$ are suppressed in part; they are included in the definitions, which are used appropriately,

$$\begin{aligned} &\tilde{a}\tilde{b} = a_0 b_0 - a_1 b_1 - a_2 b_2, \qquad && \partial^{xy} = g^{\rho\lambda} \, \partial^x_\rho \partial^y_\lambda, \qquad && \partial^{\overline{xy}} = {}^{(\mu)}h^{\rho\lambda} \, \partial^x_\rho \partial^y_\lambda \\ &\partial^{\widetilde{xy}} = g^{\widetilde{\rho\lambda}} \, \partial^x_{\tilde{\rho}} \partial^y_{\tilde{\lambda}}, && \partial^{\widehat{xy}} = {}^{(\mu)}h^{\widetilde{\rho\lambda}} \, \partial^x_{\tilde{\rho}} \partial^y_{\tilde{\lambda}}, && \partial^{x\underline{\sigma}} = \partial^x_\rho \, {}^{(\mu)}h^{\sigma\rho}, \end{aligned}$$

whereby the matrix ${}^{(\mu)}h_{ij}$ reads

$${}^{(\mu)}h_{\alpha\beta} = \begin{cases} -g_{\alpha\beta} \, g_{\mu\mu}, & \alpha \neq \mu \text{ or } \beta \neq \mu \\ +g_{\alpha\beta} \, g_{\mu\mu}, & \alpha = \beta = \mu. \end{cases}$$

Note that in accordance with the conventions (2.5) the index μ runs from 0 to 2 only. The notation $a^{\underline{\sigma}}$ is an abbreviation for $a^{\underline{\sigma}} = h^{\sigma\rho} a_\rho$. Taking into account (3.6) the product $T_{\mu\mu}(x, y)\, T_{\mu\mu}(x', y')$ appears as a product of four field operators,

$$
\begin{aligned}
&T_{\mu\mu}(x, y)\, T_{\mu\mu}(x', y') \\
&\quad = \lim_{y \to x} \lim_{y' \to x'} \tfrac{1}{2}[\partial^{\overline{xy}\,(\mu)} h^{\sigma\tau} - \partial^{x\underline{\tau}} \partial^{y\underline{\sigma}}][\partial^{\overline{x'y'}\,(\mu)} h^{\sigma'\tau'} - \partial^{x'\underline{\tau'}} \partial^{y'\underline{\sigma'}}] \\
&\qquad \times A_\sigma(x)\, A_\tau(y)\, A_{\sigma'}(x')\, A_{\tau'}(y').
\end{aligned} \tag{3.7}
$$

Because we restrict the consideration to free field theory the Wick theorem can be applied immediately:

$$
\begin{aligned}
&\langle 0|\, A(x)\, A(y)\, A(x')\, A(y')\, |0\rangle \\
&\quad = \langle 0|\, A(x)\, A(y)\, |0\rangle \langle 0|\, A(x')\, A(y')\, |0\rangle \\
&\qquad + \langle 0|\, A(x)\, A(x')\, |0\rangle \langle 0|\, A(y)\, A(y')\, |0\rangle \\
&\qquad + \langle 0|\, A(x)\, A(y')\, |0\rangle \langle 0|\, A(y)\, A(x')\, |0\rangle.
\end{aligned} \tag{3.8}
$$

Therefore the correlation function is reduced to a sum of products of elementary Wightman functions. Due to the subtracted structure of the correlation function $\langle 0|\, T_{\mu\mu}(x, y)\, T_{\mu\mu}(x', y')\, |0\rangle'$ the first term of the r.h.s. of Eq. (3.8) drops out and the point splitting can be removed in principle.

In the following we apply covariant quantization and write down all formulae for thin plates. In this case the Wightman function (2.16) can be rewritten in the form

$$
\langle 0|\, A_\mu(x)\, A_\nu(y)\, |0\rangle = i g_{\mu\nu} D^-(x - y) + i \left(\tilde{g}_{\mu\nu} - \frac{\tilde{\partial}^x_\mu \tilde{\partial}^y_\nu}{\tilde{\partial}^x \tilde{\partial}^y} \right) \bar{D}^-(x, y), \tag{3.9}
$$

whereby the function $\bar{D}$ defined by $\bar{D}^- = {}^sD^- - D^-$ takes care of the boundary condition [9]. It describes for scalar functions the deviation from the free space Wightman function. Inserting Eqs. (3.7), (3.8), and (3.9) into the correlation functions for the stress tensor we obtain

$$
\begin{aligned}
&\langle 0|\, T_{\mu\mu}(x)\, T_{\mu\mu}(x')\, |0\rangle \\
&\quad = \{ -\partial^{\overline{xy}} \partial^{\overline{x'y'}} [D^-(x, x')\, D^-(y, y') + D^-(x, x')\, \bar{D}^-(y, y') \\
&\qquad + \bar{D}^-(x, x')\, D^-(y, y')] \\
&\qquad - \tfrac{1}{2}[\partial^{yy'} \partial^{\widetilde{xx'}} P D^-(x, x')\, \bar{D}^-(y, y') + \partial^{xx'} \partial^{\widetilde{yy'}} P D^-(y, y')\, \bar{D}^-(x, x')] \\
&\qquad - \tfrac{1}{2}[(2 - P)\, \partial^{\overline{xy}} \partial^{\overline{x'y'}} + \partial^{\widetilde{xx'}} \partial^{\widetilde{yy'}} PP + (\partial^{\overline{xy}} \partial^{\widehat{x'y'}} + \partial^{\overline{x'y'}} \partial^{\widehat{xy}}) P] \\
&\qquad \times \bar{D}^-(x, x')\, \bar{D}^-(y, y') \} |_{y \to x,\, y' \to x'},
\end{aligned} \tag{3.10}
$$

where

$$
P = \left(1 - \frac{\partial^{\widehat{xy}} \partial^{\widehat{x'y'}}}{\partial^{\widetilde{xy}} \partial^{\widetilde{x'y'}}} \right).
$$

Now we take into account the special properties of the Wightman functions considered here, namely the reduction of these functions into a translation invariant part D_- and an "anti" translation invariant part D_+, according to

$$\bar{D}^-(x, y) = \bar{D}^-_-(\tilde{x} - \tilde{y}, x_3 - y_3) + \bar{D}^-_+(\tilde{x} - \tilde{y}, x_3 + y_3), \tag{3.11}$$

where each part satisfies the field equation

$$\Box \bar{D}^-_-(\tilde{x} - \tilde{y}, x_3 - y_3) = 0, \qquad \Box \bar{D}^-_+(\tilde{x} - \tilde{y}, x_3 + y_3) = 0.$$

So we obtain finally

$$\begin{aligned} &\langle 0|\, T_{\mu\mu}(x)\, T_{\mu\mu}(x')\, |0\rangle \\ &\quad = -\partial^{\overline{xy}} \partial^{\overline{x'y'}} [(D^-(x, x') + \bar{D}^-_-(x, x'))(D^-(y, y') + \bar{D}^-_-(y, y')) \\ &\qquad + \bar{D}^-_+(x, x')\, \bar{D}^-_+(y, y')]|_{y \to x,\, y' \to x'}. \end{aligned} \tag{3.12}$$

The fluctuation of the Casimir pressure on a plate located at $x_3 = a$ can be reduced to the correlation function (3.10) due to the relation (3.4). One obtains

$$\begin{aligned} \langle 0|\, p(x)\, p(x')\, |0\rangle' |\, |_{x_3 = y_3 = a} &= \langle 0|\, T_{33}(x)\, T_{33}(x')\, |0\rangle' |_{x_3 = x_3' = a + \varepsilon} \\ &\quad + \langle 0|\, T_{33}(x)\, T_{33}(x')\, |0\rangle' |_{x_3 = x_3' = a - \varepsilon} \end{aligned} \tag{3.13}$$

for ideally conducting plates. The reason for the absence of mixed terms originates from the fact that physical modes cannot propagate across the plates for ideal conductors.

6. Fluctuation of the Casimir Pressure

In this section we study the correlation functions for the stress tensor and the Casimir pressure for different physical situations. In general we assume the covariant quantization procedure for infinitely thin plates. Modifications for thick plates arising in the case of two plates are given additionally.

Correlation functions in the unsymmetrized version are built up from Wightman functions of the photon field and reflect the properties of these functions. The Wightman function of the photon field corresponding to free space given by

$$\langle 0|\, A_\mu(x)\, A_\nu(y)\, |0\rangle = i g_{\mu\nu} D^-(x - y)$$

and (2.17), (2.19) has to be compared with the corresponding functions for the two-plate system (2.18), (2.21) and for the case of one plate (2.26) derived in Section 2.

The simple power behaviour of the free function D^- (2.19) has to be contrasted with the more complicated structure of the function ${}^sD_2^-$ given in (2.21). This function is the sum of four periodic functions with a reduced Lorentz symmetry in the space perpendicular to the x_3-axis and a power behaviour according to $1/\zeta$. The infinite pole structure (see Eq. (2.22)) causes the correlations to be periodically enhanced like a resonance series with running time differences between arbitrary fixed space positions inside the plates. This is a quite different configuration space behaviour compared with the free case or the case of one plate. The simplest correlation functions showing these properties are the correlation functions of the electromagnetic field strength which are usually considered in momentum space [1] only.

In the following we study the stress-tensor fluctuations. It seems to be a general result that fluctuations at the position of the plate are enhanced in comparison with the free space case.

4.1. *Electromagnetic Field in Free Space*

As a first example we consider the correlation function of the stress tensor for the free electromagnetic field in free space. In this case Eq. (3.12) reads

$$\langle 0|\, T_{\mu\mu}(x)\, T_{\mu\mu}(x')\, |0\rangle' = -\partial^{\overline{xy}}\partial^{\overline{x'y'}} D^-(x, x')\, D^-(y, y')|_{x=y, x'=y'}. \tag{4.1}$$

Inserting the explicit expression for the free field Wightman function we obtain

$$\langle 0|\, T_{\mu\mu}(x)\, T_{\mu\mu}(x')\, |0\rangle' = \frac{1}{\pi^4}\frac{[4({}^{(\mu)}h^{\sigma\tau}(x-x')_\sigma\,(x-x')_\tau)^2 - ((x-x')^2)^2]}{[(x-x')^2]^6}. \tag{4.2}$$

As to be expected the correlation function is translational invariant and has a strong Wightman-type singularity for coinciding points and a power law decreasing behaviour for large distances.

For later use we explicitly write down the momentum space representation of the fluctuation of the 33-component of the stress tensor at coinciding x_3 coordinates

$$\langle 0|\, T_{33}(x)\, T_{33}(x')\, |0\rangle'|_{x_3=x_3'} = \frac{1}{15(2\pi)^2}\int \frac{d\tilde{k}}{(2\pi)^3}\, e^{-i\tilde{k}\tilde{z}}\Theta(k_0)(\tilde{k}^2)_+^{5/2}. \tag{4.3}$$

This describes the fluctuations of the pressure in a fixed plane perpendicular to the x_3-axis. Because of the homogeneity of the free space it does not depend on the position of the plane.

4.2. *Electromagnetic Field in the Presence of One Plate*

Here we study the fluctuation of the stress tensor disturbed by one plate at $x_3 = 0$. Again we have to apply the formula (3.12), whereby the Wightman functions D^- and $\bar{D}_1^- = -D^-(\tilde{x}-\tilde{y}, x_3+y_3)$ have to be taken into account:

$$\begin{aligned}\langle 0|\, T_{\mu\mu}(x)\, T_{\mu\mu}(x')\, |0\rangle' &= -\partial^{\overline{xy}}\partial^{\overline{x'y'}}[D^-(x, x')\, D^-(y, y') \\ &\quad + D^-(\tilde{x}-\tilde{x}, x_3+x_3')\, D^-(\tilde{y}-\tilde{y}', y_3+y_3')]|_{x=y, x'=y'}. \end{aligned}\tag{4.4}$$

An explicit evaluation leads to

$$\begin{aligned}
&\langle 0|\, T_{\mu\mu}(x)\, T_{\mu\mu}(x')\,|0\rangle' \\
&= \frac{1}{\pi^4}\{[4({}^{(\mu)}h^{\sigma\tau}(x-x')_\sigma\,(x-x')_\tau-((x-x')^2)^2]\,\frac{1}{[(x-x')^2]^6} \\
&\quad+[4({}^{(\mu)}h^{\bar\sigma\bar\tau}(x-x')_{\bar\sigma}\,(x-x')_{\bar\tau}+{}^{(\mu)}h^{33}(x_3+x_3')^2)^2 \\
&\quad-(g^{\bar\sigma\bar\tau}(x-x')_{\bar\sigma}\,(x-x')_{\bar\tau}-(x_3+x_3')^2)^2] \\
&\quad\times[g^{\bar\sigma\bar\tau}(x-x')_{\bar\sigma}\,(x-x')_{\bar\tau}-(x_3+x_3')^2]^{-6}\}.
\end{aligned} \tag{4.5}$$

In a physical picture we see contributions corresponding to the free propagation from the point x to the point x' and the propagation via a reflection at the plate x_3. However, there is no superposition between both types of waves. For fixed times and large values of $(x_3+x_3')^2$ the second term vanishes so that the fluctuations reduce to those of the free field case. We remark that this is not the case if we simultaneously consider large time differences. At last one should note that the fluctuations of the 33-component of the energy-momentum tensor but that all are enhanced by the presence of the plate. If we consider the fluctuations near one plate i.e. $x_3\to 0$ and $x_3'\to 0$, we obtain

$$\begin{aligned}
\langle 0|\, T_{33}(x)\, T_{33}(x')\,|0\rangle'|_{x_3=x_3'=0} &= \frac{2}{15(2\pi)^2}\int\frac{d\tilde k}{(2\pi)^3}\,e^{-i\tilde k\tilde z}\Theta(k_0)(\tilde k^2)_+^{5/2} \\
&= \frac{6}{\pi^4}\,\frac{1}{[(x-x_0'-i\eta)^2-(\mathbf{x}-\mathbf{x}')_\perp^2]^4},
\end{aligned} \tag{4.6}$$

which coincides with the result of Barton [2]. It is twice the amount of the fluctuation of the free field at $x_3=0$.

4.3. *Electromagnetic Field between Two Parallel Plates*

In the case of two plates the correlation functions for the inner and the exterior regions can be treated separately, because in the lowest order of perturbation theory there is no correlation between the two regions. The Wightman functions for one plate at $x_3=0$, $\bar D_1^-$ and for two plates $\bar D_2^-$ at $x_3=0$ or $x_3=d$ are identical for the exterior region, $x_3<0$; therefore the corresponding correlation functions coincide too.

The investigation of the inner region is more complicated. Again we start from the general expression (3.12)

$$\begin{aligned}
&\langle 0|\, T_{\mu\mu}(x)\, T_{\mu\mu}(x')\,|0\rangle' \\
&= -\partial^{\overline{xy}}\partial^{\overline{x'y'}}[(D^-(x,x')+\bar D_{2-}^-(x,x'))(D^-(y,y')+\bar D_{2-}^-(y,y')) \\
&\quad+\bar D_{2+}^-(x,x')\,\bar D_{2+}^-(y,y')]|_{x=y,x'=y'}.
\end{aligned} \tag{4.7}$$

Besides D^{-} we have to insert here the functions (see (3.11), (2.21))

$$ {}^{s}D_{2+}(x, y) = \bar{D}_{2+}^{-} = -\frac{1}{8\pi\, d\zeta} \left\{ \frac{1}{e^{(i\pi/d)(\zeta - x_3 - y_3)} - 1} + \frac{1}{e^{(i\pi/d)(\zeta + x_3 + y_3)} - 1} \right\} \tag{4.8} $$

and

$$ D\ \ (x, y) + \bar{D}_2\ \ (x, y) = \frac{1}{8\pi\, d\zeta} \left\{ \frac{1}{e^{(i\pi/d)(\zeta - x_3 + y_3)} - 1} + \frac{1}{e^{(i\pi/d)(\zeta + x_3 - y_3)} - 1} \right\} \tag{4.9} $$

An explicit calculation in x-space for general positions x and x' is possible and leads to a very long expression. As is to be expected it contains infinitely many poles (corresponding to the reflection principle) which are already contained in the Wightman functions. At the position of the poles—corresponding to world distances of definite length—the fluctuations are enhanced.

Let us now discuss the fluctuation of the Casimir force. Here we investigate the stress tensor fluctuations from the inner side of the plate at first. The fixation of the coordinates $x_3 = x_3' = 0$ to the position of the right plate and the restriction to the 33-component of the energy-momentum tensor simplifies all the calculations considerably. Here we will start with a momentum space representation of the Wightman function (2.20). Taking into account (3.9), (3.11), and (4.7) we obtain, after a straightforward calculation,

$$ \begin{aligned} &\langle 0|\, T_{33}(x)\, T_{33}(x')\, |0\rangle' |_{x_3 = x_3' = 0_+} \\ &= 2 \int \frac{d\tilde{p}}{(2\pi)^3} \int \frac{d\tilde{p}'}{(2\pi)^3}\, e^{i\widetilde{(p+p')}\widetilde{(x-x')}} [\Theta(-p_0)\, \Theta(\tilde{p}^2)\, \Theta(-p_0')\, \Theta((\tilde{p}')^2)] \\ &\quad \times (\tilde{p}^2 (\tilde{p}')^2 + (\tilde{p}p')^2) \frac{\cos \Gamma d \cos \Gamma' d}{\Gamma \Gamma'} \\ &\quad \times \left\{ \frac{1}{2i \sin(\Gamma d)} - \frac{1}{2i \sin(\Gamma^* d)} \right\} \left\{ \frac{1}{2i \sin(\Gamma' d)} - \frac{1}{2i \sin(\Gamma'^* d)} \right\}. \end{aligned} \tag{4.10} $$

With

$$ \frac{\cos \Gamma d}{\Gamma} \left\{ \frac{1}{2i \sin(\Gamma d)} - \frac{1}{2i \sin(\Gamma^* d)} \right\} \bigg|_{\Gamma d = n\pi} = -\frac{\pi}{d} \frac{1}{|p_0|} \delta\left(p_0 - \sqrt{(p_\perp)^2 + (\pi n/d)^2}\right), $$

it results

$$ \begin{aligned} W_2(\zeta, d) &\equiv \langle 0|\, T_{33}(x)\, T_{33}(x')\, |0\rangle' |_{x_3 = x_3' = 0_-} \\ &= \frac{1}{2d^2} \sum_{n=1}^{\infty} \sum_{n'=1}^{\infty} \int \frac{d^2 p_\perp}{(2\pi)^2} \int \frac{d^2 p_\perp'}{(2\pi)^2}\, e^{-i(p_0 + p_0')(x_0\ \ x_0') + i(p+p')_\perp (x - x')_\perp} \\ &\quad \times \left[\left(\frac{\pi n}{d}\right)^2 \left(\frac{\pi n'}{d}\right)^2 + (p\widetilde{p}')^2 \right] \frac{1}{p_0\, p_0'}. \end{aligned} \tag{4.11} $$

Further evaluation will be much simplified if we exploit the Lorentz invariance in the (x_0, x_1, x_2)-subspace. This allows us to put $z_\perp = 0$ and to use, in addition, rotation invariance in the (p_1, p_2)-plane. This leads to

$$\langle 0|\, T_{33}(x)\, T_{33}(x')\, |0\rangle'|_{x_3 = x'_3 = 0_+, z_\perp = 0}$$
$$= \frac{1}{(2\pi)^2\, 2d^2} \{A_1^2(z_0, d) + A_2^2(z_0, d) + 2A_3^2(z_0, d)\} \tag{4.12}$$

with

$$A_1 = \sum_{n=1}^{\infty} \int \frac{d^2 p_\perp}{2\pi}\, e^{-i o(x_0 - x'_0)} \frac{1}{p_0} \left(\frac{n\pi}{d}\right)^2,$$
$$A_2 = \sum_{n=1}^{\infty} \int \frac{d^2 p_\perp}{2\pi}\, e^{-i p_0(x_0 - x'_0)} p_0,$$
$$A_3 = \sum_{n=1}^{\infty} \int \frac{d^2 p_\perp}{2\pi}\, e^{-i p_0(x_0 - x'_0)} \frac{1}{p_0} \frac{p_\perp^2}{2} \tag{4.13}$$

and $p_0 = \sqrt{p_\perp^2 + (\pi n/d)^2}$.

Taking into account the analytic properties of the Wightman functions for $z_0 \to z_0 - i\eta$, $(\eta > 0)$, the integrations and summations can be carried out without problems. The final result can be written in terms of the variable $\zeta = \sqrt{(z_0 - i\eta)^2 - (z_\perp^2)}$,

$$A_1(\zeta, d) = \frac{1}{i\zeta}\left(\frac{\pi}{d}\right)^2 \frac{e^{i(2\pi\zeta/d)} + e^{i(\pi\zeta/d)}}{(e^{i(\pi\zeta/d)} - 1)^3},$$
$$A_2(\zeta, d) = \frac{2}{(i\zeta)^3} \frac{1}{(e^{i(\pi\zeta/d)} - 1)} + \frac{2}{(i\zeta)^2}\left(\frac{\pi}{d}\right) \frac{e^{i(\pi\zeta/d)}}{(e^{i(\pi\zeta/d)} - 1)^2}$$
$$+ \frac{1}{i\zeta}\left(\frac{\pi}{d}\right)^2 \frac{e^{i(2\pi\zeta/d)} + e^{i(\pi\zeta/d)}}{(e^{i(\pi\zeta/d)} - 1)^3},$$
$$A_3(\zeta, d) = \frac{1}{(i\zeta)^3} \frac{1}{(e^{i(\pi\zeta/d)} - 1)} + \frac{1}{(i\zeta)^2}\left(\frac{\pi}{d}\right) \frac{e^{i(\pi\zeta/d)}}{(e^{i(\pi\zeta/d)} - 1)^2}, \tag{4.14}$$

so that

$$W_2(\zeta, d) \equiv \langle 0|\, T_{33}(x)\, T_{33}(x')\, |0\rangle'|_{x_3 = x'_3 = 0_+}$$
$$= \frac{1}{(2\pi)^2\, 2d^2}\{A_1^2(\zeta, d) + A_2^2(\zeta, d) + 2A_3^2(\zeta, d)\}. \tag{4.15}$$

So we have two equivalent expressions (4.11) and (4.15) for the fluctuation of the Casimir pressure on the inner side of the plate. This result generalizes the corresponding investigation of Barton [2]. It is very interesting that the correlation function $W_2(\zeta, d)$ contains infinitely many multiple poles at

$$(\zeta)^2 = (x_0 - x_0')^2 - (x_\perp - x'_\perp)^2 = 4n^2 d^2.$$

In a physical interpretation these values of ζ^2 correspond to pairs of events $(x_0, x_\perp, x_3 = 0)$ and $(x_0', x'_\perp, x_3' = 0)$ connected by n-times reflected light signals. This implies a resonance behaviour of the fluctuations for such distances ζ^2. For the limiting case $d \to \infty$ the results for one mirror can be recovered. We use

$$\begin{aligned} A_1(\zeta, d))|_{d \to \infty} &= 2d/(\pi \zeta^4), \\ A_2(\zeta, d))|_{d \to \infty} &= 6d/(\pi \zeta^4), \\ A_3(\zeta, d))|_{d \to \infty} &= 2d/(\pi \zeta^4). \end{aligned} \tag{4.16}$$

The correlation function W_2 has the following scaling and limiting properties (W_1 denotes the correlation function $\langle 0 | T_{33}(x) \, T_{33}(x') | 0 \rangle$, corresponding to one plate; see Eq.(4.6)):

$$W_2(\lambda \zeta, \lambda d) = \frac{1}{\lambda^8} W_2(\zeta, d)$$

$$\lim_{d \to \infty} W_2(\zeta, d) = W_1(\zeta),$$

$$\lim_{\zeta \to 0} \frac{W_2(\zeta, d)}{W_1(\zeta)} = 1,$$

$$W_2(\zeta, d)_{\frac{|\zeta|}{d} \ll 1} = \frac{1}{d^8} \left(\frac{d}{\zeta}\right)^2 f\left(\frac{\zeta}{d}\right).$$

The function $f(\zeta/d)$ is an analytic and integrable function with poles at $\zeta/d = 2n$.

Besides the already given physical interpretations we see that the correlations at $\zeta^2 \approx 0$ approximately coincide with those of the one mirror problem. It can be understood in a simple physical picture: there is not enough time to receive the reflected signals.

As a simple consequence of the Wightman structure of the correlation functions (the poles are located in the upper z_0-plane) we conclude that

$$\int_{-\infty}^{\infty} dx_0 \int_{-\infty}^{\infty} dx_0' \, W_i(\zeta, d) = 0.$$

If we apply this procedure to the (symmetrized) correlation function by closing the integration path for the two contributing terms in the opposite half planes then we see that the fluctuations of observables measured over an infinite time interval tend to zero.

The fluctuations of the Casimir pressure is the sum (3.13) of the fluctuations of the stress tensor from both sides of the plates. Correlations between different sides of the plates are unessential because the only possible mode extended over both sides of the plate (the mode propagating parallel to the plates) gives a vanishing contribution. On the other hand in the case of thick plates we have to take into account this wave as a discrete mode propagating parallel to the plates. Again starting from Eq. (3.7) and using now the representation (2.29) we obtain instead of (3.12)

$$\begin{aligned}\langle 0|\, T_{\mu\mu}(x)\, T_{\mu\mu}(x')\, |0\rangle' &= -\partial^{\overline{xy}}\partial^{\overline{x'y'}} \\ &\times [(\bar{D}_+^-(x, x') - \tfrac{1}{2}\tilde{D}^-(x, x'))(\bar{D}_+^-(y, y') - \tfrac{1}{2}\tilde{D}^-(y, y')) \\ &+ (D^-(x, x') + \bar{D}_-^-(x, x') + \tfrac{1}{2}\tilde{D}^-(x, x')) \\ &\times (D^-(y, y') + \bar{D}_-^-(y, y') + \tfrac{1}{2}\tilde{D}^-(y, y'))]|_{y \to x,\, y' \to x'}.\end{aligned}$$

Inserting the momentum space representations of the involved functions we obtain for the Casimir pressure on the plate (we set $x_3 = x_3'$) the momentum space representation (4.11), where now the summation includes the modes with $n = 0$ (see [2])

$$\begin{aligned}&\langle 0|\, T_{33}(x)\, T_{33}(x')\, |0\rangle'|_{x_3 = x_3' = 0_+} \\ &= \frac{1}{8d^2} \sum_{n=-\infty}^{+\infty} \sum_{n'=-\infty}^{+\infty} \int \frac{d^2p_\perp}{(2\pi)^2} \int \frac{d^2p'_\perp}{(2\pi)^2}\, e^{-i(p_0 + p_0')(x_0 - x_0') + i(p + p')_\perp (x - x')_\perp} \\ &\times \left[\left(\frac{\pi n}{d}\right)^2 \left(\frac{\pi n'}{d}\right)^2 + (p\tilde{p}')^2 \right] \frac{1}{p_0 p_0'}.\end{aligned} \tag{4.17}$$

This additional mode is in principle observable, as will be discussed in the next subsection.

4.4. *Fluctuations of the Casimir Force and the Measuring Process*

Let us in conclusion of this section combine our results on correlation functions with measuring processes, which according to (3.2) make recourse to specific functions which characterize the measuring procedure. We factorize the characteristic function $f(\tilde{x})$ according to $f(\tilde{x}) = g(x_0)\, h(x_\perp)$. As an example we choose

$$g(x_0) = \frac{\tau}{\pi} \frac{1}{x_0^2 + \tau^2}, \qquad \int dx_0\, e^{-ip_0 x_0} g(x_0) = e^{-|p_0|\, \tau}$$

and $h(x_\perp)$ is implicitly defined by

$$\int dx_\perp\, e^{i\mathbf{p}_\perp \mathbf{x}_\perp} h(x_\perp) = e^{a\pi/d} e^{-a\sqrt{p_\perp^2 + (\pi/d)^2}}.$$

Both functions $g(x_0)$ and $h(x_\perp)$ are normalized to 1 and its Fourier transforms are dimensionless. The parameters τ and a describe the duration and the spatial extension of the measurement. With the help of these functions the fluctuation $(\Delta T)^2$ is expressed by means of the correlation function $W(\tilde{x}, \tilde{x}') = \langle 0| T_{33}(\tilde{x})\, T_{33}(\tilde{x}')|0\rangle'$ as

$$(\Delta T)^2 = \int d\tilde{x}\, d\tilde{x}'\, f(\tilde{x})\, f(\tilde{x}')\, W(\tilde{x}, \tilde{x}'). \tag{4.18}$$

At first we consider the case of two thin plates. Here we are interested in the fluctuations of T_{33} on the inner side of the plate at $x_3 = 0$. Combining the foregoing equations with Eq. (4.11) we obtain

$$(\Delta T)^2 = \frac{1}{2d^2} e^{2a\pi/d} \sum_{n=1}^{\infty} \sum_{n'=1}^{\infty} \int \frac{d^2p_\perp}{(2\pi)^2 p_0} \int \frac{d^2p'_\perp}{(2\pi)^2 p'_0} \left[(\tilde{p}\tilde{p}')^2 + \left(\frac{n\pi}{d}\right)^2 \left(\frac{n'\pi}{d}\right)^2 \right]$$
$$\times \exp(-2p_0\tau - 2p'_0\tau) \exp(-a(2\sqrt{(\mathbf{p}_\perp + \mathbf{p}'_\perp)^2 + (\pi/d)^2}) \tag{4.19}$$

with $p_0 = \sqrt{p_\perp^2 + (n\pi/d)^2}$ and $p'_0 = \sqrt{p_\perp'^2 + (n'\pi/d)^2}$. According to realistic possibilities the characteristic time τ of a measuring process is large in comparison with the time interval necessary for a light signal to traverse the plate distance d,

$$d \ll \tau. \tag{4.20}$$

Additionally we restrict us to local measurements $(a = 0)$. Accordingly $(\Delta T)^2$ is dominated by the term with $n = n' = 1$ with the corresponding modifications of p_0 and p'_0. This yields, taking into account rotation invariance,

$$(\Delta T)^2 = \frac{1}{2d^2} \int \frac{d^2p_\perp}{(2\pi)^2\, p_0} \int \frac{d^2p'_\perp}{(2\pi)^2\, p'_0}$$
$$\left\{ p_0^2 p_0'^2 + p_1^2 p_1'^2 + p_2^2 p_2'^2 + \left(\frac{\pi}{d}\right)^4 \right\} e^{-(2\tau)(p_0 + p'_0)}. \tag{4.21}$$

In the limit (4.20) considered here the contributions from $p_1^2 p_1'^2 + p_2^2 p_2'^2$ in the bracket are non-leading, whereas the remaining contributions are equal. The final result reads

$$(\Delta T)^2 = \frac{\pi^2}{4d^6(2\tau)^2} e^{-4\pi\tau/d}. \tag{4.22}$$

We underline that this result is based essentially on the absence of the modes with $n = 0$. The presence of such modes would change (4.22) to a power-like behaviour which coincides with the result obtained in [2]. These results are consequences of different models for the plates.

This should be compared with the fluctuations in the case of the one-plate system. Performing the same integrations by invoking Eq. (4.6) instead of Eq. (4.11), we obtain (for simplicity taking $a=0$)

$$(\Delta T)^2 = \frac{2}{15(2\pi)^5} e^{-i\tilde{q}(\tilde{x}-\tilde{x}')} \int d\tilde{q}\, \theta(q_0)(\tilde{q}_+^2)^5/2 \int d\tilde{x}\, d\tilde{x}'\, f(\tilde{x})\, f(\tilde{x}') = \frac{6}{(4\pi)^4}\left(\frac{1}{\tau}\right)^8. \quad (4.23)$$

Of course this expression describes also the fluctuations of T_{33} on the outer side of the two-plate system. Because of the inequality (4.20) the fluctuations of the inner sides of the plates are exponentially suppressed in comparison with the fluctuation of T_{33} on the outer sides of the plates. As a consequence the fluctuations of the Casimir pressure (defined by the same characteristic functions) for the one-plate system are twice as large as the fluctuations for the two-plate system. Note that this does not lead to a contradiction if the second plate is removed to infinity because the inequality (4.20) cannot be maintained in this limit.

APPENDIX A: The Photon Wightman Function between Plates

Our aim is to derive alternative representations of the expression (2.18)

$$\begin{aligned} {}^sD_2^-(\tilde{x}-\tilde{y}, x_3, y_3) &= \frac{2i}{(2\pi)^2 d} \sum_{n=1}^{\infty} \int \frac{d^2k_\perp}{2k_0} e^{-i\tilde{k}(\tilde{x}-\tilde{y})} \sin\frac{n\pi}{d} x_3 \sin\frac{n\pi}{d} y_3 \\ &= \frac{i}{(2\pi)^2 d} \sum_{n=-\infty}^{+\infty} \int d^4k\, e^{+i\tilde{k}(\tilde{x}-\tilde{y})} \Theta(-k_0)\, \delta(k^2)\, \delta\left(k_3 - \frac{n\pi}{d}\right) \sin k_3 x_3 \sin k_3 y_3 \end{aligned}$$

with $k_0 = \sqrt{k_\perp^2 + (n\pi/d)^2}$.

With the help of

$$\sum_{n=-\infty}^{+\infty} \delta\left(k_3 - \frac{n\pi}{d}\right) = 2d \sum_{n=-\infty}^{+\infty} \delta(2k_3 d - 2n\pi) = \frac{d}{\pi} \sum_{l=-\infty}^{+\infty} e^{i2k_3 dl}$$

this can be rewritten as

$$\begin{aligned} {}^sD_2^-(\tilde{x}-\tilde{y}, x_3, y_3) &= \frac{2i}{(2\pi)^3} \sum_{l=-\infty}^{\infty} \int d^4k\, e^{-i\tilde{k}(\tilde{x}-\tilde{y})} \Theta(-k_0)\, \delta(k^2) \\ &\quad \times \left(\frac{1}{2i}\right)^2 \left[e^{ik_3(x_3+y_3)} + e^{-ik_3(x_3+y_3)} - e^{ik_3(x_3-y_3)} - e^{ik_3(-x_3+y_3)}\right] e^{i2k_3 l} \\ &= \sum_{l=-\infty}^{+\infty} \left[D^-(\tilde{z}, x_3 - y_3 + 2ld) - D^-(\tilde{z}, x_3 + y_3 + 2dl)\right]. \end{aligned}$$

In order to derive the representation (2.21) we exploit the restricted Lorentz invariance in the $\tilde{x}$ subspace which allows us to represent $\tilde{z}=\tilde{x}-\tilde{y}$ by the vector $(z_0, 0, 0)$. Now the $p_\perp$ integration can be carried out:

$$\int \frac{d^2 p_\perp}{p_0} e^{-ip_0 z_0} = \frac{2\pi}{iz_0} e^{-i(z_0 n\pi/d)}.$$

Convergence is guaranteed by the Wightman prescription $z_0 \to z_0 - i\eta$, $\eta > 0$ (analyticity in the forward tube). This property also assures the convergence of the following infinite sum:

$$\sum_{n=1}^{\infty} e^{-i(z_0\pi/d)n} \sin\frac{n\pi}{d} x_3 \sin\frac{n\pi}{d} y_3$$
$$= -\frac{1}{4}\left\{\frac{1}{e^{(i\pi/d)(z_0 - x_3 - y_3)} - 1} + \frac{1}{e^{(i\pi/d)(z_0 + x_3 + y_3)} - 1}\right.$$
$$\left. - \frac{1}{e^{(i\pi/d)(z_0 - x_3 + y_3)} - 1} - \frac{1}{e^{(i\pi/d)(z_0 + x_3 - y_3)} - 1}\right\}.$$

Taking into account this formula and using once more the restricted Lorentz invariance by substituting $z_0 - i\eta \to \zeta = \sqrt{(z_0 - i\eta)^2 - z_\perp^2}$ we obtain directly the representation (2.21).

APPENDIX B: Special Wightman Function for Thin and Thick Plates

As an instructive example concerning the result of different models for the plates in the case of two plates we consider the correlation function of the field strength F_{03}. It is given by the photon Wightman functions by

$$\langle 0|\, F_{03}(x)\, F_{03}(y)\, |0\rangle$$
$$= \partial_0^x \partial_0^y \langle 0|\, A_3(x)\, A_3(y)\, |0\rangle + \partial_3^x \partial_3^y \langle 0|\, A_0(x)\, A_0(y)\, |0\rangle$$
$$- \partial_3^x \partial_0^y \langle 0|\, A_0(x)\, A_3(y)\, |0\rangle - \partial_0^x \partial_3^y \langle 0|\, A_3(x)\, A_0(y)\, |0\rangle. \tag{B.1}$$

Using at first the expression (2.16) we obtain (for $0 < x_3,\ y_3 < d$)

$$\langle 0|\, F_{03}(x)\, F_{03}(y)\, |0\rangle$$
$$= i(\partial_0^z)^2 D^-(z) + i\partial_3^x \partial_3^y \left(1 - \frac{(\partial_0^z)^2}{(\tilde{\partial}^z)^2}\right) {}^s D_{2D}^- - i\frac{(\partial_0^z)^2}{(\tilde{\partial}^z)^2}\right) (\partial_3^z)^2 D^-(z) \tag{B.2}$$

and finally with (2.18)

$$\langle 0|\, F_{03}(x)\, F_{03}(y)\, |0\rangle = \frac{2}{(2\pi)^2 d} \sum_{n=1}^{\infty} \int \frac{d^2k_\perp}{2k_0}\, k_\perp^2\, e^{-i\tilde{k}(\tilde{x}-\tilde{y})} \cos\frac{n\pi}{d} x_3 \cos\frac{n\pi}{d} y_3. \tag{B.3}$$

Next we also apply the polarization vectors e^i_μ but choose as physical space the interval $0 < x_3 < L$ ($x_\perp$ unrestricted, $L > d$). This amounts to imposing periodicity conditions on the solutions. For points between the plates we have the old solution (2.10). Instead of (2.11) the solutions f_0, f_3 now read (2.28)

$$f^{\pm}_{0,3} = \frac{1}{2\pi} \sqrt{1/(2Lk_0)}\, e^{\pm ikx}, \qquad k_3 = \frac{2n\pi}{L}, \qquad n = 0, \pm 1, \pm 2, \ldots.$$

Correspondingly, D^- is replaced by

$$D^-(x-y) = \frac{i}{(2\pi)^2 L} \int d\tilde{k} \sum_{n=-\infty}^{+\infty} e^{i[\tilde{k}(\tilde{x}-\tilde{y}) - (x_3-y_3)(2n\pi)/L]}\, \delta(k^2)\, \Theta(-k_0). \tag{B.4}$$

Also, in this case the correlation function is given by (B.2) with D^- replaced by the expression (B.4). Whereas in the foregoing case the first and the third terms in (B.2) compensate each other completely, now the term with $n = 0$ survives, so that

$$\langle 0|\, F_{03}(x)\, F_{03}(y)\, |0\rangle = \frac{1}{(2\pi)^2 d} \int \frac{d^2k_\perp}{2k_0}\, k_\perp^2\, e^{-i\tilde{k}\tilde{z}} \left\{ \frac{d}{L} + 2 \sum_{n=1}^{+\infty} \cos\frac{n\pi}{d} x_3 \cos\frac{n\pi}{d} y_3 \right\}. \tag{B.5}$$

This formula interpolates between the first case ($L \to \infty$) and the case $L = d$ which we interpret as the case of thick plates.

The calculation of the same correlation function using the polarization vectors (2.32) has to take into account the structure of the tensor $h^i_\mu h^i_\nu$ (compare (2.36)). Obviously we have $\langle 0|\, A_0(x)\, A_0(y)\, |0\rangle = \langle 0|\, A_0(x)\, A_3(y)\, |0\rangle = \langle 0|\, A_3(x)\, A_0(y)\, |0\rangle = 0$ and

$$\langle 0|\, A_3(x)\, A_3(y)\, |0\rangle = -i \frac{\Delta_\perp^2}{\Delta_\perp \partial_0^2}\, {}^sD^-_{2N},$$

where ${}^sD^-_{2N}$ is given by (2.37). Insertion into (B.1) leads to

$$\langle 0|\, F_{03}(x)\, F_{03}(y)\, |0\rangle = \frac{1}{(2\pi)^2 d} \sum_{n=-\infty}^{+\infty} \int \frac{d^2k_\perp}{2k_0}\, k_\perp^2\, e^{-i\tilde{k}\tilde{z}} \cos\frac{n\pi}{d} x_3 \cos\frac{n\pi}{d} y_3. \tag{B.6}$$

Whereas this result differs from (B.3) (infinitely thin plates, four polarization vectors e^i_μ), it coincides with the expression (B.5) for the special choice $d = L$ (thick plates). The result (B.6) is in accordance with that given in [1] if an obvious error in their Eq. (2.17) is corrected.

Acknowledgments

For interesting discussions D.R. is indebted to his colleagues K. Scharnhorst, A. Uhlmann, W. Weller, and M. Bordag, as well as D. Welsch and K.-H. Lotze (Jena), and especially G. Barton (Sussex).

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