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Is the Chiral Phase Transition in Non-Compact Lattice QED Driven by Monopole Condensation?

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Abstract

We investigate the recent conjecture that the chiral phase transition in non-compact lattice QED is driven by monopole condensation. The comparison of analytic and numerical results shows that we have a quantitative understanding of monopoles in both the quenched and dynamical cases. We can rule out monopole condensation.

1 Introduction

In a series of papers [1-10] we have investigated strongly coupled QED, both on the lattice in the non-compact formulation and in the continuum using Schwinger-Dyson equations. The strong coupling region is of interest because of the existence of a second order chiral phase transition. This implies a continuum limit, and the interesting question is whether or not it describes an interacting theory. Our calculations of the renormalized charge, e_R, and fermion mass, m_R, demonstrated that whenever m_R goes to zero in lattice units (i.e. the ultraviolet cut-off is removed) then e_R goes to zero. This suggests that the theory is non-interacting in the continuum limit in accordance with the general belief that non-asymptotically free theories are trivial. It is encouraging that the two approaches, namely lattice and Schwinger-Dyson, agree with each other. Further support of this picture comes from other authors [11].

However, this picture has been queried by Hands, Kocić, Kogut and collaborators [12-20] who investigated the behaviour of magnetic monopoles near the phase transition. Using a new monopole 'order parameter' first introduced by Hands and Wensley [20], they conclude that monopoles condense in the chirally broken phase. The occurrence of this proposed second order monopole phase transition is important because such a transition would imply the existence of monopoles in the lattice model's continuum limit. This would cast doubt on conclusions about continuum physics drawn from lattice calculations, as monopoles are presumably absent in QED. Furthermore, dual superconductivity and charge confinement are to be expected whenever monopoles condense [15]. This is in conflict with our picture [10] where we have found free electrons and massless photons in the broken phase.

In the strong coupling limit $\beta \to 0$, or the limit when we have a large number of flavours, the action is dominated by the fermion determinant, which is a compact object in the sense that it only depends on the compactified link variables e^{iA_μ}, A_μ being the gauge field. It was found that the compact U(1) Wilson action has a first order phase transition [21] which is driven by monopole condensation [21, 22]. Hence, it is conceivable that monopoles play a role in the non-compact case as well. Indeed, simulations with very large numbers of fermions [23] suggest that the phase transition becomes first order.

In this paper we shall investigate the relevance of monopoles for the phase transition. We solve the quenched case analytically [24] and look at the dynamical fermion case numerically. We find no evidence of monopole condensation.

2 Lattice monopoles

The action for non-compact lattice QED with dynamical staggered fermions is

$$S = \frac{\beta}{2} \sum_{x,\mu<\nu} F_{\mu\nu}^2(x) + \sum_{x,y} \bar{\chi}(x)[D_{x,y} + m\delta_{x,y}]\chi(y), \tag{2.1}$$

with

$$F_{\mu\nu}(x) = \Delta_\mu A_\nu(x) - \Delta_\nu A_\mu(x), \tag{2.2}$$

$$D_{x,y} = \frac{1}{2}\sum_\mu (-1)^{x_1+\cdots+x_{\mu-1}}[\mathrm{e}^{iA_\mu(x)}\delta_{y,x+\hat{\mu}} - \mathrm{e}^{-iA_\mu(y)}\delta_{y,x-\hat{\mu}}], \tag{2.3}$$

where Δ_μ is the lattice forward derivative and $\beta = 1/e^2$.

To define monopoles [21, 20] we decompose $F_{\mu\nu}$ into an integer valued string field $N_{\mu\nu}$ and a compact field $f_{\mu\nu}$ which lies in the range $(-\pi, \pi]$:

$$F_{\mu\nu} = 2\pi N_{\mu\nu} + f_{\mu\nu}. \tag{2.4}$$

The Bianchi identity tells us that $F_{\mu\nu}$ summed over any closed surface always gives zero. This does not apply to the $N_{\mu\nu}$ and $f_{\mu\nu}$ fields separately. This allows the common definition of a monopole current

$$M_\mu(x) = \frac{1}{4\pi}\epsilon_{\mu\nu\rho\sigma}\Delta_\nu f_{\rho\sigma}(x+\hat{\mu}) = -\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\Delta_\nu N_{\rho\sigma}(x+\hat{\mu}), \tag{2.5}$$

which lives on the elementary cubes of the lattice (or equivalently on the links of the dual lattice). Equation (2.5) shows that a string can only end on a monopole or antimonopole. Each component of M_μ can take the values $0, \pm 1, \pm 2$. The current M_μ is conserved: $\bar{\Delta}_\mu M_\mu(x) = 0$, where $\bar{\Delta}_\mu$ is the lattice backward derivative.

Later on we shall be interested in the monopole susceptibility. This is defined by [25]

$$\chi_m = \frac{1}{6}\sum_x \langle f_{\mu\nu}(x) f_{\mu\nu}(0)\rangle = \frac{4\pi^2}{6}\sum_x \langle N_{\mu\nu}(x) N_{\mu\nu}(0)\rangle, \tag{2.6}$$

where we have made use of the fact that $\sum_x F_{\mu\nu}(x) = 0$. In the infinite volume limit further manipulations lead to the equivalent form

$$\chi_m = -\frac{4\pi^2}{12}\sum_x \langle x^2 M_\mu(x) M_\mu(0)\rangle\,. \tag{2.7}$$

The physical interpretation of eq. (2.7) is that it measures the fluctuations of the total dipole moment, whereas eq. (2.6) can be regarded as the residue of the photon pole in

the compact photon propagator. We have used eq. (2.6) for our measurements to avoid ambiguities in defining x^2 on a finite lattice. If the monopoles condense we would expect χ_m to diverge.

At a second order phase transition fluctuations of physical quantities become large and should show up in other quantities as well. We have looked at fluctuations in monopole charge. The corresponding susceptibility is

$$\chi_q(p) = \frac{1}{4} \sum_{x,\mu} \langle M_\mu(x) M_\mu(0) \rangle \mathrm{e}^{ip \cdot x} \, . \tag{2.8}$$

Figure 1: The susceptibility (2.8) for an ideal Bose-Einstein gas, at various temperatures above and below the condensation temperature T_c.

It has long been known that when Bose condensation occurs the tendency of bosons to occupy the same state causes long range density correlations, which in turn leads to a divergence of χ_q at small momentum. For an example see 'Distance Correlations and Bose-Einstein Condensation' [26] which finds that in a Bose gas with a condensation temperature T_c the density-density correlation drops off exponentially if $T > T_c$ (giving a finite χ at $p = 0$), drops off like r^{-2} at T_c (giving a χ diverging like p^{-1}) and drops off like r^{-1} below T_c (giving a χ diverging like p^{-2}). These behaviours are illustrated for the case of the ideal Bose gas in fig. 1. We see that condensation gives a spectacular signal.

One subtlety is that for periodic boundary conditions $\chi_q(p)$ vanishes for $p = 0$ on finite lattices and so one must extrapolate to zero momentum. (Note that the limits have to be taken in the correct order: volume goes to infinity before momentum goes to zero.)

Other quantities we shall also look at are the monopole density

$$\rho = \frac{1}{4V} \sum_{x,\mu} |M_\mu(x)| \tag{2.9}$$

(V: lattice volume) and the string density [1]

$$\sigma = \frac{1}{4V} \sum_{x,\mu<\nu} |N_{\mu\nu}(x)|. \tag{2.10}$$

We would expect that these quantities would show non-analytic behaviour (though not necessarily a divergence) around a phase transition.

3 Analytic results

The authors of [20, 17] claim that even in quenched non-compact QED there is a phase transition at which the magnetic monopoles condense. Because in the quenched case the action is Gaussian, we can derive analytic formulae for most quantities [24]. The simplest quantity is the string density. The probability distribution for a single $F_{\mu\nu}$ field is a Gaussian:

$$\Psi(F) = \pi^{-\frac{1}{2}} \beta^{\frac{1}{2}} \mathrm{e}^{-\beta F^2}. \tag{3.1}$$

[1] The normalization is chosen to agree with ref. [20].

The distribution is completely determined because we know the width $\langle F^2 \rangle = 1/(2\beta)$. This gives (cf. eq. (2.10))

$$\begin{aligned} \sigma(\beta) &= \frac{3}{2}\langle |N(F)| \rangle \\ &= \frac{3}{2}\int_{-\infty}^{\infty} dF\, \Psi(F)\, |N(F)| \\ &= \frac{3}{2}\sum_{n=0}^{\infty} \mathrm{erfc}\left((2n+1)\pi\beta^{\frac{1}{2}}\right) . \end{aligned} \tag{3.2}$$

On a finite lattice with volume V the only change is that the width of the Gaussian is reduced to $\langle F^2 \rangle = (V-1)/(2V\beta)$. This means that σ on a finite lattice can be found by making the replacement $\beta \to \beta V/(V-1)$ in eq. (3.2). One sees that finite size effects are already negligible on rather small lattices (which holds for all the other quantities in this section as well).

The next quantity of interest is the monopole density. To find the monopole density we need the probability distribution for the six F fields on the faces of a cube. The outwardly directed 'plaquettes' are labelled using the dice convention, namely that F_n and F_{7-n} are on opposite faces. We find

$$\begin{aligned} \Psi(F_1, \ldots, F_6) &= \pi^{-\frac{5}{2}} 6^{\frac{1}{2}} (a-b)^{\frac{3}{2}} (a+b) \beta^{\frac{5}{2}} \delta(F_1 + \cdots + F_6) \\ &\quad \times \exp\{-\beta a (F_1^2 + \cdots + F_6^2) - 2\beta b (F_1F_6 + F_2F_5 + F_3F_4)\}. \end{aligned} \tag{3.3}$$

This is the most general Gaussian form consistent with cubic symmetry and the Bianchi identity. (Because $F_1 + \cdots + F_6 = 0$ we can add an arbitrary multiple of $(F_1 + \cdots + F_6)^2$ to the exponent without changing the distribution Ψ at all. This freedom has been used to eliminate terms of the form F_1F_2 etc.) The parameters a and b are fixed by the known expectation values

$$\begin{aligned} \langle F_1^2 \rangle &= \frac{2}{\beta}\int \frac{d^4k}{(2\pi)^4} \frac{1 - c_1}{4 - c_1 - c_2 - c_3 - c_4} &&= \frac{1}{2\beta}, \\ \langle F_1F_2 \rangle &= \frac{1}{2\beta}\int \frac{d^4k}{(2\pi)^4} \frac{-1 + c_1 + c_2 - c_1c_2}{4 - c_1 - c_2 - c_3 - c_4} &&= -\frac{\gamma}{2\beta}, \\ \langle F_1F_6 \rangle &= \frac{2}{\beta}\int \frac{d^4k}{(2\pi)^4} \frac{-c_1 + c_1c_2}{4 - c_1 - c_2 - c_3 - c_4} &&= \frac{4\gamma - 1}{2\beta}, \end{aligned} \tag{3.4}$$

where $c_\mu = \cos k_\mu$. (On a finite lattice the integrals are to be replaced by sums over allowed non-zero momenta.) On an infinite lattice

$$\gamma = 0.215\,563\ldots . \tag{3.5}$$

(Note that the value of γ is calculated from the correlation between two F fields, which is a gauge invariant quantity. Therefore Ψ is gauge invariant.) To give the correct expectation values, a and b must take the values

$$\begin{aligned} a &= \frac{1}{12}\frac{1+\gamma}{\gamma(1-2\gamma)} &&= 0.826\,049\ldots\,, \\ b &= \frac{1}{12}\frac{1-5\gamma}{\gamma(1-2\gamma)} &&= -0.052\,879\ldots\,. \end{aligned} \tag{3.6}$$

The monopole density $\rho(\beta)$ is

$$\begin{aligned} \rho(\beta) &= \langle |M_\mu| \rangle \\ &= \int_{-\infty}^{\infty} dF_1 \cdots dF_6 \, \Psi(F_1, ..., F_6) \, |N_1 + \cdots + N_6|. \end{aligned} \tag{3.7}$$

The fact that $|M_\mu|$ is bounded is enough to show that all derivatives of the monopole density $\rho(\beta)$ are finite at all β values. If $\rho(\beta)$ is expanded as a series of the form

$$\rho(\beta) = \beta^{\frac{5}{2}} \sum_{n=0}^{\infty} a_n (\beta_0 - \beta)^n \tag{3.8}$$

Figure 2: The monopole density ρ and the σ to ρ ratio as a function of β for quenched QED. The symbols represent the data from ref.[20] together with our data, while the curves show the analytic results on an infinite lattice.

about an arbitrary point β_0, then the bound on $|M_\mu|$ leads to bounds on the a_n:

$$0 < a_n < \frac{1}{3}\frac{(2n+3)!}{n!(n+1)!}\frac{1}{4^n}\beta_0^{-\frac{5}{2}-n}. \tag{3.9}$$

These bounds are strong enough to show that the series in eq. (3.8) is convergent with a radius of convergence of (at least) β_0. A convergent series expansion rules out the existence of any essential singularities in ρ. There is certainly no sign of a phase transition in $\rho(\beta)$. A similar proof holds for correlation functions involving a finite number of f's and N's.

In fig. 2 our formulae are checked against the Monte Carlo data [20]. The agreement

Figure 3: The same as fig. 2 for a larger range of β on a logarithmic scale. The data points at the smaller and larger β values are our measurements.

is excellent. In fig. 3 we show the monopole density over a larger β range. This enables us to show the asymptotic limits of our formulae. At $\beta = 0$, $\rho(\beta)$ goes to $7/15$, while for large β the density drops off like $6\,\mathrm{erfc}(\pi\beta^{\frac{1}{2}})$, i.e. approximately exponentially. Also in this extended range we find good agreement with the data. The ratio $\sigma(\beta)/\rho(\beta)$ goes to $1/4$ at large β. This must be so because at large β the only topological excitations that occur are isolated plaquettes with $|F| > \pi$, and these are surrounded by a monopole loop of length 4. At small β the ratio diverges like $\beta^{-\frac{1}{2}}$ (cf. fig. 3). In the interesting region around $\beta = 0.24$, $\sigma(\beta)/\rho(\beta)$ has only grown to ≈ 0.34, indicating that monopoles and antimonopoles are on a short leash (i.e. are tightly bound). If we look at a timeslice the average length of string joining each monopole-antimonopole pair is simply $4\sigma/\rho$, and is only about 1.4 lattice units.

We now turn to the discussion of the susceptibilities. To calculate these we need to know the general two-'plaquette' distribution $\Psi(F_i, F_j)$ and so the compact photon propagator $\langle f_i f_j \rangle$. If

$$\langle F_i^2 \rangle = \langle F_j^2 \rangle = \frac{1}{2\beta} \quad \text{and} \quad \langle F_i F_j \rangle = \frac{p_{ij}}{2\beta} \tag{3.10}$$

with p_{ij} given by the non-compact photon propagator [27],

$$\begin{aligned} \langle F_{\alpha\beta}(0) F_{\mu\nu}(x) \rangle \;=\; & \frac{1}{2\beta} \int \frac{d^4k}{(2\pi)^4} \frac{1}{4 - c_1 - c_2 - c_3 - c_4}\, \mathrm{e}^{ik\cdot x} \times \\ & \Big(\delta_{\alpha\mu}(1 - \mathrm{e}^{-ik_\beta})(1 - \mathrm{e}^{ik_\nu}) - \delta_{\alpha\nu}(1 - \mathrm{e}^{-ik_\beta})(1 - \mathrm{e}^{ik_\mu}) \\ & - \delta_{\beta\mu}(1 - \mathrm{e}^{-ik_\alpha})(1 - \mathrm{e}^{ik_\nu}) + \delta_{\beta\nu}(1 - \mathrm{e}^{-ik_\alpha})(1 - \mathrm{e}^{ik_\mu}) \Big) \end{aligned} \tag{3.11}$$

then

$$\Psi(F_i, F_j) = \frac{\beta}{\pi}(1 - p_{ij}^2)^{-\frac{1}{2}} \exp\{ -\frac{\beta}{1 - p_{ij}^2}(F_i^2 + F_j^2) + \frac{2\beta p_{ij}}{1 - p_{ij}^2} F_i F_j \}. \tag{3.12}$$

With the help of the Fourier series

$$f = -2 \sum_{k=1}^{\infty} \frac{(-1)^k}{k} \sin(kF) \tag{3.13}$$

we derive the series

$$\langle f_i f_j \rangle = 4 \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{(-1)^{n+m}}{nm} \sinh\left(\frac{nm p_{ij}}{2\beta} \right) \exp\left(-\frac{n^2 + m^2}{4\beta} \right) \tag{3.14}$$

for the compact propagator. Knowing $\langle f_i f_j \rangle$ for any pair of plaquettes we can calculate χ_m from its definition eq. (2.6).

In fig. 4 we show the theoretical curve for χ_m as a function of β. At finite β there is no divergence: $\chi_m(\beta)$ is a correlation function involving f's (or N's) for which the proof

Figure 4: The monopole susceptibility χ_m for quenched QED as a function of β on an infinite lattice.

of analyticity applies. At $\beta = 0$, χ_m has the value $\frac{2}{3}\pi^2$ because the f fields are completely uncorrelated and evenly distributed in $(-\pi, \pi]$. It is striking that after $\beta \approx 0.1$ the curve drops exponentially in β because the monopole density is dropping so quickly.

If we had monopole condensation we would expect that the monopole correlation function ($x_4 \equiv t$)

$$C(t) = \sum_{\vec{x}} \langle M_\mu(\vec{x}, x_4) M_\mu(0) \rangle \tag{3.15}$$

would exhibit long-range order [26]. $C(t)$ can be evaluated using the general form (3.14) for $\langle f_i f_j \rangle$. In fig. 5 we show $C(t)$ on an infinite lattice at the β value ($\beta = 0.244$) where Kocić et al. [17] place a phase transition. We find that the correlation function drops by

approximately three orders of magnitude between $t = 1$ and 2.

The Fourier transform of $C(t)$ gives the charge susceptibility $\chi_q(p)$ (see eq. (2.8)) when the momentum p is directed along a lattice axis. In fig. 6 we show $\chi_q(p)$ as a function of p for three different β values. Because we can work on an infinite volume we can go all the way to $p = 0$. We never see the enhanced long wavelength fluctuations characteristic of Bose-Einstein condensation (see fig. 1), on the contrary long wavelength fluctuations are strongly suppressed. The charge susceptibility does not diverge but vanishes like p^2 for all β values. As usual the finite size effects are very small, and the curve measured on a finite lattice is already very close to that seen on an infinite lattice.

Figure 5: The analytic result for the monopole correlation function $C(t)$ for quenched QED at $\beta = 0.244$ on an infinite lattice. The solid (open) symbols represent positive (negative) values of $C(t)$.

Figure 6: The susceptibility $\chi_q(p)$ as a function of momentum on an infinite lattice for three values of β for quenched QED. Kocič et al. place a phase transition at $\beta = 0.244$. The momentum was chosen along a lattice axis.

Many other quantities are also calculable. Particularly simple to calculate are the compact Wilson loops, which again are analytic and give a potential between charges which is Coulombic at all $\beta > 0$, inconsistent with confinement.

The authors of ref. [15] claim that strongly coupled dynamical non-compact QED undergoes monopole condensation and confines electric charge, the only evidence they present being the existence of a percolation threshold. Their case is greatly weakened by the fact that the quenched theory has a similar percolation threshold [17] (in fact, the evidence for the divergence of the Hands and Wensley cluster susceptibility is strongest for the quenched theory), but as we have seen there is no monopole condensation or confinement.

4 The dynamical case

We shall now investigate what happens if dynamical fermions are included. For the action, which corresponds to four fermion flavours, see eqs. (2.1-2.3).

If, as seems likely from the last section, the monopole properties are determined by very short distance fluctuations of the electromagnetic fields, we could expect that these

Figure 7: The monopole density as a function of the plaquette energy. The data are for four flavours of dynamical staggered fermions. The solid symbols represent our data, while the open symbols represent data from ref.[19]. The curve is the analytic result for the quenched case on an infinite lattice.

properties are determined by the plaquette energy values, because the plaquette energy is a good measure of the fluctuation strength.

Therefore we have plotted the monopole density against $P \equiv \frac{1}{12}\sum_{\mu<\nu}\langle F_{\mu\nu}^2\rangle$ in fig. 7 using data from refs. [19, 10] and our results given in Table 1. We also show the analytic curve calculated in the quenched case. We find surprisingly good agreement between the data and the analytic result, indicating that the inclusion of dynamical fermions does not change our previous conclusions. The data comes from a wide range of bare masses,

Figure 8: The σ to ρ ratio as a function of the plaquette energy. The data are for four flavours of dynamical staggered fermions as given in Table 1. The triangle is $m = 0.04$, the square is $m = 0.02$ and the circle is $m = 0.01$. The curve is the analytic result for the quenched case on an infinite lattice.

$m = 0.005 - 0.16$, and plotting against P has brought them all on a universal curve. (Note that the quenched case can also be viewed as the $m \to \infty$ limit.)

In fig. 8 we show the ratio σ/ρ. The measured values are about 2% higher than the quenched calculation. Plotting against P has again brought measurements at different masses onto the same curve. The fact that σ is a little different in the four flavour case than in the quenched case shows that the single F distribution deviates slightly from a Gaussian form.

Figure 9: The monopole susceptibility χ_m as a function of the plaquette energy. The data symbols are the same as in the previous figure. The curve is the analytic result for the quenched case on an infinite lattice.

The monopole susceptibility χ_m is plotted against P in fig. 9. Again, we compare the data with the analytic result. As has been remarked before [21], this quantity is hard to measure accurately because of large cancellations between positive and negative charges. Within the errors we find agreement with the analytic quenched result, and do not see any divergence of χ_m. (The 'phase transition' reported in [19] is at $P \approx 1.025$.)

In fig. 10 we show $\chi_q(p)$ as measured on our four-flavour configurations compared with curves calculated in the quenched case at the same value of P. Measurement and calculation are in excellent agreement. Even at small β (the upper curve) χ_q vanishes at small momenta, showing that the monopoles have not condensed. Consider for example χ_q measured at the momentum $p = 2\pi/L$. This measures charge density fluctuations with a wavelength equal to the size of our lattice. Since a wave with wavelength equal to the lattice size is positive in one half of the lattice and negative in the other we are measuring the difference between the monopole concentrations in the two halves of our lattice. If the monopoles have truly condensed the tendency of bosons to occupy the same quantum state would cause large differences between the number found in the right-hand and left-hand sides of our lattice. This happens for a condensed boson gas (see fig. 1), but for monopoles we see that long wavelength fluctuations are strongly suppressed, showing that condensation has not taken place.

Figure 10: The susceptibility $\chi_q(p)$ as a function of momentum for three values of β. The data are for four flavours of dynamical staggered fermions at $m = 0.04$. The curves are analytic results for the quenched case on an infinite lattice.

The surprising agreement between all the theoretical curves (with no adjustable parameters) and the Monte Carlo data confirms our hypothesis that the monopole properties are determined by short-distance fluctuations.

β	m	P	ρ	σ	χ_m
0.17	0.04	1.2832(6)	0.2087(2)	0.07621(9)	2.729(107)
	0.02	1.2669(6)	0.2041(2)	0.07395(9)	2.816(89)
0.18	0.04	1.2022(6)	0.1865(3)	0.06533(13)	2.371(150)
	0.02	1.1881(6)	0.1826(3)	0.06355(12)	2.813(146)
0.19	0.04	1.1343(5)	0.1673(2)	0.05662(7)	2.320(82)
	0.02	1.1194(5)	0.1626(2)	0.05459(7)	2.198(88)
	0.01	1.1106(9)	0.1603(3)	0.05355(11)	2.050(122)
0.20	0.04	1.0739(3)	0.1493(2)	0.04901(7)	2.110(85)
	0.02	1.0617(4)	0.1457(2)	0.04745(6)	1.969(77)
	0.01	1.0548(6)	0.1437(2)	0.04666(10)	2.095(121)
0.21	0.04	1.0234(4)	0.1342(2)	0.04289(6)	1.890(63)
	0.02	1.0133(4)	0.1314(2)	0.04176(6)	1.806(64)
	0.01	1.0068(4)	0.1292(2)	0.04091(9)	1.899(89)
0.22	0.04	0.9779(3)	0.1205(2)	0.03766(6)	1.595(56)
	0.02	0.9692(3)	0.1179(2)	0.03670(5)	1.476(51)

Table 1: The monopole density ρ, the string density σ and the monopole susceptibility χ_m on a 12^4 lattice with four flavours of dynamical staggered fermions. Also given are the plaquette energy values $P = \frac{1}{12}\sum_{\mu<\nu}\langle F_{\mu\nu}^2\rangle$.

5 Discussion

In this work we have investigated monopoles in quenched and dynamical non-compact lattice QED. In the quenched case we have derived analytic formulae which we have checked against numerical data. Here we can prove that there are no singularities and divergences in the quantities we have looked at. The same formulae describe the dynamical case when quantities are plotted against the plaquette energy, which measures the strength of the electromagnetic field. Thus we have arrived at a quantitative understanding of monopoles in both the quenched and dynamical case.

The similarity of the percolation threshold in the quenched, $N_f = 2$ and $N_f = 4$ cases [24], occurring at almost the same monopole density and with almost identical

critical exponents, lends further support to the idea that the monopole behaviour is the same in the quenched and dynamical case. This picture may, however, change when N_f is so large that the phase transition becomes first order [23].

We have also looked at the distribution of monopoles and antimonopoles in a time slice. We saw that 60% of all monopoles have an antimonopole on adjacent sites and only 10% a monopole. So actually what we see looks more like a gas of dipoles than a condensate of monopoles. (One can also see this from the low σ/ρ value; see figs. 2,3, 8.)

In dynamical QED we have already checked [10] that the potential is Coulombic and the photon does not acquire a mass. This is also inconsistent with confinement at low β.

It is not surprising that when the monopole density becomes large ($\rho \gtrsim 0.15$) percolation takes place. However, percolation is not necessarily connected with condensation or with any other field-theoretic or thermodynamic property of the theory. Indeed, it is rather easy to find examples where the percolation threshold and the "authentic" phase transition are at different couplings. One example is the Ising model of higher dimension, where the percolation threshold lies at higher β than the phase transition [28]. Also, if one looks at the 3d Ising model at $\beta = 0$ and non-zero magnetic field, h, one finds percolation thresholds at certain values of h, because if $\beta = 0$ the spins are randomly distributed with the concentration controlled by the magnetic field. It is well known that randomly distributed sites on a cubic lattice first percolate when the concentration has reached $\approx 32\%$ [29]. So for strong negative h the up-spins do not percolate, while the down-spins do. There is a percolation threshold at about 32% ($h \approx -0.38$) and then both spins percolate. At a concentration of $\approx 68\%$ ($h \approx +0.38$) there is a second threshold above which only the up-spins percolate. Despite the occurrence of these percolation thresholds there are certainly no phase transitions at non-zero h in the Ising model.

The cluster susceptibility of Hands and Wensley [20] is

$$\chi_c = \langle \frac{(\sum_{n=4}^{n_{max}} g_n n^2) - n_{max}^2}{\sum_{n=4}^{n_{max}} g_n n} \rangle, \tag{5.1}$$

where n is the number of dual sites in a cluster linked together by monopole world lines, g_n is the number of clusters of size n and n_{max} is the size of the largest cluster. This susceptibility has long been used to find percolation thresholds (it is essentially the $S(p)$ of [29] or the χ^f of [30]). However it can lead to misleading results if it is used to locate phase transitions due to the following defects. First of all, χ_c is not a Green's function, so a divergence of χ_c does not imply an infinite correlation length and so does not indicate a second order phase transition. Furthermore, χ_c counts monopoles and antimonopoles with the same sign, whereas physically they should contribute with opposite sign.

Moreover, in order to compute χ_c one must treat monopoles differently depending on whether or not they belong to the same cluster. No physical operator can do this because

by Bose symmetry all physical operations treat indistinguishable particles identically. If χ_c is not a physically realizable quantity the fermions cannot couple to it and the divergence of χ_c cannot cause a chiral phase transition.

It is worth noticing that in dynamical non-compact QED χ_c diverges in places where there is no phase transition. The chiral phase transition takes place only at $m = 0$, while at finite m quantities such as $\langle\bar{\chi}\chi\rangle$ are smooth functions of β. However, χ_c diverges not only at $m = 0$ but for all m including $m = \infty$, (see ref. [18, 19]).

In conclusion, the papers [12-20] on monopoles in non-compact QED do not prove that monopoles are relevant in the continuum limit of the lattice theory, and so do not invalidate the picture of the chiral phase transition presented in refs. [1-11].

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