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HIGGS PHENOMENOLOGY AT ONE LOOP IN THE STANDARD MODEL†

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ABSTRACT

We present a synopsis of Higgs-boson phenomenology at one loop in the standard model. Two-loop effects are reported on as far as present knowledge goes. The phenomena are classified according to the number of external particles participating in the interaction. The relevant classes correspond to two-, three-, four-, and five-point functions. Two-point functions are the source of indirect evidence for the Higgs boson; three-point functions control its two-body decays and its production through fusion processes at hadron and photon colliders; four-point functions trigger its three-body decays and its associated production with another particle that balances the quantum numbers or serves as a signal; five-point functions mediate Higgs-boson four-body decays and production from intermediate-boson fusion at hadron, ep, and e^+e^- colliders. Of course, the study of quantum corrections to these processes is interesting in its own right, since it provides a deeper insight into the completely untested Higgs sector of the electroweak theory. However, in some cases the effects are very significant and their neglect leads to rather inadequate predictions, *e.g.*, in the case of the $H \to b\bar{b}$ decay or the gluon-fusion mechanism. The purpose of this article is to deliver a broad review of the quantum effects related to the Higgs boson, to highlight their phenomenological relevance, to list the most important formulae for quick reference, and to generate a rich bibliography, so that the interested reader can find the source of the results. We shall illustrate the effects in numerous figures and investigate various limiting cases, *e.g.*, light or heavy virtual fermions, a heavy Higgs boson, *etc.* For the reader's convenience, we shall collect the relevant two-, three-, and four-point loop integrals as well as the boson vacuum polarizations and fermion self-energies in a number of appendices.

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1. Introduction

One of the great puzzles of contemporary elementary particle research is whether nature indeed makes use of the Higgs mechanism [1] in connection with spontaneous symmetry breaking to generate the observed particle masses. In the minimal standard model of electroweak interactions [2] this concept is implemented by means of one weak-isospin doublet of complex scalar fields with weak hypercharge $Y = 1$. During the spontaneous breakdown of the $\mathrm{SU}(2)_I \otimes \mathrm{U}(1)_Y$ gauge symmetry, three of these four degrees of freedom are absorbed to create the longitudinal polarization states of the intermediate bosons, $W^\pm$ and Z. At the same time, the quarks and charged leptons acquire masses through their Yukawa-type interaction with this complex scalar doublet. As an aftermath of this miracle, there remains one neutral scalar boson with positive parity and charge conjugation, the so-called physical Higgs boson, H.

Most of the Higgs-boson properties are fixed by the intrinsic structure of the standard model, *e.g.*, the vacuum expectation value, $v = 2^{-1/4} G_F^{-1/2} \approx 246\,\mathrm{GeV}$, and its couplings to the gauge bosons, $g_{VVH} = 2^{5/4} G_F^{1/2} M_V^2$ $(V = W, Z)$, and fermions, $g_{f\bar{f}H} = 2^{1/4} G_F^{1/2} m_f$. However, its mass, M_H, and its self-couplings, which depend on M_H, are essentially unspecified. The requirement that the amplitude of intermediate-boson scattering at high energies does not violate unitarity establishes an M_H upper bound at $(8\pi\sqrt{2}/3G_F)^{1/2} \approx 1\,\mathrm{TeV}$ [3]. A similar number follows from considerations concerning the practicability of perturbation theory [4, 5]. Lattice computations suggest a somewhat lower value, viz. $M_H/M_W \lesssim 9$ [6].

The direct experimental discovery of the Higgs boson would deliver a crucial confirmation of the standard model and much effort and resources have been invested towards this goal, so far in vain. The search for the Higgs boson is primarily aggravated by the lack of knowledge of its mass and by the smallness of its couplings to stable matter and light. Search strategies depend largely on the suspected value of M_H. It has become customary call a Higgs boson with $M_Z \lesssim M_H \lesssim 2M_W$ intermediate-mass, otherwise low-mass or high-mass. The branching fractions of the major decay modes of low- and intermediate-mass Higgs bosons are summarized in Fig. 1. High-mass Higgs bosons decay almost exclusively into W^+W^-, ZZ, and $t\bar{t}$ pairs; in the very upper M_H range, however, tree-level three-body decays may reach branching fractions of a few percent [7].

At present, the Higgs boson is being searched at the CERN Large Electron Positron Collider (LEP) and SLAC Linear Collider (SLC) via the Bjorken process [8], $e^+e^- \to Z \to f\bar{f}H$. The absence of genuine Higgs-boson signals in the steadily growing sample of observed

Z bosons has been shifting the M_H lower bound upwards. Having observed zero signal events, it is possible to exclude at the 95% confidence level those M_H values which would lead to more than three predicted events. At this time, this is the case for $M_H < 63.5\,\mathrm{GeV}$ [9]. The search potential of LEP 1 will be exhausted at 65 GeV or so [10].

At LEP 2, operating somewhere around $\sqrt{s} = 190\,\mathrm{GeV}$, the M_H range up to 90 GeV will be covered using the bremsstrahlung process [11], $e^+e^- \to ZH$, where the Z boson is tagged by its $Z \to \mu^+\mu^-$ decay. The optimum laboratory for discovering and studying the intermediate-mass Higgs boson would arguably be an e^+e^- linear collider with $300\,\mathrm{GeV} \lesssim \sqrt{s} \lesssim 500\,\mathrm{GeV}$ [12]. Various locations and designs have been suggested for such a machine and the generic term Next Linear Collider (NLC) will be used in the following. The intermediate-mass Higgs boson would show up as a spectacular peak in the distribution over the recoil mass of the bremsstrahung process, defined by $\left(p_{e^+} + p_{e^-} - p_{\mu^+} - p_{\mu^-}\right)^2$ [12]. The W^+W^--fusion process [13, 14], $e^+e^- \to \nu_e\bar{\nu}_e(W^+)^*(W^-)^* \to \nu_e\bar{\nu}_e H$, where the stars indicate virtual particles, gains significance as $\sqrt{s}$ increases. The cross section of ZZ fusion [13, 14], $e^+e^- \to e^+e^-Z^*Z^* \to e^+e^-H$, is about one order of magnitude smaller due to weaker couplings. The process $e^+e^- \to t\bar{t}H$ [15] has an even smaller cross section but is interesting because it can be used to measure the coupling of the intermediate-mass Higgs boson to the top quark. As for the high-mass Higgs boson, this coupling may be probed by observing $e^+e^- \to ZH^* \to Zt\bar{t}$ [16]. WWH, ZZH, and $Z\gamma H$ associated production [17, 18] and the higher-order electroweak processes $e^+e^- \to \nu eWH, \nu\bar{\nu}ZH, \nu\bar{\nu}\gamma H$ [17], where the photons carry high transverse momenta, are generally less suited for the Higgs-boson search; however, they include many electroweak couplings simultaneously and could thus provide tests of the theory, since gauge cancellations occur among the diagrams. With an integrated luminosity of $50\,\mathrm{fb}^{-1}$ at $\sqrt{s} = 500\,\mathrm{GeV}$, the discovery limit of the NLC should be somewhere around $M_H = 350\,\mathrm{GeV}$ in the $\nu\bar{\nu}W^+W^-$ and $\nu\bar{\nu}ZZ$ channels [19]. A 1-TeV collider with $200\,\mathrm{fb}^{-1}$ should be able to cover the M_H range way up to 700 GeV [20].

It is a well-known fact that the laser conversion method allows one to transform an electron beam into a photon beam with almost the same characteristics [21]. This renders it possible to operate the NLC as $e\gamma$ and $\gamma\gamma$ colliders. The dominant Higgs-boson production mechanisms will then be $e\gamma \to \nu WH$ [22, 23], $e\gamma \to eZH$ [22], $e\gamma \to e\gamma\gamma \to eH$ [24], and $\gamma\gamma \to H$ [25]. The process $\gamma\gamma \to t\bar{t}H$ [26] is sensitive to the $t\bar{t}H$ coupling but is greatly phase-space suppressed.

At future multi-TeV hadron colliders such as the CERN Large Hadron Collider (LHC) and the Superconducting Super Collider (SSC), gluon fusion [27], $gg \to H$, which proceeds via

massive-quark triangle graphs, will, in general, be the most copious source of Higgs bosons. However, W^+W^- and ZZ fusion, where the intermediate bosons are radiated off the quark lines [14, 28], may come to dominate the yield towards the upper end of the M_H range provided that m_t is moderate. The production of a Higgs boson in association with a $t\bar{t}$ pair [29], $gg \to t\bar{t}H$, or an intermediate boson [30, 31], $q\bar{q}' \to W^{\pm}H$ and $q\bar{q} \to ZH$, have been found to be throughout less yielding than the other mechanisms. For $M_H \lesssim 140\,\mathrm{GeV}$, it may be possible, under favourable experimental conditions, to discern the $H \to 2\gamma$ signal from the continuum $\gamma\gamma$ background [32, 33]. In the window $140\,\mathrm{GeV} \lesssim M_H \lesssim 2M_Z$, the Higgs boson may be recognized, after selective cuts, by its $H \to (W^+)^*(W^-)^* \to \ell^-\bar{\nu}_\ell\,\ell'^+\nu_{\ell'}$ decay [34]. For $140 \lesssim M_H \lesssim 800\,\mathrm{GeV}$, the legendary gold-plated $H \to Z^{(*)}Z^{(*)} \to \ell^+\ell^-\ell'^+\ell'^-$ events, with $\ell, \ell' = e, \mu$, appear as a prominent peak in the four-lepton invariant mass distribution above a small background [32].

At future ep colliders, intermediate-boson fusion will be the dominant source of Higgs bosons over the full M_H range [14, 35].

Numerous comprehensive—and in some cases even encyclopaedic—reviews of Higgs-boson phenomenology [36] have been published in the recent few years, including self-training guides for Higgs hunters [37]. The logistics of detecting the Higgs boson and of investigating its characteristics was on the agenda of many workshops and conferences [38], and even a flow chart for Higgs hunting has been elaborated [39]. In the present article we shall focus attention on an interesting aspect of this flourishing field of research, which has not yet been reviewed: namely on the loop effects connected with the Higgs boson in the standard model.

Prior to the direct experimental discovery of the Higgs boson, the only source of information on its mass is in the analysis of Higgs-boson-induced quantum corrections to precisely measured observables. Due to its weak couplings to stable particles, the Higgs boson can manifest itself only through the intermediate-boson self-energies. However, the sensitivity of such corrections to M_H is screened, so that meaningful constraints on M_H can be obtained only when at least the dominant effects beyond one loop are taken into account. These include two-loop corrections of the orders $G_F^2 M_W^2 M_H^2$, $G_F^2 m_t^4$, and $G_F \alpha_s m_t^2$ as well as $t\bar{t}$ threshold effects, which can be estimated from realistic assumptions on the $e^+e^- \to t\bar{t}$ excitation curve via properly subtracted dispersion relations. We shall utilize a generalized formulation of the popular S, T, and U parameters that correctly takes into account the physics at the weak scale and incorporates the dominant higher-order corrections enumerated above. This will provide a powerful tool to reliably estimate the m_t-M_H interdependence and to update the allowed ranges for both masses from a global analysis of high-precision data. After more

accurate M_W measurements and the discovery of the top quark, this kind of analysis will eventually allow us to pin down the value of M_H in the standard model.

After the Higgs boson has been discovered and its mass is known, the study of its properties will receive a high priority. In order to distinguish the Higgs boson of the minimal standard model from the Higgs sectors of more extended theories, such as supersymmetry, technicolor, *etc.*, it is necessary to have the radiative corrections to its production and decay processes well under control. One is thus led to consider the renormalization of three-, four-, and five-point functions with one external Higgs-boson line. Some of these Green functions are themselves loop-induced, so that their radiative corrections involve two-loop diagrams. The set of three-point vertices consists of ZZH, $Z\gamma H$, $\gamma\gamma H$, ggH, W^+W^-H, and $f\bar{f}H$, where f represents a generic massive fermion. Large corrections arise from QCD, from the top quark, and in the high-M_H limit. Most notably, QCD corrections reduce the $H \to b\bar{b}$ width by 50–60%, while they enhance the $H \to gg$ width by more than 60%. Similarly, they amplify the cross section of $pp \to H + X$ by a factor of 1.5–1.7.

In order for a four-point function to be relevant for Higgs-boson production at high-energy colliders, two of its external lines must be e^+e^- or any pairing of e^-, γ, q, and g. The fourth line then represents a particle that is produced in association with the Higgs boson. When the electron and parton masses are neglected, the only possibilities at tree level are e^+e^-ZH, $q\bar{q}ZH$, and $q\bar{q}'W^\pm H$. At future e^+e^- colliders, ZH associated production is the dominant source of Higgs bosons for centre-of-mass (CM) energy $\sqrt{s} \lesssim 500$ GeV and, depending on the value of M_H, the electroweak corrections may enhance the cross section by up to 25%. At future hadron supercolliders, high-transverse-momentum Higgs bosons can be produced from $pp \to ZH + X$ and $pp \to W^\pm H + X$ (via quark-antiquark annihilation) and QCD corrections are needed to reduce the scale dependence of the cross sections. Although these processes are not dominant, they are nevertheless relevant, since the leptonic decays of the intermediate bosons provide a spectacular signature for the Higgs-boson. Loop-induced four-vertices of interest are $e^+e^-\gamma H$, $ggZH$, $gggH$, and $q\bar{q}gH$. They mediate $e^+e^- \to \gamma H$, $pp \to ZH + X$ (via gluon fusion), and $pp \to jH + X$, where the Higgs boson acquires a finite transverse momentum. Vice versa, all these four-point functions may trigger three-body decays of the Higgs boson provided the latter are kinematically allowed. Finally, if $M_H < M_Z$ then the Higgs boson can be produced at LEP 1 and SLC through the Bjorken process, $Z \to f\bar{f}H$, and radiative corrections induce a shift in the M_H lower bound.

The most important five-point functions in Higgs-boson phenomenology are those connected with intermediate-boson fusion, which is the dominant production mechanism in e^+e^-

and pp collisions for high values of M_H and in ep collisions for all values of M_H, and those related to $\ell^+\ell^- H$ associated production via a Z^*H intermediate state. The leading QCD corrections to the pp and ep processes can be evaluated by means of the structure-function approach. The QED corrections to $e^+e^- \to \mu^+\mu^- H$ including the initial-final state interference, which involve genuine five-point loop integrals, are also available. Full electroweak corrections have not been calculated for any process of this category.

This article is organized as follows. Chapter n ($n = 2, \ldots, 5$) is dedicated to loop effects connected with n-point processes involving the Higgs boson. The relevant two-, three-, and four-point one-loop integrals are collected in Appendix A. The boson and fermion self-energies are listed in Appendices B and C, respectively.

2. Two-point functions: oblique corrections

So far the Higgs boson has eluded direct experimental discovery and the only way to extract from data information on its characteristics, *e.g.*, its mass, has been to analyze Higgs-boson-induced quantum corrections to precisely measured processes. Since the Higgs boson couples only feebly to stable matter and, at tree level, does not interact with the photon, it dominantly manifests itself through quantum corrections to the propagators of the weak gauge bosons.

Radiative corrections to the gauge-boson propagators are customarily called *oblique* [40]. In electroweak physics oblique radiative corrections can be parameterized in terms of four independent quantities which have the property that they are automatically ultraviolet finite. Such quantities are sometimes called *observable radiative corrections* [41]. One of them resides in the purely electromagnetic part of the standard electroweak theory, so that there remain three variables to characterize the more exciting genuinely weak phenomena and, in particular, to probe the Higgs sector. In addition, they provide a convenient tool to systematically classify deviations due to new physics beyond the standard model. Such three-dimensional parameterizations have gained vogue some time ago and meanwhile there exits a variety of definitions and notations [42–47], which have been nicely catalogued in Ref. 48. Perhaps, the most popular language is the one by Peskin and Takeuchi [44], who coined the (S, T, U) terminology. However, these authors were mainly concerned with new physics at energy far above the weak scale and their original definitions of S and U lead to mass singularities when they are naïvely applied to the standard model itself. The same is true of the $(\varepsilon_1, \varepsilon_2, \varepsilon_3)$ set introduced by Altarelli and Barbieri [47]. To properly accommodate the physics at the weak

scale, one eliminates the derivatives, evaluated at zero momentum transfer, of the non-QED vacuum-polarization functions by suitably re-introducing the weak-boson masses [45, 46, 49].

2.1 One-loop formalism

As explained in Ref. 50, observable radiative corrections can be identified quite systematically in the on-mass-shell (OMS) scheme, which uses the fine-structure constant, α, and the physical particle masses as basic parameters. Toward this end, one considers the transverse gauge-boson vacuum-polarization functions in their renormalized forms,

$$\begin{aligned}
\hat{\Pi}_{WW}(q^2) &= \Pi^{WW}(q^2) - \mathcal{R}e\,\Pi^{WW}(w) + (q^2 - w)\left(\frac{c_{\rm w}^2}{s_{\rm w}^2}R - \Pi^{AA\prime}(0)\right), \\
\hat{\Pi}_{ZZ}(q^2) &= \Pi^{ZZ}(q^2) - \mathcal{R}e\,\Pi^{ZZ}(z) + (q^2 - z)\left[\left(\frac{c_{\rm w}^2}{s_{\rm w}^2} - 1\right)R - \Pi^{AA\prime}(0)\right], \\
\hat{\Pi}_{ZA}(q^2) &= \Pi^{ZA}(q^2) - \Pi^{ZA}(0) - q^2\,\frac{c_{\rm w}}{s_{\rm w}}R, \\
\hat{\Pi}_{AA}(q^2) &= \Pi^{AA}(q^2) - q^2\,\Pi'_{AA}(0),
\end{aligned} \tag{2.1}$$

where q denotes the external four-momentum and

$$R = \mathcal{R}e\left(-\frac{\Pi^{WW}(w)}{w} + \frac{\Pi^{ZZ}(z)}{z}\right) - 2\,\frac{s_{\rm w}}{c_{\rm w}}\,\frac{\Pi^{ZA}(0)}{z}. \tag{2.2}$$

Here and in the following, we use the short-hand notation $w = M_W^2$, $z = M_Z^2$, $h = M_H^2$, and $c_{\rm w}^2 = 1 - s_{\rm w}^2 = w/z$. The standard-model expressions in $\mathcal{O}(\alpha)$ for the unrenormalized Π functions may be found in Ref. 51; for completeness, they are listed also in Appendix B. Perturbative $\mathcal{O}(\alpha\alpha_s)$ corrections to these functions are available from Refs. 52, 53. As per construction, Eq. (2.1) satisfies the on-shell conditions,

$$\mathcal{R}e\,\hat{\Pi}_{WW}(w) = \mathcal{R}e\,\hat{\Pi}_{ZZ}(z) = \hat{\Pi}_{ZA}(0) = \hat{\Pi}_{AA}(0) = 0. \tag{2.3}$$

On the other hand, $\hat{\Pi}_{WW}(0)$, $\hat{\Pi}_{ZZ}(0)$, $\mathcal{R}e\,\hat{\Pi}_{ZA}(z)$, and $\mathcal{R}e\,\hat{\Pi}_{AA}(z)$ represent nontrivial and manifestly finite combinations of unrenormalized two-point functions, which are clearly renormalization-scheme independent. They bear the following physical meanings:

- (i) $\Delta r_{\rm oblique} = \hat{\Pi}_{WW}(0)/w$ embodies the propagator corrections to the muon lifetime [54];
- (ii) $\Delta\rho = \hat{\Pi}_{ZZ}(0)/z - \hat{\Pi}_{WW}(0)/w$ measures the quantum corrections to the ratio, $\rho = 1/(1-\Delta\rho)$, of the neutral- and charged-current amplitudes at low energy [55];

(*iii*) $\Delta\kappa = -(c_{\rm w}/s_{\rm w})\,\mathcal{R}e\,\hat{\Pi}_{ZA}(z)/z$ controls the effective weak mixing angle, $\bar{s}^2_{\rm w} = s^2_{\rm w}(1+\Delta\kappa)$, that occurs in the vector coupling of the $Z \to f\bar{f}$ decay rate [56];

(*iv*) $\Delta\alpha = -\,\mathcal{R}e\,\hat{\Pi}_{AA}(z)/z$ determines the running fine-structure constant at the Z-boson scale, $\bar{\alpha} = \alpha/(1-\Delta\alpha)$, where α is the corresponding value at the electron scale [57].

The term $\Delta\alpha$ receives significant contributions only from charged particles with masses below the Z-boson scale, while high-mass particles are known to decouple from it [58]. In particular, the standard-model Higgs boson does not contribute to $\Delta\alpha$ at one loop. Thus, $\Delta\alpha$ is well under control and we shall not be concerned with it in the remainder of this section. Furthermore, we subtract from $\Delta r_{\rm oblique}$ its pure QED portion by setting $(1-\Delta r_{\rm oblique}) = (1-\Delta\alpha)(1-\Delta r_{\rm w})$, so that $\Delta r_{\rm w}$ is devoid of mass singularities when charged-particle masses are sent to zero.

The conventional parameters S, T, and U—or, equivalently, ε_1, ε_2, and ε_3—may be recovered as linear combinations of $\Delta r_{\rm w}$, $\Delta\rho$, and $\Delta\kappa$, namely

$$\begin{aligned}
\alpha S &= 4s^2_{\rm w}\varepsilon_3 = 4s^2_{\rm w}\left[c^2_{\rm w}(\Delta r_{\rm w} + \Delta\rho) + (c^2_{\rm w} - s^2_{\rm w})\Delta\kappa\right] , \\
\alpha T &= \varepsilon_1 = \Delta\rho , \\
\alpha U &= -4s^2_{\rm w}\varepsilon_2 = 4s^2_{\rm w}\left[s^2_{\rm w}(\Delta r_{\rm w} + 2\Delta\kappa) - c^2_{\rm w}\Delta\rho\right] .
\end{aligned} \tag{2.4}$$

Conversely,

$$\begin{aligned}
\Delta r_{\rm w} &= \frac{\alpha}{s^2_{\rm w}}\left[\frac{S}{2} - c^2_{\rm w}T - \frac{1}{4}\left(\frac{c^2_{\rm w}}{s^2_{\rm w}} - 1\right)U\right] = -\frac{c^2_{\rm w}}{s^2_{\rm w}}\,\varepsilon_1 + \left(\frac{c^2_{\rm w}}{s^2_{\rm w}} - 1\right)\varepsilon_2 + 2\varepsilon_3 , \\
\Delta\kappa &= \frac{\alpha}{s^2_{\rm w}}\left[-\frac{S}{4} + c^2_{\rm w}T + \frac{c^2_{\rm w}}{4s^2_{\rm w}}\,U\right] = \frac{c^2_{\rm w}}{s^2_{\rm w}}\,(\varepsilon_1 - \varepsilon_2) - \varepsilon_3 .
\end{aligned} \tag{2.5}$$

It is well known that T quantifies the violation of the custodial isospin symmetry [55], while S is an isospin-symmetric observable that measures the size of the Higgs sector and counts the total number of weak isodoublets [44]. In many new-physics scenarios, U plays a fairly unimportant rôle and it is explicitly fixed to zero by some authors. However, models with anomalous W-boson interactions provide a counter-example to this rule [47].

We are now ready to analyze in (S,T,U) space one-loop oblique corrections due to the

Higgs boson. For a first orientation, we consider the leading high-M_H behaviour:

$$\begin{aligned} S &= \frac{1}{12\pi} \ln \frac{h}{z} + \mathcal{O}(1) \,, \\ T &= -\frac{3}{16\pi c_{\rm w}^2} \ln \frac{h}{z} + \mathcal{O}(1) \,, \\ U &= \mathcal{O}\left(\frac{z}{h} \ln \frac{h}{z} \right) \,. \end{aligned} \tag{2.6}$$

In compliance with the screening theorem [4, 59], the M_H dependence is at most logarithmic. Equation (2.6) can be obtained also directly from the high-energy asymptotic formalism of Refs. 44, 47. It allows to quickly estimate the variations with M_H of S, T, and U if their values are known at some high reference value of M_H. However, it is clear that, for realistic values of M_H, Eq. (2.6) poorly predicts the absolute sizes of S, T, and U mainly because it does not include the sizeable contributions due to the (t, b) doublet. Neglecting m_b, the latter read

$$\begin{aligned} S &= \frac{1}{3\pi} \left[-\left(1 + \frac{11}{4r}\right) \sqrt{\frac{1}{r} - 1} \, \arcsin \sqrt{r} + \ln\left(2\sqrt{r}\,\right) + \frac{11}{4r} \right] \\ &= -\frac{1}{6\pi} \left(\ln \frac{m_t^2}{z} + \frac{1}{6} - \frac{7}{20} \, \frac{z}{m_t^2} + \mathcal{O}\left(\frac{z^2}{m_t^4} \right) \right) \,, \\ T &= \frac{3 m_t^2}{16\pi s_{\rm w}^2 w} \,, \\ U &= \frac{1}{\pi} \left[\left(1 + \frac{1}{2x}\right) \left(1 - \frac{1}{x}\right)^2 \ln(1-x) + \frac{1}{4x} + \frac{1}{2x^2} \right. \\ &\qquad \left. + \left(-1 + \frac{1}{4r}\right) \sqrt{\frac{1}{r} - 1} \, \arcsin \sqrt{r} - \ln\left(2\sqrt{r}\,\right) - \frac{1}{4r} \right] \\ &= \frac{1}{2\pi} \left[\ln \frac{m_t^2}{z} + \frac{1}{2} + \frac{3}{4} \left(-c_{\rm w}^2 + \frac{1}{5} \right) \frac{z}{m_t^2} + \mathcal{O}\left(\frac{z^2}{m_t^4} \right) \right] \,, \end{aligned} \tag{2.7}$$

where $x = w/m_t^2$ and $r = \left(z/4m_t^2\right)$. Setting also $m_t = 0$ in Eq. (2.7), one obtains the contributions per light-quark doublet,

$$\begin{aligned} S &= T = 0 \,, \\ U &= \frac{1}{\pi} \ln c_{\rm w}^2 \,. \end{aligned} \tag{2.8}$$

The corresponding contributions per light-lepton doublet emerge from Eq. (2.8) through division by three. Note that Eqs. (2.7), (2.8) cannot be derived from the formulation of Refs. 44, 47.

The exact predictions of S, T, and U at one loop in the standard model are displayed in Fig. 2 as functions of M_H for $m_t = 150\,\mathrm{GeV}$. Throughout this report, we set $M_Z = 91.187\,\mathrm{GeV}$ [60], adopt the parameterization of the hadronic contribution to the photon self-energy and the light-quark masses from Ref. 61, and take the residual input parameters from Ref. 57, unless stated otherwise. The individual contributions due to bosons (dotted lines), leptons (dashed lines), and quarks (dot-dashed lines) as well as the combined results (solid lines) are shown. The m_t dependence can easily be estimated using the asymptotic expansions of Eq. (2.7). The abscissa being logarithmic, the dotted and solid lines are approximately straight in the upper M_H range in the case of S and T and their slopes can be gleaned from Eq. (2.6). On the other hand, U varies only feebly with M_H, which again follows from Eq. (2.6). The leptonic contributions are indeed very well described by Eq. (2.8) and, hence, so are the ones due to the (u, d) and (c, s) doublets. In the case of S and T the quarkonic contributions are essentially generated by the (t, b) doublet alone. Obviously, S is strongly sensitive to M_H, it even changes sign as M_H runs from 50 GeV to 1 TeV. It is interesting to observe that the absolute size of this change is almost twice as large as the one due to the variation of m_t between 91 and 250 GeV. Unfortunately, the absolute size of S is more than five times smaller than the one of T, so that the observable radiative corrections that are directly extracted from current experiments, namely Δr, $\Delta\rho$, and $\Delta\kappa$, are much less sensitive to M_H. Moreover, it is evident from Eq. (2.6) that it is rather the logarithm of M_H than M_H itself that is extracted from data. On the other hand, the dominant quantity, T, depends in leading order quadratically on m_t, which explains why m_t can be pinned down through oblique corrections much more efficiently than M_H. It is, therefore, not surprising that the value of M_H determined by fine-tuning observable radiative corrections so as to fit best M_W, LEP, and other data depends delicately on whether radiative corrections beyond one loop are taken into account or not [62].

2.2 Higher-order corrections

A first step towards the considerable challenge of computing higher-order oblique corrections can be taken by concentrating on those two-loop diagrams which are expected to contribute dominantly. They involve virtual Higgs bosons, top quarks, and gluons. Bosonic two-loop diagrams involving ultraheavy Higgs bosons generate terms proportional to M_H^2 in

the gauge-boson vacuum polarizations [63]. In the language of S, T, and U these terms read

$$\begin{aligned} S &= \frac{G_F h}{16\pi^3\sqrt{2}}\left(\frac{14}{9}\zeta(2) - \frac{\pi}{4}\sqrt{3} - \frac{35}{36}\right) \\ &\approx 3.224\cdot 10^{-4}\, G_F h\,, \\ T &= \frac{G_F h}{64\pi^3\sqrt{2}c_{\rm w}^2}\left(-9\sqrt{3}\,{\rm Cl}_2\left(\frac{\pi}{3}\right) + \frac{9}{2}\zeta(2) + \frac{9}{4}\pi\sqrt{3} - \frac{21}{8}\right) \\ &\approx 4.272\cdot 10^{-4}\,\frac{G_F h}{c_{\rm w}^2}\,, \\ U &= 0\,. \end{aligned} \tag{2.9}$$

We observe that the M_H dependence of S and T is formally reinforced as compared to Eq. (2.6), while U continues to be insensitive to M_H. However, due to the smallness of the numerical coefficients, Eq. (2.9) is insignificant phenomenologically for $M_H \lesssim 1\,\text{TeV}$. In fact, assuming $M_H = 1\,\text{TeV}$, one finds $S \approx 3.8\cdot 10^{-3}$ and $T \approx 6.4\cdot 10^{-3}$, which has to be compared with Fig. 2. We remark in passing that, in contrast to some of the vacuum-polarization contributions of Ref. 63, the terms of Eq. (2.9) have a simple, overall dependence on the weak mixing angle.

Recently, the M_H-dependent two-loop corrections to the leading heavy-top-quark term of T in Eq. (2.7) have been computed with the result that [64]

$$\alpha T = N_c x\left(1 + x\rho^{(2)}\left(\frac{h}{4m_t^2}\right)\right)\,, \tag{2.10}$$

where $N_c = 3$, $x = (G_F m_t^2/8\pi^2\sqrt{2}\,)$,

$$\begin{aligned} \rho^{(2)}(r) &= -16r + 25 + 4(4r^2 - 12r - 3)\gamma - 3\left(1 - \frac{1}{2r}\right)\left[(4r-1)^2\,{\rm Li}_2(1-4r) - \zeta(2)\right] \\ &\quad - 16r(r-1)A(r) - 12(8r^2 - 12r + 5)B(r)\,, \\ A(r) &= \begin{cases} \sqrt{\dfrac{1}{r} - 1}\left(\Phi - \dfrac{\pi}{2}\right)\,, & \text{if } 0 < r \le 1\,, \\ \sqrt{1 - \dfrac{1}{r}}\,\phi\,, & \text{if } r > 1\,, \end{cases} \\ B(r) &= \begin{cases} \dfrac{1}{\sqrt{1/r - 1}}\,{\rm Cl}_2(2\Phi)\,, & \text{if } 0 < r < 1\,, \\ 2\ln 2\,, & \text{if } r = 1\,, \\ \dfrac{1}{\sqrt{1 - 1/r}}\left({\rm Li}_2\left(-\rho_-^2\right) + \dfrac{\zeta(2)}{2} + \phi^2\right)\,, & \text{if } r > 1\,, \end{cases} \end{aligned} \tag{2.11}$$

$r = \left(h/4m_t^2\right)$, $\Phi = \arcsin\sqrt{r}\,$, $\rho_\pm = \sqrt{r} \pm \sqrt{r-1}\,$, $\phi = \ln\rho_+$, and $\gamma = \ln(\rho_+ + \rho_-)$. A useful expansion of Clausen's function, ${\rm Cl}_2\,$, in terms of the Bernoulli numbers may be found in

Ref. 65. In the limiting cases one has [64]

$$\rho^{(2)}(r) = \begin{cases} -12\zeta(2) + 19 - 8\pi\sqrt{r} + \mathcal{O}(r\gamma)\,, & \text{if } r \ll 1\,, \\ 6\gamma^2 - 27\gamma + 6\zeta(2) + \dfrac{49}{4} + \mathcal{O}\left(\dfrac{\gamma^2}{r}\right)\,, & \text{if } r \gg 1\,. \end{cases} \tag{2.12}$$

For $M_H = 0$, one recovers the well-known result [66],

$$\rho^{(2)}(0) = 19 - 2\pi^2\,. \tag{2.13}$$

QCD corrections to gauge-boson propagators arise from virtual-gluon exchanges within quark-loop insertions. The results to $\mathcal{O}(\alpha\alpha_s)$ in perturbation theory may be found in Refs. 52, 53. It is straightforward to derive the corresponding contributions to S, T, and U. Neglecting all quark masses, except for m_t, one obtains

$$\begin{aligned} S &= \frac{\alpha_s(m_t)}{\pi^2}\left(-\frac{1}{6}\ln\frac{m_t^2}{z} + \frac{5}{12}\,\frac{V_1(r)}{r} - \frac{1}{4}\,\frac{A_1(r) - A_1(0)}{r}\right) \\ &= \frac{\alpha_s(m_t)}{\pi^2}\,\frac{1}{3}\left(-\frac{1}{2}\ln\frac{m_t^2}{z} + 2\zeta(3) + 1 + \mathcal{O}\left(\frac{z}{m_t^2}\right)\right)\,, \end{aligned} \tag{2.14}$$

$$\begin{aligned} T &= \frac{\alpha_s(m_t)}{\pi^2}\,\frac{m_t^2}{s_{\rm w}^2 w}\left(\frac{A_1(0)}{4} - F_1(0)\right) \\ &= -\frac{\alpha_s(m_t)}{\pi^2}\,\frac{m_t^2}{4s_{\rm w}^2 w}\left(\zeta(2) + \frac{1}{2}\right)\,, \end{aligned} \tag{2.15}$$

$$\begin{aligned} U &= \frac{\alpha_s(M_Z)}{\pi^2}\,2\ln c_{\rm w}^2 + \frac{\alpha_s(m_t)}{\pi^2}\left(\frac{1}{2}\ln\frac{m_t^2}{z} - 4\frac{F_1(x) - F_1(0)}{x} + \frac{1}{4}\,\frac{V_1(r) + A_1(r) - A_1(0)}{r}\right) \\ &= \frac{\alpha_s(M_Z)}{\pi^2}\,2\ln c_{\rm w}^2 + \frac{\alpha_s(m_t)}{\pi^2}\left(\frac{1}{2}\ln\frac{m_t^2}{z} - 2\zeta(3) + \frac{4}{9}\zeta(2) + \frac{1}{2} + \mathcal{O}\left(\frac{z}{m_t^2}\right)\right)\,, \end{aligned} \tag{2.16}$$

where $r = (z/4m_t^2)$, $x = w/m_t^2$, and $V_1(r)$, $A_1(r)$, and $F_1(x)$ are listed in Ref. 53. Here $\alpha_s(\mu)$ is evaluated at renormalization scale $\mu = M_Z$ for contributions due to the (u,d) and (c,s) doublets and at $\mu = m_t$ for those due to the (t,b) doublet. This choice can be motivated independently by direct inspection of the relevant two-loop integrals [67], by arguments based on an effective field theory [68], and by considerations involving dispersion relations [41]. As for the light-quark contributions to S, T, and U, first-order QCD corrections emerge from the free-field-theory results of Eq. (2.8) through multiplication by the universal factor $\alpha_s(M_Z)/\pi$ [53]. At present, this factor is known to third order [69], $[\alpha_s(M_Z)/\pi + 1.4092(\alpha_s(M_Z)/\pi)^2 - 12.7673(\alpha_s(M_Z)/\pi)^3]$, where the modified minimal-subtraction ($\overline{\rm MS}$) scheme with five active

quark flavours is assumed and numerically insignificant contributions due to certain singlet-type diagrams with three-gluon intermediate states are omitted. The effect of Eqs. (2.14)–(2.16) is visualized, too, in Fig. 2 (dot-dot-dashed lines). The T parameter is affected most strongly by QCD corrections [52]. It is interesting to observe that in the case of S, T, and U the one-loop effects related to the top quark are always weakened by their higher-order corrections; see Eqs. (2.10)–(2.16). As a consequence, the familiar nondecoupling of the top quark is screened and the m_t upper bound is relaxed to higher values.

So far we have confined ourselves to strictly perturbative phenomena. Although loop amplitudes involving the top quark are mathematically well behaved, it is evident that interesting and possibly significant features associated with the $t\bar{t}$ threshold cannot be accommodated when the perturbation series is truncated at finite order. In particular, perturbation theory up to $\mathcal{O}(\alpha\alpha_s)$ predicts a discontinuous steplike threshold behaviour for the cross section of $e^+e^- \to t\bar{t}$. To obtain a more realistic description, it is necessary to allow for the formation of toponium resonances [70], which for $m_t \gtrsim 130\,\mathrm{GeV}$ are smeared out to a coherent threshold structure [71]. By Cutkosky's rule [72], $\sigma(e^+e^- \to t\bar{t})$ corresponds to the absorptive parts of the photon and Z-boson vacuum polarizations and its enhancement at threshold induces additional contributions in the corresponding real parts, which can be computed via suitably subtracted dispersion relations [73]. These contributions in turn lead to shifts in observable radiative corrections. For a comprehensive discussion of these effects we refer to Ref. [41]; the cases $m_t \approx M_Z/2$ and $m_t \approx M_W - m_b$, which require a special treatment [74], are now excluded experimentally [75]. In the following, we shall briefly review the crucial points of the procedure. Decomposing the vacuum-polarization tensor generated by the insertion of a top-quark loop into a gauge-boson line as

$$\Pi^{V,A}_{\mu\nu}(q) = \Pi^{V,A}(q^2)\, g_{\mu\nu} + \lambda^{V,A}(q^2)\, q_\mu q_\nu\,, \tag{2.17}$$

where V and A label the vector and axial-vector components and q is the external four-momentum, one can show [41] that close to the $t\bar{t}$ threshold only $\mathcal{I}m\,\Pi^V(q^2)$ and $\mathcal{I}m\,\lambda^A(q^2)$ receive significant contributions and $\mathcal{I}m\,\lambda^A(q^2) \approx -\,\mathcal{I}m\,\Pi^V(q^2)/q^2$, while $\mathcal{I}m\,\Pi^A(q^2)$ is strongly suppressed due to centrifugal barrier effects. Of course, $\lambda^V(q^2) = -\Pi^V(q^2)/q^2$ by the relevant Ward identity [73]. Using subtracted dispersion relations that manifestly comply with the underlying Ward identities [73], one finds that the corresponding shifts in S, T, and U can

be written as [41]

$$\begin{aligned} \frac{3}{5}S = U &= \int \frac{dq^2}{q^2}\,\frac{\mathcal{I}m\,\Pi^V(q^2)}{q^2-z}\,, \\ T &= \frac{1}{4s_{\rm w}^2 w}\int dq^2\;\mathcal{I}m\,\lambda^A(q^2) \\ &\approx -\frac{1}{4s_{\rm w}^2 w}\int \frac{dq^2}{q^2}\;\mathcal{I}m\,\Pi^V(q^2)\,, \end{aligned} \tag{2.18}$$

where the integration extends across the threshold region. Consequently, the threshold enhancement of $\mathcal{I}m\,\Pi^V(q^2)$ beyond the perturbative prediction to $\mathcal{O}(\alpha\alpha_s)$ induces a negative shift in T, *i.e.*, it acts in the same direction as the perturbative QCD corrections of Eq. (2.15). On the other hand, since the length of the integration interval in Eq. (2.18) is small as compared to the scale m_t^2, one has $(3/5)S = U \approx -4s_{\rm w}^2 wT/\left(4m_t^2 - z\right)$, which is positive and, for large m_t, suppressed by the factor $s_{\rm w}^2 w/m_t^2$ relative to T.

To illustrate the interplay of the various contributions to S, T, and U discussed above and their relevance for the prediction of M_H prior to the direct experimental discovery of the Higgs boson itself, we study the radiative corrections to the muon decay embodied by Δr [54]. As is well known, the requirement that the theoretical prediction of the muon lifetime agrees with its precisely measured value establishes a mass relation among the particles of the standard model. In the literature this relation has often been employed to predict M_W for given M_H and m_t; see Ref. 76 and references therein. Conversely, inputting the current experimental value of M_W [57], M_H can be evaluated as a function of m_t. The results displayed in Fig. 3 are based on the on-shell formulation of Ref. 77. To disentangle the effects of the electroweak, perturbative-QCD, and $t\bar{t}$-threshold corrections, we have included them one after another on top of the usual one-loop analysis for the M_W central value. It is clear that any prediction of M_H is essentially meaningless, unless all dominant corrections beyond one loop are properly taken into account. The (m_t, M_H) points outside the strip marked by the contours for $M_W = 79.96\,\text{GeV}$ and 80.48 GeV are excluded at the 68% confidence level by the CDF and UA2 experiments, provided, of course, that the standard model is correct. At present, a prediction of M_H is out of reach. However, the detection of the top quark is believed to be impending and, in the long run, an experimental uncertainty of $\delta M_W \approx 50\,\text{GeV}$ is envisaged. Then this type of analysis will provide a powerful tool for pinning down M_H and, thus, for facilitating the direct search for the Higgs boson.

In general, the calculation of a physical observable to finite order in perturbation theory depends to a certain extent on the adopted renormalization scheme. In Ref. 77 the on-shell

and $\overline{\rm MS}$ calculations of Δr including the dominant higher-order corrections are compared and found to be in excellent agreement over the wide ranges $60\,\mathrm{GeV} < M_H < 1\,\mathrm{TeV}$ and $M_Z < m_t < 250\,\mathrm{GeV}$. This indicates that the above results should be rather reliable, *i.e.*, insensitive to residual unknown higher-order corrections. Of course, the m_t-M_H interdependence can be further constrained by extending the set of measurables to be analyzed. For a global analysis of the latest low-energy, M_W, and LEP data we refer to Ref. 62.

Besides oblique corrections, there are other tools to probe the Higgs sector prior to observing the Higgs-boson resonance. In fact, the Higgs boson can contribute in higher orders also through direct corrections, *i.e.*, process-specific vertex- and box-diagram contributions. The two-loop vertex corrections to the $Z \to b\bar{b}$ decay width involving virtual top quarks and Higgs bosons [64] provide an example for that.

Finally, we should address potential limitations of the above procedure. Very recently, it has been pointed out that the existence of a triviality bound for the Higgs sector of the standard model may influence the M_H dependence of radiative corrections [78]. It has been argued that the lowering of the Landau pole with increasing M_H implies that the sensitivity to M_H values beyond 500 GeV or so is lost due to the sizeable cutoff dependence of the theory [78]. In other words, in that regime oblique radiative corrections should receive significant contributions from unknown physics and calculations performed within the standard model may be totally unreliable [78].

3. Three-point functions: fusion processes and two-body decays

So far we have considered virtual Higgs bosons circulating in loop diagrams. At present, this is the only window to gain information on the characteristics of the Higgs boson. At future high-energy colliders the Higgs boson is expected to occur as a resonance in some inclusive channel, *i.e.*, phase-space configurations where the Higgs boson is close to its mass shell will be greatly favoured. In the following, we shall, therefore, treat the Higgs boson as a real particle whenever we calculate quantum corrections to its production cross sections and partial decay widths—or deal with loop-induced production and decay processes. It is clear, however, that this picture becomes gradually doubtful for increasing M_H and is bound to break down for $M_H \lesssim 1\,\mathrm{TeV}$, where the particle property of the Higgs boson fades. Furthermore, in the upper M_H range special care must be exercised not to violate S-matrix unitarity. This problem was tackled by Willenbrock and Valencia [79], who advocated to apply the complex-pole definition of mass and width to the Higgs-boson propagator. A

complementary strategy is to absorb typical high-M_H corrections to the W^+W^-H and ZZH couplings into an effective mass parameter [80].

In this chapter we shall be concerned with one-loop three-point functions involving the Higgs boson as an external line. This includes both the loop-induced $Z\gamma H$, $\gamma\gamma H$, and ggH couplings and radiative corrections to the tree-level W^+W^-H, ZZH, and $f\bar{f}H$ vertices, where f represents a massive fermion. Note that, due to helicity conservation, the $\nu\bar{\nu}H$ coupling vanishes for massless neutrinos to all orders of perturbation theory. In the case of the W^+W^-H and $f\bar{f}H$ vertices virtual-photon exchanges lead to infrared singularities when the charged particles are on-mass-shell. To obtain a finite result, one then needs to take soft-photon bremsstrahlung into account. A similar statement applies to gluonic corrections to the $q\bar{q}H$ vertex.

3.1 ZZH COUPLING

A comprehensive discussion of the one-loop renormalization of the ZZH vertex may be found in Refs. 51, 81. Since this vertex constitutes a central ingredient for the radiative corrections to the $f\bar{f}ZH$ four-point function, to be discussed in Chapter 4, and since the $Z\gamma H$, $\gamma\gamma H$, and ggH couplings may be derived from subsets of the ZZH loop diagrams simply by adjusting some coupling constants and the counterterm, we collect the central results below.

Let (μ, p) and (ν, k) denote the Lorentz indices and incoming four-momenta of the two Z bosons and

$$a = p^2\,, \qquad b = k^2\,, \qquad c = (p+k)^2\,. \tag{3.1}$$

For the time being, we allow for all external particles to be off their mass shells. In the Born approximation the ZZH vertex is represented by (Fig. 4a)

$$V_0^{\mu\nu} = \rho\, g^{\mu\nu}\,, \qquad \rho = \frac{eM_Z}{c_{\rm w}s_{\rm w}}\,. \tag{3.2}$$

In first order it receives contributions from fermion loops (Fig. 4b), W-boson loops (Fig. 4c), and diagrams involving virtual Z and/or Higgs bosons including tadpoles (Fig. 4d). The one-loop corrected ZZH vertex may be written in the form

$$V^{\mu\nu} = \rho(g^{\mu\nu} + T^{\mu\nu})\,, \tag{3.3}$$

where $T^{\mu\nu}$ exhibits the following tensorial structure:

$$T^{\mu\nu} = A\, p^\mu p^\nu + B\, k^\mu k^\nu + C\, p^\mu k^\nu + D\, k^\mu p^\nu + \hat{E}\, g^{\mu\nu}\,. \tag{3.4}$$

The caret indicates that the respective quantity has been rendered finite by including its OMS counterterm (Fig. 4e). Note that, due to charge-conjugation invariance, a term proportional to $\epsilon^{\mu\nu\rho\sigma} p_\rho k_\sigma$ does not occur. By Bose symmetry, one has $A \leftrightarrow B$ and $\{C; D; E\} \leftrightarrow \{C; D; E\}$ under $a \leftrightarrow b$ reflection.

In all cases of interest in this article the Z bosons are real or coupled to conserved currents, so that only D and E in Eq. (3.4) are relevant. One finds [51]

$$\begin{aligned} D &= \frac{\alpha}{4\pi}\left[\frac{4}{z}\sum_f N_f m_f^2\left(v_f^2 A_f^v + a_f^2 A_f^a\right) + \frac{1}{s_{\rm w}^2}A_W + \frac{1}{c_{\rm w}^2 s_{\rm w}^2}A_{ZH}\right]\,, \\ \hat{E} &= \frac{\alpha}{4\pi}\left[\frac{4}{z}\sum_f N_f m_f^2\left(v_f^2 B_f^v + a_f^2 B_f^a\right) + \frac{1}{s_{\rm w}^2}B_W + \frac{1}{c_{\rm w}^2 s_{\rm w}^2}B_{ZH} + B_{CT}\right]\,, \end{aligned} \tag{3.5}$$

where $v_f = \left(I_f - 2s_{\rm w}^2 Q_f\right)/(2c_{\rm w}s_{\rm w})$, $a_f = I_f/(2c_{\rm w}s_{\rm w})$, $N_f = 1(3)$ for lepton (quark) flavours, f, and

$$\begin{aligned} A_f^v &= \left(C_0 + 4C_2\right)\left(a, b, c, m_f^2, m_f^2, m_f^2\right)\,, \\ A_f^a &= \left(C_0 + 2C_1 + 4C_2\right)\left(a, b, c, m_f^2, m_f^2, m_f^2\right)\,, \\ B_f^v &= \left(\frac{a+b-c}{2}C_0 - 4C_3\right)\left(a, b, c, m_f^2, m_f^2, m_f^2\right)\,, \\ B_f^a &= B_0\left(a, m_f^2, m_f^2\right) + B_0\left(b, m_f^2, m_f^2\right) \\ &\quad + \left[\left(\frac{a+b+c}{2} - 4m_f^2\right)C_0 - 4C_3\right]\left(a, b, c, m_f^2, m_f^2, m_f^2\right)\,, \\ A_W &= \left\{2\left(-8c_{\rm w}^4 + 4c_{\rm w}^2 - 1\right)C_0 - 2C_1 \right. \\ &\quad \left. - \left[24c_{\rm w}^4 - 8c_{\rm w}^2 + 2 + \left(2c_{\rm w}^2 - 1\right)^2\frac{h}{w}\right]C_2\right\}\left(a, b, c, w, w, w\right)\,, \\ B_W &= \left(2c_{\rm w}^2 - 1\right)\left(B_0(a, w, w) + B_0(b, w, w)\right) + \frac{1}{2}B_0(c, w, w) \\ &\quad + \left\{\left[c_{\rm w}^4(-5a - 5b + 7c) + c_{\rm w}^2(a + b - 2c - 6w) + 2w + s_{\rm w}^4 h\right]C_0\right. \\ &\quad \left. + \left[24c_{\rm w}^4 - 8c_{\rm w}^2 + 2 + \left(2c_{\rm w}^2 - 1\right)^2\frac{h}{w}\right]C_3\right\}\left(a, b, c, w, w, w\right)\,, \end{aligned}$$

$$
\begin{aligned}
A_{ZH} &= -\left[C_0 + C_1 + \left(1 + \frac{h}{2z}\right) C_2\right](a,b,c,z,h,z) - \frac{3h}{2z} C_2(a,b,c,h,z,h)\,, \\
B_{ZH} &= -\frac{1}{2}\left(B_0(a,z,h) + B_0(b,z,h)\right) + \frac{1}{4} B_0(c,z,z) \\
&\quad + \left[zC_0 + \left(1 + \frac{h}{2z}\right) C_3\right](a,b,c,z,h,z) + \frac{3h}{2z}\left(zC_0 + C_3\right)(a,b,c,h,z,h)\,. \qquad (3.6)
\end{aligned}
$$

Here

$$
\begin{aligned}
C_1(a,b,c,\alpha,\beta,\alpha) &= C_{11}(a,b,c,\alpha,\beta,\alpha) - C_{12}(a,b,c,\alpha,\beta,\alpha)\,, \\
C_2(a,b,c,\alpha,\beta,\alpha) &= C_{12}(a,b,c,\alpha,\beta,\alpha) + C_{23}(a,b,c,\alpha,\beta,\alpha)\,, \\
C_3(a,b,c,\alpha,\beta,\alpha) &= C_{24}(a,b,c,\alpha,\beta,\alpha) - \frac{1}{4} B_0(c,\alpha,\alpha)
\end{aligned} \qquad (3.7)
$$

are ultraviolet finite and $a \leftrightarrow b$ symmetrical combinations of the standard two- and three-point functions defined in Appendix A. Notice that also $C_0(a,b,c,\alpha,\beta,\alpha)$ possesses these properties. The counterterm can be expressed in terms of the unrenormalized vacuum-polarization functions listed in Appendix B, viz. [51]

$$
\begin{aligned}
\frac{\alpha}{4\pi} B_{CT} = \frac{1}{2}\,\mathcal{R}e\,\Bigg[&-\left(\frac{c_{\rm w}^2}{s_{\rm w}^2} - 1\right)\frac{\Pi^{WW}(w)}{w} + \frac{c_{\rm w}^2}{s_{\rm w}^2}\,\frac{\Pi^{ZZ}(z)}{z} - \Pi^{HH'}(h)\Bigg] \\
&- \frac{3c_{\rm w}^2 - 1}{c_{\rm w} s_{\rm w}}\,\frac{\Pi^{ZA}(0)}{z} - \frac{1}{2}\Pi^{AA'}(0)\,.
\end{aligned} \qquad (3.8)
$$

It should be reminded that the above results refer to the 't Hooft-Feynman gauge and are gauge invariant only if all external lines are on their mass shells, *i.e.*, if $a = b = z$ and $c = h$. When they are to be utilized as a building block in the context of some more extensive calculation, one has to make sure that the residual loop diagrams are evaluated in the 't Hooft-Feynman gauge, too. Furthermore, it is important to note that the renormalization of the external vector-boson lines has not been included yet.

$\underline{H \to ZZ \text{ decay}}$

We are now ready to write down the one-loop corrected $H \to ZZ$ decay width. Including the wave-function renormalization for the outgoing Z bosons, it takes the form

$$
\Gamma(H \to ZZ) = \Gamma_0(H \to ZZ)(1 + \delta)\,, \qquad (3.9)
$$

where

$$
\Gamma_0(H \to ZZ) = \frac{G_F M_H^3}{16\pi\sqrt{2}}\sqrt{1 - \frac{1}{r}}\left(1 - \frac{1}{r} + \frac{3}{4r^2}\right) \qquad (3.10)
$$

is the tree-level result,

$$\delta = -\Delta r + 2\,\mathcal{R}e\left(\hat{E} - \hat{\Pi}'_{ZZ}(z)\right) + \frac{(1-1/r)\left[1-(1/2r)\right]}{1-1/r+(3/4r^2)} h\,\mathcal{R}e\,D \tag{3.11}$$

is the one-loop correction, and $r = (h/4z)$. The renormalized gauge-boson vacuum-polarization functions have been introduced in Eq. (2.1). Equation (3.9) is formulated in the so-called modified on-mass-shell (MOMS) scheme, *i.e.*, the relation [54]

$$G_F = \frac{\pi\alpha}{\sqrt{2}s_{\rm w}^2 w}\,\frac{1}{1-\Delta r}\,, \tag{3.12}$$

where

$$\Delta r = \frac{\hat{\Pi}_{WW}(0)}{w} + \frac{\alpha}{4\pi}\,\frac{1}{s_{\rm w}^2}\left[\left(\frac{7}{2s_{\rm w}^2}-2\right)\ln c_{\rm w}^2 + 6\right]\,, \tag{3.13}$$

has been employed to parameterize the lowest-order result terms of G_F rather than α. In turn, the term $-\Delta r$ appears in Eq. (3.11). The advantage of this procedure is that large logarithmic terms associated with light charged fermions are removed from the radiative corrections. In principle, their masses could now be sent to zero. In fact, this limit yields [51]

$$\delta_{(u,d)} = \frac{G_F z}{3\pi^2\sqrt{2}} N_{(u,d)}\left(1 - 2c_{\rm w}^2 s_{\rm w}^2 + 2s_{\rm w}^4 Y_{(u,d)}^2\right)\,, \tag{3.14}$$

where (u,d) represents a generic light-fermion doublet with weak hypercharge $Y_{(u,d)}$. Note that the diagrams of Fig. 4b do not contribute in this limit, since they are proportional to the square of the loop-fermion mass—one power stems from the Yukawa coupling constant and another one from the trace connected with the fermion loop. Equation (3.14) gives rise to a small positive correction independent of M_H.

At the other extreme, an ultraheavy-fermion doublet, (U,D), with masses $m_U, m_D \gg M_Z$, contributes [51]

$$\delta_{(U,D)} = \Delta\rho_{(U,D)} + \frac{G_F}{2\pi^2\sqrt{2}} N_{(U,D)} \sum_{i=U,D} m_i^2\,\mathcal{R}e\,F\left(\frac{h}{4m_i^2}\right)\,, \tag{3.15}$$

where

$$\Delta\rho_{(U,D)} = \frac{G_F}{8\pi^2\sqrt{2}} N_{(U,D)}\left(m_U^2 + m_D^2 - \frac{2m_U^2 m_D^2}{m_U^2 - m_D^2}\ln\frac{m_U^2}{m_D^2}\right) \tag{3.16}$$

is the corresponding contribution to the ρ parameter [55] and

$$\begin{aligned} F(r) &= \frac{3}{2} - \frac{1}{2r} - \left(1 - \frac{1}{2r}\right) f(r) + \frac{g(r)}{r} \\ &= -\frac{2}{3} - \frac{r}{15} + \mathcal{O}(r^2)\,. \end{aligned} \tag{3.17}$$

Here and in the following,

$$f(r) = \begin{cases} \sqrt{1 - \dfrac{1}{r}}\, \operatorname{arsinh}\sqrt{-r}\,, & \text{if } r < 0\,, \\ \sqrt{\dfrac{1}{r} - 1}\, \arcsin\sqrt{r}\,, & \text{if } 0 < r < 1\,, \\ \sqrt{1 - \dfrac{1}{r}}\, \left(\operatorname{arcosh}\sqrt{r} - i\dfrac{\pi}{2}\right)\,, & \text{if } r > 1\,, \end{cases} \tag{3.18}$$

$$g(r) = \begin{cases} \operatorname{arsinh}^2\sqrt{-r}\,, & \text{if } r < 0\,, \\ -\arcsin^2\sqrt{r}\,, & \text{if } 0 < r < 1\,, \\ \left(\operatorname{arcosh}\sqrt{r} - i\dfrac{\pi}{2}\right)^2\,, & \text{if } r > 1\,. \end{cases}$$

It is interesting to observe that the terms involving the $Zf\bar{f}$ vector coupling vanish in the ultraheavy-fermion limit. For $m_U, m_D \gg M_H$, Eq. (3.15) simplifies to [82]

$$\delta_{(U,D)} = -\frac{G_F}{8\pi^2\sqrt{2}} N_{(U,D)} \left[\frac{5}{3}\left(m_U^2 + m_D^2\right) + \frac{2m_U^2 m_D^2}{m_U^2 - m_D^2} \ln\frac{m_U^2}{m_D^2}\right]\,. \tag{3.19}$$

Obviously, the presence of an additional fermion generation would manifest itself in a strong reduction of the $H \to ZZ$ decay width, even if the doublets were mass degenerate. An alternative method of deriving the ultraheavy-fermion contributions to the ZZH coupling is provided by the Goldstone-boson equivalence theorem [83]. This theorem states that at high energies $\left(s \gg M_W^2\right)$, S-matrix amplitudes involving external longitudinal components of the $W^\pm$ and Z bosons, $W_L^\pm$ and Z_L, are equivalent, up to $\mathcal{O}(M_W/\sqrt{s}\,)$, to the corresponding amplitudes in the Higgs-Goldstone scalar theory with the respective Goldstone bosons replacing $W_L^\pm$ and Z_L. This avenue was taken by Dawson and Willenbrock [84] and, by modifying the renormalization scheme, their result may be transformed into Eq. (3.15) [51, 82].

The leading term of the high-M_H expansion of the bosonic contribution is quadratic in M_H with a positive sign, viz. [51]

$$\delta_{\rm bos} = \frac{G_F h}{8\pi^2\sqrt{2}} \left(5\zeta(2) - 3\pi\sqrt{3} + \frac{19}{2}\right)\,. \tag{3.20}$$

Incidentally, a strong numerical cancellation takes place among the terms contained within the parentheses of Eq. (3.20); their sum is of order unity. It is worth mentioning that Eq. (3.20)

can be traced to those loops in which only the physical Higgs boson, H, or the unphysical Goldstone modes, $\phi^{\pm}$ and ϕ_Z, circulate. Also this contribution can be derived [85] by means of the Goldstone-boson equivalence theorem [83].

Figure 5 visualizes the one-loop radiative corrections to the $H \to ZZ$ decay width as a function of M_H; see Eq. (3.11). Figure 5a disentangles the contributions due to bosons, leptons, and quarks for $m_t = 150\,\mathrm{GeV}$. The bosonic contribution is throughout positive and grows quadratically with M_H at large M_H; see Eq. (3.20). The lepton loops give rise to an M_H-independent increase by about 0.5%; see Eq. (3.14). The quarkonic correction oscillates around zero. It exhibits a striking spike located at $M_H = 2m_t$. This is due to the fact that by crossing the $t\bar{t}$ open-flavour threshold a new imaginary part is switched on in the top-quark triangle amplitude, which is accompanied by a sharp bend in the real part. This is not associated with any kind of singularity. The combined effect is always positive and has a global minimum at $M_H = 2m_t$. In Fig. 5b, the full correction is shown for $m_t = 100$, 150, and 200 GeV. It does not exceed 7% for $M_H \lesssim 500\,\mathrm{GeV}$, while it may become as large as 15% at $M_H = 1\,\mathrm{TeV}$.

3.2 $Z\gamma H$ COUPLING

The $Z\gamma H$ coupling is mediated by massive-fermion and W-boson loop diagrams similar to those of Figs. 4b and c, respectively. Assigning (μ, p) to the Z boson and (ν, k) to the photon, the amplitude may be written as

$$V^{\mu\nu} = \rho T^{\mu\nu}\,, \tag{3.21}$$

where ρ is defined in Eq. (3.2) and $T^{\mu\nu}$ has a decomposition similar to Eq. (3.4). For the coefficients of practical interest one finds [86]

$$\begin{aligned} D &= \frac{\alpha}{4\pi}\left(-\frac{4}{z}\sum_f N_f m_f^2 v_f Q_f A_f^v + \frac{c_\mathrm{w}}{s_\mathrm{w}} A_W\right)\,, \\ \hat{E} &= \frac{\alpha}{4\pi}\left(-\frac{4}{z}\sum_f N_f m_f^2 v_f Q_f B_f^v + \frac{c_\mathrm{w}}{s_\mathrm{w}} B_W + B_{CT}\right)\,, \end{aligned} \tag{3.22}$$

where A_f and B_f are listed in Eq. (3.6) and

$$\begin{aligned}
A_W &= \left\{ 4\left(4c_{\rm w}^2-1\right) C_0 + 2\left[12c_{\rm w}^2 - 2 + \left(2c_{\rm w}^2-1\right)\frac{h}{w}\right] C_2 \right\} (a,b,c,w,w,w)\,, \\
B_W &= -2B_0(b,w,w) + \left\{ \left[c_{\rm w}^2(5a+5b-7c) - 2a + b + c + 3w + s_{\rm w}^2 h\right] C_0 \right. \\
&\qquad \left. - 2\left[12c_{\rm w}^2 - 2 + \left(2c_{\rm w}^2-1\right)\frac{h}{w}\right] C_3 \right\} (a,b,c,w,w,w)\,, \\
B_{CT} &= \frac{4\pi}{\alpha}\frac{\Pi^{ZA}(0)}{z} = 2\frac{c_{\rm w}}{s_{\rm w}} B_0(0,w,w)\,.
\end{aligned} \tag{3.23}$$

The renormalization of the external Z-boson line does not yet enter at the one-loop level. The $Z\gamma H$ vertex is another ingredient for the radiative corrections to the $f\bar{f}ZH$ four-point function, to be discussed in Chapter 4.

<u>$H \to Z\gamma$ decay</u>

Similarly to the case of the ZZH coupling, the above results are gauge invariant only if the mass-shell conditions, $a = z$, $b = 0$, and $c = h$, are imposed [87]. Only then $(a-c)D/2 = \hat{E}$ holds, which is a necessary condition for $T^{\mu\nu}k_\nu = 0$. In fact, one easily verifies that [32, 87, 88]

$$\begin{aligned}
\frac{z-h}{2} A_f^v &= B_f^v \\
&= -1 + \frac{2\zeta_f}{\zeta_f - \eta_f}\left(f(\zeta_f) - f(\eta_f)\right) - \left(1 + \frac{1}{\zeta_f - \eta_f}\right)\left(g(\zeta_f) - g(\eta_f)\right), \\
\frac{z-h}{2} A_W &= B_W + \frac{s_{\rm w}}{c_{\rm w}} B_{CT} \\
&= \left[1 - 6c_{\rm w}^2 + 2\left(1 - 2c_{\rm w}^2\right)\eta\right]\left[1 - \frac{2}{1 - 4c_{\rm w}^2\eta}\left(f\left(\frac{1}{4c_{\rm w}^2}\right) - f(\eta)\right)\right] \\
&\quad + 2\left(1 - 6c_{\rm w}^2 + \frac{1 - 12c_{\rm w}^4}{1 - 4c_{\rm w}^2\eta}\right)\left(g\left(\frac{1}{4c_{\rm w}^4}\right) - g(\eta)\right),
\end{aligned} \tag{3.24}$$

where $\zeta_f = \left(z/4m_f^2\right)$, $\eta_f = \left(h/4m_f^2\right)$, $\eta = (h/4w)$, and the functions f and g are defined in Eq. (3.18). Equipped with these expressions, one calculates the $H \to Z\gamma$ decay width from

$$\Gamma(H \to Z\gamma) = \frac{G_F z^2}{\pi\sqrt{2}M_H}\left(1 - \frac{z}{h}\right)\left|\hat{E}\right|^2 . \tag{3.25}$$

It is instructive to consider some limiting cases. Clearly, a massless fermion does not participate in the $Z\gamma H$ coupling. On the other hand, an ultraheavy fermion, with mass

$m_f \gg M_Z$, contributes

$$\begin{aligned} B_f^v &= -1 + \left(1 - \frac{1}{\eta_f}\right) g(\eta_f) \\ &= -\frac{2}{3}\eta_f - \frac{7}{45}\eta_f^2 + \mathcal{O}\left(\eta_f^3\right) \ . \end{aligned} \tag{3.26}$$

In other words, the $Z\gamma H$ amplitude counts the number of such fermions according to the prescription

$$\hat{E} = \frac{\alpha}{6\pi}\,\frac{h}{z} \sum_f N_f v_f Q_f \left(1 + \mathcal{O}\left(\frac{h}{m_f^2}\right)\right) . \tag{3.27}$$

Finally, for $M_H \gg M_Z$ the bosonic correction becomes

$$\hat{E} = -\frac{\alpha}{4\pi}\,\frac{c_{\rm w}^2 - s_{\rm w}^2}{2 c_{\rm w} s_{\rm w}} \left(\frac{h}{z} + \mathcal{O}\left(\ln^2 \frac{h}{z}\right)\right) . \tag{3.28}$$

Comparison of Eqs. (3.27), (3.28) reveals that the top-quark and W-boson amplitudes interfere destructively. The $H \to Z\gamma$ decay rate increases with the third power of M_H.

Figure 6a displays the $H \to Z\gamma$ decay width to lowest order as a function of M_H for $m_t = 91$ and 200 GeV. For $M_H < 2m_t$, the result increases monotonically with m_t, approaching asymptotically its large-m_t limit. However, the sensitivity to m_t is rather weak. For comparison, the W-boson contribution is shown separately. As anticipated above, the bosonic and fermionic amplitudes interfere destructively. However, the influence of the fermion loops is rather insignificant, which is partly due to the smallness of the $f\bar{f}Z$ vector coupling.

<u>$Z \to \gamma H$ decay</u>

The $Z \to \gamma H$ decay width emerges from Eq. (3.25) through crossing,

$$\Gamma(Z \to \gamma H) = -\frac{1}{3}\,\frac{M_H^3}{M_Z^3}\Gamma(H \to Z\gamma)\,, \tag{3.29}$$

where the minus sign is due to the fact that, formally, Eq. (3.25) becomes negative for $M_H < M_Z$, the factor of 1/3 stems from the spin average, and the factor M_H^3/M_Z^3 adjusts the flux and the phase space.

The $Z \to \gamma H$ decay width is shown in Fig. 6b and comments similar to those concerning Fig. 6a apply. This decay is too rare to be of practical relevance for Higgs hunting at LEP and SLC [10].

QCD corrections to the $Z\gamma H$ coupling are well under control [89]. Depending on the values of M_H and m_t, they may lead to both positive and negative shifts of $\Gamma(H \to Z\gamma)$ and $\Gamma(Z \to \gamma H)$. However, the effect is below 0.3% in the case of intermediate-mass Higgs-boson decay and below 0.5% in the case of Z-boson decay.

3.3 $\gamma\gamma H$ COUPLING

The $\gamma\gamma H$ coupling is generated by massive-fermion and W-boson loops similar to those of Figs. 4b and c, respectively. The situation is quite analogous to the $Z\gamma H$ case and we merely list the results. The relevant off-shell form factors to be inserted in Eqs. (3.4), (3.21) read

$$\begin{aligned} D &= \frac{\alpha}{4\pi}\left(\frac{4}{z}\sum_f N_f m_f^2 Q_f^2 A_f^v + c_{\rm w}^2 A_W\right) , \\ E &= \frac{\alpha}{4\pi}\left(\frac{4}{z}\sum_f N_f m_f^2 Q_f^2 B_f^v + c_{\rm w}^2 B_W\right) , \end{aligned} \tag{3.30}$$

where A_f and B_f are defined in Eq. (3.6) and

$$\begin{aligned} A_W &= -4\left[4C_0 + \left(6+\frac{h}{w}\right)C_2\right](a,b,c,w,w,w) , \\ B_W &= \left[(-5a-5b+7c+h)C_0 + 4\left(6+\frac{h}{w}\right)C_3\right](a,b,c,w,w,w) . \end{aligned} \tag{3.31}$$

The C_2 and C_3 functions are listed in Eq. (3.7). There is no $\gamma\gamma H$ counterterm at one loop.

<u>$H \to \gamma\gamma$ decay</u>

In the on-shell case, $a = b = 0$ and $c = h$, gauge invariance implies $-hD/2 = E$, the fermionic contribution is given by Eq. (3.26) and the bosonic one by

$$-\frac{h}{2}A_W = B_W = 6 + 4\eta + 6\left(-2+\frac{1}{\eta}\right)g(\eta) , \tag{3.32}$$

where $\eta = (h/4w)$ and the function g is defined in Eq. (3.18). The $H \to \gamma\gamma$ decay width is then evaluated as

$$\Gamma(H \to \gamma\gamma) = \frac{G_F z^2}{2\pi\sqrt{2}M_H}|E|^2 . \tag{3.33}$$

This process was originally considered in Ref. 11 assuming $M_H \ll M_W$; the general result was obtained in Ref. 90. QCD corrections for the two-photon decay of an intermediate-mass

Higgs boson were computed in Ref. 91 and found to be small, well below 3%, confirming previous expectations [90].

Analogously to the $Z\gamma H$ case, light fermions decouple, while ultraheavy fermions contribute

$$E = -\frac{\alpha}{6\pi}\,\frac{h}{z}\sum_f N_f Q_f^2 \left(1 + \mathcal{O}\left(\frac{h}{m_f^2}\right)\right) . \tag{3.34}$$

In the high-M_H limit, the boson loops yield

$$E = \frac{\alpha}{4\pi}\left(\frac{h}{z} + \mathcal{O}\left(\ln^2\frac{h}{z}\right)\right) . \tag{3.35}$$

Again, fermion and boson amplitudes interfere destructively [11] and the $H \to \gamma\gamma$ decay rate grows like M_H^3.

Figure 7 displays the $H \to \gamma\gamma$ decay width to lowest order as a function of M_H for $m_t = 91$ and 200 GeV. The situation is quite similar to the case of $H \to Z\gamma$ (see Fig. 6a), except that now the fermionic contribution is more significant.

$\underline{\gamma\gamma \to H \text{ fusion}}$

At future Compton colliders the Higgs boson will be copiously produced via the inverse process, photon fusion [25]. Such experiments will make use of extremely focussed beams of very energetic photons produced through backscattering of laser light from high-energy electron beams [21]. Allowing for polarized photons, the cross section of $\gamma\gamma \to H$ is related to the $H \to \gamma\gamma$ decay width via

$$\sigma_{\gamma\gamma\to H}(\hat{s}) = \sigma^0_{\gamma\gamma\to H}\,\delta(\hat{s} - h)\,, \tag{3.36}$$

where

$$\sigma^0_{\gamma\gamma\to H} = \frac{8\pi^2}{M_H}\Gamma(H \to \gamma\gamma)(1 + \lambda_1\lambda_2) \tag{3.37}$$

and $\lambda_1, \lambda_2 = \pm 1$ label the photon helicities. Note that the delta function in Eq. (3.36) is an artifact of treating the Higgs boson as a stable particle. By convolution with the $\gamma\gamma$ differential luminosity of the Compton collider, $d\mathcal{L}_{\gamma\gamma}(\tau)/d\tau$, one obtains the effective cross

section as

$$\begin{aligned}\sigma(e^+e^- \to H + X) &= \int_{h/s}^{1} d\tau \, \frac{d\mathcal{L}_{\gamma\gamma}(\tau)}{d\tau} \sigma_{\gamma\gamma\to H}(\tau s) \\ &= \frac{1}{s}\sigma^0_{\gamma\gamma\to H} \left.\frac{d\mathcal{L}_{\gamma\gamma}(\tau)}{d\tau}\right|_{\tau=h/s} ,\end{aligned} \tag{3.38}$$

where s is the primary e^+e^- invariant mass squared. An analytic expression of $d\mathcal{L}_{\gamma\gamma}(\tau)/d\tau$ for arbitrary helicities may be found in Ref. 92.

3.4 ggH COUPLING

The ggH coupling is realized by massive-quark triangle diagrams similar to the one depicted in Fig. 4b. The relevant off-shell form factors emerge from Eq. (3.30) by replacing $\alpha N_f Q_f^2 \to \alpha_s \delta^{ab}/2$, where $a, b = 1, 8$ denote the colour indices of the gluons.

$\underline{H \to gg \text{ decay}}$

Mutatis mutandis, one finds the $H \to gg$ partial width to be

$$\Gamma(H \to gg) = \frac{N_c C_F \alpha_s^2 G_F M_H^3}{64\pi^3\sqrt{2}} \left| \sum_q \frac{B_q^v}{\eta_q} \right|^2 , \tag{3.39}$$

where $C_F = \left(N_c^2 - 1\right)/(2N_c)$, $\eta_q = \left(h/4m_q^2\right)$, and B_q^v is given by Eq. (3.26). Note that the summation in Eq. (3.39) extends over flavour but not colour. As before, the light-quark contributions are strongly suppressed. The $H \to gg$ decay channel has a phenomenologically interesting branching fraction only for $M_H \ll 2m_t$. Recalling Eq. (3.26), Eq. (3.39) may in this limit be approximated by [93]

$$\Gamma(H \to gg) = \frac{\alpha_s^2 G_F M_H^3}{36\pi^3\sqrt{2}} \left(1 + \frac{7}{60}\frac{h}{m_t^2} + \mathcal{O}\left(\frac{h^2}{m_t^4}\right)\right) . \tag{3.40}$$

The gg final states will be observed as dijet events. Such events are also produced via the $H \to q\bar{q}$ channels. Thus, these processes must be added up unless it is possible to discriminate experimentally gluon jets from quark jets. For example, bottom quarks can possibly be tagged with the help of mini-vertex detectors. QCD corrections to $H \to gg$ have been computed in the high-m_t limit with neglect of the other quark masses [94]. They involve virtual-gluon exchanges as well as the three-parton decays $H \to ggg$ and $H \to gg^* \to gq\bar{q}$,

where g^* represents a virtual gluon. The $gq\bar{q}$ final states are also generated through gluon bremsstrahlung in $H \to q\bar{q}$ decays. However, the latter processes are suppressed by light-quark masses, which are neglected in Ref. 94 altogether. Thus, the $H \to gg^* \to gq\bar{q}$ and $H \to q\bar{q} \to gq\bar{q}$ amplitudes do not interfere in that approximation and the QCD-corrected rates of $H \to gg$ and $H \to q\bar{q}$ are still to be added incoherently. The latter will be reviewed in Sec. 3.6, the former has been found to be [94]

$$\Gamma\big(H \to gg(g), gq\bar{q}\big) = \Gamma(H \to gg)\left[1 + \frac{\alpha_s}{\pi}\left(\frac{95}{4} - \frac{7}{6}N_F\right)\right], \tag{3.41}$$

where $N_F = 5$ denotes the number of active quark flavours and $\Gamma(H \to gg)$ is given by Eq. (3.40). Assuming $\alpha_s(M_H) \approx 0.11$, this amounts to an increase of the lowest-order result by 63% [94].

Figure 8 shows the $H \to gg$ decay width as a function of M_H for $m_t = 100$, 150, and 200 GeV with and without the QCD corrections of Eq. (3.41). In contrast to the $H \to Z\gamma$ and $H \to \gamma\gamma$ channels considered in Figs. 6 and 7, respectively, it decreases with increasing m_t. The sensitivity to m_t is appreciable only in the lower range of the currently favoured values.

Notice that the $H \to gg$ decay rate is extremely sensitive to novel high-mass quarks. In fact, it would be approximately $(1 + 2N)^2$ times as large should the standard model be extended by N additional fermion generations with similar interactions and quantum numbers as the first three. We thus recover the notion that the ggH coupling counts the number of heavy quarks [95]. In contrast to the ρ parameter, this effect is does not depend on the isospin breaking in the quark doublets as long as the pair-production thresholds are located sufficiently far above the Higgs-boson mass.

<u>$gg \to H$ fusion</u>

At future multi-TeV hadron colliders such as the LHC and the SSC, due to the overwhelming gluon luminosity, gluon fusion [27], $gg \to H$, will be the dominant source of Higgs bosons with $M_H \lesssim 700$ TeV [29]. The parton-level cross section is proportional to Eq. (3.39), viz.

$$\sigma_{gg\to H}(\hat{s}) = \sigma^0_{gg\to H}\,\delta(\hat{s} - h)\,, \tag{3.42}$$

with

$$\sigma^0_{gg\to H} = \frac{8\pi^2}{N_A^2 M_H}\Gamma(H \to gg)\,. \tag{3.43}$$

Here $N_A = N_c^2 - 1 = 8$ is the number of gluons. Similarly to Eq. (3.36), the delta function in Eq. (3.42) is a pathological consequence of ignoring subsequent Higgs-boson decays. The total cross section of inclusive Higgs-boson production via gluon fusion in collisions of hadrons, h_1 and h_2, is then given by

$$\begin{aligned}\sigma(h_1 h_2 \to H + X) &= \int\limits_{h/s}^{1} d\tau \, \frac{d\mathcal{L}_{gg}(\tau)}{d\tau} \sigma_{gg\to H}(\tau s) \\ &= \frac{1}{s}\sigma^0_{gg\to H} \left.\frac{d\mathcal{L}_{gg}(\tau)}{d\tau}\right|_{\tau=h/s},\end{aligned} \tag{3.44}$$

where s is the $h_1 h_2$ invariant mass squared and

$$\frac{d\mathcal{L}_{gg}(\tau)}{d\tau} = \int\limits_{\tau}^{1} \frac{dx}{x} F_{g/h_1}(x) F_{g/h_2}\left(\frac{\tau}{x}\right) \tag{3.45}$$

is the gg differential luminosity of the $h_1 h_2$ collider. Here $F_{g/h}(x)$ is the distribution of the gluons inside the hadron h with respect to their longitudinal-momentum fractions, x. Obviously, the Higgs boson cannot acquire a nonvanishing transverse momentum in this lowest-order approximation.

The production of high-transverse-momentum Higgs bosons at hadron colliders proceeds in $\mathcal{O}\left(\alpha_s^3\right)$ via the subprocesses $gg \to gH$, $q\bar{q} \to gH$, and $qg \to qH$ [96]. Non-negligible contributions originate also from the $gg \to ggH$ subprocess in $\mathcal{O}\left(\alpha_s^4\right)$ [97]. At the lower end of the p_T spectrum, the leading logarithms due to multiple-gluon emission must be resummed to obtain a finite result [98–101]. The QCD corrections to the total cross section of $pp \to H + X$ have been found in Refs. 94, 102, 103 by including also the virtual contributions. Their effect is to increase the lowest-order prediction by a factor which is roughly approximated by a constant [102],

$$1 + \frac{\alpha_s}{\pi}\left(\pi^2 + \frac{11}{2}\right) \approx 1.5\,. \tag{3.46}$$

The large π^2 term is generated by analytic continuation.

The exact K factor ($K_{\rm tot}$), *i.e.*, the ratio of the next-to-leading-order cross section to the leading-order one, is shown in Fig. 9 for the SSC as a function of M_H for $m_t = 140\,\text{GeV}$ [103]. Here the parton densities by Diemoz et al. [104] are used in the $\overline{\rm MS}$ scheme with $\Lambda = 207\,\text{MeV}$ for five quark flavours and the factorization and renormalization scales are set equal to M_H.

Also shown are the regularized virtual corrections ($K_{\rm virt}$) and the real corrections connected with the gg, gq, and qq hard-scattering processes (K_{gg}, K_{gq}, and K_{qq}) relative to the lowest-order cross section. Notice that $K_{\rm tot} - K_{\rm virt} - K_{gg} - K_{gq} - K_{qq}$ differs from one because in the numerator of the K factor the lowest-order parton-level cross section is dressed with next-to-leading coupling and parton distribution functions, while the denominator is evaluated strictly to lowest order. At the LHC, the K factor will be slightly larger, at about 1.7 [103].

The logistics of discriminating the jH signal with subsequent $H \to VV \to 4f$ decays from the VVj and $V + 3j$ backgrounds under realistic LHC and SLC conditions has been carefully assessed in Ref. 105.

Higgs bosons will be produced so copiously at future hadron supercolliders that it might be possible to study their self-interactions. The candidate subprocess would be $gg \to HH$, which is mediated by massive-quark triangles and box diagrams [106]. The triangle diagrams involve the Higgs-boson cubic self-coupling. Unfortunately, their contribution is hidden beneath the one of the box diagrams, so that moderate deviations of the triple Higgs-boson coupling from its standard-model value will not entail a large increase in the cross section [106].

3.5 W^+W^-H COUPLING

The renormalization of the W^+W^-H vertex proceeds in analogy to the ZZH case discussed in Sec. 3.1 and we need not dwell on technicalities here. First numerical data on the one-loop corrected $H \to W^+W^-$ decay width were presented in Ref. 81. Full analytic results may be found in Ref. 107.

The Feynman diagrams pertinent to the W^+W^-H vertex in the one-loop approximation are depicted in Fig. 10. They include the Born diagram (Fig. 10a), tadpoles (Fig. 10b), loops involving bosons and ghosts (Fig. 10c), and fermion loops (Fig. 10d). Denoting the Lorentz indices and incoming four-momenta of the W^+ and W^- bosons by (μ, p) and (ν, k), respectively, and taking all external legs to be off-shell, the one-loop corrected W^+W^-H vertex exhibits a structure similar to Eq. (3.3) with $\rho = eM_W/s_{\rm w}$ and

$$T^{\mu\nu} = A\, p^\mu p^\nu + B\, k^\mu k^\nu + C\, p^\mu k^\nu + D\, k^\mu p^\nu + \hat{E}\, g^{\mu\nu} + iF\, \epsilon^{\mu\nu\rho\sigma} p_\rho k_\sigma\,. \tag{3.47}$$

As before, $\hat{E}$ includes the counterterm in the OMS scheme; see Fig. 10e. Notice that the last term of Eq. (3.47) is absent in the ZZH case; see Eq. (3.4). It originates in the diagrams of Fig. 10d due to the fact that there are two different fermion flavours in each loop. A

mass-degenerate fermion doublet does not contribute to F. The coefficients $A, \ldots, E$ possess the same $p^2 \leftrightarrow k^2$ reflection properties as in the ZZH case, while $F \leftrightarrow -F$. Thus F vanishes if $p^2 = k^2$ and, in particular, if the W bosons are on-shell. The photon-exchange diagrams of Fig. 10c are plagued by infrared singularities when both W bosons are on-shell. To obtain an infrared-finite answer, one has to include also the W-boson wave-function renormalizations and the contributions due to soft-photon bremsstrahlung. Photon bremsstrahlung proceeds through the diagrams of Fig. 10f.

$\underline{H \to W^+W^-\text{ decay}}$

The complete one-loop result for the $H \to W^+W^-$ decay width in the MOMS scheme is found to be [107]

$$\Gamma\big(H \to W^+W^-(\gamma)\big) = \Gamma_0(H \to W^+W^-)(1 + \delta_{\rm ew} + \delta_{\rm brem})\,, \tag{3.48}$$

where

$$\Gamma_0(H \to W^+W^-) = \frac{G_F M_H^3}{8\pi\sqrt{2}}\sqrt{1-\frac{1}{r}}\left(1 - \frac{1}{r} + \frac{3}{4r^2}\right) \tag{3.49}$$

is the Born result,

$$\delta_{\rm ew} = -\Delta r + 2\,\mathcal{R}e\left(\hat{E} - \hat{\Pi}'_{WW}(w)\right) + \frac{(1-1/r)\big[1-(1/2r)\big]}{1-1/r+(3/4r^2)}h\,\mathcal{R}e\,D \tag{3.50}$$

is the electroweak loop correction,

$$\begin{aligned}\delta_{\rm brem} = \frac{\alpha}{\pi}\Bigg\{&\left(\frac{2-1/r}{\sqrt{1-1/r}}\phi - 1\right)\ln\frac{w}{\mu^2} - 2\gamma - 4\chi + \frac{14}{3} + \frac{2\phi}{\sqrt{1-1/r}} \\ &+ \frac{2-1/r}{\sqrt{1-1/r}}\left[\mathrm{Li}_2\left(\rho_-^2\right) + \mathrm{Li}_2\left(\rho_-^4\right) - 2\zeta(2) + \phi(5\phi - 2\gamma - 2\chi) - \frac{2\phi}{4r^2-4r+3}\right]\Bigg\}\end{aligned} \tag{3.51}$$

is the bremsstrahlung correction, $r = (h/4w)$, $\rho_\pm = \sqrt{r} \pm \sqrt{r-1}$, $\phi = \ln\rho_+$, $\gamma = \ln(\rho_+ + \rho_-)$, and $\chi = \ln(\rho_+ - \rho_-)$. $\hat{\Pi}_{WW}(s)$ and Δr are defined in Eqs. (2.1) and (3.13), respectively. The W^+W^-H form factors D and $\hat{E}$ are listed in Eqs. (2.7) and (2.8) of Ref. 107, respectively. Here the infrared singularities have been regularized by assigning an infinitesimal mass, μ, to the photon. The combination $2\,\mathcal{R}e\left(\hat{E} - \hat{\Pi}'_{WW}(w)\right) + \delta_{\rm brem}$ is finite for $\mu \to 0$. Notice that the photonic loop diagrams do not form an ultraviolet finite subset. Thus the separation of electromagnetic and weak corrections is not meaningful. The case is quite the reverse for the $H \to f\bar{f}$ decay, to be discussed in Sec. 3.6.

By virtue of the MOMS scheme, $\delta_{\rm ew}$ is free of mass singularities associated with light charged fermions. In fact, a doublet, (u,d), of zero-mass fermions contributes

$$\delta_{(u,d)} = \frac{G_F w}{3\pi^2\sqrt{2}} N_{(u,d)} \,. \tag{3.52}$$

At the other extreme, the contribution due to a doublet, (U,D), with masses $m_U, m_D \gg M_Z$, reads

$$\begin{aligned}\delta_{(U,D)} = -\Delta\rho_{(U,D)} + \frac{G_F}{2\pi^2\sqrt{2}} N_{(U,D)} \left\{ m_U^2 \left[\frac{3}{2} - \frac{1}{\eta_U} + \frac{1}{2\eta_D} + \left(-1 + \frac{3}{2\eta_U} - \frac{1}{\eta_D} \right) f(\eta_U) \right.\right. \\ \left.\left. - \frac{1}{2} \left(\frac{1}{\eta_D} + \frac{1}{4} \left(\frac{1}{\eta_U} - \frac{1}{\eta_D} \right)^2 \right) h C_0 \left(0, 0, h, m_U^2, m_D^2, m_U^2\right) \right] + (m_U \leftrightarrow m_D) \right\} ,\end{aligned} \tag{3.53}$$

where $\Delta\rho_{(U,D)}$ is defined by Eq. (3.16), $\eta_i = \left(h/4m_i^2\right)$ $(i = U, D)$, the function f is listed in Eq. (3.18), and [107]

$$\begin{aligned}&C_0 \left(0, 0, h, m_U^2, m_D^2, m_U^2\right) \\ &= \frac{1}{h} \left[\mathrm{Li}_2 \left(\frac{m_U^2 - m_D^2 - h}{m_U^2 - m_D^2 - h u_+} \right) - \mathrm{Li}_2 \left(\frac{m_U^2 - m_D^2}{m_U^2 - m_D^2 - h u_+} \right) + (u_+ \leftrightarrow u_-) \right. \\ &\quad \left. + \mathrm{Li}_2 \left(\frac{\left(m_U^2 - m_D^2\right)^2}{\left(m_U^2 - m_D^2\right)^2 + h\left(m_D^2 - i\epsilon\right)} \right) - \mathrm{Li}_2 \left(\frac{\left(m_U^2 - m_D^2\right)\left(m_U^2 - m_D^2 - h\right)}{\left(m_U^2 - m_D^2\right)^2 + h\left(m_D^2 - i\epsilon\right)} \right) \right] ,\end{aligned} \tag{3.54}$$

with

$$u_\pm = \frac{1}{2} \left(1 \pm \sqrt{1 - \frac{4}{h}\left(m_U^2 - i\epsilon\right)} \right) . \tag{3.55}$$

Equation (3.53) was originally obtained in Ref. 84 by means of the Goldstone-boson equivalence theorem [83]. For $m_U, m_D \gg M_H$, Eq. (3.53) coincides with the corresponding correction to the $H \to ZZ$ decay rate given by Eq. (3.19) and similar comments apply. Since a high-mass Higgs boson decays preferably into W^+W^- and ZZ pairs, its lifetime would be appreciably prolonged by the existence of a new generation of fermions which are too heavy to be pair-produced in Higgs-boson decays.

On the other hand, the bosonic correction to the $H \to W^+W^-$ decay rate is throughout positive. It is interesting to observe that the enhancement factor in the high-M_H limit is the same as in the case of the $H \to ZZ$ decay; see Eq. (3.20). This may be understood by recalling that the leading term of the bosonic loop-contribution is controlled by the Higgs-Goldstone scalar theory, which is invariant under SO(3) rotations of the Goldstone bosons corresponding to $W_L^\pm$ and Z_L [85].

Figure 11 shows the one-loop radiative corrections to the $H \to W^+W^-$ decay width in the MOMS scheme as a function of M_H. Figure 11a itemizes the contributions due to bosons, leptons, and quarks for $m_t = 150\,\mathrm{GeV}$. Photon bremsstrahlung is reckoned among the bosonic contribution, which is represented by the dotted curve. The drastic incision in this curve at $M_H = 2m_Z$ is due to a threshold singularity of the Higgs-boson wave-function renormalization in the one-loop approximation. In fact, $\Pi^{HH'}(h) \propto 1/\sqrt{4z - h}$ as $M_H \to 2M_Z$ from below [107]. The fact that $\Pi^{HH'}(h)$ is singular at $M_H = 2M_Z$ while it is regular at $M_H = 2m_t$ may be related by means of dispersive methods to the different threshold behaviours of the bosonic and fermionic decay rates of the Higgs boson at tree level [107]; the former corresponds to an S wave, the latter to a P wave. This defect originates in the underlying approximation of treating an unstable particle such as the Higgs boson as an asymptotic state. For an illuminative discussion of particles at threshold we refer to Ref. 108. On the other hand, the bosonic correction strongly enhances the probability of the $H \to W^+W^-$ decay at its threshold, $M_H = 2M_W$. This is similar to the $H \to f\bar{f}$ decay, to be discussed in Sec. 3.6. At large M_H, the bosonic contribution grows quadratically with M_H; see Eq. (3.20). The lepton loops give rise to an M_H-independent enhancement by some 0.5%; see Eq. (3.52). The quarkonic correction may be of either sign and exhibits a finite spike at $M_H = 2m_t$, which is familiar from Fig. 5a. Leaving aside the pathological scenario at $M_H = 2M_Z$, the total correction is always positive and has a global minimum at $M_H = 2m_t$. In Fig. 11b, the full correction is shown for $m_t = 100$, 150, and 200 GeV.

The W^+W^-H and ZZH couplings can be tested also by Higgs-boson production via W^+W^- and ZZ fusion, which is the most copious source of very heavy Higgs bosons in high-energy e^+e^- [13, 14], e^-p [14, 35], and pp collisions [14, 28]. Since longitudinal vector bosons make the dominant contribution to these fusion processes, the leading bosonic correction of Eq. (3.20) applies also here and leads to an increase of the cross sections.

3.6 $f\bar{f}H$ COUPLING

One-loop radiative corrections to the $H \to \tau^+\tau^-$ decay width were originally presented in Ref. 81 in numerical form. The electroweak radiative corrections to the $H \to f\bar{f}$ decay width, where f represents a generic fermion, are listed in Refs. 109, 110. QCD corrections to the $H \to q\bar{q}$ decay width were calculated to leading order for massive fermions in Ref. 111 and to next-to-leading order up to terms of $\mathcal{O}(m_f^2/M_H^2)$ in Ref. 112. To start with, we shall review the electroweak corrections. In contrast to the $H \to W^+W^-$ decay, there is a natural decomposition into a weak and an electromagnetic part, either of which is finite and gauge-invariant.

First-order QCD corrections emerge from the QED corrections by adjusting the coupling constant and the colour factor. The QCD corrections are then refined by resummation of the leading logarithms and by inclusion of the next-to-leading-order corrections.

The Feynman diagrams relevant for the $f\bar{f}H$ coupling in electroweak perturbation theory to one loop are shown in Fig. 12. They include the Born diagram (Fig. 12a), the photon-exchange diagram (Fig. 12b), and the diagrams involving virtual W, Z, and Higgs bosons (Fig. 12c). In the following, N, Q, v, a, and m are the colour multiplicity, electric charge, vector coupling, axial-vector coupling, and mass of f, respectively, m' is the mass of its isopartner, and p and k are the outgoing four-momenta of f and $\bar{f}$, respectively. The one-loop corrected transition-matrix element of $H \to f\bar{f}$ is found to be [110]

$$\mathcal{T} = \mathcal{T}_0 \left[1 + \frac{\alpha}{4\pi}\left(\hat{\delta}_{\rm weak} + Q^2\hat{\delta}_{\rm em}\right)\right] , \tag{3.56}$$

where

$$\mathcal{T}_0 = -\frac{em}{2s_{\rm w}M_W}\bar{u}(p)v(k) , \tag{3.57}$$

is the Born result and $\hat{\delta}_{\rm weak}$ and $\hat{\delta}_{\rm em}$ are the weak and electromagnetic corrections, respectively, including their OMS counterterms. To be specific,

$$\hat{\delta}_{\rm weak} = \delta_{\rm weak} + \delta_{CT} , \tag{3.58}$$

where $\delta_{\rm weak}$ is the sum of the diagrams in Fig. 12c given by Eq. (2.6) of Ref. 110 and

$$\begin{aligned}\frac{\alpha}{4\pi}\delta_{CT} = \frac{1}{2}\,\mathcal{R}e\left[\left(\frac{c_{\rm w}^2}{s_{\rm w}^2} - 1\right)\frac{\Pi^{WW}(w)}{w} - \frac{c_{\rm w}^2}{s_{\rm w}^2}\,\frac{\Pi^{ZZ}(z)}{z} - \Pi^{HH'}(h)\right] - \frac{s_{\rm w}}{c_{\rm w}}\,\frac{\Pi^{ZA}(0)}{z} + \frac{1}{2}\Pi^{AA'}(0) \\ + \left[\Sigma_S(m^2) - 2m^2\left(\Sigma'_V(m^2) + \Sigma'_S(m^2)\right)\right]_{\rm weak}\end{aligned} \tag{3.59}$$

is the weak part of the counterterm; see Fig. 12d. Here the Π functions are the unrenormalized boson vacuum-polarizations listed in Appendix B, $\Sigma_{V,S}$ are the vector and scalar parts of the unrenormalized fermion self-energy defined in Appendix C, and the subscript *weak* indicates that the photonic contributions, which can be recognized in Eq. (C.2) by the prefactor Q^2, are to be removed explicitly. These photonic contributions are combined with the diagram

of Fig. 12b to give

$$\hat{\delta}_{\rm em} = 2\left(-\frac{2-1/r}{\sqrt{1-1/r}}f+1\right)\ln\frac{m^2}{\mu^2} - 2 \\ + \frac{1}{\sqrt{1-1/r}}\left\{\left(2-\frac{1}{r}\right)\left[2\,{\rm Li}_2\left(-r_-^2\right)+\zeta(2)+2f\left(f-2\tilde{h}\right)\right]+\frac{4}{r}f\right\}\,, \tag{3.60}$$

where $r = (h+i\varepsilon)/(4m^2)$, $r_\pm = \sqrt{1-r}\pm\sqrt{-r}$, $f = \ln r_+$, and $\tilde{h} = \ln(r_+ + r_-)$. A table of handy transformation rules for the analytic continuation in r may be found in Ref. 65. As before, the fictitious photon mass, μ, regulates the infrared divergence. This divergence will be cancelled when the contribution due to soft-photon bremsstrahlung is added.

<u>$H \to f\bar{f}$ decay</u>

Adopting the MOMS scheme, the one-loop corrected $H \to f\bar{f}$ decay width takes the form [110]

$$\Gamma\left(H\to f\bar{f}(\gamma)\right) = \Gamma_0\left(H\to f\bar{f}\,\right)\left(1+\Delta_{\rm weak}+\frac{\alpha}{\pi}Q^2\Delta_{\rm em}\right)\,, \tag{3.61}$$

where [113]

$$\Gamma_0\left(H\to f\bar{f}\,\right) = \frac{NG_FM_Hm^2}{4\pi\sqrt{2}}\left(1-\frac{4m^2}{h}\right)^{3/2}\,, \tag{3.62}$$

is the tree-level result,

$$\Delta_{\rm weak} = -\Delta r + \frac{\alpha}{2\pi}\,\mathcal{R}e\,\hat{\delta}_{\rm weak}\,, \tag{3.63}$$

is the weak correction and

$$\Delta_{\rm em} = \frac{1}{2}\,\mathcal{R}e\,\hat{\delta}_{\rm em} + \delta_{\rm brem}\,, \tag{3.64}$$

the electromagnetic one. Here Δr, $\hat{\delta}_{\rm weak}$, and $\hat{\delta}_{\rm em}$ are given by Eqs. (3.13), (3.58), and (3.60), respectively, and

$$\delta_{\rm brem} = \left(\frac{2-1/r}{\sqrt{1-1/r}}\phi-1\right)\ln\frac{m^2}{\mu^2} - 2\gamma - 4\chi + \frac{26r-29}{8(r-1)} + \frac{3(8r^2-8r+1)}{8r(r-1)}\,\frac{\phi}{\sqrt{1-1/r}} \\ + \frac{2-1/r}{\sqrt{1-1/r}}\left[{\rm Li}_2\left(\rho_-^2\right)+{\rm Li}_2\left(\rho_-^4\right)-2\zeta(2)+\phi(5\phi-2\gamma-2\chi)\right]\,, \tag{3.65}$$

where the notation of Eq. (3.51) is used along with $r = (h/4m^2)$. It is plain to see that $\Delta_{\rm em}$ is indeed μ independent.

In the discussion of $\Delta_{\rm weak}$ it suggests itself to distinguish between universal and flavour-dependent effects. Universal effects arise from the Higgs-boson wave-function renormalization and the gauge-boson vacuum polarizations contained in the counterterm. The residual contributions depend explicitly on the masses and quantum numbers of f and its isopartner. In the standard model, the flavour-dependent effects come in three classes, depending on the values of m and m': (i) $m, m' \ll M_W$, (ii) $m \ll M_W < m'$, and (iii) $m' \ll M_W < m$. Classes (ii) and (iii) apply to the $b\bar{b}$ and $t\bar{t}$ modes, respectively, while class (i) refers to the residual quarkonic modes and to the $\ell^+\ell^-$ modes, where ℓ denotes a charged lepton flavour.

Let us first consider the universal effects. A light-fermion doublet completely decouples from the radiative corrections to the $H \to f\bar{f}$ partial width. In this case, adopting the MOMS scheme not only removes the would-be mass singularities but also the finite terms altogether.

On the other hand, an ultraheavy-fermion doublet, (U, D), with masses $m_U, m_D \gg M_Z$, contributes [110]

$$\Delta_{(U,D)} = \Delta\rho_{(U,D)} + \frac{G_F}{2\pi^2\sqrt{2}} N_{(U,D)} \sum_{i=U,D} m_i^2 \, \mathcal{R}e\, G\left(\frac{h}{4m_i^2}\right) , \tag{3.66}$$

where $\Delta\rho_{(U,D)}$ is given by Eq. (3.16) and

$$\begin{aligned} G(r) &= -\frac{1}{2}\left(1+\frac{1}{r}\right) + \left(1+\frac{1}{2r}\right) f(r) \\ &= \frac{1}{3} - \frac{2}{5} r + \mathcal{O}(r^2) \, . \end{aligned} \tag{3.67}$$

The function f is listed in Eq. (3.18). In the limit $m_U, m_D \gg M_H$, Eq. (3.66) becomes [82]

$$\Delta_{(U,D)} = \frac{G_F}{8\pi^2\sqrt{2}} N_{(U,D)} \left[\frac{7}{3}\left(m_U^2 + m_D^2\right) - \frac{2m_U^2 m_D^2}{m_U^2 - m_D^2} \ln\frac{m_U^2}{m_D^2}\right] . \tag{3.68}$$

Obviously, the presence of a fourth generation would entail an enhancement of the fermionic decay widths. We recall that this is in contrary to the W^+W^- and ZZ channels; see Eq. (3.19).

Similarly to the W^+W^- and ZZ modes, the leading bosonic correction for $M_H \gg 2M_W$ is positive and increases quadratically with M_H [4],

$$\Delta_{\rm bos} = \frac{G_F h}{8\pi^2\sqrt{2}} \left(-\pi\sqrt{3} + \frac{13}{2}\right) . \tag{3.69}$$

Notice that this enhancement factor is somewhat smaller than Eq. (3.20). Again, a strong numerical cancellation is at work. Also Eq. (3.69) can be derived conveniently [85] with the

help of the Goldstone-boson equivalence theorem [83]. Sizeable M_H-dependent terms arise also from the flavour-specific corrections in the case of $H \to t\bar{t}$. However, for $M_H \gg 2m_t$ these terms are subleading, of $\mathcal{O}\left(G_F m_t^2 \ln^2\left(M_H^2/m_t^2\right)\right)$ [110].

We now turn to the flavour-specific effects. Classes (i) and (ii), defined above, are phenomenologically interesting only sufficiently far below the W^+W^- production threshold. In that regime, the total weak correction is well approximated by

$$\Delta_{\rm weak} = \frac{G_F}{8\pi^2\sqrt{2}}\left\{K_f m_t^2 + w\left(-5 + \frac{3}{s_{\rm w}^2}\ln c_{\rm w}^2\right) + z\left[-3\left(1 - 4s_{\rm w}^2|Q|\right)^2 + \frac{1}{2}\right]\right\}, \qquad (3.70)$$

where $K_b = 1$ and $K_f = 7$ for all other light flavours f. Equation (3.70) has been obtained by putting $M_H = m_f = 0$ for $f \neq t$. Note that for $f = b$, $\Delta_{\rm weak}$ depends seven times more feebly on m_t than in the other cases. This is due to a strong cancellation between the triangle diagrams involving virtual top quarks in Fig. 12c and the m_t-dependent part of the counterterm, which emerges from Eq. (3.66) for $U = t$ and $D = b$. For $f \neq b, t$, the top quark does not participate in any triangles and the m_t dependence is determined entirely by Eq. (3.66). A similar situation has been encountered in the context of the $Z \to f\bar{f}$ decays [114].

The $H \to t\bar{t}$ decay, corresponding to class (iii), is particularly interesting, since it combines the two missing links of the standard model. In particular, chances are that the Higgs boson is roughly mass degenerate with a spin-zero $t\bar{t}$ bound state, in which case there should be mixing. It turns out that the weak corrections to the $H \to t\bar{t}$ decay width are finite and small at the threshold, $M_H = 2m_t$ [110]. We shall see below that the case is quite the reverse for the electromagnetic and the QCD corrections. On the other hand, as anticipated above, Eq. (3.69) gives the asymptotic result for $M_H \gg 2m_t$. In the interjacent mass range, $\Delta_{\rm weak}$ has a complicated dependence on M_H and m_t. For $m_t \gg M_W$, the expression can be simplified by setting $M_W = M_Z = 0$. Then it can be expanded around threshold and for high M_H. The following analytic fit smoothly interpolates between these two expansions [110]:

$$\Delta_{\rm weak} = \frac{G_F m_t^2}{2\pi^2\sqrt{2}}\left[\left(1 + \frac{5}{2r}\right)\gamma(\gamma - 2) + \sum_{n=0}^{4} a_n r^{1-n}\right] \qquad (3.71)$$

where $\gamma = \ln(2\sqrt{r}\,)$, $r = \left(h/4m_t^2\right)$, and

$$\begin{aligned}
a_0 &= -\pi\sqrt{3} + \frac{13}{2} && \approx \quad 1.059\,, \\
a_1 &= \frac{15}{4}\zeta(2) - \pi\sqrt{3} + \frac{11}{4} && \approx \quad 3.477\,, \\
a_2 &= \frac{37}{8}\zeta(2) - \frac{11}{8}\pi\sqrt{3} + \frac{7}{48} && \approx \quad 0.272\,, \\
a_3 &= -\frac{41}{2}\zeta(2) + \frac{171}{20}\pi\sqrt{3} - 8\ln^2 2 + \frac{847}{30}\ln 2 - \frac{1193}{40} && \approx -1.296\,, \\
a_4 &= \frac{97}{8}\zeta(2) - \frac{207}{40}\pi\sqrt{3} + \frac{9}{2}\ln^2 2 - \frac{677}{30}\ln 2 + \frac{1721}{80} && \approx -0.182\,.
\end{aligned} \tag{3.72}$$

This reproduces the full result within an error of one percent or so.

Numerical results for the weak corrections to the partial widths of the most relevant fermionic Higgs-boson decays are presented in Fig. 13 for $m_t = 100$, 150, and 200 GeV; see Eq. (3.63). In the case of $H \to \tau^+\tau^-$ (Fig. 13a) the effect is almost independent of M_H for $M_H \lesssim 140\,\mathrm{GeV}$ and Eq. (3.70) provides an excellent approximation; see dotted lines. The spikes at $M_H = 2M_V$, with $V = W, Z$, are attributed to the threshold singularities of the Higgs-boson wave-function renormalization at one loop; see the discussion of Fig. 11a. As explained above, the weak corrections to the $H \to b\bar{b}$ width (Fig. 13b) are almost insensitive to m_t. The weak corrections to the $H \to t\bar{t}$ width (Fig. 13c) range from small negative values at threshold to some 15% at $M_H = 1\,\mathrm{TeV}$. The approximation by Eq. (3.71) is visualized for $m_t = 200\,\mathrm{GeV}$ by the dotted line.

Next we examine the limiting behaviours of Δ_{em}. For $M_H \to 2m$, Δ_{em} exhibits a threshold singularity with a positive sign,

$$\Delta_{\mathrm{em}} = \frac{3\zeta(2)}{\beta} - 1 + 3\zeta(2)\beta + \mathcal{O}\left(\beta^2 \ln\beta\right)\,, \tag{3.73}$$

where $\beta = \sqrt{1 - 1/r}$, with $r = (h/4m^2)$, is the velocity of either fermion in the CM frame. Although this divergence is tamed by the overall threshold factor β^3 of Eq. (3.62), it, nevertheless, indicates the breakdown of naïve perturbation theory in the proximate vicinity of the threshold. The reliability of the prediction may be improved by resumming terms of the form $(\alpha/\beta)^n$ $(n = 1, 2, \ldots)$ as elucidated in Eq. (5–4.135) of Ref. 115. This is equivalent to using a non-relativistic Coulomb potential to describe the electromagnetic interactions between the decay products. Δ_{em} changes sign at $r \approx 4.590$. For $M_H \gg 2m$, Δ_{em} turns into a monomial

in the characteristic logarithm $\gamma = \ln(M_H/m)$,

$$\Delta_{\rm em} = -3\gamma + \frac{9}{4} + \frac{1}{2r}\left(9\gamma + \frac{15}{4}\right) + \mathcal{O}\left(\frac{\gamma}{r^2}\right) , \tag{3.74}$$

which is in compliance with Sudakov's theorem [116]. Notice that the factor m^2 of Eq. (3.62) prevents Eq. (3.61) from being divergent for $m \to 0$.

QCD corrections to the $H \to q\bar{q}$ decay width

In addition to the electroweak corrections discussed so far, there are strong-interaction effects in the case of $H \to q\bar{q}$. They may be taken into account to first order by including in Eq. (3.61) the term $\alpha_s(M_H)C_F\Delta_{\rm em}/\pi$. For quarks with $m \ll M_H/2$ the leading logarithm of Eq. (3.74) may eventually render $\Gamma\big(H \to q\bar{q}(g)\big)$ negative. The obvious preventive measure is to resum the leading logarithms to all orders according to the underlying renormalization group equation [111].

The two-loop QCD corrections to the $H \to q\bar{q}$ decay width are known for $M_H \gg 2m$ [112]. In the $\overline{\rm MS}$ the result is [112, 117]

$$\begin{aligned}\Gamma\left(H \to q\bar{q}\,\right) = {} & \frac{3G_F M_H m^2(M_H)}{4\pi\sqrt{2}}\left(1 - \frac{4m^2(M_H)}{h}\right)^{3/2} \\ & \times \left[1 + K_1\frac{\alpha_s(M_H)}{\pi} + K_2\left(\frac{\alpha_s(M_H)}{\pi}\right)^2 + \mathcal{O}\left(\frac{\alpha_s(M_H)}{\pi}\,\frac{m^2(M_H)}{h}\right)\right] ,\end{aligned} \tag{3.75}$$

where

$$\begin{aligned} K_1 &= \frac{17}{3} \approx 5.6667\,, \\ K_2 &= -\frac{39}{2}\zeta(3) - \frac{19}{2}\zeta(2) + \frac{10801}{144} + \left(\frac{2}{3}\zeta(3) + \frac{\zeta(2)}{3} - \frac{65}{24}\right)N_F \approx 35.9399 - 1.3586N_F\,, \end{aligned} \tag{3.76}$$

$N_F = 5$ is the number of active quark flavours, and $m(\mu)$ is the gauge-invariant quark mass of the $\overline{\rm MS}$ scheme, the so-called current mass, renormalized at a scale μ. For heavy flavours, such as charm and bottom, the non-relativistic bound-state picture is expected to be valid and the relevant mass to be used, for instance in the Balmer formula, is the pole mass, M, which is manifestly gauge-invariant, infrared-finite, and renormalization-scheme-independent [118]. Despite of the confinement of colour, M lends itself to a physically meaningful parameter, as long as the heavy quark is not exactly on-shell. It may be estimated by using QCD sum

rules; for a review see Ref. 119. Typical values of these so-called constituent masses are $M_c \approx 1.46\,\mathrm{GeV}$ and $M_b \approx 4.7\,\mathrm{GeV}$ [120]. To obtain $m(M_H)$, which enters Eq. (3.75), from the empirical value M, one proceeds in two steps. Firstly, one evaluates $m(M)$ via [121]

$$\frac{M}{m(M)} = 1 + \frac{4}{3}\frac{\alpha_s(M)}{\pi} + K\left(\frac{\alpha_s(M)}{\pi}\right)^2 + \mathcal{O}\left(\left(\frac{\alpha_s(M)}{\pi}\right)^3\right), \tag{3.77}$$

with

$$K \approx 16.11 - 1.04\sum_{i=1}^{N_F-1}\left(1 - \frac{M_i}{M}\right), \tag{3.78}$$

appropriate for $N_F - 1$ quarks with masses $M_i < M$ $(i = 1, N_F - 1)$. Specifically, $K_c = 13.3$ and $K_b = 12.4$ [121]. Secondly, one relates $m(M_H)$ to $m(M)$ by means of the scaling law [112, 121]

$$\frac{m(M_H)}{m(M)} = \frac{c\big(\alpha_s(M_H)/\pi\big)}{c\big(\alpha_s(M)/\pi\big)}, \tag{3.79}$$

where

$$\begin{aligned} c(x) = (2\beta_0 x)^{\gamma_0/\beta_0}\Bigg\{1 + \left(\frac{\gamma_1}{\beta_0} - \frac{\beta_1\gamma_0}{\beta_0^2}\right)x \\ + \frac{1}{2}\left[\left(\frac{\gamma_1}{\beta_0} - \frac{\beta_1\gamma_0}{\beta_0^2}\right)^2 + \left(\frac{\gamma_2}{\beta_0} + \frac{\beta_1^2\gamma_0}{\beta_0^3} - \frac{\beta_1\gamma_1}{\beta_0^2} - \frac{\beta_2\gamma_0}{\beta_0^2}\right)\right]x^2 + \mathcal{O}(x^3)\Bigg\}. \end{aligned} \tag{3.80}$$

Here β_i $(i = 0, 1, \ldots)$ are the coefficients of the Gell-Mann-Low function of QCD [122],

$$\begin{aligned} \beta_0 &= \frac{1}{4}\left(11 - \frac{2}{3}N_F\right), \\ \beta_1 &= \frac{1}{16}\left(102 - \frac{38}{3}N_F\right), \\ \beta_2 &= \frac{1}{64}\left(\frac{2857}{2} - \frac{5033}{18}N_F + \frac{325}{54}N_F^2\right), \end{aligned} \tag{3.81}$$

and γ_i $(i = 0, 1, \ldots)$ are the coefficients of the quark-mass anomalous dimension [123],

$$\begin{aligned} \gamma_0 &= 1, \\ \gamma_1 &= \frac{1}{16}\left(\frac{202}{3} - \frac{20}{9}N_F\right), \\ \gamma_2 &= \frac{1}{64}\left[1249 - \left(\frac{2216}{27} + \frac{160}{3}\zeta(3)\right)N_F - \frac{140}{81}N_F^2\right]. \end{aligned} \tag{3.82}$$

For strange, charm, and bottom Eq. (3.80) becomes

$$\begin{aligned} c_s(x) &= \left(\frac{9}{2}x\right)^{4/9}(1+0.895x+1.371x^2)\,, \\ c_c(x) &= \left(\frac{25}{6}x\right)^{12/25}(1+1.014x+1.389x^2)\,, \\ c_b(x) &= \left(\frac{23}{6}x\right)^{12/23}(1+1.175x+1.501x^2)\,, \end{aligned} \tag{3.83}$$

respectively.

Figure 14 illustrates the significance of higher-order QCD corrections to the $H \to q\bar{q}$ decay widths for $q = c$ (Fig. 14a) and $q = b$ (Fig. 14b). The dashed lines correspond to the naïve evaluation of Eq. (3.62) using the pole mass, $m = M$. The dot-dashed lines are obtained from this by multiplication with the next-to-leading-order QCD correction factor, $\left(1+\alpha_s(M_H)C_F\Delta_{\rm em}/\pi\right)$, where $\Delta_{\rm em}$ is given by Eq. (3.64) with $m = M$. The solid lines represent the next-to-next-to-leading-order result given by Eq. (3.75), where $m(M_H)$ is determined from M according to the two-step procedure described above. The values $M_c = 1.49\,\mathrm{GeV}$ and $M_b = 4.7\,\mathrm{GeV}$ [120] have been used as input. For $M_H = (60, 120, 180)\,\mathrm{GeV}$ the QCD corrections reduce the naïve Born result by (77,80,81)% in the case of the $c\bar{c}$ mode and by (50,58,61)% in the case of the $b\bar{b}$ mode.

The two-loop result of Ref. 112 is valid for $M_H \gg 2m$ only, while the one-loop formula (3.64) holds in general, except for $2m \lesssim M_H$; see Eq. (3.73). The $1/\beta$ singularity encountered in the context of the QED corrections is, of course, much more severe in case of QCD. Here it can be understood as coming from colour Coulomb interactions between the quark and antiquark. A realistic treatment of the heavy-quark threshold requires the inclusion of the mixing of the Higgs boson with zero-spin quarkonium states both below and above the open-flavour threshold. A comprehensive discussion of the $b\bar{b}$ threshold may be found in Ref. 124, which has, however, become obsolete due to the LEP 1 bound on M_H [9]. Toponium still comes into question for a similar scenario. However, for $m_t \gtrsim 130\,\mathrm{GeV}$ the lifetime a $t\bar{t}$ bound state becomes shorter than its revolution period [125] and alternative approaches are in demand [71].

4. Four-point functions: associated production and three-body decays

In this chapter we shall consider four-point functions with one external Higgs-boson line at the quantum level. This includes radiative corrections to the tree-level e^+e^-ZH, $q\bar{q}ZH$, and $q\bar{q}'W^\pm H$ vertices as well as the loop-induced $e^+e^-\gamma H$, $ggZH$, $gggH$, and $q\bar{q}gH$ vertices. Specifically, we shall discuss electroweak corrections to $e^+e^- \to ZH$ [86, 126–128] and QCD corrections to $pp \to ZH + X$ (via quark-antiquark annihilation) and $pp \to W^\pm H + X$ [129–131]. The loop-induced vertices mediate $e^+e^- \to \gamma H$ [87], $pp \to ZH + X$ (via gluon fusion) [132, 133], and $pp \to jH + X$, where the Higgs boson carries finite transverse momentum [96]. On the contrary, all these four-point functions may trigger three-body decays of the Higgs boson if these are allowed by kinematics. Finally, a Higgs bosons with $M_H < M_Z$ can be produced at LEP 1 and SLC via Bjorken's $Z \to f\bar{f}H$ process [8] and $Z \to ggH$ [134, 135]. In principle, radiative corrections to the Bjorken process induce a shift in the M_H lower bound; however, the effect turns out to be of the order of 100 MeV only [136]. The $Z \to ggH$ decay was found to be truly rare [134, 135].

4.1 $f\bar{f}ZH$ COUPLING

Numerical results on the soft-photonic and weak corrections to the cross section of $e^+e^- \to ZH$ were originally presented in Ref. 126. In Ref. 127, it was shown that the first-order QED corrections to the initial state of $e^+e^- \to \mu^+\mu^-H$ factorize and a Monte Carlo event generator was developed. The electroweak corrections to the $f\bar{f}ZH$ vertex were first listed in Ref. 86. The weak corrections to the helicity amplitudes of $e^+e^- \to ZH$ were found in Ref. 128 with the help of a computer-algebraic manipulation program. The results of Refs. 86, 128 were found to agree with each other to within computer precision and with those of Ref. 126 up to a constant term of $1.2 \times 10^{-3}\%$ [137].

We shall now review the radiative corrections to the $f\bar{f}ZH$ vertex in the one-loop approximation assuming all four legs to be on their mass shells and the fermion f to be light. To obtain compact expressions, we consider the transition-matrix element in its squared form. Our results are fully analytic so that they apply to all possible transitions between the external particles. To start with, we introduce our notation. We denote the incoming four-momenta of f, $\bar{f}$, Z, and H by p_-, p_+, p, and k, respectively. In the massless-fermion approximation the mass-shell conditions read $p_-^2 = p_+^2 = 0$, $p^2 = z$, and $k^2 = h$. Momentum conservation leads to $s + t + u = z + h$, where $s = (p + k)^2$, $t = (p + p_-)^2$, and $u = (p + p_+)^2$ are the usual

Mandelstam variables. It is convenient to introduce $n = tu - zh$ and $\lambda = \lambda(s,z,h)$, where $\lambda(a,b,c) = a^2+b^2+c^2-2(ab+bc+ca)$. N, I, Q, and m stand for the colour multiplicity, weak isospin, electric charge, and mass[#] of f. The isopartner of f is called f' and its quantum numbers are indicated by a prime. The infrared divergences are regularized by assigning a small mass, μ, to the photon (gluon).

In the following, we shall focus attention on electroweak phenomena. There are additional corrections due to strong-interaction effects when f represents a quark. To first order they simply follow from the electromagnetic corrections through multiplication by $\alpha_s(s)C_F/(\alpha Q^2)$.

We write the perturbation expansion of the renormalized T-matrix element under consideration as

$$\mathcal{T} = \mathcal{T}_0 + \mathcal{T}_{\rm em} + \mathcal{T}_{\rm weak} + \mathcal{O}(\alpha^2)\,, \tag{4.1}$$

where $\mathcal{T}_0$ is the Born amplitude and $\mathcal{T}_{\rm em}$ and $\mathcal{T}_{\rm weak}$ represent the electromagnetic and weak loops; see Figs. 86a–l. Squaring and summing over all polarization, spin, and colour states yields

$$|\mathcal{T}\,|^2 = |\mathcal{T}_0|^2\left[1 + 2\,\mathcal{R}e(\delta_{\rm weak}+\delta_{\rm em}) + \mathcal{O}(\alpha^2)\right]\,, \tag{4.2}$$

with $\delta_{\rm weak} = \mathcal{T}_0^*\mathcal{T}_{\rm weak}/|\mathcal{T}_0|^2$ and $\delta_{\rm em} = \mathcal{T}_0^*\mathcal{T}_{\rm em}/|\mathcal{T}_0|^2$. In the MOMS scheme we find

$$\begin{aligned}
|\mathcal{T}_0|^2 &= 2G_F^2 z^3 N\left[\left(1-4s_{\rm w}^2|Q|\right)^2+1\right]\frac{c_1}{(s-z)^2}\,,\\
\delta_{\rm weak} &= -\Delta r - \frac{1}{2}\,\mathcal{R}e\,\hat{\Pi}'_{ZZ}(z) + \hat{E}_{ZZH}(z,s,h) + \frac{c_2}{c_1}D_{ZZH}(z,s,h) - \frac{\hat{\Pi}_{ZZ}(s)}{s-z}\\
&\quad + \frac{1}{v^2+a^2}\left[-vQ\frac{s-z}{s}\left(\hat{E}_{ZAH}(z,s,h) + \frac{c_2}{c_1}D_{ZAH}(z,s,h) - \frac{\hat{\Pi}_{ZA}(s)}{s-z}\right)\right.\\
&\quad \left. + v\hat{V}_{Zf\bar{f}}(s) + a\hat{A}_{Zf\bar{f}}(s) + (s-z)\left(T+\frac{B}{c_1}\right)\right]\,,\\
\delta_{\rm em} &= \frac{\alpha}{4\pi}Q^2F_1\left(\frac{s+i\varepsilon}{m^2}\right)\,,
\end{aligned} \tag{4.3}$$

with $c_1 = 2(2s+n/z)$ and $c_2 = n(-s-z+h)/z$ being kinematical factors. Here the hatted quantities have been renormalized by including their OMS counterterms. This is true also for F_1. The residual terms were ultraviolet finite from the outset.

[#] Note that a finite value of m is needed to regulate the collinear divergences associated with the photonic and gluonic corrections.

We shall now explain the various pieces of Eq. (4.3) one by one. The renormalized Z-boson self-energy, $\hat{\Pi}_{ZZ}$, and Z-photon mixing amplitude, $\hat{\Pi}_{ZA}$, are defined in Eq. (2.1). The term $-(1/2)\,\mathcal{R}e\,\hat{\Pi}'_{ZZ}(z)$ is due to the renormalization of the Z-boson wave function. The ZZH form factors, $D_{ZZH}(a,b,c)$ and $\hat{E}_{ZZH}(a,b,c)$, are listed in Eq. (3.5) for arbitrary invariant masses of the two Z bosons and the Higgs boson, a, b, and c, respectively. The general ZAH form factors, $D_{ZAH}(a,b,c)$ and $\hat{E}_{ZAH}(a,b,c)$, where b is the photon invariant mass, are specified in Eq. (3.22). The renormalized vector and axial-vector $Zf\bar{f}$ form factors, $\hat{V}_{Zf\bar{f}}$ and $\hat{A}_{Zf\bar{f}}$, are given by

$$\begin{aligned}
\hat{V}_{Zf\bar{f}}(s) &= \frac{\alpha}{4\pi}\left[v(v^2+3a^2)F_2\left(\frac{s+i\varepsilon}{z}\right)+\frac{v'+a'}{4s_{\rm w}^2}F_2\left(\frac{s+i\varepsilon}{w}\right)+\frac{c_{\rm w}^2}{s_{\rm w}^2}aF_3\left(\frac{s+i\varepsilon}{4w}\right)\right],\\
\hat{A}_{Zf\bar{f}}(s) &= \frac{\alpha}{4\pi}\left[a(3v^2+a^2)F_2\left(\frac{s+i\varepsilon}{z}\right)+\frac{v'+a'}{4s_{\rm w}^2}F_2\left(\frac{s+i\varepsilon}{w}\right)+\frac{c_{\rm w}^2}{s_{\rm w}^2}aF_3\left(\frac{s+i\varepsilon}{4w}\right)\right],
\end{aligned} \tag{4.4}$$

where s is the Z-boson invariant mass. In each line of Eq. (4.4), the first (second) term arises from Z-boson (W-boson) exchange between the fermions, while the third term is due to the configuration where the Z boson is linked to the the fermion line by two virtual W bosons involving a triple-gauge-boson coupling. The functions F_i $(i=1,2,3)$ are defined by

$$\begin{aligned}
F_1(x) &= 2\big(-\ln(-x)+1\big)\ln\frac{m^2}{\mu^2}-\ln^2(-x)+3\ln(-x)+2\zeta(2)-4\,,\\
F_2(x) &= 2\left(1+\frac{1}{x}\right)^2\big(\mathrm{Li}_2(1+x)-\zeta(2)\big)+\left(3+\frac{2}{x}\right)\ln(-x)-\frac{7}{2}-\frac{2}{x}\,,\\
F_3(r) &= \frac{1}{r}\left(4+\frac{1}{2r}\right)\mathrm{arsinh}^2\sqrt{-r}+\left(2+\frac{1}{r}\right)\sqrt{1-\frac{1}{r}}\,\mathrm{arsinh}\sqrt{-r}+\frac{5}{2}-\frac{1}{2r}\,.
\end{aligned} \tag{4.5}$$

The photonic form factor, F_1, is plagued by an infrared singularity, which will cancel against the one produced by the soft-photon bremsstrahlung. Radiative corrections to the $f\bar{f}H$ vertex have been discussed in Sec. 3.6 assuming all three legs to be on their mass shells. In that case, due to helicity conservation, the Higgs boson is bound to decouple from the fermionic current in the massless-fermion limit. However, this argument fails if at least one fermion is taken off its mass shell. In fact, the set of diagrams represented by Figs. 15e and f contribute the term

$$T=\frac{\alpha}{4\pi}\left[\frac{c_{\rm w}^2}{s_{\rm w}^2}(v+a)^2T_1(t,h,w)+4(v^4+6v^2a^2+a^4)T_1(t,h,z)+(t\leftrightarrow u)\right], \tag{4.6}$$

where

$$T_1(t,h,w)=\frac{1}{2(t-h)}\big[B_0(t,w,0)-B_0(h,w,w)-wC_0(0,t,h,w,0,w)\big]\,. \tag{4.7}$$

The B_0 and C_0 functions are listed in Appendix A. Finally, the $f\bar{f}ZH$ box diagrams yield

$$
\begin{aligned}
B = \frac{\alpha}{4\pi}\Bigg\{ & \frac{c_{\rm w}^2}{s_{\rm w}^2}(v+a)(v'+a')B_1(t,u,z,h,w) + 4(v^4+6v^2a^2+a^4)B_1(t,u,z,h,z) \\
& + c_{\rm w}^2 a(v+a)\left[\frac{c_{\rm w}^2}{s_{\rm w}^2}B_2(t,u,z,h,w) + B_3(t,u,z,h,w) + (t \leftrightarrow u)\right]\Bigg\}\,,
\end{aligned}
\tag{4.8}
$$

where

$$
\begin{aligned}
& B_1(t,u,z,h,w) \\
& = -2\big(B_0(z,0,0) - B_0(h,w,w)\big) + \left[2 - \frac{n}{z(t-h)}\right]\big(B_0(t,w,0) - B_0(h,w,w)\big) \\
& \quad + \left[2 - \frac{n}{z(u-h)}\right]\big(B_0(u,w,0) - B_0(h,w,w)\big) + 2(-t+z+w)C_0(0,t,z,0,w,0) \\
& \quad + 2(-u+z+w)C_0(0,u,z,0,w,0) + \left[2(u-z-w) + \frac{wn}{z(t-h)}\right]C_0(0,t,h,w,0,w) \\
& \quad + \left[2(t-z-w) + \frac{wn}{z(u-h)}\right]C_0(0,u,h,w,0,w) \\
& \quad + 2\big[-tu + (z+w)(-s+h-w)\big]D_0(0,z,0,h,t,u,w,0,0,w)\,, \\
& B_2(t,u,z,h,w) \\
& = 4\left[2 - \frac{n}{z(u-h)}\right]\big(B_0(u,w,0) - B_0(h,w,w)\big) + \frac{8s}{\lambda}(-t+u)\big(B_0(s,w,w) - B_0(h,w,w)\big) \\
& \quad + \frac{8}{\lambda}\big[-2n + (t-u)(-t+z)\big]\big(B_0(z,w,w) - B_0(h,w,w)\big) \\
& \quad + \left[2(-2s+h) - \frac{u}{z}(s+2t)\right]C_0(0,0,s,w,0,w) \\
& \quad + \left[8(t-z-w) + \frac{u}{z}(-u+h) + \frac{4wn}{z(u-h)}\right]C_0(0,u,h,w,0,w) \\
& \quad + \left\{2(-2t+4z-h+4w) + \frac{u}{z}(-3t+u) + \frac{8s}{\lambda}\big[-2n + t(-t+u)\big]\right\}C_0(s,z,h,w,w,w) \\
& \quad + \left(5u - 4z - 8w - \frac{u^2}{z}\right)C_0(0,u,z,w,0,w) \\
& \quad + \left\{2u(-2s+h) + w(7s-5t+9z-5h+8w) + \frac{u}{z}\big[-u(s+2t) + w(-s+h)\big]\right\} \\
& \quad \times D_0(0,0,z,h,s,u,w,0,w,w)\,, \\
& B_3(t,u,z,h,w) \\
& = \Big[2(-s+h) - \frac{u}{z}(s+2t)\Big]C_0(0,0,s,w,0,w) + (u-h)\left(2 - \frac{u}{z}\right)C_0(0,u,h,w,0,w) \\
& \quad + \left(-2z + h + \frac{u^2}{z}\right)C_0(s,z,h,w,w,w) + \left(-u + 2z - \frac{u^2}{z}\right)C_0(0,u,z,w,0,w) \\
& \quad + \left\{2u(-s+h) + w(2s+u-2z) + \frac{u}{z}\big[-u(s+2t) + w(-s+h)\big]\right\}
\end{aligned}
$$

$$\times D_0(0,0,z,h,s,u,w,0,w,w)\,. \tag{4.9}$$

The relevant D_0 functions are given in Appendix A. In Eq. (4.8), the terms proportional to B_1 refer to Figs. 15k and l, the B_2 terms to Figs. 15g and i, and the B_3 terms to Figs. 15h and j. Note that $B_1(t,u,z,h,w) = B_1(u,t,z,h,w)$, so that Eq. (4.2) is symmetrical in t and u as required by charge-conjugation invariance.

It is instructive to elaborate on the fermionic contributions to $\delta_{\rm weak}$. In the MOMS scheme, the contribution due to a light-fermion doublet, (u,d), is devoid of mass singularities,

$$\begin{aligned}\delta_{(u,d)} = \frac{G_F z}{6\pi^2\sqrt{2}} N_{(u,d)} \Bigg\{ \left[c_{\rm w}^4 + s_{\rm w}^4\left(1+2Y_{(u,d)}^2\right)\right]\left(\frac{s}{s-z}\ln\frac{-s-i\epsilon}{z}+\frac{1}{2}\right) \\ + c_{\rm w}s_{\rm w}\frac{vQ}{v^2+a^2}\left[\left(c_{\rm w}^2 - s_{\rm w}^2\left(1+2Y_{(u,d)}^2\right)\right)\ln\frac{-s-i\epsilon}{z} - \frac{c_{\rm w}^2}{s_{\rm w}^2}\ln c_{\rm w}^2\right]\Bigg\}\,.\end{aligned} \tag{4.10}$$

This is independent of M_H and feebly varies with s.

At the other extreme, a doublet, (U,D), of fermions which are heavy as compared to the largest energy scale associated with the $f\bar{f}ZH$ interaction contributes

$$\delta_{(U,D)} = \left(1 - \frac{c_{\rm w}}{s_{\rm w}}\frac{vQ}{v^2+a^2}\right)\Delta\rho_{(U,D)} - \frac{G_F}{6\pi^2\sqrt{2}} N_{(U,D)}\left(m_U^2+m_D^2\right)\,, \tag{4.11}$$

where $\Delta\rho_{(U,D)}$ is defined by Eq. (3.16). The OMS-scheme version of Eq. (4.11) was found in Ref. 138. Obviously, ultraheavy fermions generate negative corrections proportional to the squares of their masses, no matter if weak isospin is conserved or badly broken. A similar trend was observed in Secs. 3.1 and 3.5 for the radiatively corrected ZZH and W^+W^-H couplings, respectively. Contrariwise, the $f\bar{f}H$ coupling is enhanced in the presence of ultraheavy fermions; see Sec. 3.6.

<u>$e^+e^- \to ZH$ bremsstrahlung process</u>

Perhaps, the most important application of Eq. (4.2) is to calculate the electroweak corrections to the cross section of $e^+e^- \to ZH$. To obtain an infrared-finite result, one needs to include the contribution due to real-photon radiation off the initial state; see Figs. 15m and n. It is technically convenient to introduce a cutoff parameter, Δ, with $\mu \ll \Delta \ll \left(s-(M_Z+M_H)^2\right)/(2\sqrt{s}\,)$ such that for ZH invariant masses, s', with $\sqrt{s}\,(\sqrt{s}-2\Delta) < s' < s$ the kinematics can be considered as practically unaffected by the emission of the extra photon. Those events are conventionally called soft, while events with $(M_Z+M_H)^2 < s' < \sqrt{s}\,(\sqrt{s}-$

2Δ) are called hard. In the CM system, Δ bears the meaning of a photon-energy cutoff. Soft photons give rise to a universal factor which multiplies the uncorrected cross section,

$$\delta_{\rm soft} = \frac{\alpha}{\pi} Q^2 \left[(L-1) \ln \frac{4\Delta^2}{\mu^2} - \frac{L^2}{2} + L - 2\zeta(2) \right] , \tag{4.12}$$

where $L = \ln(s/m^2)$ and terms of order $(m^2/s)L$ have been neglected.

Collecting the above results, we can now write the differential cross section of $e^+e^- \to ZH$ including the weak and soft-photonic corrections as

$$\frac{d\sigma}{dt} = \frac{d\sigma_0}{dt} \left[1 + 2\,\mathcal{R}e(\delta_{\rm weak} + \delta_{\rm em}) + \delta_{\rm soft} + \mathcal{O}(\alpha^2) \right] , \tag{4.13}$$

where

$$\frac{d\sigma_0}{dt} = \frac{G_F^2 z^3}{32\pi s^2} \left[\left(1 - 4s_{\rm w}^2\right)^2 + 1 \right] \frac{c_1}{(s-z)^2} \tag{4.14}$$

is the Born result. Usually, e^+e^- colliders are operated in such a way that the CM system coincides with the laboratory frame. In terms of the cosine, c, of the angle between the flight directions of the incoming electron and the outgoing Z boson, then one has $t = -\big(s - z - h - \sqrt{\lambda}\, c\big)/2$.

The combined electromagnetic correction in Eq. (4.13),

$$2\,\mathcal{R}e\,\delta_{\rm em} + \delta_{\rm soft} = \frac{\alpha}{\pi} Q^2 \left[(-L+1) \ln \frac{s}{4\Delta^2} + \frac{3}{2} L + 2\zeta(2) - 2 \right] , \tag{4.15}$$

is μ independent. However, there remains a strong dependence on Δ, which is now to be related to the energy resolution of the photon detector. The effect of hard-photon events on the differential cross section is most conveniently included by means of Monte Carlo event generators [127].

We now turn to the total cross section and define

$$\begin{aligned} C_1 &= \int\limits_{-\left(s-z-h+\sqrt{\lambda}\right)/2}^{-\left(s-z-h-\sqrt{\lambda}\right)/2} dt\, c_1 = \sqrt{\lambda} \left(4s + \frac{\lambda}{3z} \right) , \\ C_2 &= \int\limits_{-\left(s-z-h+\sqrt{\lambda}\right)/2}^{-\left(s-z-h-\sqrt{\lambda}\right)/2} dt\, c_2 = \frac{\lambda^{3/2}}{6z} (-s - z + h) . \end{aligned} \tag{4.16}$$

Integration of Eq. (4.14) yields

$$\sigma_0 = \frac{G_F^2 z^3}{32\pi s^2}\left[\left(1-4s_{\rm w}^2\right)^2+1\right]\frac{C_1}{(s-z)^2}\,. \tag{4.17}$$

The hard-photon contribution can be obtained by convoluting Eq. (4.17) with a photon radiator,

$$\sigma_{\rm hard} = \frac{\alpha}{\pi}Q^2(L-1)\int\limits_{(M_Z+M_H)^2/s}^{1-2\Delta/\sqrt{s}} dx\frac{1+x^2}{1-x}\sigma_0(xs)\,, \tag{4.18}$$

which can be solved analytically [86]. Putting everything together, the radiatively corrected total cross section is found to be

$$\sigma = \sigma_0\left(1+2\,\mathcal{R}e\,\Delta_{\rm weak}+\delta_{\rm QED}+\mathcal{O}(\alpha^2)\right), \tag{4.19}$$

where σ_0 is given by Eq. (4.17),

$$\begin{aligned}\Delta_{\rm weak} = {} & -\Delta r - \frac{1}{2}\,\mathcal{R}e\,\hat{\Pi}'_{ZZ}(z) + \hat{E}_{ZZH}(z,s,h) + \frac{C_2}{C_1}D_{ZZH}(z,s,h) - \frac{\hat{\Pi}_{ZZ}(s)}{s-z} \\ & + \frac{1}{v^2+a^2}\left[-vQ\frac{s-z}{s}\left(\hat{E}_{ZAH}(z,s,h)+\frac{C_2}{C_1}D_{ZAH}(z,s,h)-\frac{\hat{\Pi}_{ZA}(s)}{s-z}\right)\right. \\ & \left. + v\hat{V}_{Zf\bar{f}}(s) + a\hat{A}_{Zf\bar{f}}(s) + \frac{s-z}{C_1}\int\limits_{-(s-z-h+\sqrt{\lambda})/2}^{-(s-z-h-\sqrt{\lambda})/2} dt\,(c_1T+B)\right],\end{aligned} \tag{4.20}$$

and

$$\delta_{\rm QED} = \frac{\alpha}{\pi}Q^2\left[(L-1)\left(\ln\frac{\lambda^2}{s^2zh}+\frac{z^2}{C_1}F\left(\frac{z}{s},\frac{h}{s}\right)\right)+\frac{3}{2}L+2\zeta(2)-2\right]. \tag{4.21}$$

is the full QED correction to first order. The function F reads

$$\begin{aligned}F(a,b) = {} & G(a,b) + \frac{(a-b)(1-a)^2}{3a^6}\left[-a^2(11+a)+ab(7+2a)-b^2(2+a)\right] \\ & \times\left(\ln\frac{1+a-b+\sqrt{\lambda}}{1-a+b+\sqrt{\lambda}}-\frac{1}{2}\ln\frac{a}{b}\right)+\frac{(1-a)^2}{3a^3}(-1-11a+3b)\ln\frac{1-a-b+\sqrt{\lambda}}{2\sqrt{ab}} \\ & + \frac{2\sqrt{\lambda}}{3a^5}\left[a^2(-7+2a-7a^2)+ab(3-2a+3a^2)+b^2(-1+a-a^2)\right],\end{aligned} \tag{4.22}$$

where $\lambda = \lambda(1,a,b)$, $\Lambda = \lambda(a,a,b)$, and

$$
\begin{aligned}
G(a,b) = \frac{1}{a^3}\Bigg\{ & -\frac{\sqrt{-\Lambda}}{3a^3}\left[12a^2(1-2a-a^2)+ab(-5+9a+3a^2+a^3)+b^2(2-3a-a^3)\right] \\
& + \frac{4b}{\sqrt{-\Lambda}}(1-a)(1+a^2)\Bigg\}\left(2\arctan\frac{\sqrt{-\Lambda}}{1-a+\sqrt{\lambda}} - \arccos\left(\frac{1}{2}\sqrt{\frac{b}{a}}\right)\right), \quad 4a > b\,, \\
= {} & \frac{4(1-a)(1+a^2)}{a^3}\left(\frac{8a}{1-a+\sqrt{(1-a)(1-9a)}}-1\right), \qquad 4a = b\,, \\
= {} & \frac{1}{a^3}\Bigg\{\frac{\sqrt{\Lambda}}{3a^3}\left[12a^2(1-2a-a^2)+ab(-5+9a+3a^2+a^3)+b^2(2-3a-a^3)\right] \\
& + \frac{4b}{\sqrt{\Lambda}}(1-a)(1+a^2)\Bigg\}\left(\ln\frac{1-a+\sqrt{\lambda}+\sqrt{\Lambda}}{1-a+\sqrt{\lambda}-\sqrt{\Lambda}} - \ln\frac{b+\sqrt{\Lambda}}{2\sqrt{ab}}\right), \qquad 4a < b\,.
\end{aligned}
\tag{4.23}
$$

Notice that Eq. (4.21) is linear in L as demanded by Sudakov's theorem [116].

Initial-state corrections for e^+e^- annihilation are available up to $\mathcal{O}(\alpha^2)$ in QED. This includes contributions due to photons [139], charged-lepton pairs [140], and hadronic matter [141]. The purely photonic part is known to constitute the bulk of these corrections. The well-known technique of exponentiating the infrared-sensitive parts [142] provides a further refinement. Higher-order initial-state corrections are known to be indispensible when cross sections strongly vary with $\sqrt{s}$, *e.g.*, in the case of the Z-boson line shape. However, they are expected to be less significant in the case of ZH associated production in the continuum.

Figure 16 shows versus M_H the one-loop weak correction, $2\,\mathcal{R}e\,\Delta_{\text{weak}}$, to the total cross section of $e^+e^- \to ZH$; see Eq. (4.20). Figure 16a disentangles the contributions due to bosons, leptons, and quarks at $\sqrt{s} = 500\,\text{GeV}$ for $m_t = 150\,\text{GeV}$. The bosonic contribution is throughout negative and is plagued by the well-understood threshold singularities at $M_H = 2M_V$, with $V = W, Z$; see the discussion of Fig. 11a. The lepton loops give rise to an M_H-independent increase by about 2%; see Eq. (4.10). The quarkonic correction is somewhat larger and exhibits the familiar spike at $M_H = 2m_t$; see the discussion of Fig. 5a. Leaving aside pathological points, the combined effect ranges between -5% and 3% for the considered choice of parameters. In Fig. 16b, the sensitivity to m_t of the total weak correction investigated. Note that for $m_t = 90\,\text{GeV}$ the dent at $M_H = 2m_t$ is overwhelmed beneath the incision at $M_H = 2M_Z$, while it does not show up for $m_t = 210\,\text{GeV}$. The present lack of knowledge of the precise value of m_t induces a considerable uncertainty in the prediction at the quantum level. However, this source of incertitude is likely to be removed before the

NLC goes into operation. In Fig. 16c, the M_H dependence of the weak correction is studied at $\sqrt{s} = 200$, 500, 1000, and 1500 GeV assuming $m_t = 150\,\mathrm{GeV}$. For $M_H < 2m_t$, there is an almost M_H independent reduction of the production rate, which is more dramatic at higher energies. For $M_H > 2m_t$, the corrections increase monotonically with M_H and may take sizeable positive values, of the order of 25%.

Figure 17 shows the photonic corrections to second order for the total cross section of $e^+e^- \to ZH$ as a function of M_H at $\sqrt{s} = 200$, 500, 1000, and 1500 GeV. The infrared-sensitive parts are exponentiated, so that the probability for the emission of a photon with energy ω goes to zero for $\omega \to 0$. Consequently, the correction equals -100% at $\sqrt{s} = M_Z + M_H$. Initial-state radiation may considerably enhance the cross section in the continuum. This may be understood by observing that the energy available for the hard-scattering process is reduced by the emitted quanta and that the undressed cross section is a decreasing function of $\sqrt{s}$. For comparison, first-order QED corrections without exponentiation as given by Eq. (4.21) are shown also. As expected, the agreement is fairly good, except for $\sqrt{s} \gg M_Z + M_H$. At LEP 2, the QED effect will be throughout negative. At the NLC, there may be an enormous enhancement should M_H be settled close to the lower end of the currently favoured mass range.

The so-called beamstrahlung phenomenon [143] is a novel feature to be faced at the NLC, as an unavoidable consequence of the quest for luminosities that exceed current achievements by three orders of magnitude. It is evoked when particles in one bunch undergo bremsstrahlung upon entering the the electromagnetic field of the other bunch, so that sizeable parts of the opposite bunches interact coherently with each other. In contrast to usual bremsstrahlung, beamstrahlung can be controlled by adjusting the machine design and operation. A comprehensive discussion of the impact of beamstrahlung on Higgs-boson production at the NLC may be found in Refs. 19, 144.

$\underline{Z \to f\bar{f}H \text{ decay}}$

Current Higgs searches at LEP 1 and SLC rely on the Bjorken process [8], $Z \to f\bar{f}H$. The Higgs bosons would decay almost exclusively into hadrons or $\tau^+\tau^-$ pairs (see Fig. 1) and the channels where f represents a neutrino or a charged lepton would yield the clearest signatures [10]. For increasing M_H, the available phase space is quenched and the expected rate drops. As yet, the mass range $M_H < 63.5\,\mathrm{GeV}$ has been excluded at the 95% CL level [9]. Further progress will be made much more slowly. In the next few years, with two or

three million events collected per LEP 1 experiment, one may estimate that this procedure will come to a grinding halt at 65 GeV or so [10]. The relative increments of the M_H lower bound will be comparable to usual quantum corrections in electroweak perturbation theory. It is, therefore, obvious that such a procedure makes sense only if loop effects are properly taken into account. The point is that the inclusion of radiative corrections to the $Z \to Hf\bar{f}$ partial width induces an equivalent shift of the M_H lower bound.

Neglecting external-fermion masses, the $Z \to f\bar{f}H$ decay width, differential with respect to the $f\bar{f}$ invariant mass, s, is in the Born approximation given by

$$\frac{d\Gamma_0}{ds} = \frac{G_F^2 M_Z^3}{384\pi^3} N \left[\left(1 - 4s_{\rm w}^2 |Q|\right)^2 + 1 \right] \frac{C_1}{(s-z)^2} , \qquad 0 \le s \le (M_Z - M_H)^2 , \tag{4.24}$$

where C_1 is defined in Eq. (4.16). Note that the internal Z boson is far away from its mass shell, so that it would be incorrect to include the total width in its propagator; the appropriate Z-boson self-energy will enter at the one-loop level. The integrated result reads [135]

$$\Gamma_0 = \frac{G_F^2 M_Z^5}{384\pi^3} N \left[\left(1 - 4s_{\rm w}^2 |Q|\right)^2 + 1 \right] F\left(\frac{h}{z}\right) , \tag{4.25}$$

where

$$\begin{aligned} F(x) = {} & \frac{1-x}{3}\left(-\frac{47}{2} + \frac{13}{2}x - x^2\right) + \left(-2 + 3x - \frac{x^2}{2}\right)\ln x \\ & + \left(10 - 4x + \frac{x^2}{2}\right)\sqrt{\frac{x}{4-x}}\left(\pi - 6\arcsin\frac{\sqrt{x}}{2}\right) . \end{aligned} \tag{4.26}$$

Complete first-order electroweak corrections can be accommodated in Eq. (4.24) by multiplication with

$$\left(1 + \frac{\alpha}{\pi}\frac{3}{4}Q^2 + 2\,\mathcal{R}e\,\Delta_{\rm weak}\right) , \tag{4.27}$$

where $\Delta_{\rm weak}$ emerges by analytic continuation of Eq. (4.20). Consequently, the correction factor for Eq. (4.25) reads

$$\left(1 + \frac{\alpha}{\pi}\frac{3}{4}Q^2 + \frac{1}{zF(h/z)} \int_0^{(M_Z-M_H)^2} ds\, \frac{C_1}{(s-z)^2} 2\,\mathcal{R}e\,\Delta_{\rm weak}\right) . \tag{4.28}$$

Note that the term proportional to Q^2, which represents the electromagnetic correction, is not enhanced by large logarithms of the form $\ln(s/m^2)$. This is a manifestation of the Kinoshita-Lee-Nauenberg theorem [145], which guarantees that such logarithms cancel when all $f\bar{f}(\gamma)$

configurations with invariant mass s are summed up. There are additional corrections due to strong-interaction effects if f represents a quark. They are similar as in the case of $Z \to q\bar{q}$. To first order they can be taken into account by including within the parentheses of Eqs. (4.27), (4.28) the term $\left(3C_F\alpha_s(s)/4\pi\right)$. We remark that the relevant QCD corrections are fully known in second order [65, 146] and partly in third order [69].

A heavy top quark is the only potential source of large corrections to the $Z \to f\bar{f}H$ decay width in the standard model. The leading high-m_t term may be gleaned from Eq. (4.11),

$$\Delta_{\rm weak} = -\frac{3G_F m_t^2}{8\pi^2\sqrt{2}}\left(\frac{1}{3} + \frac{c_{\rm w}}{s_{\rm w}}\,\frac{vQ}{v^2+a^2}\right), \tag{4.29}$$

which is negative for all flavours f. Equation (4.11) gives the contribution to $\Delta_{\rm weak}$ due to a doublet, (U, D), of fermions with $m_U, m_D \gg M_Z$. Obviously, the existence of such fermions would lead to an abnormal suppression of all $Z \to Hf\bar{f}$ modes and could, in fact, entail the non-observation of the Higgs boson in spite of its production being kinematically possible. In other words, the current lower bound on M_H can be relaxed to lower values if one assumes that the minimal standard model is extended by an additional family of high-mass fermions. Note that, in contrast to the departure of the ρ parameter from one, isospin breaking is not necessary for this mechanism to work. In fact, for given m, with $m \gg M_Z$, the effect would be more dramatic in the case $m_U = m_D = m$ than in the cases $m_D \ll m_U = m$ or $m_U \ll m_D = m$, irrespective of the final-state flavour f.

Figure 18a shows for the $Z \to \nu\bar{\nu}H$ mode the one-loop radiative corrections given by Eq. (4.28) as a function of M_H for several values of m_t. The effect is at the 1% level. The value of m_t being increased by constant steps, the distance from one curve to the next is proportional to the current value of m_t, which illustrates the dominant influence of the $G_F m_t^2$ term of Eq. (4.29). However, using just that term would lead to a poor description, since it is throughout negative and M_H independent. Figure 18b visualizes first-order radiative corrections to the $Z \to \ell^+\ell^- H$ decay rate, where $\ell = e$, μ, or τ. The strong enhancement towards the upper end of the kinematically allowed M_H range can be traced to those diagrams where the $\ell^+\ell^-$ pair is created from a virtual photon. These diagrams are not affected by the small-s behaviour of the Z-boson propagator, which leads to a suppression of the Born graph. This class of diagrams is, of course, absent in the case of $\nu\bar{\nu}H$ final states.

$\underline{H \to f\bar{f}Z \text{ decay}}$

In Sec. 3.1, we discussed the radiative corrections to the $H \to ZZ$ decay width for real Z bosons. This mode will have a significant branching fraction also below threshold, where one Z boson will be forced to be off-shell. By analogy to Eq. (3.29), the tree-level result for the $H \to f\bar{f}Z$ decay rate [147] emerges from Eq. (4.25) through multiplication by $-3M_Z^3/M_H^3$. The relative correction factor is given by Eq. (4.28) with the sign of the last term changed.

In Fig. 19, the full one-loop corrections to the $H \to f\bar{f}Z$ decay widths, where $f = \nu, \ell$, are shown as a function of M_H for $m_t = 150\,\mathrm{GeV}$. The spikes at $M_H = 2M_W$ are due to the usual threshold singularity in the Higgs-boson wave-function renormalization at one loop; see the discussion of Fig. 13a. Apart from the vicinity of this point, the corrections increase monotonically with M_H. In the $f = \ell$ case, the corrections take large negative values for $M_H \to M_Z$. Similarly to Fig. 19b, this is related to the s-channel pole of the photon propagator in the amplitude involving the $Z\gamma H$ coupling, which is absent in the $f = \nu$ case. However, due to a sign flip of $\hat{E}_{ZAH}(z, s, h)$, the overall effect acts in the opposite direction.

4.2 $f\bar{f}\gamma H$ COUPLING: $e^+e^- \to \gamma H$

In Sec. 3.2.2, we discussed the $Z \to \gamma H$ decay taking the Z boson to be on-shell. When M_H is sufficiently close to M_Z, the rate of this process exceeds the one of the Bjorken process, since the latter is more strongly affected by phase-space limitation. On top of the Z-boson resonance, the γH production cross section can be approximated by

$$\begin{aligned} \sigma(e^+e^- \to \gamma H) &= \frac{12\pi}{M_Z^2} \frac{\Gamma(Z \to e^+e^-)\Gamma(Z \to \gamma H)}{\Gamma_Z^2} \\ &\approx \left[\Gamma(Z \to \gamma H) \text{ in keV}\right] \times 23.9\,\mathrm{fb}\,, \end{aligned} \tag{4.30}$$

where $\Gamma_Z = 2.487\,\mathrm{GeV}$ and $\Gamma(Z \to e^+e^-)/\Gamma_Z = 3.345\%$ [57] have been used and $\Gamma(Z \to \gamma H)$ may be read off from Fig. 6b for given M_H. Note that the tree-level amplitude for this process is suppressed by a factor of m_e/M_Z and can be safely neglected [87].

Equation (4.30) corresponds to the square of the Z-boson exchange diagram of Fig. 15d with the external Z boson replaced by a photon. Also the photon-exchange diagram of Fig. 15d and the diagrams of Figs. 15e–l contribute to the one-loop amplitude. However, at $\sqrt{s} = M_Z$, their effect is expected to be suppressed by a factor of Γ_Z/M_Z. In fact, explicit calculation [87] reveals that the resonant term is about 500 times greater than the largest of the other terms. However, it is important to notice that, for $\sqrt{s} \neq M_Z$, the full set of diagrams must be included to obtain a gauge-invariant result [87].

Figure 20 shows the total cross section of $e^+e^- \to \gamma H$ in the one-loop approximation assuming $s_{\rm w}^2 = 0.21$, $M_W = 80.8\,\text{GeV}$, and $m_t = 40\,\text{GeV}$ [87]. As expected, the cross section rises sharply when $\sqrt{s}$ approaches M_Z. At $M_H = 60\,\text{GeV}$, the peak cross section is about 20 fb, which nicely agrees with the estimate by Eq. (4.30) in connection with Fig. 6b. It is clear that, a few GeV away from the peak, the cross section becomes too small for the reaction to be observed. ¿From Fig. 7 of Ref. 87 it may be seen that, for $m_t \gtrsim 80\,\text{GeV}$, the peak cross section is practically m_t independent and that its value at $m_t = 40\,\text{GeV}$, which is assumed in Fig. 20, almost coincides with its asymptotic value.

4.3 $q\bar{q}'W^\pm H$ AND $q\bar{q}ZH$ COUPLINGS: $pp \to W^\pm H + X$ AND $pp \to ZH + X$

The intermediate-mass Higgs boson will be difficult to detect at future hadron supercolliders. In the window $140\,\text{GeV} \lesssim M_H \lesssim 2M_Z$, it may be recognized by its $H \to (W^+)^*(W^-)^* \to \ell^-\bar{\nu}_\ell\,\ell'^+\nu_{\ell'}$ [34] and $H \to Z^*Z^* \to \ell^+\ell^-\ell'^+\ell'^-$ [32] decays. For $M_H \lesssim 140\,\text{GeV}$, the $H \to 2\gamma$ decay may yield an observable signal if excellent photon energy resolution and photon/jet discrimination are achieved [32, 33]. In contrast to gluon fusion, $W^\pm H$ associated production with subsequent photonic Higgs-boson and leptonic W-boson decays provides a nearly background-free signal even with modest photon energy resolution and photon/jet discrimination [117]. The same is true of ZH associated production, which, however, has a five-times smaller cross section and will be experimentally relevant only if very high luminosities can be achieved. $W^\pm H$ and ZH associated production proceed at tree-level through quark-antiquark annihilation [30, 31], $q\bar{q}' \to W^\pm H$ and $q\bar{q} \to ZH$.

An alternative mechanism of ZH associated production is provided by gluon fusion, $gg \to ZH$ [132, 133], to be discussed in the next subsection. Although this process is loop-induced, it nevertheless competes with the tree-level reaction, due to the absence of the s-channel suppression and the enhancement by the large gluon luminosity.

Due to the modest number of $W^\pm H$ and ZH signal events, it is mandatory to calculate the production cross sections as accurately as possible. The bulk of the radiative corrections is expected to be attributed to QCD effects in the initial state. Moreover, the lowest-order prediction suffers from the usual normalization uncertainty connected with the choice of the factorization and renormalization scales, which, as a rule, is reduced in the next-to-leading-order formalism.

For fixed VH invariant mass, the QCD corrections to the subprocesses $q\bar{q}' \to V^* \to VH$, where $V = W^\pm, Z$, are essentially the same as those to the Drell-Yan process [148], which are

known even in next-to-next-to-leading order [149]. In fact, the QCD-corrected cross section of $pp \to VH + X$, differential with respect to the VH invariant mass, q^2, factorizes as [129]

$$\frac{d\sigma}{dq^2}(pp \to VH + X) = \frac{G_F M_V^4}{2\pi^2\sqrt{2}} \frac{\sqrt{\lambda}}{q^2\left(q^2 - M_V^2\right)^2} \left(1 + \frac{\lambda}{12 M_V^2 q^2}\right) \sigma(pp \to V^* + X)\,, \qquad (4.31)$$

where $\lambda = \lambda\left(q^2, M_V^2, M_H^2\right)$ and it is understood that $\sigma(pp \to V^* + X)$ includes its well-known QCD corrections [148, 149]. The total cross section is obtained by integrating Eq. (4.31) over $(M_V + M_H)^2 \leq q^2 \leq s$, where $\sqrt{s}$ is the pp CM energy.

The cross section depends on the factorization scale, μ, both in the parton distribution functions and explicitly in the matrix element at $\mathcal{O}(\alpha_s)$. Since the explicit dependence is proportional to $\ln(\mu^2/q^2)$, the choice $\mu = \sqrt{q^2}$ is natural to avoid large logarithms. The cross section also depends weakly on the renormalization scale in the running coupling α_s. In the following, the factorization and renormalization scale will be identified. In Fig. 21, the μ dependence of the total cross section of $pp \to W^{\pm}H + X$ at the SSC is analyzed in leading and next-to-leading order in the deep-inelastic-scattering (DIS) scheme, assuming $M_H = 100\,\text{GeV}$ [129]. Set B1 of the Morfin-Tung [150] parton distribution functions in the DIS scheme is used. The results in the $\overline{\text{MS}}$ scheme are very similar. As expected, the scale dependence of the prediction is considerably reduced by the inclusion of QCD corrections. The residual variation is usually interpreted as the theoretical uncertainty due to higher-order corrections. For $\mu = \sqrt{q^2}$, the corrections amount to a 15% increase of the cross section. At the LHC, the effect is slightly larger, 17%.

Figure 22 shows versus M_H the total cross sections of $W^{\pm}H$ and ZH associated production at the SSC in next-to-leading order for three different sets of parton distribution functions and the choice $\mu = \sqrt{q^2}$ [129]. The contribution due to the $gg \to ZH$ subprocess is not included. Comparison of Figs. 21, 22 reveals that, at present, the uncertainty in the prediction due to the parton distribution functions is larger than the one due to higher orders. It turns out that the size of the corrections, relative to the lowest-order cross section, depend only weakly on the parton distribution functions and the value of M_H [129].

The above structure-function approach cannot be applied when transverse-momentum and rapidity distributions are to be evaluated. In particular, it does not allow one to impose acceptance cuts on the decay products of the V and Higgs bosons. In Refs. 130, 131, the completely differential cross section of $pp \to WH \to \ell\nu\gamma\gamma + X$ has been computed using Monte Carlo methods.

Figure 23 shows the transverse-momentum distribution of the Higgs boson in leading and next-to-leading order for $pp \to W^{\pm}H + X$ at the SSC assuming $M_H = 100\,\mathrm{GeV}$ [130]. It is summed over W^+, W^- and the $W \to e\nu$ and $W \to \mu\nu$ decays are included. We see that the overall correction is small and increases slowly with increasing p_T.

4.4 $ggZH$ COUPLING

The $ggZH$ coupling is generated in lowest order via the quark triangle and box diagrams depicted in Fig. 24. An analytic expression may be found in Ref. 135.

By charge-conjugation invariance, the Z boson couples only axially to the internal quarks, so that the contribution due to a mass-degenerate quark doublet vanishes. The contribution due to a light quark circulating in the box diagrams is suppressed by two powers of its mass: one power stems from its Yukawa coupling to the Higgs field and another one from the trace. There is no such suppression in the case of the triangle diagram. At the other extreme, there is a conspiracy between the triangle and box diagrams in the limit that the quark mass, m, is much larger than any other energy scale associated with the $ggZH$ interaction: the leading terms of both amplitudes are proportional to $1/m^2$ but they cancel each other leaving behind terms proportional to $1/m^4$. Consequently, the $ggZH$ interaction of the standard model is dominated by the bottom-quark triangle diagram whenever its characteristic energy is small against m_t.

$gg \to ZH$ subprocess

As anticipated in Sec. 4.3, the $gg \to ZH$ subprocess contributes appreciably to ZH associated production at hadron colliders. Formally, this subprocess is suppressed by a factor of $(\alpha_s/\pi)^2$ relative to the tree-level $q\bar{q} \to ZH$ subprocess. However, this suppression will be partly compensated by the overwhelming gluon luminosity of future supercolliders. Furthermore, for increasing $q\bar{q}$ invariant mass, $\hat{s}$, the cross section of $q\bar{q} \to ZH$ falls off like $1/\hat{s}$, due to the s-channel Z-boson exchange. As a consequence of the Landau-Yang theorem [151], which prohibits that two massless vector bosons constitute a spin-one state, this feature is absent in the $ggZH$ triangle amplitude; the box graphs do not involve virtual Z bosons anyway. It is well known that the triangle contribution to ZH associated production can, in fact, substantially exceed that from the usual light-quark sources, if the top quark is very heavy [152]. At first hand, one would expect the triangle and box diagrams to be essentially incoherent, since the former contributes only to the S wave, while the latter can contribute

to all partial waves. However, explicit calculation reveals that the interference between the triangle and box amplitudes is chiefly destructive [133].

Figure 25a disentangles the transverse-momentum distributions of a Higgs boson with $M_H = 100\,\mathrm{GeV}$ for $pp \to ZH + X$ at the SSC arising from $q\bar{q} \to ZH$ at tree level and from $gg \to ZH$ with $m_t = 80$, 140, and 200 GeV. Set 1 of the Duke-Owens [153] parton distribution functions with $\Lambda = 200\,\mathrm{GeV}$ is used. Taking into account just the triangle diagrams greatly overestimates the gg-fusion component, especially at high p_T, which nicely illustrates the destructive triangle-box interference. While $q\bar{q}$ annihilation is insensitive to m_t, gg fusion attains relative importance as m_t increases. However, $q\bar{q}$ annihilation is clearly dominant in the high-p_T range, $p_T \gtrsim 400\,\mathrm{GeV}$, independently of M_H. The only chance for gg fusion to be competitive is in the window $200\,\mathrm{GeV} \lesssim p_T \lesssim 400\,\mathrm{GeV}$ if m_t is close to 200 GeV and $M_H \lesssim 300\,\mathrm{GeV}$. The spikes in the p_T spectra of gg fusion are located at $p_T = \sqrt{\lambda\,(4m_t^2, M_Z^2, M_H^2)}/(4m_t)$, where $\hat{s}$ is just even sufficiently large to create a $t\bar{t}$ pair provided that no energy is deposited in the longitudinal motion of the ZH system. Figure 25b shows versus M_H the contributions due to $q\bar{q}$ annihilation and gg fusion to the total cross section of $pp \to ZH + X$ at the SSC. The m_t values are chosen as in Fig. 25. Roughly speaking, the corresponding predictions for the LHC may be obtained by scaling down the cross sections of Fig. 25b by an order of magnitude. The triangle contribution by itself leads to a poor description as to both size and line shape. The full gg-fusion contribution drops rapidly for increasing M_H until it enters a plateau at the threshold $M_H = 2m_t$. In conclusion, the support from gg fusion is comparable to that of $q\bar{q}$ annihilation only for very large m_t and for M_H around 200 GeV or beyond 800 GeV. In general, however, it exceeds the next-to-leading-order corrections to $q\bar{q}$ annihilation, discussed in Sec. 4.3.

$\underline{H \to ggZ \text{ decay}}$

At first sight, $H \to ggZ$ is a potentially interesting decay mode of the intermediate-mass Higgs boson. After fragmentation, it has the same signature as the $H \to q\bar{q}Z$ decay: one leptonically or hadronically decaying Z boson accompanied by two jets, the common invariant mass of which is below M_Z.

The $H \to ggZ$ decay rate can be computed straightforwardly from the results of Ref. 135 after appropriate analytic continuation [154]. In the approximation that the top quark is infinitely heavy while the other quarks are massless, the three-body phase-space integration

can be performed analytically and yields [154]

$$\Gamma(H \to ggZ) = \left(\frac{\alpha_s}{\pi}\right)^2 \frac{G_F^2 M_H^5}{1024\pi^3}(1 - 8r + 8r^3 - r^4 - 12r^2 \ln r)\,, \tag{4.32}$$

where $r = M_Z^2/M_H^2$.

It turns out that the branching fraction of $H \to ggZ$ is below 2×10^{-6}, *i.e.*, more than one order of magnitude smaller than the one of $H \to \mu^+\mu^-$.

$\underline{Z \to ggH \text{ decay}}$

The mechanism $Z \to ggH$ contributes to the $Z \to H + 2j$ signal and can, in principle, be used to look for a low-mass Higgs boson at LEP 1 and SLC [134]. However, it has been proven [135] that the $Z \to ggH$ decay has a branching fraction of less than 2×10^{-10} for $M_H > 60\,\text{GeV}$ and is, therefore, far to rare to be of practical relevance for Higgs hunting at existing Z factories. An approximation formula for the $Z \to ggH$ decay width follows from Eq. (4.32) by multiplication with $-\left(M_H^2/3M_Z^2\right)$; see Eq. (3.29).

4.5 $gggH$ AND $q\bar{q}gH$ COUPLINGS: $pp \to jH + X$

Higgs bosons created in hadron collisions via the $gg \to H$ subprocess, discussed in Sec. 3.4.2, will have almost zero transverse momentum. To obtain finite p_T, the Higgs boson must be produced in association with another particle or a jet. It has been argued that the production of an intermediate-mass Higgs boson together with a large-p_T jet may aid in reconstructing M_H [96].

In leading order, $pp \to jH + X$ receives contributions from the subprocesses $gg \to gH$, $qg \to qH$, and $q\bar{q} \to gH$ which proceed via massive-quark triangles; see Fig. 26. The cross sections of these subprocesses may be found in Ref. 96. They simplify considerably in the limit where the internal quark becomes very large. For one single flavour of very heavy quark, they read [96]

$$\begin{aligned}
\frac{d\sigma}{dt}(gg \to gH) &= \frac{1}{16\pi s^2}\,\frac{1}{4N_A^2}\,\frac{4\sqrt{2}}{9\pi} G_F \alpha_s^3 N_c N_A \frac{s^4 + t^4 + u^4 + h^4}{stu}\,, \\
\frac{d\sigma}{dt}(qg \to qH) &= \frac{1}{16\pi s^2}\,\frac{1}{4N_c N_A}\,\frac{2\sqrt{2}}{9\pi} G_F \alpha_s^3 N_A \frac{s^2 + u^2}{-t}\,, \\
\frac{d\sigma}{dt}(q\bar{q} \to gH) &= \frac{1}{16\pi s^2}\,\frac{1}{4N_c^2}\,\frac{2\sqrt{2}}{9\pi} G_F \alpha_s^3 N_A \frac{t^2 + u^2}{s}\,,
\end{aligned} \tag{4.33}$$

where $N_c = 3$, $N_A = N_c^2 - 1 = 8$, and the Mandelstam variables, s, t, and u, are defined canonically. The fact that Eq. (4.33) does not depend on the mass, m, of the loop quark may

be understood as follows: There is one power of m from the Yukawa coupling and a second from the trace over the internal quark loop. These are cancelled by a factor of $1/m^2$ from the loop integral. The latter is absent in the case of light quarks, with $m \ll M_H$, so that their contribution is suppressed by m^2/M_H^2. Consequently, only the top-quark contribution is relevant for jH associated production at hadron colliders.

Figure 27 shows the $d^2\sigma/dy\,dp_T$ distributions of Higgs bosons with $M_H = 110$ and 160 GeV from $pp \to jH + X$ at the SSC as a function of p_T at $y = 0$ for $m_t = 160\,\mathrm{GeV}$ [96]. Set 2 of the parton distribution functions of Eichten et al. [31] is used with $\Lambda = 290\,\mathrm{MeV}$ and the factorization and renormalization scales are identified with the Higgs-boson transverse mass, $\sqrt{M_H^2 + p_T^2}$. The pointlike approximation by Eq. (4.33) is also visualized. It is satisfactory at intermediate values of p_T but seriously overestimates the true rate at large p_T. For smaller values of m_t, the p_T spectrum looks similar in the low-p_T range but falls off more steeply at high p_T [96]. At the LHC, the cross sections will be generally smaller by a factor of 3–5, depending on the value of p_T [96].

The $gg \to ggH$ subprocess contributes in $\mathcal{O}\left(\alpha_s^4\right)$ to Higgs-boson-plus-two-jet production and has been investigated in the infinite-m_t approximation by means of an effective Lagrangian [97].

As is evident from Fig. 27, the lowest-order evaluation of the $pp \to jH + X$ differential cross section diverges as p_T tends to zero. This can already be verified on the basis of the ultraheavy-quark approximation by Eq. (4.33). In fact, using $p_T^2 = tu/s$ and $s+t+u = h$, one sees that the cross sections of $gg \to gH$ and $qg \to qH$ diverge in this limit. To obtain a reliable prediction in the low-p_T region, it is necessary to take into account multiple-gluon emission, which generates terms of the orders $\alpha_s^n \ln^m\left(M_H^2/p_T^2\right)/p_T^2$, with $m \le 2n-3$. The largest of these terms, those in the double leading-logarithmic approximation, with $m = 2n-3$, can be resummed to all orders. This was originally done in the $m_t \to \infty$ limit normalizing the p_T spectrum to the lowest-order cross section of $pp \to H + X$ [98]. Subsequently, this analysis was generalized to arbitrary m_t values [99]. Knowing the total cross section of $pp \to H + X$ to next-to-leading order in the $m_t \to \infty$ limit [94, 102], one can adjust the normalization of the p_T spectrum perturbatively [100]. Similar results can be obtained by means of a rigorous renormalization-group-invariant approach [101]. The resulting predictions for the SSC are shown in Fig. 28 for $y = 0$, $M_H = 100$, 150, and 200 GeV [100]. The parton distribution functions by Harriman et al. [155] are used with $\Lambda = 190\,\mathrm{MeV}$ for five quark flavours and the factorization and renormalization scales are set equal to M_H.

The Feynman diagrams of Fig. 26 also contribute in next-to-leading order to the total cross section of $pp \to H + X$ and to the hadronic decay width of the Higgs boson; see Sec. 3.4.

5. Five-point functions: intermediate-boson fusion and $e^+e^- \to \mu^+\mu^- H$

This chapter is devoted to loop effects in connection with five-point functions with one external Higgs-boson line. Perhaps, the couplings of the Higgs boson to four fermions via two intermediate bosons play the most important rôle in this context. They mediate, for example, intermediate-boson fusion, which is the dominant Higgs-boson production mechanism at e^+e^- [13, 14] and hadron [14, 28] colliders for high M_H and at ep [14, 35] colliders for all values of M_H. Furthermore, the production of a Higgs boson with $M_H > M_Z$ via the bremsstrahlung process in e^+e^- annihilation below the ZH threshold, *i.e.*, when $M_Z < M_H < \sqrt{s} < M_Z + M_H$, makes use of a kinematically irreducible five-point vertex. Also four-fermion decays of low-mass Higgs bosons proceed through such vertices [154, 156].

The computation of electroweak corrections to genuine five-point processes is rather involved and has not been tackled yet. As for QCD corrections, the problem is considerably facilitated by the fact that the coloured part of the process consist at most of two isolated quark lines. The diagrams where one gluon is exchanged between these lines do not interfere with the tree diagram after colour summation. Thus, one just needs to renormalize each quark-intermediate-boson vertex separately. The use of the structure-function approach provides a further simplification in the case of inclusive cross sections [157]. This strategy was adopted in Refs. 157, 158 for W^+W^- and ZZ fusion at pp colliders and in Ref. 159 for W^+W^- fusion at ep colliders. In the latter reference also a Monte Carlo technique was elaborated, which allows one to include selection cuts. In the case of four-fermion decays of the Higgs boson, QCD corrections may be accommodated by simply including a factor $\left(1 + \alpha_s(s)/\pi\right)$ for each quark pair, where $\sqrt{s}$ is the respective quark-pair invariant mass.

The mechanism that prohibits one-gluon exchanges between the two fermion lines is not in effect in the case of photons. Interference effects between the photon radiation off the initial and final states of $e^+e^- \to \mu^+\mu^- H$ have been computed in Ref. 160.

5.1 QCD CORRECTIONS TO INTERMEDIATE-BOSON FUSION IN pp AND ep COLLISIONS

The canonical method of calculating the inelastic contribution to intermediate-boson scattering in hadron or ep collisions is to convolute the parton-level cross sections with the usual quark distribution functions [14, 28, 35]. An alternative formulation [157] makes use of the hadronic tensor, $W_{\mu\nu}$, that characterizes the emission of an intermediate boson off the proton. $W_{\mu\nu}$ may be expressed in terms of three Lorentz scalars. The standard decomposition reads

$$MW_{\mu\nu}(x,Q^2) = F_1(x,Q^2)\left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right)+\frac{F_2(x,Q^2)}{P\cdot q}\left(P_\mu-\frac{P\cdot q}{q^2}q_\mu\right)\left(P_\nu-\frac{P\cdot q}{q^2}q_\nu\right) - i\frac{F_3(x,Q^2)}{2P\cdot q}\epsilon_{\mu\nu\rho\sigma}P^\rho q^\sigma\,, \tag{5.1}$$

where M is the proton mass, P is the proton four-momentum, q is the intermediate-boson four-momentum, $Q^2 = -q^2$, $x = (Q^2/2P\cdot q)$, and $F_i(x,Q^2)$ $(i = 1,2,3)$ are the so-called hadronic structure functions. The canonical results are recovered as the quark-parton-model limit of this structure-function approach. However, this approach offers the advantages that it readily carries over to both elastic-scattering configurations and next-to-leading order.

The elastic-scattering contribution is concentrated at very low Q^2 and can be eliminated by imposing a lower cut on the Higgs-boson transverse momentum. In principle, though, it enters the total cross section. In order to evaluate it, one just needs to substitute $F_i(x,Q^2)$ in Eq. (5.1) by the analogous expressions in terms of the three elastic form factors, $G_e(Q^2)$, $G_m(Q^2)$, and $G_a(Q^2)$, of the proton-intermediate-boson vertex [157]. Since these form factors effectively cut off the phase-space integral at $Q^2 \approx M^2$, the contribution per elastic vertex is suppressed by a factor of $M^2/M_V^2 \approx 10^{-4}$ relative to the inelastic case. This estimate has been verified numerically for the single- and double-elastic components of W^+W^- fusion in pp collisions [157].

As explained at the outset of this chapter, the factorization implicit in the structure-function approach continues to work for intermediate-boson fusion at hadron and ep colliders in next-to-leading order of QCD. The full $\mathcal{O}(\alpha_s)$ corrections may be incorporated in the cross sections by simply employing the expressions for the $F_i(x,Q^2)$ functions corrected up to that order [148].

Figure 29 shows versus M_H the total cross sections of $pp \to H + X$ via W^+W^- and ZZ fusion at the SSC through $\mathcal{O}(\alpha_s)$ [158]. Sets SL and S1-DIS of the Morfin-Tung [150]

parton distribution functions are used in leading and next-to-leading order, respectively, with $Q^2 = M_V^2$ $(V = W, Z)$. Inclusion of the QCD corrections leads to an increase by $(12, 6)\%$ at $M_H = (100, 800)\,\mathrm{GeV}$. The corresponding numbers for the LHC are $(8, 6)\%$ [158]. Most of the increase is due to the parton distribution functions and not the explicit $\mathcal{O}(\alpha_s)$ corrections to the structure functions, since the total cross section is dominated by $F_2(x, Q^2)$ [157], which receives no QCD corrections in the DIS scheme.

First-order QCD corrections to the total cross section of $e^- p \to \nu_e H + X$ via W^+W^- fusion at LEP$\otimes$LHC, operating at $\sqrt{s} = 1360\,\mathrm{GeV}$, were found to be smaller than 1% in the range $80\,\mathrm{GeV} < M_H < 150\,\mathrm{GeV}$ [159].

5.2 QED corrections to $e^+e^- \to \mu^+\mu^- H$

Analogously to $e^+e^- \to \mu^+\mu^-$ [161, 162], first-order QED corrections to the cross section of $e^+e^- \to \mu^+\mu^- H$ may be decomposed into three groups, which are separately finite and gauge invariant: initial-state radiation, final-state radiation, and their interference. While the initial- and final-state corrections are similar for both processes, the evaluation of the interference radiation is rather involved in the latter case due to the occurrence of genuine five-point loop integrals.

The virtual and soft parts of this problem have been computed recently [160]. The effect due to photons whose energy does not exceed 10% of the minimal dimuon energy was found to be smaller than one tenth of a percent for $M_Z + M_H < \sqrt{s} \leq 250\,\mathrm{GeV}$ and $M_H \lesssim M_W$. The smallness of this effect could be traced to an accidental cancellation which is maximal when the $\ell^+\ell^- Z$ vector coupling vanishes.

6. Conclusions

In the previous chapters, we have tried to compile the presently available information on quantum effects related to the Higgs boson in the standard model. At this point, we should take a look back and assess the significance of these effects.

As a rule, the size of radiative corrections to a given process is determined by the discrepancy between the various energy and mass scales involved in that process. As for weak corrections, the central scale is connected with the intermediate bosons. In general, weak corrections are minimized when the Born formulae are expressed in terms of the Fermi constant. Then large logarithms due to loops with light charged fermions are automatically resummed

and ambiguities in the definition of the light-quark masses and low-q^2 hadronic effects are inconsequential. However, there remain large corrections of $\mathcal{O}\left(G_F m_t^2\right)$ due to virtual top quarks. The effect of a heavy Higgs boson on oblique corrections is screened. Its tree-level production and decay processes receive corrections of $\mathcal{O}\left(G_F M_H^2\right)$ with positive but incidentally small coefficients. It is interesting to observe that the high-m_t and high-M_H effects tend to compensate each other in all cases considered, except for the fermionic Higgs-boson decays.

Higgs-boson production in e^+e^- collisions is unavoidably accompanied by electromagnetic radiation, mainly off the initial state. Initial-state corrections are less interesting but usually much larger than weak loop effects and it is important to have them well under control. In the case of e^+e^- annihilation, initial-state corrections are known to second order in QED and it is well understood how the infrared-sensitive parts are to be exponentiated. The pattern in which the leading logarithms arrange themselves is universal for all reactions with an e^+e^- initial state. In addition, there will be beamstrahlung at the NLC. However, this unwelcome effect can be reduced to an almost unnoticeable level by optimizing machine design and operation.

There are substantial QCD effects in the case of the hadronic decays of the Higgs boson and its production by gluon fusion in hadron collisions. In the case of the quarkonic decays, the bulk of the corrections is related to the running of the final-state quark mass and can be resummed. Most importantly, the rate of $H \to b\bar{b}$, which is the preferred decay mode for $M_H \lesssim 135\,\mathrm{GeV}$, is reduced by 50–60% as compared to the tree-level result evaluated with the pole mass. In turn, the branching fractions of the non-quarkonic decay modes are enhanced correspondingly in that window. As for the $H \to gg$ decay, there is an additional enhancement by more than 60% by specific QCD corrections. At the same time, the rate of the crossed process, gluon fusion, which will be the main source of Higgs bosons with $M_H \lesssim 700\,\mathrm{GeV}$ at future hadron supercolliders, is increased by a factor of 1.5–1.7. Furthermore, the dependence of the production cross section on the factorization and renormalization scales is considerably reduced by the inclusion of the QCD corrections, which renders the prediction more reliable.

The program of computing radiative corrections in Higgs-boson physics is not yet completed. So far, only little is known about electroweak corrections to five-point processes such as intermediate-boson fusion. Also, one should be alarmed by the size of the leading QCD corrections to gluon fusion and proceed to next-to-leading order.

It is almost unavoidable that some relevant references have not been cited. The reader

should be reassured that this has happened unintentionally and is entirely due to the author's ignorance.

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APPENDIX A: One-loop Integrals

In this paper we adopt the conventions of Refs. 161, 163 on dimensional regularization, except that we use the Minkowskian metric, $g^{\mu\nu} = \mathrm{diag}(1,-1,\ldots,-1)$. We define the one-loop integrals of interest by

$$\begin{aligned}
&A(m^2) = -\int \frac{d^nq}{i\pi^2}\,\frac{1}{q^2-m^2+i\epsilon}\,,\\
&\{B_0;B_\mu;B_{\mu\nu}\}\,(p^2,m_1^2,m_2^2) = \int \frac{d^nq}{i\pi^2}\,\frac{\{1;q_\mu;q_\mu q_\nu\}}{(q^2-m_1^2+i\epsilon)\,[(q+p)^2-m_2^2+i\epsilon]}\,,\\
&\{C_0;C_\mu;C_{\mu\nu}\}\,(p^2,k^2,(p+k)^2,m_1^2,m_2^2,m_3^2)\\
&\quad = -\int \frac{d^nq}{i\pi^2}\,\frac{\{1;q_\mu;q_\mu q_\nu\}}{(q^2-m_1^2+i\epsilon)\,[(q+p)^2-m_2^2+i\epsilon]\,[(q+p+k)^2-m_3^2+i\epsilon]}\,,\\
&D_0(p^2,k^2,l^2,(p+k+l)^2,(p+k)^2,(k+l)^2,m_1^2,m_2^2,m_3^2,m_4^2)\\
&\quad = \int \frac{d^nq}{i\pi^2}\,\left\{\left(q^2-m_1^2+i\epsilon\right)\left[(q+p)^2-m_2^2+i\epsilon\right]\left[(q+p+k)^2-m_3^2+i\epsilon\right]\right.\\
&\qquad \left.\times\left[(q+p+k+l)^2-m_4^2+i\epsilon\right]\right\}^{-1}\,,
\end{aligned} \tag{A.1}$$

in conjunction with the decompositions

$$\begin{aligned}
B_\mu &= p_\mu B_1\,,\\
B_{\mu\nu} &= p_\mu p_\nu B_{21} - g_{\mu\nu}B_{22}\,,\\
C_\mu &= p_\mu C_{11} + k_\mu C_{12}\,,\\
C_{\mu\nu} &= p_\mu p_\nu C_{21} + k_\mu k_\nu C_{22} + (p_\mu k_\nu + k_\mu p_\nu)C_{23} - g_{\mu\nu}C_{24}\,.
\end{aligned} \tag{A.2}$$

Here n represents the dimensionality of space time. Note that we set the 't Hooft mass equal to unity.

In the following, we send $n \to 4$ and neglect terms proportional to $(n-4)$. In this limit the tadpole function can be written as

$$A(\alpha) = -\alpha\big(B_0(0,\alpha,\alpha)+1\big)\,. \tag{A.3}$$

The scalar two-point function is ultraviolet divergent and reads [51]

$$\begin{aligned}
B_0(a,\alpha,\beta) = \Delta - \frac{1}{2}\ln(\alpha\beta) + 2 + \frac{1}{a}\left(-\frac{\alpha-\beta}{2}\ln\frac{\alpha}{\beta}\right.\\
\left. + \sqrt{\lambda(a+i\epsilon,\alpha,\beta)}\,\mathrm{arcosh}\,\frac{\alpha+\beta-a-i\epsilon}{2\sqrt{\alpha\beta}}\right)\,,
\end{aligned} \tag{A.4}$$

where $\Delta = 2/(4-n) - C - \ln\pi$, with C being the Euler-Mascheroni constant, regulates the divergence. Note that $n\Delta = \Delta - 2$. When Eq. (A.4) is inserted in expressions for renormalized

quantities, Δ drops out and the mass arguments of the logarithms accompanying Δ form dimensionless combinations. Here and in the following, $\lambda(x,y,z) = x^2 + y^2 + z^2 - 2(xy + yz + zx)$.

For positive $\lambda = \lambda(a,b,c)$, the scalar three-point function is given by [51]

$$C_0(a,b,c,\alpha,\beta,\gamma) = \frac{1}{\sqrt{\lambda}} \sum_{i=1}^{3} \sum_{\sigma=\pm} \left(\mathrm{Li}_2\left(\frac{x_i}{x_i - z_i^\sigma}\right) - \mathrm{Li}_2\left(\frac{x_i - 1}{x_i - z_i^\sigma}\right)\right), \tag{A.5}$$

where Li_2 denotes the dilogarithm [164] and

$$\begin{aligned} x_1 &= \frac{1}{2}\left(1 + \frac{a-b-c}{\sqrt{\lambda}}\right) + \frac{1}{2a}\left[-\alpha\left(1 + \frac{a+b-c}{\sqrt{\lambda}}\right) + \beta\left(1 - \frac{a-b+c}{\sqrt{\lambda}}\right)\right] + \frac{\gamma}{\sqrt{\lambda}}, \\ z_1^\pm &= \frac{1}{2}\left(1 - \frac{\alpha-\beta}{a} \pm \sqrt{\left(1 - \frac{\alpha-\beta}{a}\right)^2 - \frac{4}{a}(\beta - i\epsilon)}\right), \quad \text{et cycl.} \end{aligned} \tag{A.6}$$

The solution for negative λ may be obtained by substituting $\sqrt{\lambda} \to i\sqrt{-\lambda}$.

The coefficients of Eq. (A.2) can be expressed in terms of the B_0 and C_0 functions by means of the reduction algorithm of Ref. 163. In the case of the two-point function one obtains

$$\begin{aligned} B_1(a,\alpha,\beta) &= \frac{1}{2a}\left[-(a+\alpha-\beta)B_0(a,\alpha,\beta) + \alpha B_0(0,\alpha,\alpha) - \beta B_0(0,\beta,\beta) + \alpha - \beta\right], \\ B_{21}(a,\alpha,\beta) &= \frac{1}{3a^2}\left[\big(3a\alpha + \lambda(a,\alpha,\beta)\big)B_0(a,\alpha,\beta) - \alpha(a+\alpha-\beta)B_0(0,\alpha,\alpha)\right. \\ &\qquad \left. + \beta(2a+\alpha-\beta)B_0(0,\beta,\beta) + \frac{a^2}{6} - \frac{3}{2}a(\alpha-\beta) - (\alpha-\beta)^2\right], \\ B_{22}(a,\alpha,\beta) &= \frac{1}{12a}\left[\lambda(a,\alpha,\beta)B_0(a,\alpha,\beta) - \alpha(a+\alpha-\beta)B_0(0,\alpha,\alpha)\right. \\ &\qquad \left. - \beta(a-\alpha+\beta)B_0(0,\beta,\beta) + \frac{2}{3}a^2 - 3a(\alpha+\beta) - (\alpha-\beta)^2\right]. \end{aligned} \tag{A.7}$$

The three-point form factors with argument $(a,b,c,\alpha,\beta,\gamma)$ are found to be

$$\begin{aligned} C_{11} &= \frac{1}{\lambda}\left[(a+b-c)\Delta_1 - 2b\Delta_2 + (Y-\lambda)C_0\right], \\ C_{12} &= \frac{1}{\lambda}\left[2a\Delta_1 - (a+b-c)\Delta_2 - XC_0\right], \\ C_{21} &= \frac{1}{\lambda}\left\{-b - \frac{1}{2a}(a+b-c)(\alpha-\beta)\delta_1 + \frac{1}{2c}(a-b-c)(\alpha-\gamma)\delta_3\right. \end{aligned}$$

$$
\begin{aligned}
&+\left[\beta-\gamma-\frac{1}{2}(3a+b-3c)-\frac{3}{\lambda}bX\right]\Delta_1+b\left(4-\frac{3}{\lambda}Y\right)\Delta_2\\
&+\left[(\beta-\gamma-a+c)^2-4\alpha b+\frac{6}{\lambda}bZ\right]C_0\Bigg\}\,,\\
C_{22}=\frac{1}{\lambda}\Bigg\{&-a+\frac{1}{2b}(a+b-c)(\beta-\gamma)\delta_2+\frac{1}{2c}(a-b+c)(\alpha-\gamma)\delta_3-\frac{3}{\lambda}aX\Delta_1\\
&+\left[-\alpha+\beta+\frac{1}{2}(3a+b-c)-\frac{3}{\lambda}aY\right]\Delta_2+\left(\lambda(\alpha,\beta,a)+\frac{6}{\lambda}aZ\right)C_0\Bigg\}\,,\\
C_{23}=\frac{1}{\lambda}\Bigg\{&-\frac{1}{2}(a+b-c)-(\alpha-\beta)\delta_1+\frac{1}{2c}(a-b+c)(\alpha-\gamma)\delta_3\\
&+\left[\frac{1}{2}(3\alpha-3\beta-5a)+\frac{3}{\lambda}aY\right]\Delta_1+\left[-\frac{1}{2}(\beta-\gamma-b)+a-c+\frac{3}{\lambda}bX\right]\Delta_2\\
&+\left[(a+b-c)\left(\frac{3}{\lambda}Z-\alpha-\beta+a\right)-(\alpha-\beta+a)(\beta-\gamma+b)\right]C_0\Bigg\}\,,\\
C_{24}=\frac{1}{4}&\left[1+B_0(c,\alpha,\gamma)+\frac{1}{\lambda}(X\Delta_1+Y\Delta_2-2ZC_0)\right]\,,
\end{aligned}
\tag{A.8}
$$

where

$$
\begin{aligned}
\lambda&=\lambda(a,b,c)\,,\\
X&=\alpha(-a-b+c)+\beta(-a+b-c)+2\gamma a+a(a-b-c)\,,\\
Y&=2\alpha b+\beta(a-b-c)+\gamma(-a-b+c)+b(-a+b-c)\,,\\
Z&=\alpha^2b+\beta^2c+\gamma^2a+(\alpha\beta+\gamma a)(a-b-c)+(\beta\gamma+\alpha b)(-a+b-c)\\
&\quad+(\alpha\gamma+\beta c)(-a-b+c)+abc\,,\\
\delta_1&=B_0(a,\alpha,\beta)-B_0(0,\alpha,\beta)\,,\\
\delta_2&=B_0(b,\beta,\gamma)-B_0(0,\beta,\gamma)\,,\\
\delta_3&=B_0(c,\alpha,\gamma)-B_0(0,\alpha,\gamma)\,,\\
\Delta_1&=B_0(a,\alpha,\beta)-B_0(c,\alpha,\gamma)\,,\\
\Delta_2&=B_0(b,\beta,\gamma)-B_0(c,\alpha,\gamma)\,,\\
C_0&=C_0(a,b,c,\alpha,\beta,\gamma)\,.
\end{aligned}
\tag{A.9}
$$

The four-point functions occurring in Eq. (4.9) are given by

$$
\begin{aligned}
&D_0(0,z,0,h,t,u,w,0,0,w)\\
&=\frac{1}{r}\left[\ln(-z-i\varepsilon)\ln\left(1-\frac{1}{x_+}\right)-I(u,w,x_+)+I(t,w,1-x_+)+J(h,w,x_+)-(x_+\leftrightarrow x_-)\right]\,,
\end{aligned}
\tag{A.10}
$$

with

$$\begin{aligned} r &= \sqrt{n^2 + 2wn(s+z-h) + w^2\lambda - i\varepsilon 4sn}\,, \\ x_\pm &= \frac{1}{2}\left(1 + \frac{w(t-u) \pm r}{n}\right)\,, \end{aligned} \tag{A.11}$$

and further

$$\begin{aligned} &D_0(0,0,z,h,s,u,w,0,w,w) \\ &= \frac{1}{r}\left\{\ln w \ln\left(1-\frac{1}{x_+}\right) - J(s,w,x_+) + I\big(u,w,(1-\alpha)(1-x_+)\big) - J\big(z,w,(1-\alpha)(1-x_+)\big)\right. \\ &\quad - I\left(u,w,\left(1-\frac{1}{\alpha}\right)x_+\right) + J\left(h,w,\left(1-\frac{1}{\alpha}\right)x_+\right) \\ &\quad \left. + i\pi\theta(u-w)\left[\ln\big(1-(1-\alpha)(1-x_+)\big) - \ln\left(1-\left(1-\frac{1}{\alpha}\right)x_+\right)\right] - (x_+ \leftrightarrow x_-)\right\}\,, \end{aligned} \tag{A.12}$$

with

$$\begin{aligned} r &= \sqrt{s^2u^2 + 2wsu(t-u) + w^2\lambda + i\varepsilon 4su^2}\,, \\ x_\pm &= \frac{1}{2}\left(1 + \frac{w\sqrt{\lambda} \pm r}{su}\right)\,, \\ \alpha &= \frac{-s+z+h-\sqrt{\lambda}}{2z}\,. \end{aligned} \tag{A.13}$$

Here we have employed two auxiliary integrals, viz.

$$\begin{aligned} I(a,b,c\pm i\varepsilon) &= \int_0^1 \frac{dx}{x-(c\pm i\varepsilon)} \ln(-ax+b-i\varepsilon) \\ &= \ln(-ac+b-i\varepsilon)\ln\left(1-\frac{1}{c\pm i\varepsilon}\right) + \mathrm{Li}_2\left(\frac{-ac}{-ac+b-i\varepsilon}\right) \\ &\quad - \mathrm{Li}_2\left(\frac{a(1-c)}{-ac+b-i\varepsilon}\right)\,, \end{aligned} \tag{A.14}$$

$$\begin{aligned} J(a,b,c\pm i\varepsilon) &= \int_0^1 \frac{dx}{x-(c\pm i\varepsilon)} \ln\big[-ax(1-x)+b-i\varepsilon\big] \\ &= \ln\big[-ac(1-c)+b-i\varepsilon\big]\ln\left(1-\frac{1}{c\pm i\varepsilon}\right) + \mathrm{Li}_2\left(\frac{c}{c-a_+}\right) - \mathrm{Li}_2\left(\frac{c-1}{c-a_+}\right) \\ &\quad - \mathrm{Li}_2\left(\frac{1-c}{1-c-a_+}\right) + \mathrm{Li}_2\left(\frac{-c}{1-c-a_+}\right)\,, \end{aligned} \tag{A.15}$$

where

$$a_+ = \frac{1}{2}\left(1 + \sqrt{1 - \frac{4}{a}(b - i\varepsilon)}\right) . \tag{A.16}$$

Note that $D_0(0,z,0,h,t,u,w,0,0,w) = D_0(0,z,0,h,u,t,w,0,0,w)$.

APPENDIX B: Boson Self-energies

In the one-loop approximation, the boson vacuum polarization functions each decompose into a fermionic and a bosonic part,

$$\Pi^i(s) = \Pi^i_{\rm fer}(s) + \Pi^i_{\rm bos}(s) \qquad (i = WW, ZZ, ZA, AA, HH) . \tag{B.1}$$

Referring to the nomenclature introduced in Secs. 2 and 3.1, these read in detail

$$\begin{aligned}
\Pi^{WW}_{\rm fer}(s) = \frac{\alpha}{4\pi}\frac{1}{3s_{\rm w}^2}\sum_{\rm doublets} N_f \Bigg\{ & \left[s - \frac{m_+^2 + m_-^2}{2} - \frac{\left(m_+^2 - m_-^2\right)^2}{2s} \right] B_0\left(s, m_+^2, m_-^2\right) \\
& + m_+^2\left(-1 + \frac{m_+^2 - m_-^2}{2s}\right) B_0\left(0, m_+^2, m_+^2\right) \\
& + m_-^2\left(-1 + \frac{m_-^2 - m_+^2}{2s}\right) B_0\left(0, m_-^2, m_-^2\right) - \frac{s}{3} + \frac{\left(m_+^2 - m_-^2\right)^2}{2s} \Bigg\} , \\
\Pi^{WW}_{\rm bos}(s) = \frac{\alpha}{4\pi}\frac{1}{3s_{\rm w}^2} \Bigg\{ & \left[\left(-10c_{\rm w}^2 + \frac{1}{4}\right) s + \left(-4c_{\rm w}^2 - \frac{27}{2} + \frac{5}{2c_{\rm w}^2}\right) w + s_{\rm w}^4\left(2c_{\rm w}^2 + \frac{1}{4}\right)\frac{z^2}{s} \right] \\
& \times B_0(s,w,z) + 2s_{\rm w}^2\left(-5s - 2w + \frac{w^2}{s}\right) B_0(s,w,0) \\
& + \left(3w + \frac{\lambda(s,w,h)}{4s}\right) B_0(s,w,h) + w\left(5 - \frac{2w - z - h}{4s}\right) B_0(0,w,w) \\
& + \left(4c_{\rm w}^2 + \frac{1}{2}\right) z\left(1 - \frac{s_{\rm w}^2 z}{2s}\right) B_0(0,z,z) + \frac{h}{2}\left(1 + \frac{w-h}{2s}\right) B_0(0,h,h) \\
& - \frac{s}{3} - \frac{s_{\rm w}^2\left(7c_{\rm w}^2 + 1\right) z^2 + (w-h)^2}{4s} \Bigg\} , \\
\Pi^{ZZ}_{\rm fer}(s) = \frac{\alpha}{4\pi}\frac{4}{3}\sum_f N_f \Big\{ & \left[v_f^2\left(s + 2m_f^2\right) + a_f^2\left(s - 4m_f^2\right) \right] B_0\left(s, m_f^2, m_f^2\right) \\
& - \left(v_f^2 + a_f^2\right)\left(2m_f^2 B_0\left(0, m_f^2, m_f^2\right) + \frac{s}{3}\right) \Big\} , \\
\Pi^{ZZ}_{\rm bos}(s) = \frac{\alpha}{4\pi}\frac{1}{3c_{\rm w}^2 s_{\rm w}^2} \Bigg\{ & \left[\left(-9c_{\rm w}^4 - c_{\rm w}^2 + \frac{1}{4}\right) s + \left(-12c_{\rm w}^4 - 8c_{\rm w}^2 + 5\right) w \right] B_0(s,w,w) \\
& + \left(3z + \frac{\lambda(s,z,h)}{4s}\right) B_0(s,z,h) + \left(12c_{\rm w}^4 - 4c_{\rm w}^2 + 1\right) w B_0(0,w,w)
\end{aligned}$$

$$
\begin{aligned}
&+\frac{z}{2}\left(1-\frac{z-h}{2s}\right)B_0(0,z,z)+\frac{h}{2}\left(1+\frac{z-h}{2s}\right)B_0(0,h,h)\\
&-\frac{2c_{\rm w}^2-1}{3}s-\frac{(z-h)^2}{4s}\Bigg\}\,,\\
\Pi_{\rm fer}^{ZA}(s)&=\frac{\alpha}{4\pi}\frac{4}{3}\sum_f N_f v_f Q_f\left[-\left(s+2m_f^2\right)B_0\left(s,m_f^2,m_f^2\right)+2m_f^2 B_0\left(0,m_f^2,m_f^2\right)+\frac{s}{3}\right]\,,\\
\Pi_{\rm bos}^{ZA}(s)&=\frac{\alpha}{4\pi}\frac{1}{3c_{\rm w}s_{\rm w}}\Bigg\{\left[\left(9c_{\rm w}^2+\frac{1}{2}\right)s+4\left(3c_{\rm w}^2+1\right)w\right]B_0(s,w,w)\\
&-2\left(6c_{\rm w}^2-1\right)wB_0(0,w,w)+\frac{s}{3}\Bigg\}\,,\\
\Pi_{\rm fer}^{AA}(s)&=\frac{\alpha}{4\pi}\frac{4}{3}\sum_f N_f Q_f^2\left[\left(s+2m_f^2\right)B_0\left(s,m_f^2,m_f^2\right)-2m_f^2B_0\left(0,m_f^2,m_f^2\right)-\frac{s}{3}\right]\,,\\
\Pi_{\rm bos}^{AA}(s)&=\frac{\alpha}{4\pi}\big[-(3s+4w)B_0(s,w,w)+4wB_0(0,w,w)\big]\,,\\
\Pi_{\rm fer}^{HH}(s)&=\frac{\alpha}{4\pi}\frac{1}{s_{\rm w}^2w}\sum_f N_f m_f^2\left[\left(\frac{s}{2}-2m_f^2\right)B_0\left(s,m_f^2,m_f^2\right)-m_f^2\Big(B_0\left(0,m_f^2,m_f^2\right)+1\Big)\right]\,,\\
\Pi_{\rm bos}^{HH}(s)&=\frac{\alpha}{4\pi}\frac{1}{s_{\rm w}^2w}\left[\left(-ws+3w^2+\frac{h^2}{4}\right)B_0(s,w,w)+\frac{1}{2}\left(-zs+3z^2+\frac{h^2}{4}\right)B_0(s,z,z)\right.\\
&+\frac{9}{8}h^2B_0(s,h,h)+\frac{w}{2}\left(3w+\frac{h}{2}\right)B_0(0,w,w)+\frac{z}{4}\left(3z+\frac{h}{2}\right)B_0(0,z,z)\\
&\left.+\frac{3}{8}h^2B_0(0,h,h)+\frac{w}{2}\left(-3w+\frac{h}{2}\right)+\frac{z}{4}\left(-3z+\frac{h}{2}\right)+\frac{3}{8}h^2\right]\,.
\end{aligned}
\tag{B.2}
$$

Here we have dropped the tadpole contributions, since they cancel among themselves when physical observables are calculated.

APPENDIX C: Fermion Self-energy

The fermion self-energy at momentum transfer q, $\Sigma(q)$, has the following decomposition:

$$
\Sigma(q)=q\big(\Sigma_{\rm V}(q^2)+\gamma_5\Sigma_{\rm A}(q^2)\big)+m\Sigma_{\rm S}(q^2)\,. \tag{C.1}
$$

In the one-loop approximation, the coefficient functions read

$$
\begin{aligned}
\Sigma_V(s)=\frac{\alpha}{4\pi}\Bigg\{&-\frac{1}{4s_{\rm w}^2}\left[\left(2+\frac{m^2+m'^2}{w}\right)B_1\left(s,m'^2,w\right)+1\right]-\left(v^2+a^2\right)\left(2B_1(s,m^2,z)+1\right)\\
&-Q^2\left(2B_1(s,m^2,\mu^2)+1\right)-\frac{m^2}{4s_{\rm w}^2w}\big(B_1(s,m^2,z)+B_1(s,m^2,h)\big)\Bigg\}\,,
\end{aligned}
$$

$$\Sigma_A(s) = \frac{\alpha}{4\pi}\left\{\frac{1}{4s_{\rm w}^2}\left[\left(2-\frac{m^2-m'^2}{w}\right)B_1\left(s,m'^2,w\right)+1\right]+2va\big(2B_1(s,m^2,z)+1\big)\right\}\,,$$

$$\begin{aligned}\Sigma_S(s) = \frac{\alpha}{4\pi}\Bigg\{&-\frac{m'^2}{2s_{\rm w}^2 w}B_0\left(s,m'^2,w\right)-2(v^2-a^2)\big(2B_0(s,m^2,z)-1\big)\\&-2Q^2\big(2B_0(s,m^2,\mu^2)-1\big)+\frac{m^2}{4s_{\rm w}^2 w}\big(-B_0(s,m^2,z)+B_0(s,m^2,h)\big)\Bigg\}\,,\end{aligned}\qquad\text{(C.2)}$$

where Q, v, a, and m are the charge, vector coupling, axial-vector coupling, and mass of the considered fermion, respectively, and m' is the mass of its isopartner.

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FIGURE CAPTIONS

1) Branching fractions versus M_H of the Higgs-boson decays into a) W^+W^-, ZZ, $Z\gamma$, $\gamma\gamma$, b) $b\bar{b}$, $c\bar{c}$, $\tau^+\tau^-$, and gg. For the loop-induced decays $m_t = 130$ GeV has been assumed.

2) a) S, b) T, and c) U as functions of M_H in the one-loop approximation of the standard model with $M_W = 80.22\,\text{GeV}$ and $m_t = 150\,\text{GeV}$. The total results are decomposed into lepton, quark, and boson contributions and complemented by perturbative $\mathcal{O}(\alpha\alpha_s)$ corrections.

3) m_t-M_H interdependence predicted from Δr using $M_W = (80.22 \pm 0.26)\,\text{GeV}$ as input. In the case of the central value, starting from the one-loop analysis, in turn electroweak, perturbative-QCD, and $t\bar{t}$-threshold corrections beyond one loop are included.

4) Feynman diagrams pertinent to the ZZH coupling in the one-loop approximation: a) Born diagram, b) fermion loops, c) W-boson loops, d) diagrams involving virtual Z and/or Higgs bosons, and e) counterterm. $\phi^\pm$ and ϕ_Z are the Higgs-Kibble ghosts associated with the $W^\pm$ and Z bosons, respectively, and $u^\pm$ and $\bar{u}^\pm$ are the charged Faddeev-Popov ghosts.

5) One-loop radiative corrections to the $H \to ZZ$ decay width in the MOMS scheme as a function of M_H. a) The contributions due to bosons, leptons, quarks, and their combined effect are shown for $m_t = 150\,\text{GeV}$. b) The combined effect is shown for $m_t = 100$, 150, and 200 GeV.

6) a) $H \to Z\gamma$ and b) $Z \to \gamma H$ decay widths to lowest order as a function of M_H for $m_t = 91$ and 200 GeV. For comparison, the contribution due to the W-boson loops is shown separately.

7) $H \to \gamma\gamma$ decay width to lowest order as a function of M_H for $m_t = 91$ and 200 GeV. For comparison, the contribution due to the W-boson loops is shown separately.

8) $H \to gg$ decay width to leading order as a function of M_H for $m_t = 100$, 150, and 200 GeV. The dotted lines represent the respective results in next-to-leading order of QCD.

9) QCD K factor for the total cross section of $pp \to H + X$ at the SSC as a function of M_H for $m_t = 140\,\text{GeV}$ (taken from Ref. 103). Also shown are the regularized virtual corrections and the real corrections connected with the gg, gq, and qq hard-scattering processes.

10) Feynman diagrams pertinent to the W^+W^-H coupling in the one-loop approximation: a) Born diagram, b) tadpoles, c) diagrams involving virtual bosons and ghosts, d) fermion loops, e) counterterm, and f) photon bremsstrahlung. $\phi^\pm$ and ϕ_Z are the Higgs-Kibble ghosts associated with the $W^\pm$ and Z bosons, respectively, and $u^\pm$, u_Z, and u_γ are the Faddeev-Popov ghosts associated with the $W^\pm$, Z bosons, and the photon, respectively.

11) One-loop radiative corrections to the $H \to W^+W^-$ decay width including photon bremsstrahlung in the MOMS scheme as a function of M_H. a) The contributions due to bosons, leptons, quarks, and their combined effect are shown for $m_t = 150\,\text{GeV}$. b) The combined effect is shown for $m_t = 100$, 150, and 200 GeV.

12) Feynman diagrams pertinent to the $f\bar{f}H$ coupling in the one-loop approximation: a) Born diagram, b) virtual-photon exchange, c) diagrams involving virtual bosons and ghosts, d) counterterm, and f) photon bremsstrahlung. $\phi^\pm$ and ϕ_Z are the Higgs-Kibble ghosts associated with the $W^\pm$ and Z bosons, respectively.

13) One-loop weak radiative corrections to the a) $H \to \tau^+\tau^-$, b) $H \to b\bar{b}$, and c) $H \to t\bar{t}$ decay widths in the MOMS scheme as a function of M_H for $m_t = 100$, 150, and 200 GeV. The dotted lines visualize the approximations by Eq. (3.70) in the case of a) and b) and by Eq. (3.71) in the case of c).

14) a) $H \to c\bar{c}$ and b) $H \to b\bar{b}$ decay widths as functions of M_H in leading and next-to-leading order of QCD using the pole mass and in next-to-next-to-leading order using the running mass explicitly derived from the pole mass.

15) Feynman diagrams pertinent to the $f\bar{f}ZH$ coupling in the one-loop approximation: a) Born diagram, b)–l) electroweak loops, m) and n) photon bremsstrahlung. ϕ is the Higgs-Kibble ghost associated with the W boson. The blobs of b), c), d), e), and f) represent 4, 13 (8), 26 (15), 2, and 2 nonvanishing diagrams in the 't Hooft-Feynman gauge, respectively. Thereby one generic fermion flavour is assumed and counterterm diagrams are not counted.

16) One-loop weak corrections to the total cross section of $e^+e^- \to ZH$ in the MOMS scheme as a function of M_H. a) The contributions due to bosons, leptons, quarks, and their combined effect are shown at $\sqrt{s} = 500\,\text{GeV}$ for $m_t = 150\,\text{GeV}$. b) The combined effect is shown at $\sqrt{s} = 500\,\text{GeV}$ for $m_t = 90$, 150, and 210 GeV. c) The combined effect is shown at $\sqrt{s} = 200$, 500, 1000, and 1500 GeV for $m_t = 150\,\text{GeV}$.

17) Photonic initial-state corrections up to $\mathcal{O}(\alpha^2)$ to the total cross section of $e^+e^- \to ZH$ as a function of M_H at $\sqrt{s} = 200$, 500, 1000, and 1500 GeV. The infrared-sensitive parts are exponentiated. The dotted lines correspond to the full QED corrections in $\mathcal{O}(\alpha)$ without exponentiation.

18) One-loop radiative corrections to the a) $Z \to \nu\bar{\nu}H$ and b) $Z \to \ell^+\ell^-H$ decay widths in the MOMS scheme as a function of M_H for $m_t = 90$, 130, 170, and 210 GeV.

19) One-loop radiative corrections to the $H \to f\bar{f}Z$ decay widths, where $f = \nu, \ell$, in the MOMS scheme as a function of M_H for $m_t = 150\,\text{GeV}$.

20) Total cross section of $e^+e^- \to \gamma H$ in the one-loop approximation assuming $s_{\text{w}}^2 = 0.21$, $M_W = 80.8\,\text{GeV}$, and $m_t = 40\,\text{GeV}$ (taken from Ref. 87).

21) Total cross section of $pp \to W^{\pm}H + X$ (sum of W^+H and W^-H) at the SSC, for $M_H = 100\,\text{GeV}$, as a function of the factorization-scale parameter n, where $\mu = n\sqrt{q^2}$ (taken from Ref. 129). The lowest-order cross section, the QCD-corrected cross section, and the separate contributions of the $gq \to W^*q$ subprocess (gq order α_s) and the real and virtual gluon corrections to the $q\bar{q} \to W^*$ subprocess ($q\bar{q}$ order α_s) are shown.

22) QCD-corrected total cross sections of $pp \to W^{\pm}H + X$ (sum of W^+H and W^-H) and $pp \to ZH + X$ at the SSC versus M_H for three different sets of parton distribution functions (taken from Ref. 129).

23) Transverse-momentum distribution of the Higgs boson at leading order (dashed curve) and at next-to-leading order (solid curve) for $pp \to W^{\pm}H + X$ at $\sqrt{s} = 40\,\text{TeV}$ and $M_H = 100\,\text{GeV}$ (taken from Ref. 130). Decay branching fractions $B(W \to e\nu, \mu\nu)$ and $B(H \to \gamma\gamma)$ are included.

24) Feynman diagrams pertinent to the $ggZH$ coupling to lowest order.

25) a) Transverse-momentum distribution of a Higgs boson with $M_H = 100\,\text{GeV}$ and b) M_H dependence of the total cross section for $pp \to ZH + X$ at the SSC from tree-level $q\bar{q}$ annihilation (dotted curves) and gg fusion assuming $m_t = 80$, 140, and 200 GeV (solid lines). For comparison, also the separate contribution due to the $ggZH$ triangle diagrams is shown. Lower curves correspond to smaller values of m_t.

26) Feynman diagrams pertinent to the a) $gg \to gH$, b) $qg \to qH$, and c) $q\bar{q} \to gH$ subprocesses in lowest order.

27) Transverse-momentum distribution of zero-rapidity Higgs bosons with $M_H = 110$ and

160 GeV from $pp \to jH + X$ at the SSC for $m_t = 160\,\mathrm{GeV}$ (taken from Ref. 96). For comparison, also the approximate results evaluated on the basis of Eq. (4.33) are shown.

28) Transverse-momentum distribution of zero-rapidity Higgs bosons with $M_H = 100$, 150, and 200 GeV from $pp \to jH + X$ at the SSC in the $m_t \to \infty$ limit (taken from Ref. 100). The spectrum includes multiple-gluon effects and is normalized to next-to-leading order in α_s.

29) Total cross sections of $pp \to H + X$ via W^+W^- and ZZ fusion at the SSC in leading (dashed lines) and next-to-leading (solid lines) order as a function of M_H (taken from Ref. 158).