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Gauge-invariant formulation of the S, T, and U parameters

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It is shown that the bosonic contributions to the S, T, and U parameters, defined in terms of conventional self-energies, are gauge dependent in the standard model (SM). Moreover, T and U are divergent unless a constraint is imposed among the gauge parameters. Implications of this result for renormalization schemes of the SM are discussed. A gauge-invariant formulation of S, T, and U is proposed in the pinch-technique framework. The modified S, T, and U parameters provide a gauge-invariant parametrization of leading electroweak radiative corrections in the SM and some of its extensions.

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In recent discussions of electroweak radiative corrections in the standard model (SM) and some of its extensions, it has become customary to parametrize leading contributions in terms of three amplitudes: S, T, and U [1,2]. These are suitable combinations of self-energies (frequently called oblique corrections) that describe important components of the electroweak corrections. The rationale of this approach is that in some interesting cases the dominant effects are contained in the vacuum-polarization functions. An alternative formulation has been proposed in terms of the ϵ_i $(i=1,2,3,b)$ parameters, which are closely connected with observable quantities and do not necessarily assume the dominance of self-energies [3].

In the early papers the focus was on fermionic contributions. Although there has been some variation in the precise definition of S, T, and U, recent works appear to converge to the expressions

$$\hat{\alpha}S=\frac{4\hat{e}^2}{m_Z^2}\mathrm{Re}[\Pi_{33}(m_Z^2)-\Pi_{33}(0)-\Pi_{3Q}(m_Z^2)+\Pi_{3Q}(0)]$$

$$=\frac{2\hat{e}^2}{m_Z^2}[\Pi_{3Y}(0)-\Pi_{3Y}(m_Z^2)]\ , \tag{1a}$$

$$\hat{\alpha}T=\frac{\hat{e}^2}{\hat{s}^2m_W^2}\mathrm{Re}[\Pi_{11}(0)-\Pi_{33}(0)]\ , \tag{1b}$$

$$\hat{\alpha}U=\frac{4\hat{e}^2}{m_W^2}\mathrm{Re}[\Pi_{11}(m_W^2)-\Pi_{11}(0)]-\frac{4\hat{e}^2}{m_Z^2}\mathrm{Re}[\Pi_{33}(m_Z^2)-\Pi_{33}(0)]\ , \tag{1c}$$

where the Π's are unrenormalized vacuum-polarization functions with coupling constants factored out, 1 and 3 are SU(2) indices, and Q refers to the electromagnetic current. In writing down the second equality of Eq. (1a), we have employed $J_3^\mu-J_Q^\mu=-J_Y^\mu/2$. As coupling constants we have used the modified minimal subtraction scheme ($\overline{\mathrm{MS}}$) parameters $\hat{e}^2\equiv\hat{e}^2(m_Z^2)$ and $\hat{s}^2\equiv\sin\hat{\theta}_W^2(m_Z^2)$, as these are very well suited to describe physics at the m_Z scale [4,5]. It is convenient to express Eqs. (1a)–(1c) in the mass-eigenstate basis. Writing

$$A_{ZZ}(q^2)=\frac{\hat{e}^2}{\hat{c}^2\hat{s}^2}[\Pi_{33}(q^2)-2\hat{s}^2\Pi_{3Q}(q^2)+\hat{s}^4\Pi_{QQ}(q^2)]\ , \tag{2a}$$

$$A_{\gamma Z}(q^2)=\frac{\hat{e}^2}{\hat{c}\hat{s}}[\Pi_{3Q}(q^2)-\hat{s}^2\Pi_{QQ}(q^2)]\ , \tag{2b}$$

$$A_{\gamma\gamma}(q^2)=\hat{e}^2\Pi_{QQ}(q^2)\ , \tag{2c}$$

$$A_{WW}(q^2)=\frac{\hat{e}^2}{\hat{s}^2}\Pi_{11}(q^2)\ , \tag{2d}$$

which can be understood from the relation $J_Z^\mu=J_3^\mu-\hat{s}^2J_Q^\mu$, and recalling $\Pi_{QQ}(0)=0$, we have

$$\hat{\alpha}S=\frac{4\hat{c}^2\hat{s}^2}{m_Z^2}\mathrm{Re}\left\{A_{ZZ}(m_Z^2)-A_{ZZ}(0)-\frac{\hat{c}^2-\hat{s}^2}{\hat{c}\hat{s}}[A_{\gamma Z}(m_Z^2)-A_{\gamma Z}(0)]-A_{\gamma\gamma}(m_Z^2)\right\}\ , \tag{3a}$$

$$\hat{\alpha}T=\frac{A_{WW}(0)}{m_W^2}-\frac{\hat{c}^2}{m_W^2}\left[A_{ZZ}(0)+\frac{2\hat{s}}{\hat{c}}A_{\gamma Z}(0)\right]\ , \tag{3b}$$

$$\hat{\alpha}U=4\hat{s}^2\mathrm{Re}\left[\frac{A_{WW}(m_W^2)-A_{WW}(0)}{m_W^2}-\hat{c}^2\frac{A_{ZZ}(m_Z^2)-A_{ZZ}(0)}{m_Z^2}-2\hat{c}\hat{s}\frac{A_{\gamma Z}(m_Z^2)-A_{\gamma Z}(0)}{m_Z^2}-\hat{s}^2\frac{A_{\gamma\gamma}(m_Z^2)}{m_Z^2}\right]\ . \tag{3c}$$

In Eqs. (2a)–(3c) the A's are conventional unrenormalized self-energies, defined according to Refs. [6,7]. As mentioned before, the original application of Eqs. (3a)–(3c) involved only fermionic contributions. More recently, these expressions have been generalized to incorporate the bosonic contributions of the SM and extensions that preserve the $SU(2)\times U(1)$ structure.

As the values of m_t and m_H are currently unknown, the SM contributions to S, T, and U, evaluated at judiciously chosen values of these masses, are regarded as standard. The differences between the actual values of S, T, and U and the standard ones are determined by fitting the electroweak data; they parametrize both the effect of variations of m_t and m_H within the SM and possible contributions from new physics.

We now observe that Eqs. (3a)–(3c) present a theoretical problem: although the one-loop fermionic contributions to S, T, and U are gauge invariant, their SM bosonic counterparts are not. This can be checked by using the results of Ref. [8]. It is useful to recall that the gauge dependence is specified by three parameters ξ_i $(i=W,Z,\gamma)$ [9] and to observe that, with an unimportant technical caveat [10], only $A_{WW}(q^2)$ depends on ξ_Z and ξ_γ at the one-loop level. By appropriately choosing ξ_γ and ξ_Z, it is easy to see that T and U, as defined in Eqs. (3b) and (3c), are gauge dependent. Moreover, using the results of Ref. [8], one finds that they are divergent at the one-loop level unless the gauge parameters are related by

$$\xi_W=\hat{c}^2\xi_Z+\hat{s}^2\xi_\gamma \ . \tag{4}$$

Thus, they are convergent, for example, in the $\xi_W=\xi_Z=\xi_\gamma$ gauges, but divergent over a wide class of renormalizable gauges. On the other hand, $\hat{\alpha}S$ [Eq. (3a)], which contains only neutral-current amplitudes, is gauge dependent but remains convergent in the full class of renormalizable gauges.

To the interested reader who wants to partially verify these statements, without carrying out the complete calculations of Ref. [8], we suggest a simple short cut. It is well known that S, T, and U are convergent in the $\xi_i=1$ gauge. Consider $\hat{\alpha}T$ [Eq. (3b)], which is supposed to represent leading corrections the ρ parameter, retain $\xi_W=\xi_Z=1$, but choose $\xi_\gamma\neq 1$. As only diagrams with a photon propagator are affected, A_{ZZ}, $A_{\gamma Z}$, and $A_{\gamma\gamma}$ remain unaltered. On the other hand, one readily finds that $A_{WW}(0)$ is modified by a divergent shift proportional to $\xi_\gamma-1$ and $\hat{\alpha}T$ is no longer finite. We conclude that, although Eq. (3b) describes correctly the fermionic corrections to the ρ parameter [7,11], it cannot be regarded as a satisfactory representation of the leading bosonic counterparts. It is, in fact, gauge dependent and, moreover, divergent unless the restrictive condition of Eq. (4) is imposed. What happens, as expected from general principles and as shown explicitly in Ref. [8] in the case of four-fermion amplitudes, is that the vertex and box diagrams have additional gauge dependences that cancel those arising from the self-energies.

The above results have interesting implications for certain renormalization schemes of the SM. In fact, in some frequently applied formulations, Eq. (3b) follows from a combination of renormalized WW and ZZ self-energies; see, for example, Ref. [12]. The fact that Eq. (3b) is divergent if Eq. (4) is not satisfied implies that such renormalization schemes implicitly assume a reduction of the three-dimensional space of renormalizable gauges to a two-dimensional subspace. Thus, they are not applicable if one considers the full class of renormalizable gauges under which the S matrix is invariant. On the other hand, they can be used to obtain renormalized self-energies and vertex parts in a restricted set of gauges, probably that defined in Eq. (4). However, when the bosonic contributions of the SM are included, linear combinations of these renormalized self-energies should not be identified, as often is done in the literature, with physical observables. Formulations that deal directly with the renormalization of the S matrix [4,6,7] or in which each self-energy is renormalized by independent subtractions [13] are not subject to the restriction of Eq. (4). The same should be true in schemes involving three independent field-renormalization constants for the vector bosons, provided these are chosen judiciously.

The above problems of gauge invariance do not apply to the ϵ_i parameters, provided they are identified with the defining observables rather than their self-energy contributions. For then they must necessarily contain the vertex and box diagrams that render them gauge invariant. It is important to emphasize that, in most cases of interest, the gauge dependence of S, T, and U, although awkward, does not pose a serious problem in the parametrization of new physics. The point is that usually the new-physics contributions to the self-energies are gauge invariant. One can then argue that, although the standard values of S, T, and U are gauge dependent, their combination with the other SM corrections are gauge invariant. Or, alternatively, one can argue that the differences of S, T, and U values corresponding to various choices of m_t and m_H are gauge invariant at the one-loop level. A possible exception is the consideration of anomalous $WW\gamma$ and WWZ couplings. If they are introduced in a way that respects the $SU(2)\times U(1)$ gauge symmetry of the S matrix, it is likely that the new-physics contributions to self-energies and vertex parts are separately gauge dependent. In that case, by retaining only the contributions to the self-energies, one would parametrize new physics in a gauge-dependent manner which, of course, is not theoretically acceptable.

Whatever is the new-physics scenario, it is clearly desirable to have a gauge-invariant formulation of S, T, and U. For then these parameters would provide a theoretically sound framework to describe leading electroweak radiative corrections, both in the SM and in some of its extensions. In order to achieve this aim, we propose to retain the structure of Eqs. (3a)–(3c) but replace the A's with the gauge-invariant self-energies of the pinch-technique (PT) approach [14–16]. We recall that in the PT, propagatorlike pinch parts are segregated from vertex and box diagrams and combined with the conventional self-energy and tadpole diagrams. The resulting modified self-energies are gauge invariant, i.e., ξ_i independent at the one-loop level. In Ref. [16] a novel interpretation of the pinch parts was given, namely their identification with contributions from equal-time commutators in relevant Ward identities. This observation can be used to show that the pinch parts are not affected by hadron dynamics and are, in this sense, universal. It was also shown that the PT self-energies in $e^+e^- \to W^+W^-$ can be identified with those occurring in four-fermion amplitudes, which gives support to the idea that they are process independent. Application of the PT to the SM leads to the following expressions for the one-loop self-energies [16]:

$$a_{\gamma\gamma}^{\rm SM}(q^2)=A_{\gamma\gamma}^{\rm SM}(q^2)|_{\xi_W=1}-4\hat{e}^2q^2I_{WW}(q^2)\ , \tag{5a}$$

$$a_{\gamma Z}^{\rm SM}(q^2)=A_{\gamma Z}^{\rm SM}(q^2)|_{\xi_W=1}-2\hat{e}\hat{g}\hat{c}(2q^2-m_Z^2)I_{WW}(q^2)\ , \tag{5b}$$

$$\left[a_{ZZ}(q^2)+\frac{\hat{g}m_WT}{\hat{c}^2m_H^2}\right]^{\rm SM}=\left[A_{ZZ}(q^2)+\frac{\hat{g}m_WT}{\hat{c}^2m_H^2}\right]^{\rm SM}_{\xi_i=1}-4\hat{g}^2\hat{c}^2(q^2-m_Z^2)I_{WW}(q^2)\ , \tag{5c}$$

$$\left[a_{WW}(q^2)+\frac{\hat{g}m_WT}{m_H^2}\right]^{\rm SM}=\left[A_{WW}(q^2)+\frac{\hat{g}m_WT}{m_H^2}\right]^{\rm SM}_{\xi_i=1}-4\hat{g}^2(q^2-m_W^2)[\hat{c}^2I_{ZW}(q^2)+\hat{s}^2I_{\gamma W}(q^2)]\ , \tag{5d}$$

where

$$I_{ij}(q^2)=i\mu^{4-n}\int\frac{d^nk}{(2\pi)^n}\frac{1}{(k^2-m_i^2)[(k+q)^2-m_j^2]}\ . \tag{5e}$$

The a's are the PT self-energies, SM denotes SM contributions, and iT represents the overall tadpole amplitude [8], which must be included to obtain gauge-invariant ZZ and WW amplitudes. We note that the expressions on the right-hand sides (RHS's) of Eqs. (5a)–(5d) are manifestly ξ_i independent. As pointed out in Ref. [16], the PT self-energies and the vertex and box corrections automatically satisfy very desirable theoretical properties. The relation of the PT self-energies to those proposed by other authors, notably Kennedy and Lynn [17] and Kuroda, Moultaka, and Schildknecht [18], was also explained in Ref. [16]. In our proposal, the S, T, and U parameters are given by Eqs. (3a)–(3c), except that the self-energies on the RHS's are replaced by the corresponding PT self-energies and associated tadpoles. They include both SM and "new-physics" contributions. In order to obtain the SM contributions to the new S, T, and U parameters, we first note that the SM tadpoles exactly cancel in these amplitudes. It is then sufficient to replace every conventional self-energy $A_{ij}^{\rm SM}$ in Eqs. (3a)–(3c) by their PT counterparts $a_{ij}^{\rm SM}$. Employing Eqs. (5a)–(5d), one obtains

$$\hat{\alpha}S_{\rm SM}=\hat{\alpha}(S_c)_{\rm SM}+8\hat{e}^2\hat{c}^2[I_{WW}(m_Z^2)-I_{WW}(0)]\ , \tag{6a}$$

$$\hat{\alpha}T_{\rm SM}=\hat{\alpha}(T_c)_{\rm SM}+4\hat{g}^2[\hat{c}^2I_{ZW}(0)+\hat{s}^2I_{\gamma W}(0)-I_{WW}(0)]\ , \tag{6b}$$

$$\hat{\alpha}U_{\rm SM}=\hat{\alpha}(U_c)_{\rm SM}+16\hat{e}^2\{\hat{c}^2[I_{WW}(0)-I_{ZW}(0)]+\hat{s}^2[I_{WW}(m_Z^2)-I_{\gamma W}(0)]\}\ , \tag{6c}$$

where S, T, and U are the gauge-invariant parameters, defined in terms of the PT amplitudes, and S_c, T_c, and U_c are the conventional ones, defined in terms of the usual self-energies [cf. Eqs. (3a)–(3c)] evaluated in the $\xi_i=1$ gauge.

As $(S_c)_{\rm SM}$, $(T_c)_{\rm SM}$, and $(U_c)_{\rm SM}$ are known to be finite and the I_{ij} are only logarithmically divergent, we see that the gauge-invariant functions $S_{\rm SM}$, $T_{\rm SM}$, and $U_{\rm SM}$ are convergent at the one-loop level. For clarity, we point out that the exact cancellation of divergences in the S, T, and U parameters defined above occurs in the on-shell scheme of renormalization, in which $\hat{c}^2$ and $\hat{s}^2$ are replaced by $c^2=1-s^2=m_W^2/m_Z^2$ [6,7]. When one employs $\overline{\rm MS}$ couplings, there are residual divergent terms proportional to $\hat{\alpha}(\hat{c}^2/c^2-1)$, which are neglected here as they represent higher-order effects. This particular "higher-order problem" can be circumvented by defining *ab initio* the S, T, and U parameters in the $\overline{\rm MS}$ scheme of renormalization [2,19].

Evaluating the contributions from the I_{ij} terms, we find

$$S_{\rm SM}=(S_c)_{\rm SM}+\frac{4\hat{c}^2}{\pi}\left[\sqrt{4c^2-1}\arcsin\left[\frac{1}{2c}\right]-1\right]\ , \tag{7a}$$

$$T_{\rm SM}=(T_c)_{\rm SM}-\frac{1}{\pi\hat{s}^2}\left[\frac{\hat{c}^2}{s^2}\ln c^2+1\right]\ , \tag{7b}$$

$$U_{\rm SM}=(U_c)_{\rm SM}+\frac{4}{\pi}\left\{\frac{\hat{c}^2}{s^2}\ln c^2+1+2\hat{s}^2\left[\sqrt{4c^2-1}\arcsin\left[\frac{1}{2c}\right]-1\right]\right\}\ . \tag{7c}$$

We recall that $a_{\gamma Z}^{\rm SM}(0)=0$[16] and, consequently, Eq. (6b) can also be expressed as

$$\hat{\alpha}T_{\rm SM}=\frac{[a_{WW}(0)-\hat{c}^2a_{ZZ}(0)]^{\rm SM}}{m_W^2}\ , \tag{8}$$

which, being gauge invariant and convergent, qualifies as a theoretically acceptable description of fermionic and dominant bosonic contributions to the ρ parameter.

If the new-physics effects on the conventional self-energies are gauge invariant, as is usually the case, they will result in additional contributions to the conventional parameters S_c, T_c, and U_c defined in Eqs. (3a)–(3c) and, via Eqs. (7a)–(7c), to S, T, and U. If the new-physics effects on the conventional self-energies happen to be gauge dependent, they will be combined with the pinch parts derived from the new contributions to vertex and box diagrams. This procedure will generate additional pinch contributions to the gauge-invariant S, T, and U parameters, analogous to the SM I_{ij} terms in Eqs. (6a)–(6c).

In summary, the S, T, and U amplitudes, constructed in the PT framework, provide a simple gauge-invariant and convergent parametrization of leading electroweak radiative corrections, in the SM and some of its extensions.

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