

## Yukawa coupling unification with supersymmetric threshold corrections

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Radiative corrections to the down-type fermion masses at the supersymmetric threshold are enhanced by the ratio of vacuum expectation values,  $\tan\beta$ . This can have a strong impact on the unification of Yukawa couplings in supersymmetric grand unified theories. We present an example of such a model with a horizontal gauge symmetry that naturally explains the fermion mass hierarchy and the small mixing angles of the Kobayashi-Maskawa matrix. The unification of the lepton and the down-quark Yukawa couplings is achieved without introducing large Higgs multiplets.

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One of the most puzzling features of the quark mass matrices is the large ratio of quark masses (e.g.,  $m_t/m_u \approx 3 \times 10^4$ ) and the small off-diagonal entries of the Kobayashi-Maskawa (KM) matrix. There have been many attempts to explain these properties by imposing additional symmetries. The light quark masses in these scenarios are generated radiatively [1] or suppressed by ratios of vacuum expectation values [2–5]. Additional problems arise if we try to embed the  $SU(3)_c \times SU(2)_L \times U(1)_Y$  standard model gauge group in the simple gauge group of a grand unified theory (GUT) [6]. In the minimal version of an  $SU(5)$  GUT [7] the right-handed down-type quarks and the left-handed leptons are different components of the same fields. As a result, the down-type quarks and the leptons have the same Yukawa couplings at  $M_{\text{GUT}}$ . By running the coupling constants from  $M_{\text{GUT}}$  to  $m_Z$  one obtains a prediction for the down-type quark masses in terms of the lepton masses.

It has been shown recently [8] that in the minimal supersymmetric model (MSSM) [9,10] the  $SU(3)_c \times SU(2)_L \times U(1)_Y$  gauge couplings unify at a scale  $M_{\text{GUT}} \approx 10^{16}$  GeV. Additionally, the unification of  $\tau$  and bottom Yukawa couplings at  $M_{\text{GUT}}$  within the MSSM is rather successful [5,11,12]. However, the prediction for the first generation is clearly incompatible with experiment.

One way out is to introduce a large Higgs representation such as the **45** under  $SU(5)$ . However, such an extension introduces many new interactions and it would be desirable to find alternatives with a smaller particle content. Another possibility to evade GUT predictions for the light fermion masses is by radiative generation at the supersymmetry (SUSY) threshold via off-diagonal entries in the squark mass matrices [13]. Such entries are inevitable at one loop if the KM matrix is different from unity. However, the smallness of the off-diagonal elements in the KM matrix suggests the existence of additional symmetries that are broken spontaneously. In such a scenario it is natural on dimensional grounds that off-

diagonal entries of the squark mass matrix are generated at the tree level, while off-diagonal entries of the KM matrix are generated in next order. In this paper we will present an explicit example of such a model. It is obtained by including a fourth family of fermions and a family of mirror fermions (a more general, and therefore less predictive, class of models has been considered in Ref. [14]). Such additional fields are naturally present in many extensions of  $SU(5)$ . According to the “survival” hypothesis [15] the additional fermions and their mirror fermions combine and acquire masses of the order of  $M_{\text{GUT}}$ . Nonetheless, their existence will affect the parameters of the low energy effective Lagrangian. In particular, we will show that the existence of a fourth family of fermions and a family of mirror fermions suffices in order to reconcile the bad GUT predictions for the first generation down-type fermion masses.

Furthermore, we will constrain the Yukawa matrices by introducing a horizontal  $U(1)_h$  [16] gauge symmetry and a discrete  $Z_3$  symmetry. This will explain naturally the quark mass hierarchy and the Kobayashi-Maskawa (KM) matrix. In Table I we list the full particle content of the theory and their transformation properties under  $SU(5)$ ,  $U(1)_h$ , and  $Z_3[\phi \rightarrow \exp(iz_\phi \pi/3)\phi]$ . This set of fields is manifestly anomaly-free. In addition, we impose  $R$  invariance in order to avoid baryon-number-violating interactions. Here the four generations of quarks and leptons are denoted by  $U_i$  and  $D_i$  ( $i=1,2,3,4$ ) and the mirror quarks and mirror leptons are denoted by  $U'$  and  $D'$ . The adjoint representation responsible for the breaking of  $SU(5)$  is denoted by  $\Phi$  and the Higgs field responsible for the electroweak breaking are denoted by  $H_1$  and  $H_2$ . In addition, we need to introduce  $SU(5)$  singlets  $N_i$  ( $i=1,2,3,4$ ) and  $N'$  to break  $U(1)_h$  and  $Z_3$ . The superpotential of this theory can be written as

$$W = W_Q + W_Y + W_H + W_N + W_\Phi, \quad (1)$$

where

TABLE I. The particle spectrum.

		SU(5)	U(1) <sub>h</sub>	Z <sub>3</sub>
Matter ( <i>i</i> = 1, 2, 3, 4)	<i>U<sub>i</sub></i>	$\mathbf{10}$	1, -1, 0, 0	-1, -1, -1, 0
	<i>U'</i>	$\overline{\mathbf{10}}$	0	0
	<i>D<sub>i</sub></i>	$\overline{\mathbf{5}}$	1, -1, 0, 0	-1, -1, -1, 0
	<i>D'</i>	$\mathbf{5}$	0	0
Higgs <i>H<sub>1</sub>, H<sub>2</sub>, Φ</i>		$\overline{\mathbf{5}}, \mathbf{5}, \mathbf{24}$	0	1
Singlet ( <i>i</i> = 1, 2, 3, 4)	<i>N<sub>i</sub></i>	$\mathbf{1}$	-1, 1, 0, 0	1
	<i>N'</i>	$\mathbf{1}$	0	-1

$$\begin{aligned}
W_Q &= \sum_{Q=U,D} [\lambda_{Q1} N_1 Q_1 + \lambda_{Q2} N_2 Q_2 \\
&\quad + (\lambda_{Q3} N_3 + \lambda_{Q4} N_4 + \lambda_Q \Phi) Q_3 + m_Q Q_4] Q', \\
W_Y &= y_D H_1 U_3 D_4 + y'_D H_1 U_4 D_3 + y_U H_2 U_3 U_4, \\
W_H &= H_1 (h_3 N_3 + h_4 N_4 + h_\Phi \Phi) H_2, \\
W_N &= \frac{1}{3!} \kappa_{ijk} N_i N_j N_k + m_3 N_3 N' + m_4 N_4 N' + \frac{1}{3!} \kappa' N'^3, \\
W_\Phi &= \frac{1}{2!} (\lambda_3 N_3 + \lambda_4 N_4) \Phi^2 + \frac{1}{3!} \lambda_\Phi \Phi^3.
\end{aligned} \tag{2}$$

All the mass parameters are assumed to be  $\gtrsim M_{\text{GUT}}$ . The entries of  $\kappa_{ijk}$  are constrained by U(1)<sub>h</sub> and Z<sub>3</sub>. In addition, the fields *N<sub>3</sub>* and *N<sub>4</sub>* have the same quantum numbers and can be rotated such that  $\kappa_{123} = 0$ . the potential  $\mathcal{V}$  is minimized if the *D* terms and the *F* terms vanish:

$$D_h = \frac{g_h^2}{2} (N_2^* N_2 - N_1^* N_1) = 0, \quad F_\phi^* = -\frac{dW}{d\phi} = 0. \tag{3}$$

where  $\phi = N_i, N', \Phi$  and  $g_h$  is the horizontal gauge coupling. In this basis the potential has a minimum at

$$\begin{aligned}
\langle N_i \rangle &= n_i \neq 0 \quad (i = 1, 2, 3), \\
\langle N' \rangle &= n' \neq 0, \\
\langle \Phi \rangle &= a \text{diag}(-\frac{2}{3}, -\frac{2}{3}, -\frac{2}{3}, 1, 1) \neq 0,
\end{aligned} \tag{4}$$

and all other fields equal to zero.<sup>1</sup> Nonzero values of *a* break SU(5) to SU(3)<sub>c</sub> × SU(2)<sub>L</sub> × U(1)<sub>Y</sub> under which the representations decompose as

$$U(10) \rightarrow q(3, 2, \frac{1}{3}) + u^c(\overline{3}, 1, -\frac{4}{3}) + e^c(1, 1, 2), \tag{5}$$

$$D(\overline{5}) \rightarrow d^c(\overline{3}, 1, \frac{2}{3}) + l(1, 2, -1).$$

The low energy effective Lagrangian is now obtained by diagonalizing the fermion mass matrices  $m_i^f = M_f v_i^f$  and the sfermion mass matrices  $m_{ij}^{f2} = m_i^f m_j^f$

[here,  $M_f^2 \equiv (\lambda_{f1} n_1)^2 + (\lambda_{f2} n_2)^2 + (\lambda_{f3} n_3 + \lambda_f a)^2 + m_f^2$ ,  $v_1^f \equiv \lambda_{f1} n_1 / M_f$ ,  $v_2^f \equiv \lambda_{f2} n_2 / M_f$ ,  $v_3^f \equiv (\lambda_{f3} n_3 + Y_f \lambda_f a) / M_f$  and  $v_4^f \equiv m_f / M_f$ ;  $Y_f = \frac{1}{3}, -\frac{2}{3}, -\frac{4}{3}, 1, 2$  for  $f = q, d, u, l, e$ ] and removing all the fields that acquire masses at  $M_{\text{GUT}}$ . The unitary matrices  $\mathcal{U}^f$ , defined by

$$v_i^f \rightarrow v_j^f \mathcal{U}_{ji}^f = (0, 0, 0, 1), \tag{6}$$

are given by

$$\mathcal{U}^f = \begin{pmatrix} c_f'' & s_f'' c_f' & s_f'' s_f' c_f & s_f'' s_f' s_f \\ -s_f'' & c_f'' c_f' & c_f'' s_f' c_f & c_f'' s_f' s_f \\ 0 & 0 & -s_f & c_f \\ 0 & -s_f' & c_f' c_f & c_f' \end{pmatrix}. \tag{7}$$

Here we have defined

$$c_f \equiv \cos \theta_f = v_3^f, \quad c_f' \equiv \cos \theta_f' = \frac{v_4^f}{s_f}, \quad c_f'' \equiv \cos \theta_f'' = \frac{v_2^f}{s_f' s_f}, \tag{8}$$

and  $s_f \equiv \sin \theta_f$ , etc. After decoupling the superheavy mass eigenstates [with masses  $O(M_{\text{GUT}})$ ] we obtain a constrained version of the MSSM. The Higgs boson mass parameter of the superpotential is<sup>2</sup>  $\mu = h_3 n_3 - h_\Phi a$  and the quark and lepton Yukawa couplings are given by

$$\begin{aligned}
y_{ij}^u &= y_U \mathcal{U}_{3i}^u \mathcal{U}_{4j}^u + y_U \mathcal{U}_{4i}^u \mathcal{U}_{3j}^u, \\
y_{ij}^d &= y_D \mathcal{U}_{3i}^d \mathcal{U}_{4j}^d + y_D \mathcal{U}_{4i}^d \mathcal{U}_{3j}^d, \quad i, j = 1, 2, 3. \\
y_{ij}^e &= y_D \mathcal{U}_{3i}^e \mathcal{U}_{4j}^e + y_D \mathcal{U}_{4i}^e \mathcal{U}_{3j}^e,
\end{aligned} \tag{9}$$

This equation yields the following texture for the down Yukawa matrix,

$$y^d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & y_D s_d s_d' \\ 0 & y_D s_q s_d' & -y_D s_q c_d' c_d - y_D s_d c_q' c_q \end{pmatrix}, \tag{10}$$

<sup>1</sup>Supersymmetric theories often have multiple degenerate minima with  $\mathcal{V} = 0$ . Here we simply pick the minimum that is phenomenologically viable.

<sup>2</sup>The fine-tuning required in SUSY GUT in order to obtain  $\mu \lesssim 1$  TeV is a well-known problem [17] and we shall not attempt to solve it within the framework of the model considered here.

and analogous expression for  $y^u$  and  $y^e$ . From here the correct  $m_t/m_c$  ratio is obtained by requiring that  $s'_q/c_q = 2\sqrt{m_c/m_t} \simeq 1/6$ . All other quantities ( $s_q, s_d$ , etc.) can be of order one and require no fine-tuning (the ratio  $m_s/m_b$  is already suppressed by  $s'_q/2$ ). From grand unification we obtain  $c'_d = c'_l, c''_d = c''_l, c'_q = c'_u = c'_e$  and  $c''_q = c''_u = c''_e$ . However, if the  $U(1)_h$  breaking scale is not much above the GUT scale we find in general that  $c_d \neq c_l$  and  $c_q \neq c_u \neq c_e$ . As a result, the eigenvalues of  $y^d$  and  $y^e$  are independent. However, it is clear that this mechanism will affect the unification of the third generation quark and lepton masses less than of the second generation since the latter are given by a product of the two off-diagonal divided by the diagonal matrix elements in Eq. (10). This possibility is suppressed by powers of  $\lambda_f a/M_f$  and will be ignored in the following discussion.

In this model, the KM matrix  $V$  is very close to unity (i.e., the only nonzero off-diagonal matrix elements are  $|V_{cb}|, |V_{ts}| \lesssim \sqrt{m_c/m_t}$ ). Thus, at the tree level this model is in good agreement with the masses and mixing angles of the second and third generation. However, we still need masses and mixing angles for the first generation.

In any realistic low energy model supersymmetry (SUSY) must be broken. This breaking is assumed to occur in a "hidden" sector. Gravitational coupling then induces explicit soft SUSY-breaking terms at a scale  $m \gtrsim m_z$  into the "visible" sector. In minimal  $N=1$  supergravity models those terms are [18]

$$\mathcal{V}_{\text{soft}} = (AmW_3 + BmW_2 + \text{H.c.}) + m^2\phi^\dagger\phi, \quad (11)$$

where  $W_2(W_3)$  is the quadratic (trilinear) part of the superpotential [Eq. (1)].  $A$  and  $B$  are dimensionless constants of order one and  $m$  is the universal SUSY-breaking mass parameter for all the scalars  $\phi$ . If we minimize the potential including  $\mathcal{V}_{\text{soft}}$  we find in general, that the  $D$  term with  $D_h = O(m) \neq 0$  give rise to additional squark mass terms. Other squark mass terms are derived from the  $A$  and  $B$  terms:

$$\mathcal{V}_{\text{soft}} = Am \sum_{f,i} M_f v_{fi} \tilde{f}_i \tilde{f}' + Bm \sum_f M_f v_{f4} \tilde{f}_4 \tilde{f}' + \text{H.c.}, \quad (12)$$

where the summation is over  $f = q, u, d, l, e$  and  $i = 1, 2, 3$ . If we decouple the superheavy states, we find the sfermion mass matrices below  $M_{\text{GUT}}$ :

$$m_{ij}^f = m^2 \delta_{ij} + m_h^2 (U_{1i}^f U_{1j}^f - U_{2i}^f U_{2j}^f) + m_A^2 U_{4i}^f U_{4j}^f, \quad (13)$$

where  $m_h^2 \equiv g_h^2 D_h / 2$  and  $m_A^2 \equiv -(A-B)^2 m^2$ . Clearly, the mass matrices in Eq. (13) are not diagonal in flavor space. The low energy mass parameters are obtained by renormalization group evolution from  $M_{\text{GUT}}$  to  $m$ . While this running of the mass parameters will cause a rather significant splitting of the squark and slepton masses, it will only have a very moderate effect on the off-diagonal sfermion mass matrix elements. In the following discussion, we will assume that these off-diagonal mass matrix elements at the scale  $m$  are of the same order

as at  $M_{\text{GUT}}$ . With these off-diagonal matrix elements we can generate the masses for the fermions of the first generation at the one-loop level [14,13]. We will now consider the case of large  $\tan\beta$ . Here the dominant contributions to the down-type masses arise come from the radiatively generated Yukawa couplings to  $H_2$  (see Fig. 1)<sup>3</sup>

$$m_{ij}^d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & m_{s0} & 0 \\ 0 & 0 & m_{b0} \end{pmatrix} + M_{RC} \begin{pmatrix} \frac{m_{3i}^{q2} m_{3j}^{d2}}{6m^4} & \frac{m_{3i}^{q2}}{3m^2} \\ \frac{m_{3j}^{d2}}{3m^2} & 1 \end{pmatrix}, \quad (14)$$

where  $i, j = 1, 2$ . An analogous equation can be derived for the up quark mass matrix. Here we have defined

$$M_{RC} \equiv m_b f_{RG} \tan\beta \frac{\alpha_s}{3\pi} \frac{\mu m_g}{m^2} = \tan\beta \times (50-100 \text{ MeV}), \quad (15)$$

where  $\alpha_s$  is the strong coupling constant. The gluino and the Higgs mass parameters are  $m_{\tilde{g}}, \mu = O(m)$ . The subscript 0 indicates the unrenormalized quark masses. Note that in order to obtain the coupling constants at the scale  $m$  we have to run  $m_b$  to  $m$  and then run the radiatively generated masses from  $m$  to the corresponding masses. This is taken into account by the renormalization group factor  $f_{RG} \equiv 1-2$ . Since in Eq. (14) we have five parameters ( $M_{RC}$  and  $m_{3i}^{f2}, i=1,2$  and  $f=q,d$ ) to fix four experimental quantities (two quark masses and two mixing angles) we cannot make any predictions. Nonetheless, an order of magnitude calculation is at place in order to see whether any additional fine-tuning is required. First, for the ratio of up to down quark mass we find within our approximation

$$\frac{m_u}{m_d} = \frac{m_t}{m_b \tan\beta} \frac{Am}{\mu} \frac{m_{31}^{q2}}{m_{31}^{d2}}. \quad (16)$$

From here we see that the large top mass enhancement of

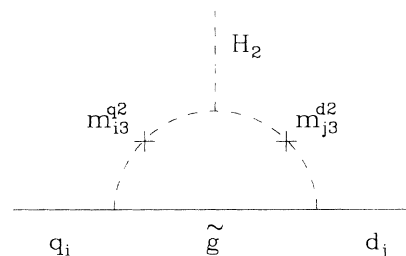


FIG. 1. The dominant contributions to the radiatively generated fermion masses.

<sup>3</sup>In this scenario the second Higgs doublet mediates flavor-changing neutral current (FCNC) and thus has to be heavier than  $\sim 1$  TeV.

the up quark mass can easily be overcompensated by a large value of  $\tan\beta$  and the ratio  $m_{31}^{q2}/m_{31}^{d2} = O(s'_q)$ . This explains why  $m_u$  is smaller than  $m_d$  [19].

The off-diagonal squark mass matrix elements are strongly constrained from above by FCNC processes such as neutral meson mixing [20] and  $b \rightarrow s\gamma$  decay [21]. On the other hand, we can derive lower bounds on the off-diagonal squark matrix elements by imposing the experimental constraints on the KM matrix. From requiring that

$$m_{12}^d = \tan\beta \frac{m_{31}^{q2} m_{32}^{d2}}{6m^4} \times 100 \text{ MeV} \approx |V_{dc}| m_s, \quad (17)$$

we can derive a constraint for the largest off-diagonal matrix elements:

$$\frac{m_{31}^{q2} m_{32}^{d2}}{m^4} \approx \frac{2}{\tan\beta}. \quad (18)$$

This imposes a lower limit on the squark masses of  $m \gtrsim$  few TeV for  $\tan\beta \gtrsim 20$  [9,22]. Note, however, that these are only crude order of magnitude constraints that can easily be avoided since the squark mass matrices have to be expressed in the basis where the radiatively corrected quark mass matrices [Eq. (14)] are diagonal. In addition, the constraints on  $m_{12}^{q2}/m^2$  and  $m_{12}^{d2}/m^2$  coming from  $K-\bar{K}$  mixing is roughly an order of magnitude stronger than the constraints on  $m_{13}^{q2}/m^2$  and  $m_{13}^{d2}/m^2$ . While these matrix elements are irrelevant for the radiative generation of the quark masses [Eq. (14)] it would require some fine-tuning to suppress them enough such that the experimental bounds are satisfied without a significant increase in  $m$ .

Note that even in the absence of the off-diagonal squark mass matrix elements the radiative corrections in Eq. (14) will be very important. Consider, e.g., the case of  $\tan\beta=30$  and the natural value for  $m_g \mu/m^2 = -1/4$  obtained by imposing radiative breaking of the  $SU(2)_L \times U(1)_Y$  gauge symmetry [23]. The the value of  $m_t$  obtained from the  $\tau$  and bottom unification [24,5,11,12] will be lowered by  $\sim 25$  GeV and thus is in much better agreement with current data from high precision experiments [25].

As stated above, in the low energy limit we obtain the MSSM with a particular set of parameters. The most characteristic but very hard to verify property of the

model is the fact that the first generation down-type fermions effectively obtain their mass from one-loop-induced coupling to  $H_2$ . As a result, the  $A^0 d\bar{d}$  coupling is proportional to  $\cot\beta$  instead of  $\tan\beta$ . From the prediction for the radiatively generated electron mass [19] we obtain predictions for the ratio:

$$\frac{m_\tau}{m_\bar{b}} \approx \left[ \frac{\frac{1}{2}\alpha_{\text{em}}}{\frac{4}{3}c_w^2\alpha_s} \frac{m_{\bar{b}}}{m_g} \frac{m_\tau m_d}{m_b m_e} \right]^{1/6} \approx \frac{1}{2}, \quad (19)$$

where the  $\alpha_{\text{em}}$  is the fine structure constant,  $c_w \equiv m_w/m_z$  and we have eliminated the gaugino mass parameter by imposing the GUT prediction  $m_g/m_{\bar{b}} = 3\alpha_s c_w^2/(5\alpha_{\text{em}})$ . This predicted value for the sfermion mass ratio, which is of course modified quantitatively by  $L/R$  splitting, electroweak corrections, etc., is quite natural within the framework of a radiatively broken electroweak gauge symmetry [24] and can be tested if SUSY partners will be found at a few TeV colliders.

While the total lepton number in this model is conserved, there are one-loop-induced  $e, \mu$ , and  $\tau$ -number-violating processes such as  $\mu \rightarrow \gamma e$  and  $\tau \rightarrow \gamma e$ . The resulting lower limits on the slepton masses are well below 1 TeV [26].

We have calculated the dominant  $\tan\beta$  radiative corrections to the fermion masses in supersymmetric theories. We find that they will have a strong impact on the unification of  $\tau$  and bottom Yukawa couplings in SUSY-GUT theories. An example of a theory is presented where the fermion masses of the first generation are generated via these corrections. In this model, the tree-level Yukawa matrices [Eq. (10)] have a simplified version of the "Fritzsch texture" [2] so that a large ratio of mass eigenvalues for the fermions of the second and third generation is natural. Furthermore, the relations that  $m_e/m_d \ll m_\mu/m_s$  and  $m_u < m_d$  are a natural consequence of our model.

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