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Heavy Quark Physics in Photo- and Leptoproduction Processes at HERA and Lower Energies[1]

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Abstract

The electron-proton collider HERA became operational in early 1992 and first results on the photoproduction cross sections and deep inelastic scattering have been published by the H1 and ZEUS collaborations. The present integrated HERA luminosity is not quite sensitive to the heavy quark physics but this is expected to change soon. Eventually, heavy quark physics will develop into one of the most exciting areas of physics research as the HERA luminosity increases and reaches its designed value. In addition to the mainstream ep collisions, which we mainly address here, there also exists the exciting possibility to use the proton beam halo and undertake a fixed-target experiment at HERA, the main interest of which lies in dedicated studies of the beauty and charm physics. We review here selected aspects of the production, decay and detection of heavy quarks at HERA and at lower energy fixed target photo- and hadroproduction experiments. A good fraction of these lectures is devoted to elucidating the production mechanisms. The topics taken up in this context include estimates of the open and bound heavy quark (charm and beauty) production cross sections, heavy quark content of the nucleon, some selected final states involving heavy quarks and their use in determining the gluon density of the proton at HERA energies. In doing this, we also show comparisons of the existing theoretical framework with the fixed target photoproduction results. We also review some selected aspects of the charmed and beauty hadron decays and mixing; in particular, rare D- and B-hadron decays and particle-antiparticle mixings $D^0 - \overline{D}^0$ and $B_s^0 - \overline{B}_s^0$ are emphasized. Measurements of some of these transitions would mark precision tests of the Standard Model in the flavour sector and in some cases may signal physics beyond the Standard Model.

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1 Introduction

Heavy quark physics in lepto- and hadroproduction processes has a tortuous history! Apart from the large-$p_\perp$ domain, these processes are not yet under the complete quantitative control of QCD. A confluence of imprecise experiments in this area (in particular from the early vintage fixed target experiments) and a not yet fully developed QCD framework is responsible for this state of affairs. It is not uncommon to find large relative normalization factors among experiments themselves and in comparison with QCD-based theory (often referred to as K-factors). In many cases, it is not even clear how to use perturbative QCD methods to explain heavy quark production. This, for example, is the case with diffractive production mechanisms. Likewise, in some theoretical interpretations bigger roles are ascribed to the higher-twist contributions and the so-called intrinsic heavy quark content of the nucleon. There exist several *prima facie* charges on the "evidence" of the present data against the usual leading-twist QCD estimates of heavy quark production off a nucleon, though neither the data in question nor the encumbered phenomenology is resting on solid footing.

These circumstances by themselves make it *mandatory* for experiments at HERA to understand lepto- and photoproduction mechanisms of heavy quarks. In addition, with the estimated cross sections $\sigma(ep \to c\bar{c}X) = O(1\ \mu b)$ and $\sigma(ep \to b\bar{b}X) = O(10\ nb)$ and its high energy $\sqrt{s} = 314\ GeV$, HERA provides increased sensitivity in hitherto undiscovered heavy hadrons, particle-antiparticle mixings invoving the neutral heavy mesons, and rare decays of the charmed and beauty hadrons. Consequently, dedicated studies of heavy quark physics at HERA in both the ep and fixed target pA modes (having higher cross sections) have the potential of paying very high dividends. We discuss some of the potentially promising aspects of heavy quark physics here updating the present limits, wherever available, from the ongoing lower energy experiments.

The experimental physics programme at HERA is well under way. Apart from the photoproduction processes [1]-[3] and truely deep inelastic events [4]-[8], including first measurements of the proton structure function, $F_2(x, Q^2)$, and jets, also diffractive processes [9], as well as, elastic photoproduction of vector mesons [10] have been measured at HERA. All these classes of events have their specific heavy quark content, though no published results are yet available on this aspect of the HERA data. As discussed by several speakers at this meeting [11]-[13], experiments at HERA will measure quite precisely the proton structure functions, $F_2(x, Q^2)$ and $F_L(x, Q^2)$, over a wide range in (x, Q^2). Studies of final states involving heavy quarks will allow to measure the charm and beauty quark content of these structure functions. Such measurements will be very instructive since theoretical opinions about the heavy quark content of the nucleon and the contribution of the leading-twist operators in the evolution of this content are rather divided.

Heavy quark physics at HERA has been studied at some length in the context of two HERA workshops at DESY [14]-[18], and elsewhere [19]-[25]. We review and update these studies. In discussing the inelastic production of heavy quarks, we shall discuss only the leading-twist QCD contributions to the photo- and electroproduction processes and take into account the considerable theoretical progress made in the last several years concerning

the next-to-leading order corrections, as well as, the renormalization group resummation of the large logarithmic terms. The diffractive processes are likely to be interpreted as due to the pomeron exchange. Such processes will also contribute to heavy quark production through the mechanism $\gamma + \mathbb{P} \rightarrow Q\bar{Q} + X$ [26]. The theory of such diffractively produced quarks is both model dependent and not quite developed yet. We will discuss some model estimates for this class of events here [18, 20].

We shall not entertain top quark physics at HERA in these lectures. The reason is by now obvious: The existing lower bound on the top quark mass m_t from $p\bar{p}$ collisions obtained by the CDF collaboration at Fermilab, giving $m_t > 113\ GeV$ [27], and indirect estimates from the precision electroweak data at LEP for which a recent compilation quotes $m_t = 147^{+23+19}_{-27-22}$ GeV [28], have pushed away the top quark frontier practically beyond the reach of HERA. This statement can be quantified by noting that at HERA (with $\sqrt{s} = 314$ GeV), the inclusive top quark production cross section is estimated to be $\sigma(ep \rightarrow tX) \leq 0.02\ pb$ for $m_t \geq 100$ GeV [19]. Cross sections and top quark search strategies at the proposed higher energy ($\sqrt{s} = 1.3$ TeV) LEP + LHC ep collider at CERN can be seen in [29].

These lectures are organized as follows. In section 2, we review the next-to-leading order QCD perturbation theory applied to the inelastic photo- and electroproduction of heavy quarks. Section 3 contains comparisons of this framework with the data from low energy fixed-target photproduction experiments. Estimates of the electroproduction of charm and beauty quarks at HERA are reviewed in section 4, which also contains estimates of the heavy quark content of the nucleon at HERA. The $K_\perp$-factorization method to resum the large logarithms in the photproduction and electroproduction cross sections are discussed in section 5. In section 6, we discuss the determination of the gluon density using open $(D, D^*, ...)$ and bound $(J/\psi, ...)$ charmed hadron production in the inelastic production processes. We also discuss various diffractive and elastic J/ψ-production processes at lower energies and HERA, making use of the existing phenomenological models. Some selected aspects of the precision physics in charmed hadron decays and $D^0 - \overline{D}^0$-mixing are reviewed in section 7. With the Standard Model rates in many of the rare decays and mixing being negligibly small, we illustrate the induced effects of leptoquarks which may enhance these transitions greatly. In section 8, we summarize the main interest in B-physics in ep collisions at HERA. The potential of a fixed target experiment at HERA is also mentioned here.

2 QCD Perturbation Theory for Inelastic Photo- and Electroproduction of Heavy Quarks

2.1 Premise

It is expected that the cross sections for $ep \rightarrow QX$ for $Q = c, b$ at HERA, are dominated by the photoproduction processes, with the photon being almost real ($Q^2 \simeq 0$). We shall, therefore, concentrate on the photon-induced heavy quark production processes. The contributions from the weak processes have been worked out in HERA studies and in ref. [19]

but since their contribution to the total cross section is rather small and we will not discuss the issue of the Cabibbo-Kobayashi-Maskawa (CKM) [30] matrix element determination from the charged current (CC) processes at HERA, we will not consider the $W^{\pm}$- and Z^0-mediated processes in these lectures.

We maintain in these lectures that the renormalization group improved leading-twist contributions to the leptoproduction of heavy quarks at HERA will be adequate in accounting for the rates and distributions of heavy quarks, produced inelastically. This optimism is partly based on the satisfactory comparison of the next-to-leading order QCD-calculations [21, 22], which have been used in the interpretation of the data from low energy fixed target charm photoproduction experiments. It should, however, be pointed out that some dissenting notes on this issue exist in literature. In particular, the data on leptoproduction of charm reported quite some time ago by the EMC collaboration [31] seem to imply a much harder (Feynman-x) x_F- distribution than is expected from the photon-gluon fusion mechanism [21] or what has been measured in a number of photoproduction experiments [32, 33, 34]. Not entirely unrelated is the observation made by the E769 collaboration [35] that the Feynman-x_F distributions of the $D^{\pm}$ measured in the π^--Nucleon interactions using a 250 GeV π^- beam incident on the Be, Al, Cu, and W targets are much harder than expected from the next-to-leading order charm hadroproduction calculations [36], after folding in the fragmentation function effects on the partonic calculations. While the debris of the left-over nucleon may influence this distribution and rescue the leading-twist QCD framework, yet it remains a puzzle why such effects are not apparently required by the photoproduction experiments.

The mentioned x_F-distributions, coupled with the surprisingly large rates of open and hidden charm production observed [38], tend to provide some degree of credibility to the proposition of a rather significant intrinsic charm quark content of the nucleon [39]. In this context new QCD production mechanisms for hard processes at large-x, namely peripheral processes in which low-x spectator quarks interact coherently, have been proposed recently [40]. Phenomenological studies of the event topology in the charm electroproduction at HERA, based on an assumed contribution of the intrinsic-charm content of the proton, and ways to distinguish such events from the leading-twist photon-gluon fusion based processes at HERA have been carried out recently by Ingelman et al. [41], to which we refer the interested reader for details. Many of the issues mentioned in the preceding paragraph will be addressed at HERA through the electroproduction of charmed and beauty quarks. These issues will also be addressed quantitatively after the analysis of the ongoing fixed target charm hadroproduction experiments E791 and E831 at Fermilab become available.

2.2 Kinematics and Notations

We begin with the definition of the standard notation and kinematics. The neutral current deep inelastic scattering (DIS) is represented by the process:

$$e(k) + P(p) \rightarrow e'(k') + X(p_X), \tag{1}$$

where the momenta of the incoming and ougoing electron and hadron(s) have been specified. The kinematics of DIS is usually discussed in term of the standard variables

$$\begin{aligned} Q^2 &= -(k'-k)^2, \\ x &= \frac{Q^2}{2pq}, \\ y &= \frac{2pq}{2pk}, \\ s &= (p+k)^2. \end{aligned} \tag{2}$$

The two scaling variables are related through:

$$Q^2 = xys \approx 4xyE_eE_p, \tag{3}$$

where s is the square of the total centre-of-mass energy in the ep system and E_e and E_p are the energies of the electron and proton beams, respectively. The nominal values of these quantities at HERA are: $E_e = 30\ GeV$, $E_p = 820\ GeV$, $\sqrt{s} = 314\ GeV$. The variable Q^2 denotes the resolution by which the photon "sees" the proton. Likewise, the Bjorken scaling variable x is the fraction of the proton momentum carried by the struck parton. The differential cross section for the neutral current (NC) processes $ep \to e + X$ can be expressed through three structure functions:

$$\begin{aligned} \frac{d^2\sigma_{NC}(e^{\mp})}{dx\,dQ^2} &= \frac{4\pi\alpha^2}{xQ^4}\left[\left((1-y)+\frac{y^2}{2}\right)F_2(x,Q^2) - \frac{y^2}{2}F_L(x,Q^2)\right. \\ &\left. \pm\,(y-\frac{y^2}{2})xF_3(x,Q^2)\right], \end{aligned} \tag{4}$$

where $F_L(x,Q^2) = F_2(x,Q^2) - 2xF_1(x,Q^2)$ is the longitudinal structure function and xF_3 is proportional to the propagator $P_Z = Q^2/(Q^2+M_Z^2)$. At a given x, the upper limit of Q^2 is : $Q^2 \leq 4xE_eE_p$. At low x values ($x < 10^{-2}$), the Q^2 range is then below $1000(GeV)^2$, a domain where the contribution of xF_3 can be safely neglected [43]. The differential cross section at low x depends therefore only on two structure functions F_2 and F_L.

2.3 Leading and the next-to-leading order contributions

We discuss in this section photoproduction of heavy quarks in the leading order in the QED coupling constant α and in the leading and next-to-leading order in the QCD coupling constant α_S. The case of electroproduction is always treated in the Weizsäcker-Williams (WW) equivalent photon approximation, if not stated otherwise. Hence, we shall mainly concentrate on the process

$$\gamma + g \ \to \ Q + \bar{Q}. \tag{5}$$

In the next to leading order in α_S, the following processes also contribute

$$\begin{aligned} \gamma + g \ &\to \ Q + \bar{Q} + g, \\ \gamma + q(\bar{q}) \ &\to \ Q + \bar{Q} + q(\bar{q}). \end{aligned} \tag{6}$$

These processes are referred to as the direct photon processes. In addition, one has also to include the so-called resolved-photon processes

$$\begin{aligned} q + \bar{q} &\rightarrow Q + \bar{Q}, \\ g + g &\rightarrow Q + \bar{Q}, \end{aligned} \tag{7}$$

where one of the incoming partons ($q/\bar{q}$ or g) originates from the proton and the other from the photon. The parton densities in the photon, very much like the parton densities in hadrons, are described by Q^2-dependent distribution functions $f_i^\gamma(x, Q^2)$. In the case of electroproduction, one needs additionally the probability functions $f_i^e(x, Q^2)$ of finding a parton i with fractional momentum x in the electron. These can be obtained by convoluting the WW-function $f_\gamma^e(x)$ (below m_e is the electron mass)

$$f_\gamma^e(x) = \frac{\alpha}{2\pi} \frac{1 + (1-x)^2}{x} \ln \frac{(xs - 4m^2)(1-x)}{x^2 m_e^2}, \tag{8}$$

(with $s = E_{cm}^2$) with the corresponding partonic densities $f_i^\gamma(x, Q^2)$, for which there are several parametrizations available [44]-[47]. A separation of the direct and resolved-photon components is possible if one uses the topology of the heavy quark events. In the case of a highly virtual photon the resolved-photon component disappears and one has a much better theoretical and experimental control over the cross sections. One of the first goals of HERA experiments indeed is to measure the parton densities of the photon and proton. We point out that the data on the inclusive photproduction at HERA already provide a discrimination of the various parametrizations of the photon structure function [1, 2]. Eventually, data on the inclusive photoproduction cross sections and jet-production will pin down the photon structure function at HERA to a large extent. So, we shall summarize here the strategies to determine the gluon density of the proton $G(x, Q^2)$ using open and bound heavy quarks at HERA.

Continuing the discussion of the heavy quark photoproduction processes, we recall that the cross sections for the process $\gamma p \rightarrow Q\bar{Q}X$ can be decomposed as follows:

$$\begin{aligned} \sigma(s) &= \sum_j \int dx\, \hat{\sigma}_{\gamma j}(xs, m^2, \mu^2) f_j^p(x, \mu^2) \\ &+ \sum_{i,j} \int dx_1\, dx_2\, \hat{\sigma}_{ij}(x_1 x_2 s, m^2, \mu^2) f_i^\gamma(x_1, \mu^2) f_j^p(x_2, \mu^2) \end{aligned} \tag{9}$$

with m and μ being the heavy quark mass and the QCD renormalization scale, respectively; $\hat{\sigma}_{\gamma j}$ and $\hat{\sigma}_{ij}$ are the cross sections for the basic partonic processes $\gamma j \rightarrow Q\bar{Q}X$ and $i + j \rightarrow Q + X$, respectively, both of which are calculated in QCD perturbation theory. The functions $f_i^p(x, \mu)$ and $f_i^\gamma(x, \mu)$ in (9) are the parton densities of type i in the proton and photon evaluated at the scale μ, respectively. Following Ellis and Nason [21], we define the following dimensionless auxiliary functions:

$$\hat{\sigma}_{\gamma j} = \frac{\alpha_S(\mu^2)\alpha}{m^2} f_{\gamma j}(\rho, \frac{\mu^2}{m^2}),$$

$$\hat{\sigma}_{ij} = \frac{\alpha_S(\mu^2)^2}{m^2} f_{ij}(\rho, \frac{\mu^2}{m^2}). \tag{10}$$

These functions can be further decomposed into their lowest and next-to-leading order components:

$$\begin{aligned} f_{\gamma j}(\rho, \frac{\mu^2}{m^2}) &= f_{\gamma j}^{(0)}(\rho) \\ &+ g^2(\mu^2)[f_{\gamma j}^{(1)}(\rho) + \bar{f}_{\gamma j}^{(1)}(\rho) \ln(\frac{\mu^2}{m^2})] + O(g^4), \end{aligned} \tag{11}$$

with the varaiables ρ and β defined as $\rho \equiv 4m^2/s$ and $\beta \equiv \sqrt{1-\rho}$. The threshold and high energy regions correspond to the limiting values $(\rho \to 1, \beta \to 0)$ and $(\rho \to 0, \beta \to 1)$, respectively. The particular form of the next-to-leading order contribution given in eq. (11) follows the renormalization group arguments where coefficients of the term $\ln(\mu^2/m^2)$ are determined from the lowest order expressions [21],

$$\bar{f}_{\gamma j}^{(1)}(\rho) = \frac{1}{8\pi^2}[2\pi b_0 f_{\gamma j}^{(0)} - \sum_k \int_\rho^1 dz_1 f_{kj}^{(0)}(\frac{\rho}{z_1}) P_{k\gamma}(z_1) - \sum_k \int_\rho^1 dz_2 f_{\gamma k}^{(0)}(\frac{\rho}{z_2}) P_{kj}(z_2)]. \tag{12}$$

The Gribov-Lipatov-Altarelli-Parisi (GLAP) kernels [48, 49] in the lowest order are given by:

$$\begin{aligned} P_{g\gamma}(x) &= 0, \\ P_{q\gamma}(x) &= 3e_q^2[(x^2 + (1-x)^2], \\ P_{gq}(x) &= \frac{4}{3}[\frac{1 + (1-x)^2}{x}], \\ P_{gg}(x) &= 6[\frac{x}{(1-x)_+} + \frac{(1-x)}{x} + x(1-x)] + 2\pi b_0 \delta(1-x), \end{aligned} \tag{13}$$

where $b_0 = (33 - 2n_f)/12\pi$, with n_f being the number of light quarks. The meaning of $(1-x)_+$ in the above equation is to be understood in the distribution sense; the resulting expression for P_{gg} taking into account the infrared singularities in the virtual corerections to the process $\gamma p \to Q\bar{Q}X$ will be given below.

Since in the next-to-leading order contributions to the photoproduction of heavy quarks the processes where the photon couples to a light quark-antiquark pair also contribute, the electric charge of the heavy quark e_Q^2 no longer factorizes. To take such non-factorizing contributions into account it is useful to define the following functions:

$$\begin{aligned} f_{\gamma g}(\rho, \frac{\mu^2}{m^2}) &\equiv e_Q^2 C_{\gamma g}(\rho, \frac{\mu^2}{m^2}), \\ f_{\gamma q}(\rho, \frac{\mu^2}{m^2}) &\equiv e_Q^2 C_{\gamma q}(\rho, \frac{\mu^2}{m^2}) + e_q^2 d_{\gamma q}(\rho, \frac{\mu^2}{m^2}), \end{aligned} \tag{14}$$

where e_q denotes the charge of the light quark-antiquark pair coupled to the photon. Substituting these definitions in (11), one defines in an analogous fashion the corresponding

functions $C_{\gamma q}^{(0)}$ and $C_{\gamma q}^{(1)}$ etc. In the lowest order, only the function $C_{\gamma g}^{(0)}$ contributes and one has:

$$\begin{aligned} C_{\gamma g}^{(0)}(\rho) &= \frac{\pi\beta\rho}{4}[\frac{1}{\beta}(3-\beta^4)\ln(\frac{1+\beta}{1-\beta})-4+2\beta^2], \\ & C_{\gamma q}^{(0)}(\rho) = d_{\gamma q}^{(0)}(\rho) = C_{\gamma\bar{q}}^{(0)}(\rho) = d_{\gamma\bar{q}}^{(0)}(\rho) = 0. \end{aligned} \tag{15}$$

The expressions for the next-to-leading order functions are given in [21]. In order to take into account the contribution where the incoming photon fragments into a light quark-antiquark pair one needs the quantity $f_{q\bar{q}}^{(0)}(\rho)$,

$$f_{q\bar{q}}^{(0)}(\rho) = \frac{\pi\beta\rho}{27}[2+\rho]. \tag{16}$$

The functions $C_{\gamma q}^{(0)}(\rho)$, $C_{\gamma g}^{(1)}(\rho)$, $\overline{C}_{\gamma g}^{(1)}(\rho)$, are shown in Fig. 1, where they are plotted as functions of the variable $1/\rho = s/4m^2$. At asymptotic eneregies, i.e. as $\rho \to 0$, the lowest order function $C_{\gamma q}^{(0)}(\rho)$ vanishes, as follows from eq. (15) and the heavy quark photoproduction cross section is dominated by the contribution of the gluon exchange in the t-channel. These diagrams are shown in Fig. 2. At high energies, they lead to the following expressions for the coefficient functions $C_{\gamma g}^{(1)}$ and $C_{\gamma q}^{(1)}$ [21]:

$$\begin{aligned} C_{\gamma g}^{(1)} &\to N_C\frac{41}{108\pi} + O(\rho\ln^2\rho), \\ C_{\gamma q}^{(1)} &\to C_F\frac{41}{108\pi} + O(\rho\ln^2\rho). \end{aligned} \tag{17}$$

Here $N_c = 3$ and $C_F = 4/3$, representing the colour factors of the quark-quark-gluon and the gluon-gluon-gluon vertex, respectively, in the lower parts of the diagrams shown in Fig. 2. The behaviour in eq. (17) leads to a constant photoproduction cross section as $s \to \infty$. This is typical of heavy quark production cross sections in QCD involving a hadronic target. Likewise, the functions $\bar{C}_{\gamma g}^{(1)}$ and $\bar{C}_{\gamma q}^{(1)}$ asymptotically tend to constant values; again the ratio $\bar{C}_{\gamma g}^{(1)}/\bar{C}_{\gamma q}^{(1)}$ is given by the ratio of the colour factors $N_C/C_F = 9/4$, as is the case for the ratio of the coefficient functions given in eq. (17).

Before discussing the actual production cross sections at fixed-target photoproduction experiments and at HERA, we shall discuss the mass factorization schemes which are used to remove the collinear singularities.

2.4 Mass factorization schemes

We recall that the photoproduction (as well as leptoproduction) cross sections contain infrared (IR) and collinear divergences. It is customary to use the dimensional regularization method to isolate these singularities, in which case they appear as poles in the quantity $\epsilon \equiv n-4$. The IR singularities cancel upon adding the virtual and real contributions. The collinear divergences don't cancel; however, they are removed through the factorization

Figure 1: *The auxiliary functions* $C_{\gamma g}^{(0)}$, $C_{\gamma g}^{(1)}$ *and* $\bar{C}_{\gamma g}^{(1)}$ *in the photon-gluon fusion process* $\gamma + g \to Q\bar{Q}X$, *plotted as a function of the variable* $1/\rho = s/4m^2$ *(from [21]).*

procedure by absorbing them in the partonic densities. Defining the kinematic variables for the lowest order process

$$\gamma(k_1) + g(k_2) \to Q(p_1) + \bar{Q}(p_2), \tag{18}$$

as

$$\begin{aligned} \hat{s} &= 2k_1.k_2; \quad t_1 \equiv t - m^2 = (k_2 - p_2)^2 - m^2; \\ u_1 &\equiv u - m^2 = (k_1 - p_2)^2 - m^2, \end{aligned} \tag{19}$$

a typical form for a differential cross section in photo- and leptoproduction processes is as follows:

$$\begin{aligned} s'^2 \frac{d^2\sigma_{k,j}(s', t_1, u_1, q^2, \mu^2, \epsilon)}{dt_1 du_1} &= \int_0^1 \frac{dx_1}{x_1} \Gamma_{ij}(x_1, M^2, \mu^2, \epsilon) \\ &\times \hat{s}'^2 \frac{d^2\sigma_{k,i}(\hat{s}', \hat{t}_1, u_1, q^2, M^2)}{d\hat{t}_1 du_1} \end{aligned} \tag{20}$$

where $\hat{s}' = x_1\hat{s}$, $\hat{t}_1 = x_1 t_1$. The Γ_{ij} are the transition functions which refer to the incoming parton j and the outgoing parton i. These transition functions contain the collinear singularities indicated by the poles ϵ and further depend on the mass factorization scale M^2, which is of the order of the other kinematic variables present in the problem, i.e., it is of order s, t_1, u_1, m^2. To see the sheme dependence, it is sufficient to study the $O(\alpha_S)$ term in the expansion of the transition functions Γ_{ij}:

$$\Gamma_{ij} = \delta_{ij}\delta(1-x) + \frac{\alpha_S}{2\pi}[P_{ij}\frac{2}{\epsilon} + f_{ij}(x, M^2, \mu^2)], \tag{21}$$

Figure 2: *Diagrams responsible for the constant behaviour of the photproduction cross section* $\gamma p \to Q\bar{Q}X$.

where P_{ij} denote the GLAP-splitting functions, given above. The scheme dependence of the cross section enters through the functions $f_{ij}(x, M^2, \mu^2)$. In the $\overline{MS}$ scheme, these are defined as follows:

$$f_{ij}^{\overline{MS}}(x, M^2, \mu^2) = P_{ij}(x)(\gamma_E - \ln 4\pi + \ln \frac{M^2}{\mu^2}), \tag{22}$$

where γ_E is the Euler constant. The splitting function $P_{gg}(x)$ is given by:

$$\begin{aligned} P_{gg}(x) &= C_A[\theta(1-x-\delta)\{\frac{2}{1-x} + \frac{2}{x} + 4 + 2x - 2x^2\} \\ &+ \delta(1-x)(2\ln\delta + \frac{11}{6})] - \frac{1}{3}n_f\delta(1-x), \end{aligned} \tag{23}$$

where $C_A = 3$. Comparing this expression with the function P_{gg} given in (13) above, the meaning of the regularization prescription becomes manifest, namely that in the above expression the pole at $x = 1$ has been regulated by the introduction of the regulator δ, which can be expressed in terms of the relevant kinematic variables in the $2 \to 3$ subprocesses in the photoproduction of heavy quarks. Another scheme, which has also been used in the evaluation of the higher order corrections to the cross sections is the so-called DIS scheme. In this scheme, the functions $f_{ij}^{DIS}(i, j = q, g)$ are defined as follows:

$$\begin{aligned} f_{gg}^{DIS}(x, M^2, \mu^2) &= f_{gg}^{\overline{MS}}(x, M^2, \mu^2) - 2n_f f_{qg}^{DIS}(x, M^2, \mu^2), \\ f_{qg}^{DIS}(x, M^2, \mu^2) &= T_f[\{x^2 + (1-x)^2\}\ln\frac{1-x}{x} + 8x(1-x) - 1], \end{aligned}$$

$$
\begin{aligned}
f_{gq}^{DIS}(x,M^2,\mu^2) &= f_{gq}^{\overline{MS}}(x,M^2,\mu^2) - f_{qq}^{DIS}(x,M^2,\mu^2), \\
f_{qq}^{DIS}(x,M^2,\mu^2) &= C_F[\theta(1-x-\delta)\{\frac{1+x^2}{1-x}(\ln\frac{1-x}{x}-\frac{3}{4})+\frac{9}{4}+\frac{5}{4}x\} \\
&+ \delta(1-x)\{\ln^2\delta - \frac{3}{2}\ln\delta - \frac{9}{2} - 2\xi(2)\}]. \qquad (24)
\end{aligned}
$$

Here $\xi(2) = \pi^2/6$ and $T_f = 1/2$. It is a common practice to set the mass factorization scale M equal to the renormalization scale μ, though, in general, they are different parameters.

3 Charm Photoproduction at Low Energies

We discuss open charm production at fixed target experiments carried out at a number of experiments at CERN and Fermilab with the photon energy in the range $50\ GeV \leq E_\gamma \leq 400\ GeV$. Our main interest is to compare the experimental results with the next-to-leading order QCD calculations by Ellis and Nason [21], which have been checked by Smith and van Neerven [22, 23], and which have been extensively used in the interpretation of the photoproduction data. The calculations described above at the partonic level have to be supplemented with the appropriate partonic densities $f_i^\gamma(x,\mu^2)$ and $f_i^p(x,\mu^2)$. In addition, a fragmentation function for the process $c \to D + X$ has to be provided. Ellis and Nason have used the proton structure function parametrization of Diemoz et al. [50] with $\Lambda_4 = 260 \pm 100\ MeV$ being the QCD scale parameter with 4 flavours. The factorization scale was set at $\mu = 2m_c$ and the uncertainty associated with this choice was assumed to be a factor 2. The contribution from the hadronic component of the photon (also called the resolved photon contribution) at these energies has been estimated to be less than 5%, using the parametrization of the photon structure function by Drees and Grassie [44], and hence neglected in comparisons with data. For the charm quark fragmentation function, the Peterson et al. parametrization [51] was used:

$$
D(z) = \frac{1}{z}(1 - \frac{1}{z} - \frac{\epsilon}{(1-z)})^{-2}. \qquad (25)
$$

Apart from μ and Λ, the cross section and related distributions depend on the value of the charmed quark mass, and various collaborations have used their data to determine this important parameter.

A comparison of the next-to-leading order QCD calculation with the charm photoproduction cross sections from a number of experiments is shown in Fig. 3. An adequate description of the data is achieved with a value $m_c = 1.5\ GeV$. The $p_\perp^2$- and x_F-distributions have also been measured in a number of experiments [32, 33, 34]. The x_F-distribution of the D-mesons measured in the photoproduction experiments E687, NA14/2 is shown in Fig. 4 and compared with the fitted distribution of the same by the E691 collaboration in the charm photoproduction off Be target. The three x_F-distributions are in agreement with each other and with the expectations of central production process via the photon-gluon fusion mechanism. The distributions peak at small value of x_F (typically 0.2) and there is no evidence

for any appreciable leading particle effect which would lead to events in the region $x_F \to 1$. The curve shown in Fig. 4 is given by

$$\frac{d\sigma}{dx_F} = A(1 + Bx_F)(1 - x_F)^n, \tag{26}$$

and the E691 fit yields the values $B = 14 \pm 10$ and $n = 2.95 \pm 0.22$. Figs. 5 display the $p_\perp^2$-distribution of the D mesons measured in the photoproduction process off Be-target by the E691 collaboration (Fig. 5a) and off Si-target by the NA14/2 collaboration (Fig. 5b). The curve fitted by the E691 collaboration correspond to the expressions

$$\frac{d\sigma}{dp_\perp^2} \simeq \exp(-bp_\perp^2 - cp_\perp^4) \tag{27}$$

the E691 fit yields the values $b = 1.07 \pm 0.05$, $c = -0.04 \pm 0.01$, giving $< p_\perp^2 >= 1.16 \pm 0.04\ GeV^2$. The measurements by the E687 and NA14/2 collaborations are qualitatively similar, though the E687 collaboration has reported a higher mean $p_\perp^2$ for the charmed hadrons as compared to the E691 measurements. The E687 fit yields the values $b = 0.85 \pm 0.02$, $c = -0.03 \pm 0.01$, giving $< p_\perp^2 >= 1.51 \pm 0.02\ GeV^2$ [42]. The increased value for the $< p_\perp^2 >$ is probably due to the different x_F-values used in the two experiments (the so-called sea-gull effect) and to some extent to the higher photon beam energy of the E687 collaboration. The NA14/2 $p_\perp^2$-distribution is compared with the photon-gluon fusion based theoretical predictions with the indicated values of the charm quark mass, and an assumed $\exp(-6m_T)$ distribution (here m_T is the transverse mass, $m_T = \sqrt{m_c^2 + p_\perp^2}$). The corrected cross sections using the Lund /Dual parton model based monte carlo by E691/(NA14/2) and the Ellis-Nason calculation to correct for the acceptance are as follows:

$$\begin{aligned} \sigma(\gamma p \to c\bar{c}X) &= 0.58 \pm 0.01 \pm 0.06\ \mu b, \quad \text{with} < E_\gamma >= 145\ GeV\ \text{(E691)}\ [32] \\ &= 0.465 \pm 0.053 \pm 0.084\ \mu b, \ \text{with} < E_\gamma >= 100\ GeV\ \text{(NA14/2)}\ [33] \end{aligned} \tag{28}$$

Summarizing, the experimental photoproduction cross sections at fixed-target experiments, and their x_F- and $p_\perp$-distributions, are reasonably well described by the next-to-leading order QCD calculations after including the structure function and fragmentation effects. The measured cross sections have been used to determine m_c, and the resulting values are in good agreement with the ones obtained from potential models and QCD sum rules.

4 Electroproduction Cross Sections for Charm and Beauty Quarks at HERA

With the fixed target charm photoproduction in reasonable agreement with the next-to-leading order QCD calculations, it is an interesting question to ask if this will remain to be the case in the charm and beauty electroproduction at HERA. Apart from the fact

Figure 3: *The total cross section for the photoproduction of a pair of charmed quarks* $\gamma p \to c\bar{c}X$ *versus* E_γ *from a number of fixed target experiments. The curve shown represents the next-to-leading order calculation by Ellis and Nason [21] for* $m_c = 1.5\ GeV$*. (from [42]).*

that leptoproduction brings in an additional structure function as compared to the real photoproduction experiments, the obvious difference is in the big step in energy (and Q^2) in going from the fixed-target experiments to HERA *ep* collisions. For example, the mean photon energy for the E691 data corresponds to the centre of mass energy $< \sqrt{s_{\gamma p}} > \simeq 23.3\ GeV$, to be contrasted with the corresponding mean centre of mass energy $< \sqrt{s_{\gamma p}} > \simeq O(200)\ GeV$ measured in the photoproduction processes at HERA [1, 2].

In the context of the central production of heavy quarks, there are two obvious points that have to be carefully investigated:

- In the heavy quark photo- and leptoproduction processes, the effective perturbation theory parameter is $\alpha_S \ln \rho$ and not α_S by itself. As the centre of mass energy increases, the effective parameter $\alpha_S \ln \rho$ gets bigger and these large logaritmic terms have to be resummed.

- Secondly, as already established by the photoproduction measurements at HERA, the resolved photon contribution is numerically important. This brings in an additional dependence of the heavy quark cross sections on the photon structure function, which was very weak at lower energy photoproduction experiments.

In the comparison of the low energy data shown, the parametrization by Drees and Grassie [44] was used to argue that the resolved photon contribution is negligible. It has been argued in the literature that the inclusive charm electroproduction cross sections at HERA are very sensitive to the assumed photon structure function. So, either one has to use additional

Figure 4: *The x_F-dependence of D-mesons in photoproduction processes off fixed targets measured by the E687 and NA14 collaborations and compared with the corresponding distribution (fitted curve) by the E691 collaboration. The fit is discussed in the text. (from [42]).*

topological cuts to separate the resolved and direct components or else fix the photon structure function from independent measurements of, for example, the inclusive photoproduction cross sections. Luckily, in our opinion, this issue will be decided by photoproduction measurements at HERA much before data on heavy quarks become available. In that context, we remark that both the H1 and ZEUS collaborations have measured the total photoproduction cross section and final states involving jets. The photoproduction cross section and the energy-momentum profile of the final states depend sensitively on the input photon structure function. The large step in the centre of mass energy in going from fixed target experiments to HERA and the final state distributions provide very good discrimination on the various parametrizations of the photon structure function.

The calculations by Ellis and Nason described above yield the following charm photoproduction cross section at HERA [21]:

$$\sigma(ep \to c + \bar{c} + X) = 0.68^{+0.26}_{-0.18}\ \mu b \quad \text{with } m_c = 1.5\ GeV, \tag{29}$$

where the hadronic component of the photon (the resolved photon contribution) has been estimated using the Dress-Grassie photon structure function [44], which yields $< 30\%$ to the electroproduction of charm at HERA energy.

The dependence of the inclusive charm production cross section at HERA on the assumed parametrizations of the photon structure functions has been estimated in ref. [23], from where we show the results in Table 1, where the contributions from the various components are shown separately. The parton densities $f_i^p(x, \mu^2)$ have been fixed by the choice of the

Table 1: Total cross sections in nb for the process $\sigma(ep \to c + \bar{c} + X)$ at HERA ($\sqrt{s} = 314\ GeV$) in the $\overline{MS}$ renormalization scheme, assuming $m_c = 1.5\ GeV$ and the indicated parametrizations for the photon structure functions (from ref. [23]).

-	DG[44]	LAC1[45]	LAC2[45]	LAC3[45]	GRV[46]	GS[46]
$\sigma_{\gamma g}^{(0)}$	358.8	358.8	358.8	358.8	358.8	371.0
$\sigma_{\gamma g}^{(1)}$	216.0	216.0	216.0	216.0	216.0	196.7
$\sigma_{\gamma q}^{(1)}$	25.2	25.2	25.2	25.2	25.2	10.4
σ_{pt}	600.0	600.0	600.0	600.0	600.0	578.1
$\sigma_{gg}^{(0)}$	52.9	370.0	301.9	113.0	48.1	32.6
$\sigma_{q\bar{q}}^{(0)}$	3.2	2.6	3.1	3.3	4.3	2.7
σ_{tot}	656.1	972.6	905.0	716.3	652.4	613.4

Martin-Tung parametrization (set B1), with $\Lambda_4 = 194\ MeV$ and the charm quark mass has been fixed to $m_c = 1.5\ GeV$. The six different parametrizations of the photon structure function $f_i^\gamma(x, \mu^2)$ are taken from Drees and Grassie [44], the three parametrizations due to Abramowicz et al. [45], one from Glück et al. [46], and one from Gordon and Storrow [47]. It is seen that while the estimates for the direct component in all the six parametrizations almost coincide, the resolved-photon component, shown as $\sigma_{gg}^{(0)}$, is much higher in the three parametrizations of Abramowicz et al. (labelled as LAC1, LAC2 and LAC3 in Table 1) as compared to the other three parametrizations.

Judging from the total cross section estimates given in the last row of Table 1, it appears that the charm cross section at HERA is uncertain by 50% due to the photon structure function itself! The actual situation, however, is not as pessimistic. It is sufficient to remark here that the HERA data on the total photoproduction cross section have already ruled out any rapid rise of the cross sections with energy. Some of the LAC-type parametrizations predict a very rapid growth in energy and hence they are very likely not describing the photoproduction data at HERA [1, 2]. This is shown in Fig. (6). From this comparison it is not unreasonable to conclude that one should not expect any dramatic rise in the charm quark photoproduction cross section either. Very similar conclusions are reached on the jet-$E_\perp$ and related distributions from photoproduction measurements at HERA. The present data are not good enough to determine the photon structure function. However, this is expected to change in not too distant future removing an irritating source of uncertainty in the estimates of the cross sections for the photoproduction of heavy quarks. In conclusion, whereas the estimates of the charm production cross section are not completely quantitative, the inherent uncertainty due to the photon structure function parametrizations is not expected to be overwhelming–thanks to the valuable information from the photoproduction measurements already carried out at HERA.

We now turn to the estimates of the b-quark cross sections at HERA. Using the values $\Lambda_{\overline{MS}}(n_f = 4) = 260 \pm 100\ MeV$, the factorization scale μ in the range $m_b/2 < \mu < 2m_b$, the b-quark in the mass range $4.5\ GeV < m_b < 5.0\ GeV$, and the Eichten et al. parametrization

Table 2: Partial and total cross sections in nb for the process $\sigma(ep \to b + \bar{b} + X)$ at HERA ($\sqrt{s} = 314\ GeV$) in the DIS and $\overline{MS}$ renormalization schemes, assuming $m_b = 4.75\ GeV$. The values of α_S are given in the first row (from ref. [22]).

-	-	DIS	-	-	$\overline{MS}$	-
-	$Q = m_b/2$	$Q = m_b$	$Q = 2m_b$	$Q = m_b/2$	$Q = m_b$	$Q = 2m_b$
α_S	0.237	0.190	0.162	0.237	0.190	0.162
$\sigma_{\gamma g} O(\alpha_S)$	4.94	4.60	4.36	4.30	4.16	4.02
$\sigma_{\gamma g} O(\alpha_S^2)$	1.92	1.62	1.72	1.71	1.54	1.68
$\sigma_{\gamma q} O(\alpha_S^2)$	0.92	0.15	-0.21	0.98	0.20	-0.17
$\sigma_{qq} O(\alpha_S^2)$	0.14	0.12	0.11	0.15	0.12	0.11
$\sigma_{gg} O(\alpha_S^2)$	0.85	0.64	0.53	0.73	0.57	0.48
σ^{tot}	8.77	7.13	6.51	7.87	6.59	6.12

of the structure functions of the proton [59], Ellis and Nason obtained $\sigma(ep \to b\bar{b}X) = (6.0 \pm 1.1)nb$ at $\sqrt{s} = 314\ GeV$ [21]. The contribution of the hadronic component of the photon (the so-called resolved-photon contribution) to the beauty quark cross section has been estimated to be less than 30 % at HERA energies [21, 60], using the photon densities from [44].

The order $(\alpha\alpha_S^2)$ calculations for the photoproduction of heavy quarks by Ellis and Nason[21] have been checked and extended by Smith and van Neerven [22]. The results for the cross section $\sigma(ep \to eb\bar{b}X)$ at HERA ($\sqrt{s} = 314\ GeV$) in the DIS and $\overline{MS}$ renormalization schemes, assuming $m_b = 4.75\ GeV$ and $\Lambda(n_f = 4) = 0.212\ GeV$ are given in Table 2, where the individual contributions are also listed. The results from this table can be summarized as follows:

$$\begin{aligned} \sigma(ep \to b + \bar{b} + X)(\mathcal{O}(\alpha)) &= 4.60^{+0.34}_{-0.24}\ nb. \\ \sigma(ep \to b + \bar{b} + X)(\mathcal{O}(\alpha^2); \mathrm{DIS}) &= 7.13^{+1.64}_{-0.62}\ nb. \\ \sigma(ep \to b + \bar{b} + X)(\mathcal{O}(\alpha^2); \overline{MS}) &= 6.59^{+1.28}_{-0.47}\ nb. \end{aligned} \tag{30}$$

The central values correspond to the factorization scale set equal to $\mu = m_b$, and the upper and lower numbers represent the shift in $\sigma(ep \to b + \bar{b} + X)$ if one sets $\mu = m_b/2$ and $\mu = 2m_b$, respectively.

The cross sections quoted above have been obtained in the next-to-leading order perturbative QCD and they should be quite adequate at not-too-high energies, $s > m^2 \gg \Lambda^2$, where m is the generic heavy quark mass. However, there are good reasons to believe that at least for charm electroproduction at HERA, the cross section should be appreciably higher due to the resummation of the large logarithms, encountered in the calculations of the fixed order perturbation theory. There now exist a well defined theoretical framework in which the renormalization group has been used to do this resummation. We shall discuss this later.

4.1 Charm and Beauty Quark Content of the Proton

We now discuss the heavy quark content of the proton obtained by taking into account the leading-twist perturbative QCD contributions, evaluated in next-to-leading order in α_S. Following refs. [24] and [65], we define the light-quark structure functions as:

$$F_k(x,Q^2) \equiv \sum_i \int_{\xi_{min}}^1 \frac{d\xi}{\xi} f_i(\xi,M^2) \mathcal{C}_{k,i}(x/\xi,Q^2,M^2) \tag{31}$$

whereas the heavy-quark ones are defined as:

$$F_k(x,Q^2,m^2) \equiv \sum_i \int_{\xi_{min}}^1 \frac{d\xi}{\xi} f_i(\xi,M^2) \mathcal{F}_{k,i}(x/\xi,Q^2,M^2) \tag{32}$$

where $\xi_{min} = x(4m^2+Q^2)/Q^2$, the index $k = 2, L$ denotes the structure functions F_2 and F_L, and the index i ranges over the partons (quarks, antiquarks and gluons). In the above expressions M denotes the mass factorization scale, and f_i are the partonic densities. The light- and heavy-quark Wilson coefficients $\mathcal{C}_{k,i}$ and $\mathcal{F}_{k,i}$ have been computed in ref. [65] and [24], respectively. The results shown below are taken from these calculations and they correspond to using the so-called $\overline{MS}$ scheme. The renormalization scale μ, which enters also through the QCD running coupling constant in the perturbative expensions of the coefficient functions, has been set equal to the mass factorization scale M.

It is convenient to define the following ratios $K_k^{(i)}$,

$$K_k^{(j)}(x,Q^2,m^2) = \frac{F_k^{(j)}(x,Q^2,m^2)}{F_k^{(j)}(x,Q^2,m^2) + F_k^{(j)}(x,Q^2)}. \tag{33}$$

They should be interpreted as measures of the (perturbative) heavy-quark content of the nucleon, expressed as a percentage of the total quarkonic-parton content of the nucleon. In other words, the gluonic content of the nucleon is not included in the normalization of $K_k^{(j)}(x,Q^2,m^2)$. The superscript j indicates the order in α_S to which the coefficient functions $\mathcal{C}_{k,i}$ and $\mathcal{F}_{k,i}$ have been calculated. The results for the ratios of the structure functions K_2 and K_L, obtained for the choice $m_c = 1.5$ GeV and $m_b = 4.75$ GeV are shown in Fig. (7) and Fig. (8), respectively. The results shown here correspond to using the Morfin-Tung parametrization (MT) of the parton densities, taking the fit B_1 in the $\overline{MS}$ scheme, with $\Lambda_4 = 0.194$ GeV. It should be pointed out that the gluon content of the nucleon is influenced by the sum of the quarkonic contributions due to the momentum sum rule. In actual determination of the heavy quark structure functions, one should use the best fit for the gluon densities, or else fit all the parton densities iteratively using several pieces of data. An important feature of these results is the large value of the ratio $K_2(x,Q^2,m^2)$ at small x. Thus, the currently available parametrizations for the light-parton densities yield a large perturbative QCD heavy quark content in the nucleon, though this content is small for large-x, a feature typical of the photon-gluon fusion mechanism. Deviations from the behaviour depicted here will point to important non-leading-twist effects, for example the ones advocated by Brodsky et al [39].

5 Resummed Heavy Quark Cross Sections at HERA

It is known that at high energies (multiple) gluon exchanges give rise to large factors $\ln^n \rho$ (with $\rho \equiv 4m^2/s$) in the cross sections of interest. In the limit $\rho \to 0$, the leading logarithms can be summed to all orders by using the transverse-momentum factorization of the short- and long-distance physics (i.e., the hard-vertex and the structure function terms) [25, 61, 62]. Concentrating on the γg-induced processes this factorization can be expressed as:

$$4m^2\sigma_{\gamma g}\left(\rho, m^2\,; Q_0^2\right) = \int d^2k_\perp \int_0^1 \frac{dz}{z} x\hat{\sigma}\left(\rho/z, k_\perp^2/m^2\right) \mathcal{F}\left(z, k_\perp, Q_0^2\right),$$

where $\hat{\sigma}$ is the cross section for $\gamma + g(k) \to Q\bar{Q}$ involving an off-shell gluon. The function $\mathcal{F}(z, k_\perp, Q^2)$ is defined as the (unintegrated) structure function of the gluon with the longitudinal fractional momentum z and transverse momentum $k_\perp$ at the scale Q^2. The usual gluon structure function follows from doing the $k_\perp$-integration:

$$G(z, m^2, Q^2) \equiv \int_0^{m^2} d^2k_\perp \mathcal{F}(z, k_\perp, Q^2). \tag{34}$$

The resummed result can be most easily obtained in terms of the moments of the cross section in the variable ρ. Denoting these moments by σ_N, one can easily show that they satisfy

$$\begin{aligned} & 4m^2\sigma_N(m^2; Q_0^2) \\ = \;& C_N(\alpha_S(m^2))\, G_N(m^2; Q_0^2)\,, \end{aligned} \tag{35}$$

where,

$$G_N(m^2; Q_0^2) = \exp\left[\int_{Q_0^2}^{m^2} \frac{dk^2}{k^2}\, \gamma_N(\alpha_S(k^2))\right]. \tag{36}$$

The BFKL anomalous dimension $\gamma_N(\alpha_S)$ appearing in the structure function G_N [53] and the process-dependent coefficient function $C_N(\alpha_S)$ are computed in fixed order perturbation theory. In particular, $\gamma_N(\alpha_S)$ is the perturbative solution of the implicit equation ($\bar{\alpha_s} \equiv C_A\alpha_s/\pi$)

$$\begin{aligned} & 1 = \frac{\bar{\alpha_s}}{N} f(\gamma_N)\;\;, \\ & f(\gamma) \equiv 2\psi(1) - \psi(\gamma) - \psi(1-\gamma), \end{aligned} \tag{37}$$

where ψ is the Euler ψ-function. The lowest order perturbation theory expansions for the BFKL anomalous dimension and the function C_N for heavy flavour photoproduction are known [21]:

$$\begin{aligned} \gamma_N(\alpha_s) = \frac{C_A\alpha_s}{\pi N} + 2\zeta(3)\left(\frac{C_A\alpha_s}{\pi N}\right)^4 + \\ + O\left(\left(\frac{\alpha_s}{N}\right)^6\right)\;,\; (N \to 0)\,, \end{aligned} \tag{38}$$

$$C_N(\alpha_s) = \frac{28}{9}\pi\alpha\alpha_s e_Q^2 \left(1 + \frac{41}{21}\frac{C_A\alpha_s}{\pi N}\right) +$$
$$+O\left(\left(\frac{\alpha_s}{N}\right)^2\right) \ , \ (N \to 0) \ . \tag{39}$$

The resummed results for the photoproduction of heavy quarks follows if one replaces the coefficient function moments $C_N(\alpha_S(m^2))$ by $h_N(\gamma_N)$ [25]:

$$4m^2 \ \sigma_{\gamma g,N}(m^2; Q_0^2)$$
$$= h_N(\gamma_N) \ G_N(m^2; Q_0^2) \ , \tag{40}$$

where h_N, the resummed coefficient function, is given by the $k_\perp$-transform of the "hard vertex" cross section

$$h_N(\gamma) \equiv \gamma \int_0^\infty \frac{d\mathbf{k}^2}{\mathbf{k}^2} \left(\frac{\mathbf{k}^2}{m^2}\right)^\gamma \hat{\sigma}_N(\frac{\mathbf{k}^2}{m^2}). \tag{41}$$

Its explicit expression has been worked out in ref. [25]:

$$h_N(\gamma) = \pi^{\frac{3}{2}}\alpha\alpha_s e_Q^2 \ 4^\gamma[(14 + 20N +$$
$$+9N^2 + N^3) - \gamma(10 + 7N + N^2)]$$
$$\cdot\frac{\Gamma(1+\gamma)\Gamma^2(1-\gamma+N)}{\Gamma(\frac{5}{2}-\gamma+N)\Gamma(4+N)}. \tag{42}$$

The energy dependence of the cross section follows from eq. (40) by the inverse Mellin transform. We note here that the cross sections in this approach depend on the BFKL anomalous dimensions [53], which are energy-dependent. As $s \to \infty$, the anomalous dimension γ asymptotes to its maximum value $\gamma = 1/2$, giving

$$\frac{4m^2\sigma_{\gamma g}(\rho, m^2; Q^2)}{G(\rho, m^2, Q^2)} \simeq h(1/2) = \frac{27}{56}\pi^2 h(0), \tag{43}$$

which is a factor (~ 5) larger than the lowest order result for the photoproduction of heavy quarks, $h(0)$, obtained by neglecting the resummation effects,

$$h(0) = \frac{28\pi}{9}\alpha\alpha_S e_Q^2. \tag{44}$$

Note that $h(0)$ is the leading term in the expansion of the coefficient function $C_N(\alpha_S)$ and is independent of N. As far as the resummed coefficient function C_N is concerned, we obtain from eq. (40) [25]:

$$C_N(\alpha_s) = \oint \frac{d\gamma}{2\pi i}$$
$$\cdot h_N(\gamma)\left(1 - \frac{\bar{\alpha}_s}{N}f(\gamma)\right)^{-1} \left(-\frac{f'(\gamma)}{f(\gamma}\right) \ . \tag{45}$$

Here f' is the derivative of f and the γ-contour in the complex plane has to be chosen in such a way to pick up the pole $(N-\bar{\alpha}_s f(\gamma))^{-1}$ for $0 < \mathrm{Re}\,\gamma \leq 1/2$, thus reproducing $h_N(\gamma_N)$ in eq. (40). We separate the Born and the higher orders contributions,

$$C(\rho,\alpha_s) = C^{(0)}(\rho,\alpha_s) + \tilde{C}(\rho,\alpha_s) \quad , \tag{46}$$

where

$$C^{(0)}(\rho,\alpha_s) = \int \frac{dN}{2\pi i}\,\rho^{-N}\,h_N(0) \tag{47}$$

gives precisely the photo-production cross section in the Born approximation, and

$$\begin{aligned}&\tilde{C}(\rho,\alpha_s) = \\ &= \int \frac{dN}{2\pi i}\,\rho^{-N} \oint \frac{d\gamma}{2\pi i}\,h_N(\gamma)\cdot \\ &\cdot \frac{\bar{\alpha}_s(-f'(\gamma))}{N-\bar{\alpha}_s f(\gamma)}\end{aligned} \tag{48}$$

takes into account the high energy resummation. The coefficient function $\tilde{C}(\rho,\alpha_s)$ is to be compared with the exact one-loop result $C^{(1)}(\rho,\alpha_s)$ computed in Ref. [21], which we have discussed in the preceding section (cf. $C_{\gamma g}^{(1)}$ in Eq. 17). As remarked earlier, in the high energy region ($\rho \to 0$) the one-loop coefficient function is large and constant, which is to be attributed to the gluon exchange in the t-channel. This constant behaviour of $C^{(1)}$ generates, at higher orders, powers of $\ln\rho$ which are resummed by eq. (48) evaluating the N-integral at the BFKL pole $N = \bar{\alpha}_s f(\gamma)$. As the energy increases, the moment pole value decreases and reaches the minimum value $\bar{N} = 4\bar{\alpha}_s \ln 2$ at $\gamma = 1/2$. Therefore the resummed coefficient function $\tilde{C}(\rho,\alpha_s)$ has asymptotically the power-like behaviour $\rho^{-\bar{N}} = (S/4m^2)^{4\bar{\alpha}_s \ln 2}$. Care should be taken, however, of the N-dependence of the integrand in eq. (48). Since the effective asymptotic moment $\bar{N}$ is of order $\bar{N} \sim \alpha_s$ and $\alpha_s(m^2)$ is not very small, finite N effects both in $h_N(\gamma)$ and in the anomalous dimension may sizeably affect charm and beauty quark production.

The following remarks, mostly based on the work of Catani et al. [25], are in order at this stage.

- The finite-N dependence of the hard vertex function $h_N(\gamma)$ is known from eq. (42) and has been worked out numerically in [25]. As N increases $h_N(\gamma)$ becomes flatter thus leading to the reduction of the resummation effects.

- Subleading terms in the anomalous dimension (37) can be introduced by replacing $N \to N/(1-N)$ in the denominator of eq. (48). This simple model provides a good approximation of the exact one-loop anomalous dimension for $N \leq 1$ and implements the constraint $\gamma_N = 0$ for $N = 1$, thus enforcing momentum conservation in the evolution of the gluon structure function.

- Resummation effects involve both the anomalous dimension γ_N in the structure function and the coefficient function. To get the total cross section, one has to convolute

the improved coefficient function $\tilde{C}$ with the parton distributions. To the best of our knowledge this has not been done for electroproduction of heavy quarks at HERA.

- In the numerical estimates, Catani et al. use only the resummed coefficient, which for beauty quark production ($m = 5$ GeV, $\Lambda = 210$ MeV) is shown in Fig. 9 in the small ρ-region. As the energy increases the resummation of the higher order terms $(\alpha_s \ln \rho)^k$ leads to a coefficient function $\tilde{C}$ that for $\rho = 10^{-3}$ (the kinematical boundary for beauty production at HERA) is approximately a factor of two larger than $C^{(1)}$.

- The resummed coefficient function joins quite smoothly on to the one-loop coefficient function $C^{(1)}$ at intermediate values of ρ ($\rho \sim 10^{-2}$), so that they can be matched for larger values of ρ. The matching is performed subtracting from $\tilde{C}$ its one-loop contribution and adding $C^{(1)}$. Fig. 10 shows the ensuing coefficient function $\tilde{C}^{(1)}$, which reproduces the exact result to the first order in α_s and includes the resummation of the higher order terms $(\alpha_s \ln \rho)^k$. The result for the quark channel contribution obtained with the same matching procedure can be seen in Fig. 11.

- Since the high energy corrections increase the coefficient function just over a unity in rapidity out of three available for the beauty quark and approximately two units in rapidity for the charm quark production at HERA energies, the increase in the cross section will be correspondingly reduced, depending also on the form of the input structure function.

- Finally, it is to be noted that in the formalism discussed above, the argument of α_S has been frozen at m^2 because $k_\perp^2/m^2 = O(1)$ at the heavy flavour hard vertex. The corresponding expressions with a running α_S have yet to be calculated.

It should be pointed out that an accurate evaluation of the structure function evolution, taking into account the BFKL anomalous dimension, is not yet available. However simplified model calculations [61, 62] suggest that in the x-range ($x \geq 10^{-4}$), relevant for charm and beauty quark production at HERA, the resulting parton distributions will increase only marginally with respect to the standard (GLAP) evolution. Recent data from deep inelastic scatterings at HERA, covering $x \geq 10^{-4}$ and $Q^2 \geq 10\ GeV^2$, have been analysed in terms of the structure function $F_2(x, Q^2)$ which is found to increase in the small-x region. The measurements are in agreement with the structure function parametrizations based on the conventional Altarelli-Parisi evolution. The HERA data, despite their preliminary character, are good enough to distinguish among various input parton distributions, eliminating a number of models. So, as far as the small-x structure function part is concerned, this can be largely taken care of by the deep inelastic data thereby greatly reducing the uncertainty on the parametrizations of the proton structure functions in estimating heavy quark production at HERA. Since, subsequently we are going to argue that heavy quark production provides a precise method to determine the gluon distribution in the proton, one has to adopt an iterative procedure involving the DIS and heavy quark data to determine both the heavy quark cross section and the gluon distributions. In doing this one has to take into account the resummation discussed in this section.

5.1 The electroproduction case

In this section we discuss the electroproduction of heavy quarks in the $K_\perp$-formalism. Using the standard notations for lepton-hadron scattering, the differential cross section for the leptoproduction of heavy quarks can be written as

$$\begin{aligned} d\sigma^{(e)} &= \frac{\alpha}{\pi}\frac{dQ^2}{Q^2}\frac{dy}{y}\Theta(Q^2 - \frac{y^2 m_e^2}{1-y}) \\ \cdot \quad & [(1-y)\sigma_2(\frac{\rho}{y}, \frac{Q^2}{m^2}; \frac{m^2}{Q_0^2}) \\ + \quad & \frac{1}{2}y^2\sigma_1(\frac{\rho}{y}, \frac{Q^2}{m^2}; \frac{m^2}{Q_0^2})] \ , \end{aligned} \tag{49}$$

where σ_1 and σ_2 are related in the (heavy quark mass) $m \to 0$ limit to the customary DIS structure functions F_1 and F_2 by the relations

$$\sigma_1 = \frac{4\pi^2\alpha}{Q^2} 2x_B F_1 \ , \sigma_2 = \frac{4\pi^2\alpha}{Q^2} F_2 \ , (m^2 = 0). \tag{50}$$

In the high energy limit, $k_\perp$-dependent factorization formulae hold for each one of the cross sections σ_i, leading to resummed expressions of the type (40), with Q^2-dependent coefficient functions $h_N^{(i)}(\frac{Q^2}{m^2}, \gamma_N)$ [25]. As in the photoproduction case ($Q^2 = 0$), the asymptotic limit of these formulae is obtained by taking the $N \simeq 0$ approximation in $h_N^{(i)}$ and setting the anomalous dimension at its saturating value $\gamma_N = 1/2$:

$$\begin{aligned} & 4m^2\sigma_i(\rho, Q^2/m^2, m^2/Q_0^2) \to \\ & \to h^{(i)}(Q^2/m^2, 1/2) G(\rho', m^2, Q_0^2) \ , \end{aligned} \tag{51}$$

where $\rho' = \rho + x_B$. The explicit evaluation of the asymptotic K-factors $h^{(i)}$ in electroproduction gives [25]

$$h^{(i)}(Q^2/m^2, 1/2) = h(1/2) g^{(i)}(\frac{Q^2}{4m^2}) \ , \tag{52}$$

where

$$\begin{aligned} g^{(1)}(a) &= \frac{1}{1+a}[(\frac{1}{12} - \frac{1}{4a}) + \\ &+ (\frac{1}{4a} + 1 + \frac{3}{4}a)\sqrt{\frac{1}{a}} \arctan\sqrt{a}], \\ g^{(2)}(a) &= \frac{1}{4a} + (\frac{11}{12} - \frac{1}{4a})\sqrt{\frac{1}{a}} \arctan\sqrt{a}, \end{aligned} \tag{53}$$

and $h(1/2)$ is the asymptotic resummed photoproduction K-factor defined earlier.

Let us now consider the consequences of the results (51)- (53). As emphasized by Catani et al. [25], the $m^2 \ll Q^2$ limit of these formulae gives the asymptotic behaviour of the structure functions F_i in Eq. (50) thus providing the effective point-like couplings of the perturbative pomeron [53] to the longitudinal and transverse photons. In fact, in this limit

$$G(\rho', m^2, Q_0^2) \to (\frac{m^2}{Q^2})^{\frac{1}{2}} G(x_B, Q^2, Q_0^2) \ ,$$
$$h^{(i)}(Q^2/m^2, 1/2) \to (\frac{m^2}{Q^2})^{\frac{1}{2}} c^{(i)} \ , \tag{54}$$

where the square root factors provide the overall change of scale $m^2 \to Q^2$ and the constants $c^{(i)}$ represent the photon-pomeron pointlike couplings. They have been computed in a number of papers [25, 55, 57, 58], and are given by:

$$c^{(1)} = \frac{9\pi^2}{128} \alpha_s \Sigma e_q^2 \ , \quad c^{(2)} = \frac{11\pi^2}{128} \alpha_s \Sigma e_q^2 \ . \tag{55}$$

These couplings are measured by the cross section for deep inelastic + 1-jet events [54] in the fragmentation region of the proton at large rapidity gap ($x_B \ll x_1 \sim O(1)$) and large transverse momenta ($k_{1\perp}^2 \sim Q^2$) of the produced jet k_1:

$$d\sigma_{1-jet}^{(i)} = \frac{\alpha_s}{\pi} C_A \frac{dk_{1\perp}^2}{k_{1\perp}^2} \frac{dx_1}{x_1}$$
$$\cdot (f_g(x_1, k_{1\perp}^2) + \frac{C_F}{C_A} f_q(x_1, k_{1\perp}^2))$$
$$\cdot c^{(i)} \frac{1}{4\bar{\alpha}_s \ln 2} G(\frac{x_B}{x_1}, Q^2, k_{1\perp}^2) \ . \tag{56}$$

Here the factor in the first line represents the emission probability of the jet k_1, and f_g and f_q are the gluon and quark distributions of the incoming proton, respectively.

It has been argued in [25] that the effect of $k_\perp$-dynamics on the electroproduction cross sections is non-trivial. In the usual derivation of the heavy flavour production ($m^2 \gg m_e^2$) the cross section (49) is dominated by the collinear pole ($1/Q^2$) in the small Q^2 region, so that the total cross section can be obtained by the Weizsäcker-Williams approximation

$$\sigma^{(e)}(\rho, m^2/Q_0^2; m^2/m_e^2) \simeq$$
$$\simeq \frac{\alpha}{2\pi} \int_0^1 dy \frac{1 + (1-y)^2}{y} \ \ln \frac{Q_{eff}^2 (1-y)}{m^2 y^2} \cdot$$
$$\cdot \sigma_{\gamma g}(\rho/y, m^2/Q_0^2) \ , \tag{57}$$

where $\sigma_{\gamma g}$ is the photoproduction cross section , i.e. the small Q^2 limit of the cross sections σ_i in Eq. (49). Q_{eff} is an effective upper limit on the Q^2-integration in Eq. (49) and is usually chosen to be the kinematical boundary value $Q_{eff}^2 \simeq yS - 4m^2$ [14, 21, 37]. Catani et al. have argued that the value of Q_{eff} has to be determined on the basis of dynamics. To

elaborate this point, they note that Q_{eff} enters eq. (57) from eq. (49) through the small Q^2 approximation

$$\sigma_i(\frac{\rho}{y}, \frac{Q^2}{m^2}; \frac{m^2}{Q_0^2}) \simeq \sigma_{\gamma g}(\frac{\rho}{y}; \frac{m^2}{Q_0^2})\Theta(Q^2_{eff} - Q^2). \tag{58}$$

As the energy S increases, one may think that the value of Q^2_{eff}, for which eq. (58) is valid, increases too reaching eventually the kinematical boundary. Actually, the asymptotic result (52) shows that this is not the case. At high energy, the cross sections σ_i are proportional to the photoproduction cross section with a Q^2-shape as given by the functions $g^{(i)}$ in Eq. (53). These latter provide an asymptotic cut-off on Q which is of the order of the heavy quark mass m. Numerically, the effect of using the dynamical limit for Q_{eff} in place of the kinematic limit is to reduce the cross section by about 20 percent for beauty production ($m = 5$ GeV, $\Lambda = 210$ MeV) at HERA energies [25].

The actual enhancement in the beauty quark cross sections due to resummation is expected to be rather modest at HERA energies. It is expected to be more pronounced in the case of the charm quark production though the numerical details have not been worked out completetly. Catani et al. [25] estimate that the total cross section for b-quark production at HERA will be $O(20\%)$ larger than predicted by the one-loop calculations [21, 22]. Very similar conclusions have been reached in the detailed monte carlo simulation by Webber [64], based on similar considerations.

Summarizing, the cross sections $\sigma(ep \to QX)$ for $Q = c, b$ at HERA, using improved QCD perturbation theory, are estimated to be [21, 22]:

$$\begin{aligned} \sigma(ep \to cX) &\simeq O(1\ \mu b). \\ \sigma(ep \to bX) &= O(10\ nb). \end{aligned} \tag{59}$$

With a luminosity of 100 pb^{-1}/year, one expects an annual crop of $O(10^8)$ charmed hadrons and $O(10^6)$ beauty hadrons at HERA. These rates make HERA a powerful charm factory (though, unfortunately, not a Tau-factory!), providing new and unique opportunities to test perturbative QCD, measure the structure functions of the proton as well as of the photon, and search for new physics beyond the Standard Model in the flavour changing neutral current (FCNC) sector.

6 Determination of the Gluon Density Using Open and Bound Charmed Hadrons at HERA

Apart from providing quantitative tests of perturbative QCD through the production processes, charm physics at HERA also offers the following possibilities:

- Determination of the gluon density through the process $ep \to ec\bar{c}X$, which is dominated by the fusion process $\gamma + g \to c + \bar{c}$.

- An understanding of the dynamics of the leptoproduction of the heavy quark-antiquark bound states. This includes the elastic, diffractive, and inelastic J/ψ-production processes. The inelastic processes will lead eventually to a determination of the gluon density and the elastic and diffractive processes will provide valuable information on the physics related to the pomeron.

- Search for the rare decays of charmed mesons and $D^0 - \overline{D^0}$ mixings, providing possible clues to physics beyond the Standard Model, such as intermediate-mass leptoquarks.

We summarize the main points and conclusions of a HERA study conducted in 1991-92 concerning these points [16]-[18], and update them taking into account theoretical and experimental progress since then, having a direct bearing on some of the issues involved.

6.1 Determination of the gluon density using the process ($ep \to ec\bar{c}X$) at HERA

The determination of the gluon density from (open) charmed hadron events at HERA in the process $ep \to ec\bar{c}X$ has been studied by Woudenberg et al. [17] by using the traditional methods for charmed hadron tagging, namely $D^{*\pm}$ reconstruction from the decays $D^{*\pm} \to D^0\pi^\pm \to (K^\mp\pi^\pm)\pi^\pm$, and inclusive dileptons from the decays $c\bar{c} \to l^+l^-X$. The inclusive cross section for $D^{*\pm}$ events, taking into account detector simulation and trigger efficiency, has been estimated to be $O(10^3\ pb)$ giving $O(10^5)$ reconstructed $D^{*\pm}/100\ pb^{-1}$ at HERA. The cross section for the dielectron events $ep \to c\bar{c}X \to e^+e^-X$, with a beam pipe cut of 0.1 rad. and the transverse momentum of the prompt (decay) electron satisfying $p_\perp(e^\pm) > 0.8\ GeV$, is estimated to be $O(60\ pb)$ giving 6×10^3 dielectrons/ $100\ pb^{-1}$ from $c\bar{c} \to e^+e^-X$ events. The dilepton cross section with the same cuts is then a factor 4 larger than this number. The reconstruction techniques for the variable x_g, the fractional momentum carried by the gluon, is discussed in ref. [17]. In the case of D^*-tag, the variable x_g can be determined using the relation

$$x_g = \frac{\hat{s} + Q^2}{ys} \simeq \frac{\hat{s}}{ys}, \tag{60}$$

with $\hat{s} = M^2_{c\bar{c}} = M^2_{\gamma g}$. To a good approximation, one could equate

$$\hat{s} = \frac{p_\perp^2 + m_c^2}{z(1-z)}, \tag{61}$$

where $p_\perp \equiv p_\perp(D^*)$ and $z = p.p_c/y(p.l)$. With a cut $p_\perp(D^*)^{\min} = 1.4\ GeV$, a resolution $\sigma(\frac{\Delta x_g}{x_g}) \simeq 40\%$ and a mean value $< \frac{\Delta x_g}{x_g} > \simeq 0.7\%$ have been estimated in the monte carlo studies, assuming 6 pb^{-1} luminosity. The HERA study in [17] concludes that with this method x_g can be determined in the range $5 \times 10^{-4} \leq x_g \leq 10^{-1}$. With larger integrated luminosity, values of x_g as low as 10^{-4} should be reachable, as the charmed hadron fraction contributing to $F_2(x, Q^2)$ at those values of x_g is expected to be quite significant. A second

method of estimating x_g consists of determining the rapidity $\hat{y}_{cc}$ of the partonic system, giving (since $Q^2 \simeq 0$)

$$x_g = \frac{y E_e}{E_p} \exp\left(2\hat{y}_{cc}\right). \tag{62}$$

To determine $\hat{y}_{cc}$, one can either use the reconstructed $D^{*\pm}$, in which case $\hat{y}_{cc} \sim \hat{y}_{D^{*+}D^{*-}}$, or tag one charmed quark and reconstruct the hardest hadron in the opposite direction, for which $\hat{y}_{\bar{c}c} \sim \hat{y}_{D^*-\text{hadron}}$. This method, according to the analysis in [17], can be used to determine x_g in the range $6 \times 10^{-4} \leq x_g \leq 10^{-1}$ with a mean value $< \frac{\Delta x_g}{x_g} > \sim 4\%$ and a resolution $\sigma\left(\frac{\Delta x_g}{x_g}\right) \sim 37\%$.
For the dilepton events, the reconstruction of $\hat{y}(cc)$ can be well approximated by the rapidity of the lepton pair. The reconstructed value, $x_g^{\rm rec}$, can then be calculated using the following relation

$$x_g^{\rm rec} = \frac{\delta^{\rm rec} E_e}{E_p} \exp\left(2\hat{y}(e^+e^-)\right), \tag{63}$$

where $\delta^{\rm rec}$ is defined as

$$\delta^{\rm rec} \equiv \sum_i \frac{E_i - P_{Li}}{2E_e}. \tag{64}$$

With this method, x_g-values in the range $3 \times 10^{-4} \leq x_g \leq 3 \times 10^{-1}$ with $< \frac{\Delta x_g}{x_g} > \sim 4\%$ and a resolution $\sigma\left(\frac{\Delta x_g}{x_g}\right) \sim 60\%$ can be measured.

Having constructed the variable x_g, one could study the dependence of the charm cross section on x_g to measure the gluon density $G(x_g)$ in the proton at HERA, providing a much more direct and theoretically dependable extraction of this function. For the time being, one could show the sensitivity of the cross section on currently available parametrizations. To illustrate this, we remark that one would be able to distinguish the gluon densities due to Morfin and Tung (sets B1 and B2) [66] with a rather modest HERA luminosity. This is shown in Fig. 12, where the product $x_g G(x_g)$, reconstructed from the inclusive production $ep \to D^{*+}X$ at HERA, is shown as a function of x_g. The error bars indicated correspond to the statistical error in a Monte Carlo simulation assuming an integrated luminosity of 6 pb^{-1}.

First measurements on the DIS structure function $F_2(x, Q^2)$ by the H1 [10] and ZEUS [5] collaborations have been reported. Though it is still too early to draw definite conclusions, it is certain that $F_2(x, Q^2)$ is rising at the low-x values measured so far. For illustration, we show in Fig. 13 the differential cross section $d^2\sigma/dxdQ^2$ for the DIS events measured by the ZEUS collaboration. Concerning the structure functions used here, the trend of the data is that the predictions of the Morfin-Tung (set B2) lie above the data but the corresponding set B1 is in fair agreement [5]; likewise the structure function MRSD0 is in good agreement with data. Inclusive charm production will provide a very precise determination of $G(x, Q^2)$ and the data on $F_2(x, Q^2)$ could then be used to disentangle the quark contribution. However, a detailed analysis will be necessary to reduce the uncertainty due to the partonic structure function of the photon since the resolved photon contribution to $\sigma(ep \to c + X)$ is model

dependent and large at HERA. We have already discussed in the preceding section that this contribution could be well determined using the photoproduction data. Thus, precise measurements of the partonic densities at HERA will be an inter-twined exercise, and various aspects of HERA data will have to be used to disentangle all these components. In view of the fact that no bound state wave function is required to determine the cross section (as opposed to the J/ψ-leptoproduction discussed below), and the underlying QCD framework in the next-to-leading order (NLO) is at hand, final states having open charmed hadrons are well suited to measure the gluon density at HERA.

6.2 J/ψ-production processes at lower energies and at HERA

We remark that the theory of the photo- and leptoproduction of the bound heavy quark-antiquark states in general, and J/ψ in particular, is not yet developed to the required rigour. In particular, the next-to-leading order QCD corrections to the basic photon-gluon fusion mechanism are not yet available. In addition to this, an inherent dependence on the wave-function describing the $J/\psi - c\bar{c}$ metamorphosis introduces a certain degree of fuziness in theoretical estimates of the inelastic J/ψ-production. Also, the elastic and diffractive processes (which contribute to the J/ψ-production as well) can at present only be modelled, for example using the vector meson dominance (VMD) type description involving the exchange of a pomeron between the proton and the photon. Clearly, there is much room for theoretical improvements in this sector. In what follows, we review the present state-of-the art phenomenology. In particular, we review the J/ψ-production cross sections at HERA and lower energy fixed target photoproduction experiments. Subsequently, we take up the point concerning the determination of the gluon density using J/ψ production at HERA, arguing that normalized distributions could be used meaningfully for this purpose.

We denote the generic process of J/ψ-production as:

$$e(\ell) + p(p) \to e(\ell') + J/\psi(p_{J/\psi}) + X(p_X). \tag{65}$$

In the WW approximation ep cross sections are obtained from the corresponding γp cross sections for the process $\gamma(q) + p(p) \to J/\psi(p_{J/\psi}) + X(p_X)$ by the usual convolution of $f_\gamma^e(y, Q^2)$ with $\sigma_{\gamma^* p}(W^2, Q^2)$:

$$\begin{aligned} &\sigma_{ep}(s) \\ &= \frac{\alpha}{2\pi} \int \frac{dy}{y} \int \frac{dQ^2}{Q^2} \left\{ \left[1 + (1-y)^2 \right] \sigma_{\gamma^* p}(W^2, Q^2) \right\} \end{aligned}$$

with the Q^2 integration limits being

$$m_e^2 \frac{y^2}{1-y} \leq Q^2 \leq ys - W^2_{\min}, \tag{66}$$

and $W^2 = s_{\gamma p} = (q+p)^2$. As discussed in the previous section, the $K_\perp$-formalism (along the lines developed for the process $ep \to Q\bar{Q}X$) would restrict the upper bound on Q^2 to be of

$O(m^2_{J/\psi})$ instead of what is shown in the above equation. However, the $K_\perp$-formalism for the photo- and leptoproduction of J/ψ, and hence the dynamical constraints on the kinematics and energy dependence of the cross section in the high energy domain $m^2_{J/\psi}/s \to 0$, are not yet worked out. We shall, therefore, restrict ourselves to the lowest order estimates. In this approximation, the photoproduction of J/ψ at HERA has been studied in refs. [18, 74, 80]. Concentrating on the photoproduction of J/ψ, the following processes have been calculated in the HERA study by Jung et al. [18]:

1. Elastic J/ψ production, $\gamma + p \to J/\psi + p$.
2. Diffractive inelastic J/ψ production, $\gamma + p \to (J/\psi + X) + p$.
3. Diffractive dissociation, $\gamma + p \to J/\psi|\text{diff.} + X$.
4. Inelastic J/ψ production, $\gamma + p \to J/\psi + X$.
5. Direct J/ψ production in the resolved-photon processes, $g + g \to J/\psi + g$ and $g + g \to J/\psi + \gamma$.
6. J/ψ via χ-production in the resolved-photon processes,

$$\begin{aligned} g + g &\to \chi_{0,2} \to J/\psi + \gamma, \\ \{g + g, q + \bar{q}\} &\to \chi(\to J/\psi + \gamma) + g, \\ g + q &\to \chi(\to J\psi + \gamma) + q. \end{aligned}$$

7. J/ψ-production via $\gamma\gamma$ collision, $\gamma + \gamma \to \chi_{0,2} \to J/\psi + \gamma$.
8. J/ψ-production via b-production, $\gamma + g \to b\bar{b} \to J/\psi + X$.
9. J/ψ in double charm quark pair production, $(\gamma, g) + g \to J/\psi + c\bar{c}$.

In addition, double J/ψ-production using the partonic processes $q + \bar{q} \to J/\psi + J/\psi$ and $g + \bar{g} \to J/\psi + J/\psi$ have been estimated for ep collisions at HERA and elsewhere in ref. [69]. Feynman diagrams and possible models for some of the above mentioned processes are shown in Fig. 14. It is understood that in the processes (5),(6) and (9), one of the initial gluons (or quarks) stems from the photon and the other from the proton. The processes (5)-(9) are numerically unimportant at lower energies. Their contribution to J/ψ production at HERA is, however, not anticipated to be negligible and may become measurable due to the characteristic event topology. We summarize below the cross sections and then discuss x_g-determination from J/ψ-producion at HERA.

- **Inelastic J/ψ-production process**

The dominant contribution to J/ψ-production is through the inelastic process $\gamma + p \to J/\psi + X$ (reaction 4 listed above). In the Colour-Singlet (CS) model [67, 68], γg-fusion leads to a colour-singlet state in lowest order,

$$\gamma + g \to J/\psi + g. \tag{67}$$

The differential cross section for $\gamma + p \to J/\psi + X$ can be written as

$$\frac{d\sigma}{dxdt} = G(x,\mu^2)\frac{d\hat{\sigma}}{dt}, \tag{68}$$

where $G(x,\mu^2)$ is the gluon distribution function of the proton, evaluated at the mass factorization scale μ^2 (assumed $= \hat{s}$) and $d\hat{\sigma}/dt$ is the differential cross section for $\gamma g \to J/\psi g$ which can be written as [67, 68]:

$$\begin{aligned} &\frac{d\hat{\sigma}}{d\hat{t}} \\ = \;& \frac{8\pi\alpha_S^2\Gamma_{\ell\bar{\ell}}\; m_{J/\psi}^3}{3\alpha_{em}\hat{s}^2}\frac{\hat{s}^2(\hat{s}-m_{J/\psi}^2)^2 + \hat{t}^2(\hat{t}-m_{J/\psi}^2)^2 + \hat{u}^2(\hat{u}-m_{J/\psi}^2)^2}{(\hat{s}-m_{J/\psi}^2)^2(\hat{t}-m_{J/\psi}^2)^2(\hat{u}-m_{J/\psi}^2)^2}, \end{aligned} \tag{69}$$

where,

$$\begin{aligned} \hat{s} &= (q+p_1)^2 = (p_{J/\psi}+p_2)^2 \\ \hat{t} &= (p_{J/\psi}-q)^2 = (p_2-p_1)^2 = t \\ \hat{u} &= (p_{J/\psi}-p_1)^2 = (p_2-q)^2. \end{aligned} \tag{70}$$

Here $\Gamma_{\ell\bar{\ell}}$ is the leptonic decay width for the decay $J/\psi \to \ell\bar{\ell}$, which involves the ${}^3S_1(c\bar{c})$-wave function at the origin, $|\Psi(0)|^2$. This can be estimated from the data on J/ψ-decays and QCD-improved widths. A comparison of the CS model predictions compared with data from the fixed target experiments (FTPS [70], NA14 [71], EMC [72], and NMC [73]) in the differential distribution $d\sigma/dz$ (here z is the scaled J/ψ-energy, $z = E_{J/\psi}/E_\gamma$) is shown in Fig. 15, which reveals that the shapes measured in these experiments are well accounted for by the model only for $z < 0.8$. However, a K-factor between 1.8 [70] and 4.9 [73] is needed to explain the rates, if one uses the lowest order J/ψ-decay width $\Gamma_{l\bar{l}}$ [18]. Using instead the QCD corrected width $\Gamma_{l\bar{l}}$, the respective K-factors are decreased by a factor ~ 2.

Concerning the large-z region ($z \to 1$), there are several mechanisms available that contribute dominantly in this region (listed as processes 1-3 in the list given above). In the same context, it has been argued in [20] that the shape $d\sigma/dz$ from the CS model is sensitive to the relativistic corrections, which are expected to be significant for the charmed quark case, and they contrive in such a way so as to enhance the large-z part of the distribution. To illustrate this point, we show in Fig. 16 the distribution $d\sigma/dz$ calculated in [20] including the relativistic corrections as a function of the parameter ϵ/m_c (here ϵ can be thought of as

the binding energy defined by $m_{J/\psi} = 2m_c + \epsilon$). Assuming a K-factor 2 these distributions are compared with the EMC [72] and NMC data[73]. This shows that a better agreement with the data in the large-z region is obtained if the non-relativistic parameter ϵ has a value of ≥ 0.15. Very similar conclusion is reached analysing the double differential distribution $d^2\sigma/dp_\perp^2 dz$ [20]. However, there is no denying of the fact that as $z \to 1$, one has to take into account the elastic and quasi-elastic J/ψ-production, which are not included in the CS γg-fusion model, or variations of it.

Summarizing, with the parameters $m_c = 1.5\ GeV, K = 2.8$ and the Morfin-Tung parametrization (set B1) [66], the inelastic cross section for $ep \to J/\psi + X$ at HERA is estimated to be 8.5 nb in the CS model [18]. Experiments at HERA will soon provide important input here for theoretical studies. It remains an interesting and open question to quantitatively understand the fixed target results on J/ψ production in QCD. A calculation of the QCD K-factor will be a very useful contribution to this field, which would remove one of the present theoretical uncertainties.

- **Elastic-J/ψ-production at HERA and lower energies**

Taking up now the elastic J/ψ production $\gamma p \to J/\psi p$, it is usually assumed that its cross section is determined by the single-pomeron exchange in the Regge picture. This involves the proton-pomeron-proton (p-$\mathbb{P}$-p) vertex, which can be taken from analogous studies in proton-proton collisions [75, 26, 81], as well as, the photon-pomeron-J/ψ (γ-$\mathbb{P}$-J/ψ) vertex, which is less well known and hence more model dependent. In the VMD-model the vertex γ-$\mathbb{P}$-J/ψ is related to the hadronic vertex J/ψ-$\mathbb{P}$-J/ψ, which in turn can be related to the proton form factor [75]. Following [80, 18], the elastic J/ψ-differential cross section can be parametrized as:

$$\frac{d\sigma_{el}(W^2,t)}{dt} = \frac{4\pi\alpha}{f_{J/\psi}^2}\frac{1}{4\pi}(3\beta F_p(t))^2(2\beta' F_{J/\psi}(t))^2(\alpha' W^2)^{2(\epsilon+\alpha' t)}, \tag{71}$$

where $4\pi\alpha/f_{J/\psi}^2 = 1/1570$ and the other parameters have the following values:

$$\alpha' = 0.25\ GeV^{-2}, \quad \epsilon = 0.085, \quad \beta = 2\ GeV^{-1}, \quad \beta' = 0.105\ GeV^{-1} \tag{72}$$

The proton and J/ψ form factors are parametrized as:

$$F_p(t) = \frac{4m_p^2 - 2.79t}{4m_p^2 - t}\frac{1}{(1 - t/0.71)^2}, \tag{73}$$

where $F_{J/\psi}(t) = \exp(B_{J/\psi}t)$ and $B_{J/\psi} = 0.23\ GeV^{-2}$. Normalizing this model by the elastic J/ψ cross section measured by the FTPS collaboration [70], and using the energy dependent pomeron density given above, gives a cross section $\sigma(ep \to eJ/\psi p) \simeq 4.5\ nb$ at HERA [18]. An alternative approach is to determine the γ-$\mathbb{P}$-J/ψ vertex by modelling it on open $c\bar{c}$ production instead of VMD [18], giving $\sigma(ep \to eJ/\psi p) = 2.2\ nb$ at HERA energy.

Recently, the H1 collaboration has measured the elastic ρ- and ϕ-production through the process $\gamma P \to VP$, with the center of mass γp energy in the range $20\ GeV \leq W_{\gamma p} \leq 80\ GeV$ [10]. The angular distribution of the $\rho \to \pi^+\pi^-$ sample is found consistent with a $\sin^2\theta$ distribution expected for the transversely polarized photons at $Q^2 \to 0$. The slope of the t-distribution in the reaction $\gamma P \to \rho P$ is in agreement with an exponential, $d\sigma/dt = \exp(bt)$, in the interval $t \leq 0.32\ GeV^2$, as shown in Fig. 17. The exponent b depends on the energy and a fit of the H1 data gives: $b = 9.4 \pm 1.1\ GeV^{-2}$ for the interval $W_{\gamma p} = 20 - 40\ GeV$ and $b = 11.1 \pm 1.3\ GeV^{-2}$ for the interval $W_{\gamma p} = 40 - 75\ GeV$. The energy dependence of the slope is in agreement with the Donnachie-Landshoff type description given above for the elastic J/ψ cross section [75] and with the phenomenological fit proposed in ref. [77]. Using this approach, the HERA data on $\gamma p \to \rho p$ and $\gamma p \to \phi p$ could be used to predict the cross section and t-dependence of the elastic $\gamma p \to J/\psi p$ process. This would predict that the distribution $d\sigma/dt$ involving the elastic photoproduction of J/ψ could be obtained from the corresponding distribution for the ϕ (or ρ) by the ratio of the squared elastic form factors:

$$\frac{d\sigma/dt[\gamma p \to J/\psi p]}{d\sigma/dt[\gamma p \to \phi p]} \simeq [\frac{F_{J/\psi}(t)}{F_\phi(t)}]^2. \tag{74}$$

Using this relation, the estimate for the elastic J/ψ-production in the VMD-approach presented above could be made more quantitative once the elastic ρ and ϕ cross sections at HERA are consolidated.

- **Diffractive J/ψ-production and miscellaneous J/ψ- production processes**

We discuss here mainly the diffractive J/ψ-production and summarize the estimates for the cross sections at HERA from this and other numerically less important mechanisms listed above. For the diffractive dissociation process $\gamma + p \to J/\psi|_{\text{diff.}} + X$, one could generalize the single-pomeron exchange model for the elastic $\gamma + p \to J/\psi + p$ scattering by replacing the elastic form factor of the proton by a suitable integral over the inelastic structure function of the proton, $\bar{F}_2$. The function $\bar{F}_2(x, Q^2)$ differs from the standard proton structure function $F_2(x, Q^2)$ in that the latter involves a weighting according to the usual parton model description of quark charges squared due to the couplings of the quarks with the photon, while the pomeron coupling to the quarks is independent of the electric charges of the quarks. A parametrization of the function $\bar{F}_2(x, Q^2)$ is given in ref. [75], which has been used in refs. [18, 80] to get the following differential distribution:

$$\frac{d\sigma}{dtdM^2} = \frac{4\pi\alpha}{f^2_{J/\psi}} \frac{1}{4\pi M^2} (2\beta' F_{J/\psi}(t))^2 (\frac{s}{M^2})^{2(\epsilon+\alpha' t)} (1 - \frac{M^2}{s}) \beta^2 \bar{F}_2(x, |t|), \tag{75}$$

where $x = |t|/(M^2 + |t| - m_p^2)$. The parameters in this model are all fixed from the elastic J/ψ cross section given above. This model predicts $\sigma(\gamma p \to J/\psi|_{\text{diff.}} + X) = 2.6\ nb$, compared to the FTPS data for the diffractive dissociation, $\sigma(\gamma p \to J/\psi|_{\text{diff.}} + X) = (4.4 \pm 0.9(\text{stat.}) \pm 1.1(\text{syst.}))\ nb$. Extrapolating to HERA energies, a cross section $\sigma(e + p \to e + J/\psi|_{\text{diff.}} + X) = 1.55\ nb$ has been obtained in ref. [18]. The cross section for the diffractive inelastic J/ψ

production $\gamma + p \to (J/\psi + X) + p$ in the VDM approach is obtained from the corresponding elastic-J/ψ scattering by replacing the form factor $F_{J/\psi}$ by an inelastic function describing the excitation of the photon into an inelastic state containing a J/ψ. In [18, 80] this cross section is estimated to be $\sigma(e + p \to e + (J/\psi + X) + p) \simeq 0.35\ nb$ at HERA.

Finally, the resolved-photon and the two-photon production of J/ψ (reactions 5-9 above) have been evaluated using the photon densities of ref. [44] and the details can be seen in [18]. We summarize the cross sections for J/ψ-production at HERA in Table 3, where a cut on the transverse momemntum of the soft parton has been imposed in the estimates of the resolved photon processes listed as reactions 5 and 6 above. Summing up the contributions of all these processes, the inclusive J/ψ cross section at HERA is estimated to be:

$$\sigma(ep \to J/\psi X) = O(15)\ nb. \tag{76}$$

This rate would allow to undertake quantitative studies of the various competing processes in J/ψ-leptoproduction at HERA.

6.3 Determination of the gluon density using J/ψ-production at HERA

We now briefly review the determination of x_g in the process $ep \to J/\psi X$. The photon-parton CM energy $\sqrt{\hat{s}}$ for the $e + p \to J/\psi X$ events can be determined via the relation

$$\hat{s} = \frac{p_\perp^2 + (1-z)m_{J/\psi}^2 + z s_2}{z(1-z)}, \tag{77}$$

where $\sqrt{s_2}$ is the invariant mass of the partonic final state excluding the J/ψ. With Q^2 and s_2 set to 0, the determination of x_g requires the measurement of $p_\perp^{J/\psi}, z$, and y. The partonic CM energy can be well measured via the leptonic decay $J/\psi \to l^+l^-$, with $p_\perp(l^+l^-) = p_\perp^{J/\psi}$ and $z(l^+l^-) = z(J/\psi)$. The energy fraction carried by the photon y is reconstructed using the Jacquet-Blondel method

$$y_{\rm rec} = \sum_h \frac{(E + P_L)_h}{2E_e}, \tag{78}$$

where the sum runs over all the particles in the calorimeter except the scattered electron. The scaled J/ψ energy, z, can be determined in terms of $y_{\rm rec}$

$$z_{\rm rec} = \frac{p \cdot p_{J/\psi}}{y\ (p \cdot l)} = \frac{(E + P_L)_{J/\psi}}{2 y_{\rm rec}\ E_e}. \tag{79}$$

The study in ref. [18] shows that using the inelastic J/ψ production the fractional gluon momentum x_g can be measured in the range $3 \times 10^{-4} \leq x_g \leq 1 \times 10^{-1}$ at $< \mu^2 >= \hat{s}$ with about 10% accuracy at the statistical level assuming an integrated luminosity of 20 pb^{-1}. Thus, already the first generation experiments at HERA would be able to discriminate among the various gluon densities. For the previous example of the sets B_1 and B_2 of Morfin-Tung

Table 3: Estimates of the J/ψ-production cross sections in ep collisions at HERA for the various processes considered in the text. Note that a K-factor, $K = 2.8$, is used in the estimates of the inelastic J/ψ cross section at HERA and the Morfin-Tung (set B1) is used for the proton structure function; the elastic J/ψ and diffractive-J/ψ production are modelled using VMD and the Landshoff-Donnachie type parametrizations of the elastic and diffractive form factors.

Production Process	Cross section (in units of nb)
1. Elastic J/ψ production	4.5
2. Diffractive inelastic J/ψ production	0.35
3. Diffractive dissociation	1.55
4. Inelastic J/ψ production	8.5
5. Direct J/ψ production (resolved-photon processes)	0.11
6. J/ψ via χ-production (with $p_\perp(J/\psi) \geq 1.0\ GeV$)	0.11
7. J/ψ-production via $\gamma\gamma$ collision	0.001
8. J/ψ-production via b-production	0.07
9. J/ψ in double charm quark pair production	0.16
10. J/ψ-pair production	4.3×10^{-4}
Sum (1-10): Inclusive J/ψ-production	15.35

parametrizations this can be seen in Fig. 18. However, we emphasize that extracting the gluon density from the J/ψ data would require a firmer theoretical framework than what is currently available.

7 Precision Charm Physics at HERA and Elsewhere

The next several years are expected to see rapid progress in the experiments (hopefully also in theory!) concerning charm physics related with the charged current (CC) and FCNC decays and mixing. The charmed hadron data available for precision study are expected to rise by several orders of magnitude, with e^+e^- annihilation remaining very much competitive through the Beijing collider operating in the $J/\psi, \psi', \psi'', ...$-resonance region, as well as, the CLEO detector taking data at CESR and possible B-factories operating at or in the vicinity of the $\Upsilon(4S)$. Likewise, the photo- and hadroproduction experiments with fixed-targets at Fermilab, namely the experiments E687, E769 and, in particular E791, have collected an impressive amount of data with projected yields of upto a million reconstructed charmed hadron events. As we have argued, one also expects $O(10^8)$ charmed hadrons produced per 100 pb^{-1} at HERA. Of particular importance here are tagging capibilities with charged leptons, D^*, and J/ψ, and particle identification. Moreover, past experience at fixed target experiments has made it abundantly clear that the signal/background ratio for heavy quark production is a sensitive function of the vertex resolution. This also holds for the ep collisions at HERA.

Given the large charm hadron statistics, it is natural to ask what are the interesting aspects of charm physics which one could study in the next several years at HERA and elsewhere. We have discussed at some length the quantitative tests of perturbative QCD and models in the production processes. In this section we concentrate on the charmed hadron decays and summarize the interest in dedicated studies of charm physics.

- **Charmed hadron CC decays in the Standard Model**

These involve decays such as $c \to s + (q\bar{q}, \ell\nu_\ell)$. At the hadronic level, decays such as $D^0 \to (K^-, K^{*-})\ell^+\nu_\ell$ and $D^0 \to K^-\pi^+$, are expected to be measured with an error of a few percent on their respective branching ratios. This in turn would lead to a determination of the various form factors at the level of $(5-10)\%$, leading to a determination of the CKM matrix-element $|V_{cs}|$ at the level of ± 10 % or better [52]. Likewise, the singly Cabibbo-suppressed decays $D \to (\pi, \rho) + \ell + \nu_\ell$ could be used with the corresponding Cabibbo-allowed decays to determine the CKM-matrix element ratio $|V_{cd}|$, again at a level of $\pm 10\%$. Much interest has lately been aroused by the measurements of the leptonic decays, $D_s^\pm \to \mu^\pm + \nu_\mu$ in a number of experiments [82, 83, 84]. In particular, the CLEO collaboration has presented statistically impressive measurements of the decay chain, $D_s^* \to D_s + \gamma$ followed by $D_s^\pm \to \mu^\pm + \nu_\mu$, using the momenta of the μ, γ and ν, where the missing momentum technique is used to determine the ν-momentum. The CLEO measurements yield a ratio [83]:

$$\frac{\Gamma(D_s^\pm \to \mu^\pm \nu_\mu)}{\Gamma(D_s^\pm \to \phi\pi^\pm)} = 0.245 \pm 0.052 \pm 0.074, \tag{80}$$

which using the PDG numbers for the lifetime of $D_s^\pm$ and the branching ratio for the decay mode $D_s^\pm \to \phi\pi^\pm$ gives the following value for the pseudoscalar coupling constant $f(D_s)$ [83]:

$$f(D_s) = (344 \pm 37 \pm 52 \pm 42)\ MeV \quad \text{[CLEOII]}, \tag{81}$$

to be compared with the WA75 number [82]:

$$f(D_s) = (232 \pm 45 \pm 20 \pm 48)\ MeV \quad \text{[WA75]}. \tag{82}$$

A recent compilation of the lattice-QCD results for the same quantity gives [86]:

$$f(D_s) = (230 \pm 8 \pm 20)\ MeV. \tag{83}$$

While the WA75 and the lattice-QCD results are in striking agreement, the CLEOII value for $f(D_s)$ is appreciably higher. The errors are still quite large to draw a quantitative conclusion. It is, therefore, imperative to measure the branching ratio in question precisely to check the quantitative rapport between theory (in particular, lattice-QCD) and experiment. Further down the line, the doubly Cabibbo-suppressed decays (DCSD), like for example $D^{*+} \to K^{*+}\pi^0$ and $D^0 \to K^+\pi^-$, are interesting in their own right, with the latter providing also a signature for the $D^0 - \overline{D^0}$-mixing, discussed below.

- **$D^0 - \overline{D^0}$-mixing in the Standard Model and beyond**

In the Standard Model, the last word on the strength of the $D^0 - \overline{D^0}$-mixing amplitude is probably not yet in! There is consensus on the phenomenological irrelevance of the so-called short distance contribution to the mass difference $\Delta M(D_1^0 - D_2^0)$ but the magnitude of the long-distance contribution is not quite definite. It has been argued some time ago by Donoghue et al. [87] and Wolfenstein [88], that the dispersive contributions from the second order weak interactions with mesonic intermediate states,

$$\sum_I \frac{\langle D^0|H_W|I\rangle\langle I|H_W|\bar{D}^0\rangle}{m_D^2 - m_I^2 + i\epsilon}, \tag{84}$$

could easily overwhelm the short distance piece and could be measurably large. This point of view has been lately put to question by Georgi [89], who conjectured, based on the techniques of the heavy quark effective theory (HQET), that there are important cancellations among the dispersive parts of different kinds so that the contribution to the $D^0 - \overline{D}^0$ mass mixing is much smaller than the estimates of ref. [87] and [88]. In a formidable paper, Ohl, Ricciardi and Simmons [90] have followed up this suggestion and have undertaken a quantitative analysis of the $D^0 - \overline{D}^0$-mixing in HQET, including leading order QCD corrections. Adding all the contributions, their final result is [90]:

$$\Delta m(D_1^0 - D_2^0) \simeq (0.9 - 3.5) \times 10^{-8}\ eV \tag{85}$$

for $m_s = 0.2\ GeV$. This strengthens the conjecture by Georgi, predicting strong cancellations among the dispersive channels. The HQET estimate is to be contrasted with the present best experimental bound [85], giving

$$\Delta M(D_1^0 - D_2^0) < 1.3 \times 10^{-4}\ eV \tag{86}$$

The above limit on $\Delta M(D_1^0 - D_2^0)$ translates into a mixing ratio probability $r_D \leq 3.7 \times 10^{-3}$ [85], which is soon expected to be improved by the ongoing experiments by a factor 3 [52]. To be competitive, experiments at HERA have to aim at a sensitivity on r_D significantly better than $r_D \leq 1.0 \times 10^{-3}$. Since both particle identification and excellent vertex resolutions would be at hand in the proton-fixed target experiment, being contemplated presently at HERA (the so-called HERA-B experiment)[93], this experiment is also well suited for searching for $D^0 - \overline{D}^0$ mixing as there will be sufficient number of charmed hadrons available in the process $pA \to c\bar{c} + X$. It goes without saying that the serach for $D^0 - \bar{D}^0$ mixing will require a trigger different from the dileptons, which is the choice for tagging on the $J/\psi \to \ell^+\ell^-$ mode for CP-violation studies in B-decays [93].

It should be remarked here that the CLEO collaboration has recently reported preliminary measurements of the decay mode $D^0 \to K^+\pi^- + c.c.$, which could be interpreted both as a DCSD decay mode and $(D^0 - \overline{D^0})$-mixing. Using the decay chain $D^* \to D\pi$ followed by the decay $D^0 \to K^-\pi^+, K^+\pi^-$, the following ratio of the Cabibbo-allowed and the DCSD (and possibly $D^0 - \overline{D^0}$ mixed) modes of the D^0 are reported [91]:

$$R = \frac{\Gamma(D^0 \to K^+\pi^-)}{\Gamma(D^0 \to K^-\pi^+)} = (0.77 \pm 0.25)\%. \tag{87}$$

This ratio, if interpreted in terms of the DCSD mode, translates into $R = (2.92 \pm 0.95) \times \tan^4\theta_C$. The coefficient is not an unreasonably large number implying that the mode seen by CLEO is very likely a DCSD mode. At HERA and at fixed-target experiments, one could attempt to disentangle the two sources, i.e. mixing and DCSD modes, by studying the time evolution of these decays.

If the HQET estimate is in the right ball-park, then it would imply that there are four orders of magnitude improvement required before experiments could measure a signal in $D^0 - \overline{D}^0$-mixing! This is good, since it makes the search for a signal in this mixing mode all the more interesting. A mixing in the $D^0 - \overline{D}^0$-sector at a significantly larger level would imply physics beyond the Standard Model.

In the context of the loop-induced effects of physics beyond the Standard Model, leptoquark interactions have received special interest. It has been argued that if relatively low-mass leptoquarks exist (say, $m_{LQ} = O(10)TeV$), then they could induce significant mixing in the neutral meson sector, as well as, rare D^0-decays at a measurable rate. In particular, a new set of bounds due to the $D^0 - \overline{D^0}$-mixing (likewise for the neutral K- and B-meson mixings) has been proposed by Leurer [92]. Concentrating on the mixing in the neutral charmed hadron system, one could define the following couplings involving the leptons, up-type quarks and leptoquarks (denoted generically by Φ):

$$\mathcal{L} = \{\bar{l}[g_L^u P_L + g_R^u P_R]u + \bar{l}[g_L^c P_L + g_R^c P_R]c\}\Phi, \tag{88}$$

where the indices on the coupling constants indicate the flavour and the chirality of the quark, and P_L and P_R are the LH and RH projection operators. The above interaction leads to a new contribution to the $D^0 - \overline{D^0}$-mixing via a loop of leptons and leptoquarks. The present bound on $\Delta M(D^0 - \overline{D^0})$ translates into the following bound on the leptoquark mass M_{LQ} and the couplings in the Lagrangian introduced above [92]:

$$\frac{1}{M_{LQ}^2}|(a_L^D)^2 + (a_R^D)^2 + (E_D + \frac{3}{2})a_L^D a_R^D| \leq 2.2 \times 10^{-3}\ TeV^{-2}, \tag{89}$$

where $a_L^D = g_L^u (g_L^c)^*$, $a_R^D = g_R^u (g_R^c)^*$, and the parameter E_D is defined as:

$$E_D = \frac{\langle \bar{D}^0|\bar{u}\gamma_5 c|0\rangle\langle 0|\bar{u}\gamma_5 c|D^0\rangle}{\langle \bar{D}^0|\bar{u}\gamma_\mu\gamma_5 c|0\rangle\langle 0|\bar{u}\gamma_5\gamma_\mu c|D^0\rangle}. \tag{90}$$

It is stressed in ref. [92] that these leptoquark couplings provide new bounds, as all previous FCNC bounds for chirally coupled leptoquarks could be avoided by demanding that the leptoquarks couple diagonally to the lepton and down sectors. Now, with the interaction in (88), one would have to demand the diagonality of the leptoquarks in the up quark sector as well. So, if the leptoquarks couple to RH quarks, all diagonality demands can be satisfied. However, if the leptoquarks couple to the LH quarks, the CKM-mixing implies that one cannot diagonalize the couplings to the down and up quarks simultaneously and hence the FCNC bounds on the leptoquark couplings cannot be evaded.

The scenario of leptoquarks that couple to the LH quarks has been studied in ref. [92]. Though the details depend on the $SU(2)_W$- and hypercharge Y-assignments of the leptoquark, the point can be illustrated assuming that the leptoquark in question is an $SU(2)_W$-doublet and carries a hypercharge $Y = -7/6$. Concentrating on the first generation of leptons and quarks, the coupling of the leptoquark (called here D_Q, with the subscript denoting its electrical charge) can be defined through the Lagrangian [92]:

$$\mathcal{L} = \sum_i \{g_i \bar{e} u_L^i D_{-5/3} + g_i' \bar{e} d_L^i D_{-2/3}\} \tag{91}$$

where the g_i and g_i' couplings are related by the CKM-matrix

$$g_i' = g_j V_{ji}. \tag{92}$$

In the numerical exercise of deriving bounds on the masses and coupling of the leptoquarks, the coupling of the third generation V_{13} is ignored, in which case the CKM-matrix parameters reduce to the Cabibbo angle. The present bond on $\Delta M(D^0 - \overline{D^0})$ then translates into a bound: $|g_1 g_2|/M_{LQ}(TeV) \leq 0.21 \sin\theta_C$. The same bound and its effectiveness can also be expressed as:

$$\frac{M_{LQ}/(21.6\ TeV)}{g_1 g_2} \geq [\frac{1.5 \times 10^{-4}\ eV}{\Delta M(D_1^0 - D_2^0)}]^{1/2} \tag{93}$$

With $g_1 g_2 = O(1)$, already the present experimental limits imply that $M_{LQ} \geq O(20)\ GeV$. Of course, this number goes down fast as $g_1 g_2$ become small. For a comparative study of bounds on the leptoquark mass and couplings, see the recent review by Schrempp [94].

- **Searches of physics beyond the Standard Model in rare decays of charmed hadrons**

These involve decays of the type $c \to u + X$, where $X = \gamma, \ell^+\ell^-$ and $\nu\bar{\nu}$, giving rise to decays such as $D \to \rho + \gamma$, $D^0 \to \mu^+\mu^-$, and $D^+ \to \pi^+\ell^+\ell^-$. Their rates in the Standard Model are rather small but they provide windows on new physics, as discussed below. The clean leptonic decays $D^0 \to \ell_1\bar{\ell}_2$ make them ideal candidates to search for at HERA. It has been argued in [16] that these decay modes could be used to search for leptoquark induced transitions. Here, the roles of rare K and B- decays are complementary, since they involve coupling to different quarks. If the leptoquarks are scalars, these decays proceed without the usual helicity suppression and consequently the experimental sensitivity to such decays is enhanced. Writing the effective four-fermion coupling as (g_{eff}^2/M_{LQ}^2), one obtains [16]:

$$\frac{M_{LQ}/(1.8\ TeV)}{g_{eff}} \geq [\frac{10^{-5}}{BR(D^0 \to \mu^+\mu^-)}]^{1/4} \tag{94}$$

Here g_{eff} is an effective coupling and includes possible mixing effects. Similarly, the decays $D^+ \to \ell^+\nu_\ell$, $D^+ \to \pi^+\ell^+\ell^-$ can be used to set bounds on M_{LQ}.

The possibility of observing the decays $D^0 \to \mu^+\mu^-$, e^+e^-, $\mu^\pm e^\mp$ at HERA has been studied by Egli et al. [16]. Note that the last of the decays, being lepton flavour violating,

is not allowed in the Standard Model. If SM provides the dominant contribution to the FCNC processes, then the anticipated branching ratios $BR(D^0 \to \mu^+\mu^-) \simeq 10^{-19}$ and $BR(D^0 \to e^+e^-) \sim 10^{-24}$ are unreachable at HERA–and for that matter anywhere else. However, these branching ratios could be dramatically enhanced if new physics effects are present, such as leptoquarks in the TeV mass-range, causing the decays $D \to \bar{l}_1 l_2$ at a measurable rate. In the monte carlo simulation undertaken in ref. [16], the D^*-tagging technique discussed earlier, $D^* \to D^0\pi$ with the subsequent decay $D^0 \to l_1\bar{l}_2$, has been used to estimate the sensitivity to the leptonic branching ratio of D^0. With $\sigma(ep \to c\bar{c}X) \simeq 0.5\ \mu b$ and an integrated luminosity of 100 pb^{-1}, a sensitivity of $1.5 \times 10^{-6}, 5.0 \times 10^{-7}$ and 1.0×10^{-6}, for $BR(D^0 \to \mu^+\mu^-)$, $BR(D^0 \to e^+e^-)$ and $BR(D^0 \to \mu^+e^-)$, respectively, can be reached at HERA. This is one to two orders of magnitude better than the present experimental limits [85]. Likewise, the sensitivity in the FCNC semileptonic decays $D^0 \to (\bar{K}^0, \pi^0, \rho^0)l^+l^-$ and $D^+ \to (K^+, \pi^+, \rho^+)l^+l^-$, could be substantially improved at HERA. We recall here that at present typical sensitivity in these channels is at $(1-2) \times 10^{-3}$ [85], though very recently the E791 collaboration has put a much improved upper limit on the branching ratio $BR(D^+ \to \pi^+\mu^+\mu^-)$ which is quoted as $< 3 \times 10^{-4}$ at 90 % $C.L.$ [52]. This bound is expected to become a factor 3 better after the background studies and analysis are fully understood. The sensitivity to such semileptonic decays, as well as, $D^0 - \bar{D}^0$ mixing and doubly Cabibbo suppressed decays $D^0 \to K^+\pi^-$ depends crucially on the ability to measure the charges of the $K^\pm, \pi^\pm$, etc. It is clear from the foregoing discussion that HERA experiments will not have a monopoly over rare charm decay seraches, and with the stiff competition already present and imminent in future from high statistics low-energy experiments, it is not going to be a "Spaziergang" for HERA experiments!

In summary, there is a good case for HERA in doing consolidation work in charm physics (charmed hadrons and J/ψ production cross sections, gluon density measurements). Also, improved sensitivity in rare charmed hadron decays is expected.

8 B-Physics at HERA

We have already discussed the b-quark production cross section and the beauty quark content of the nucleon at HERA energy in a previous section. We recall that the inclusive b-cross section in ep collisions at HERA is estimated as $O(10)pb$, which is comparable to the corresponding cross section at the Z^0 resonance. At both LEP and HERA B hadrons are produced in the continuum, hence higher mass B-states, such as Λ_b-baryons and $B_s(\equiv \bar{b}s)$ and $B_c(\equiv \bar{b}c)$ mesons, will be produced and detected and their properties studied. While on B physics, there exists the exciting possibilty of using the HERA proton beam halo to do a fixed internal target experiment. The cross section for $pA \to bX$ with the HERA proton energy $E_p = 820$ GeV is estimated to be $O(15)$ nb/nucleon, which increases by a factor ≈ 1.8, if the proton beam energy could be increased to $E_p = 1000\ GeV$, giving $O(10^9)$ B hadrons/year with a high-A target, such as Tungsten [93]. This number of B-hadrons is approximately three orders of magnitude larger than what would be produced in an ideal year in ep collisions. A fixed target experimental facility, therefore, provides an attractive

addition to the present research programme at HERA. In particular, it may allow a possible measurement of CP violation in B physics, apart from a number of important FCNC decays and a time-dependent measurement of the mixing ratio, $x_s = \Delta M(B_s^1 - B_s^2)/\Gamma(B_s)$, in the neutral B_s^0-system. However, many new and interesting aspects of B physics could also be studied in ep collisions, with good vertex resolution and lepton identification and assuming that an integrated luminosity of $O(1) fb^{-1}$ will be accumulated over a decade.

We discuss briefly the physics that one could study with $O(10^6)$ B-events per 100 pb^{-1} in eP-collisions at HERA. A number of higher mass beauty hadrons, both mesons and baryons, remains to be discovered. In the mesonic sector there are two which deserve special attention, namely B_s and B_c. Their masses have been estimated to lie in the range $M(B_s) = 5345-5388$ MeV and $M(B_c) = 6194 - 6292$ MeV, while their spin-1 partners are expected to have the masses $M(B_s^*) = 5410 \pm 10$ MeV and $M(B_c^*) = 6340 \pm 20$ MeV [95, 96, 97]. Since the anticipated mass difference $M(1^-) - M(0^-) \simeq 50(80)$ MeV for the $B_s(B_c)$ mesons is smaller than the pion mass, the heavier 1^--mesons will decay to the lighter 0^--mesons radiatively. For the B_s-mesons the cross section at HERA can be estimated assuming a value for P_s, the probability of the fragmentation $b \to B_s X$. With $P_s = 15\%$, one obtains:

$$\sigma(e + p \to B_s + X) = \\ 2\ P_s\ \sigma(e + p \to b\bar{b} + X) = O(2\ nb).$$

This would yield some 200,000 $B_s/\bar{B}_s$ mesons per 100 pb^{-1}. The dominant semileptonic decays, $B_s \to (D_s^+, D_s^{*+})\ \ell^-\nu_\ell$ and non-leptonic decays $B_s \to J/\psi(\phi, \eta, K^+K^-, ...)$ and $B_s \to D_s^{*+}D_s^{*-}, D_s^{*+}D_s^-, ...$ are measurable at HERA. The branching ratios for the B_s-decays in a good number of modes can be estimated from the corresponding known decays and branching ratios for the B_d-mesons. Using this analogy, the inclusive semileptonic branching ratio for $B_s \to D_s + X + \ell + \nu_\ell$ should be $O(10\%)$, for $\ell = e, \mu$, each. Assuming a detection efficiency of 10% for the D_s meson by summing over the known decay modes, one gets a cross section of $O(40\ pb)$ for the tagged B_s semileptonic decays at HERA. For the determination of the B_s-mass a non-leptonic decay is needed. The decay mode $B_s \to J/\psi\phi$ should have a branching ratio comparable to the decay mode $B_d \to J/\psi K^*$, which is put at $(3.7 \pm 1.3) \times 10^{-3}$ in the PDG tables [85]. With 14% branching ratio for $J/\psi \to \ell^+\ell^-$, one gets $\sigma(e + p \to B_s X \to J/\psi\ \phi \to \ell^+\ell^-\phi) = O(1\ pb)$, providing $O(100)$ tagged B_s-mesons in the indicated decay mode.

It should be remarked here that indirect evidence of the B_s-meson through the semileptonic decay $B_s \to D_s + X + \ell\nu_\ell$, with the subsequent decay $D_s \to \phi\pi^\pm$, has been presented in a number of experiments at LEP, and lately also by the CDF collaboration. A recent compilation gives the following number for the product branching ratio [98]:

$$\begin{aligned} f(b \to B_s) BR(B_s \to D_S^\pm + X) BR(D_s^\pm \to \phi + \pi^\pm) & \\ &= (5.9 \pm 1.9 \pm 1.1) \times 10^{-3} \quad [OPAL] \\ &= (3.9 \pm 1.0 \pm 0.5) \times 10^{-3} \quad [ALEPH] \\ &= (6.1 \pm 1.9 \pm 1.2) \times 10^{-3} \quad [DELPHI] \end{aligned} \tag{95}$$

Recently, direct evidence for the decay of B_s meson in the exclusive non-leptonic decays $B_s \to J/\psi + \phi,\ \psi'\phi,\ D_s\pi$ has also been presented [98]. Already in 1992, the OPAL collaboration had reported a first candidate for B_S in the mode $B_s \to J/\psi + \phi$, with a mass $m(B_s) = 5.36 \pm 0.07\ GeV$ [99]. Statistically significant measurements of the B_s mass have been reported by the CDF collaboration with $m(B_s) = 5.3833 \pm 0.0045 \pm 0.0015\ GeV$ (reconstructed from 16 events in the mode $B_s \to J/\psi + \phi$) [98] and $m(B_s) = 5.3686 \pm 0.0056 \pm 0.0015\ GeV$ by the ALEPH collaboration (reconstructed from 2 events in the mode $B_s \to \psi' + \phi$ and $B_s \to D_s + \pi$) [100]. These and related modes should also be observable at HERA; in particular the fixed target HERA-B experiment has the potential of doing precision measurements in B_s decays, including the rare decays $B_s \to (\phi, \eta, \eta', K^+K^-) + \ell^+\ell^-$ and radiative rare B_s-decays $B_s \to (\phi, K^+K^-, ...) + \gamma$. Some of the rare B-decay branching ratios and the present limits are given in Table 4.

Next, we discuss the more massive and less copious B_c-mesons with characteristic decay signatures $B_c^+ \to J/\psi\ell^+\nu_\ell$ (semileptonic) and $B_c^+ \to J/\psi D_s^{*+},\ J/\psi\rho^+$ etc. (nonleptonic). The cross sections for B_c production at HERA and LEP are both small and model dependent. The quantitity $F(B_c)$ defined as:

$$F(B_c) \equiv \frac{\sigma(B_c^+X) + \sigma(B_c^-X)}{\sigma(b\bar{b}X)} \tag{96}$$

has been estimated by Lusignoli et al. to be $F(B_c) = (2.0 \pm 0.2) \times 10^{-3}$ and $(1.0 \pm 0.2) \times 10^{-3}$ for HERA and LEP, respectively [101]. This gives a cross section $\sigma(e+p \to B_cX) = O(10\ pb)$, yielding $O(10^3)$ B_c mesons per 100 pb^{-1} at HERA in ep collisions. The quantity $F(B_c)$ is uncertain for the fixed target HERA-B experiment because of the phase space suppression and threshold energy. Increasing the HERA proton energy here from the nominal 820 GeV to 1000 GeV would not only increase $\sigma(b\bar{b})$ but more importantly $F(B_c)$. Estimating conservatively, $F(B_c) = 5 \times 10^{-4}$, would then lead to a cross section $\sigma(pp \to B_C + X) \simeq\ O(10\ pb)$, leading to about 10^5 B_c mesons per year at the HERA fixed target experiment.

The cleanest experimental signature for the B_c^+-meson is through the semileptonic decay $B_c^+ \to J/\psi\ell^+\nu_\ell$, with the subsequent decay $J/\psi \to \ell^+\ell^-$, giving rise to three charged leptons in a jet with a branching ratio around 1% [101]. With this cross section for the B_c-meson, one expects $O(10)$ three-lepton states in a jet for an integrated luminosity of 100 pb^{-1} at HERA (ep) and $O(10^3)$ at the HERA-B. This will establish the B_c-signal. However, measuring its mass would require the detection in a non-leptonic mode, which may or may not be accessible with $O(10^6)$ B-decays at HERA(ep). However, this should be well within the reach of the HERA-B experiment. For related studies of B_c-search strategies at hadron colliders and a compilation of the dominant decay modes of B_c-mesons, we refer to a recent report [114]. Since heavy quarks in ep collisions are dominantly produced with low energy, the probability of the fusion $b\bar{c} \to B_c + X$ at HERA in the process $e + p \to c\bar{c} + b\bar{b} + X$ may not be as suppressed as the above estimates of $F(B_c)$ indicate. A more reliable calculation is needed here.

Concerning b-baryons, the UA1 collaboration [102] published a measurement concerning the lightest of them, Λ_b, which was reconstructed in the exclusive decay mode $\Lambda_b \to J/\psi\Lambda$,

with the mass $m(\Lambda_b) = (5640 \pm 50 \pm 30)\ MeV$ and the product branching ratio

$$P_{\Lambda_b} Br(\Lambda_b \to J/\psi\Lambda) = (1.8 \pm 1.0) \times 10^{-3}. \tag{97}$$

Unfortunatley, this measurement has not yet been confirmed by the LEP experiments and CDF. The 90% confidence level limits from the various experiments are [98]:

$$\begin{aligned} P_{\Lambda_b} Br(\Lambda_b \to J/\psi\Lambda) & \\ & < 1.5 \times 10^{-3} \quad [OPAL] \\ & < 0.4 \times 10^{-3} \quad [ALEPH] \\ & < 0.5 \times 10^{-3} \quad [CDF] \end{aligned} \tag{98}$$

So, the non-leptonic decays of Λ_b have yet to be measured.

Indirect evidence for the Λ_b-baryon has come from the ALEPH [103], OPAL [104] and DELPHI [105] collaborations at LEP, through the semileptonic decays and the ensueing $\Lambda\ell$ correlations. The posted product branching ratios are [98]:

$$\begin{aligned} P_{\Lambda_b} Br(\Lambda_b \to \Lambda_c^+ \ell^- \bar{\nu}_\ell) Br(\Lambda_c^+ \to \Lambda X) & \\ & = [0.47 \pm 0.11(stat.) \pm 0.10(syst.)]\% \quad [ALEPH] \\ & = [0.28 \pm 0.05(stat.) \pm 0.08(syst.)]\% \quad [OPAL] \\ & = [0.32 \pm 0.05(stat.) \pm 0.08(syst.)]\% \quad [DELPHI] \end{aligned} \tag{99}$$

Measurements along these lines are also feasible both at HERA (ep), given that $O(10^5)$ b-baryons will be produced per 100 pb^{-1} luminosity. At HERA-B, there will be enough b-baryons to study their semileptonic and non-leptonic decays and measure the mass spectra.

We now review the prospects of measuring the mixing parameter χ_s and the mass difference $x_s = (\Delta M/\Gamma)_s$ at HERA. A number of experiments in e^+e^--annihilation and $p\bar{p}$ collisions has measured the mixing parameter χ characterizing mass mixing in the neutral B-meson sector. The present world average is $\chi = 0.121 \pm 0.010$, whereas the LEP-average is $\chi = 0.117 \pm 0.010$ [109], where the quantity χ is defined as $\chi = P_d\chi_d + P_s\chi_s$, with $\chi_d(\chi_s)$ being the mixing parameter in the $B_d^0 - \bar{B}_d^0$ ($B_s^0 - \bar{B}_s^0$) sector and $P_d(P_s)$ the production probability for the $B_d(B_s)$ meson. A recent complilation of χ_d from the time-integrated measurements at the $\Upsilon(4S)$ by the ARGUS and CLEO collaboration and the recent time-dependent measurements of the same quantity at LEP by the ALEPH and DELPHI collaborations gives $\chi_d = 0.159 \pm 0.024$ [109]. This translates into the ratio $x_d = \Delta M/\Gamma(B_1^d - B_2^d) = 0.685 \pm 0.076$. With $\chi_d = 0.159 \pm 0.024$ and $\chi = 0.121 \pm 0.010$, only a model dependent determination of χ_s is possible. For example, using $P_d = 0.375$ and $P_s = 0.15$ one gets $\chi_s > 0.22$ (at 90%C.L.), consistent with the SM expectations, $\chi_s \simeq 0.5$.

In the HERA study [14], the statistical accuracy attainable for $\Delta\chi/\chi$ has been estimated to be 7% with an integrated luminosity of 200 pb^{-1} using the dilepton modes $e + p \to b + \bar{b} + X \to l^\pm l^\pm + X$ and $e + p \to b + \bar{b} + X \to l^\pm l^\mp + X$. Flavour correlations are important

in disentangling χ_s and χ_d in continuum b-production experiments and we refer to [107] for a detailed discussion on this point.

The ratio involving the mass difference and lifetime for the $B_s^0 - \bar{B}_s^0$ system, $x_s = (\frac{\Delta M}{\Gamma})_s$, is an important quantity to calibrate the strength of the $|\Delta B| = 2, \Delta Q = 0$ transitions. Since x_d, the mixing ratio for the $B_d^0 - \bar{B}_d^0$ system, is known, a measurement of x_s could be reliably used to determine the Cabibbo-Kobayashi-Maskawa (CKM) matrix element ratio $|V_{ts}/V_{td}|$ via the relation $\frac{x_s}{x_d} = |\frac{V_{ts}}{V_{td}}|^2(1+\Delta)$ [106], where $\Delta \sim O(SU(3)\text{ breaking}) \sim 0.2-0.3$. The above measurement can be combined with the unitarity constraint $|V_{ts}| = |V_{bc}| \simeq 0.045$ to provide a good determination of the (otherwise difficult to measure) CKM matrix element, V_{td}. Present CKM phenomenology gives $x_s \simeq 15$ [107]. Such a large value will make it mandatory to undertake time-dependent measurements to determine x_s. A case to case study of various experimental facilities in carrying out x_s-measurement has been done in [107], where also the HERA case is discussed. One estimates that for a proper time resolution $\sigma_t/\tau \leq 0.1$ and the B_s-tagging efficiency of $O(10^{-3})$ at least $O(10^7)$ B hadrons will be required to measure $x_s \leq 10$. This, according to the quoted b-quark cross section at HERA, would require an integrated luminosity of 1000 pb^{-1} – a long term goal for HERA ep experiments.

However, time-dependent measurements of x_s are more likely to be done in hadron colliders and fixed target experiments, such as HERA-B. Since the cross section for producing the B_s mesons at HERA-B is sufficiently large, the limitations will be set by the trigger and the resolution in proper time, which in fact are the main constraining factors. These aspects deserve a dedicated study in their own right.

Finally on the B-physics we discuss the prospects of measuring rare B-decays at HERA. They provide fertile grounds for testing the Standard Model in FCNC processes. Since the short distance contributions are dominated by the top quark, rare B-decays provide an alternative means to bound the top quark mass and constrain the CKM matrix elements V_{td}, V_{ts}, and V_{tb}. In this context, radiative decay of the b-quark $b \to s+\gamma$ have received special attention, for which one obtains an inclusive branching ratio $BR(b \to s+\gamma) = (3-4) \times 10^{-4}$ for the top quark mass in the range $100 - 200$ GeV [108]. Theoretical estimates for $B \to K^\star + \gamma$ are less certain, with the predictions dispersed over a larger range. This decay mode has been seen recently by the CLEO collaboration which has reported a branching ratio $Br(B \to K^\star + \gamma) = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$ [111]. It is important to measure the decay rate $B \to X_s + \gamma$ inclusively (here X_s denotes hadrons with total strangeness S=-1), for example via the inclusive photon energy spectrum as has been advocated in ref. [113]. Theoretical uncertainties in this case are expected to be relatively small though from an experimental point of view such measurements are quite challenging. The present upper limit quoted on this mode by the CLEO collaboration is: $BR(B \to X_s + \gamma) < 5.4 \times 10^{-4}$ at (95 $\%C.L.$) [110].

Likewise, B-decays involving a charged dilepton pair, $B \to X_s + \ell^+\ell^-$, provide precision tests of the Standard Model. The SM estimates for the short-distance (m_t-dominated) contributions are: $BR((B_d, B_u) \to X_s + \mu^+\mu^-) = 7.0 \times 10^{-6}$ and $BR((B_d, B_u) \to X_s + e^+e^-) = 1.2 \times 10^{-5}$, for $m_t = 150$ GeV [108]. Based on model calculations, one expects that the low lying states such as $(K, K^*, K^{**}, ..)$ will contribute a significant fraction to the

inclusive rare decays involving dileptons [108].

Even more interesting from a theoretical point of view are the CKM-suppressed radiative decays, $B \to X_d + \gamma$ (likewise $B \to \rho + \gamma, ...$), as well as the FCNC semileptonic decays $B \to X_d + (\ell^+\ell^-)$ (likewise $B \to (\pi, \rho, A_1, ...) + (\ell^+\ell^-)$), since they measure the CKM matrix element $|V_{td}|$. A branching ratio of $(1-2) \times 10^{-5}$ for the inclusive decay $B \to X_d + \gamma$, and $(1-2) \times 10^{-6}$ for the exclusive decay $B \to \rho + \gamma$ have been estimated in ref. [113]. These rates would be very challenging to measure in HERA ep collisions. The branching ratios for a number of CKM-allowed rare B-decays, estimated in the Standard Model, together with the present experimental limits are given in [108].

Summarizing the discussion on rare B-decays, we note that they can be searched for more successfully in a high luminosity experiment at $\Upsilon(4S)$ and in high energy hadronic collisions such as at the Fermilab $p\bar{p}$ collider but also in a fixed target experiment using the HERA proton beam. Since the present experimental limits on rare B-decays are already very stringent, the sensitivity in the ep mode at HERA is limited to the CKM-allowed decays like $B \to X_s + \gamma$ and $B \to X_s + \ell^+\ell^-$, if $O(10^7)$ B-decays could be analysed. Searches for rare B decays would certainly be an additional motivation for the high luminosity version of HERA (ep). Likewise, there is a good physics case for a fixed target dedicated B-experiment at HERA, or even better pp collisions. We have outlined here some of the advantages that such an experiment would provide concerning precision tests of the Standard Model in the flavour sector.

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References

[1] M. Derrick et al. (ZEUS) Phys. Lett. **B293** (1992) 465.

[2] T. Ahmed et al. (H1) Phys. Lett. **B299** (1993) 374.

[3] T. Ahmed et al. (H1), Phys. Lett. **B297** (1992) 205.

[4] M. Derrick et al. (ZEUS) Phys. Lett. **B297** (1992) 404.

[5] M. Derrick et al. (ZEUS) Phys. Lett. **B303** (1993) 183.

[6] M. Derrick et al. (ZEUS) Phys. Lett. **B306** (1993) 158.

[7] T. Ahmed et al. (H1) Phys. Lett. **B298** (1993) 469.

[8] T. Ahmed et al. (H1) Phys. Lett. **B299** (1993) 385.

[9] M. Derrick et al. (ZEUS) DESY Report 93-093 (1993).

[10] M. Erdmann, H. Küster, S. Levonian, Y. Sirois and C. Vallee (H1), DESY Report 93-077 (1993).

[11] A.D. Martin (these proceedings).

[12] J. Terrón (these proceedings).

[13] M.W.S. Krasny (these proceedings).

[14] A. Ali et al., in *Proceedings of the HERA Workshop*, DESY, Hamburg, R. D. Peccei, ed. (1988).

[15] A. Ali and D. Wyler, in *Proceedings of the HERA Workshop*, DESY, Hamburg, W. Buchmüller and G. Ingelman, eds. (1992).

[16] S. Egli et al., in *Proceedings of the HERA Workshop*, DESY, Hamburg, W. Buchmüller and G. Ingelman, eds. (1992).

[17] R. van Woudenberg et al., in *Proceedings of the HERA Workshop*, DESY, Hamburg, W. Buchmüller and G. Ingelman, eds. (1992).

[18] H. Jung, G.A. Schuler and J. Terrón, in *Proceedings of the HERA Workshop* DESY, Hamburg, W. Buchmüller and G. Ingelman, eds. (1992).

[19] G. Ingelman and G.A. Schuler, Z. Phys. **C40** (1988) 299; U. Baur and J.J. van der Bij, Nucl. Phys. **B304** (1988) 451; M. Glück, R.M. Godbole and E. Reya, Z. Phys. **C38** (1988) 441, Erratum **C39** (1988) 590; R.A. Eichler and Z. Kunszt, Nucl. Phys. **B308** (1988) 791; G. A. Schuler, Nucl. Phys. **B299** (1988) 21; G. Abbiendi and L. Stanco, Z. Phys. **C51** (1991) 81.

[20] H. Jung et al., DESY Report 93-072, ZU-TH 12/93 (1993).

[21] R.K. Ellis and P. Nason, Nucl. Phys. **B312** (1989) 551.

[22] J. Smith and W.L. van Neerven, Nucl. Phys. **B374** (1992) 36.

[23] W.L. van Neerven, in *Proc. of 1992 Zeuthen Workshop on Elementary Particle Theory: Deep Inelastic Scattering*, Teupitz/Brandenburg, Germany, (J. Blümlein and T. Riemann, eds.) Nucl. Phys. B (Proc. Suppl.) **29A** (1992) 199.

[24] E. Laenen, S. Riemersma, J. Smith and W.L. van Neerven, Phys. Lett. **B291** (1992) 325; ITP Stony Brook Report, ITP-SB-92-09 (1992); ITP Report ITP-SB-92-41 & Fermilab-Pub-92/271-T (1992).

[25] S. Catani, M. Ciafaloni and F. Hautmann, Phys. Lett. **B242** (1990) 97; Nucl. Phys. **B366** (1991) 135, and in *Proc. of 1992 Zeuthen Workshop on Elementary Particle Theory: Deep Inelastic Scattering*, Teupitz/Brandenburg, Germany, (J. Blümlein and T. Riemann, eds.) Nucl. Phys. B (Proc. Suppl.) **29A** (1992) 182.

[26] G. Ingelman and P.E. Schlein, Phys. Lett. **B152** (1985) 256.

[27] D. Amidei (CDF), talk at the *5th. International Symposium on Heavy Flavour Physics*, Montreal, Canada, July 6-10 (1993).

[28] L. Rolandi, Proceedings of the *XXVI International Conference on High Energy Physics*, Dallas, USA, (AIP Conference Proceedings No. 292) Vol. I, p. 56, James R. Sanford ed. (1993).

[29] A. Ali et al., in *Proceedings of the ECFA-LHC Workshop*, Aachen, FRG, CERN-90-10 & ECFA 90-133, Vol. II, p. 917, G. Jarlskog & D. Rein, eds. (1990).

[30] N. Cabibbo, Phys. Rev. Lett. **10** (1963) 531; M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49** (1973) 652.

[31] J.J. Aubert et al, Nucl. Phys. **B213** (1983) 31; E. Hoffmann and R. Moore, Z. Phys. **C20** (1983) 71; C.S. Kim, *ibid* **B353** (1991) 87.

[32] J.C. Anjos et al. (E691), Phys. Rev. Lett. **65** (1990) 2503.

[33] M.P. Alvarez et al. (NA14/2), CERN-EP-88-148 (1988) and CERN-PPE-92-028 (1992).

[34] See, for example, J. Wiss (E687), in *Proc. of Third Topical Seminar on Heavy Flavours*, San Miniato, Italy (F. Grancagnolo, F.-L. Navarria and P.G. Pelfer, eds.) Nucl. Phys. B (Proc. Suppl.) **27** (1992) 207.

[35] See, for example, P.E. Karchin and Z. Wu (E769), in *Proc. of Third Topical Seminar on Heavy Flavours*, San Miniato, Italy (F. Grancagnolo, F.-L. Navarria and P.G. Pelfer, eds.) Nucl. Phys. B (Proc. Suppl.) **27** (1992) 219.

[36] P. Nason, S. Dawson and R.K. Ellis, Nucl. Phys. **B327** (1989) 49.

[37] W. Beenakker, H. Kuijf, W.L. van Neerven and J. Smith, Phys. Rev. **D40** (1989) 54; W. Beenakker, W.L. van Neerven, R. Meng, G.A. Schuler and J. Smith, Nucl. Phys. **B351** (1991) 507; E. Laenen, S. Riemersma, J. Smith and W.L. van Neerven, ITP-SB-92-09.

[38] M. Aguilar-Benitez et al., Z. Phys. **C40** (1983) 321.

[39] G. Bertsch, S.J. Brodsky, A.S. Goldhaber and J. Gunion, Phys. Rev. Lett. **47** 297 (1981) 297; S.J. Brodsky and P. Hoyer, Phys. Rev. Lett. **63** (1989) 1566; R. Vogt, S.J. Brodsky and P. Hoyer, Nucl. Phys. **B360** (1991) 67.

[40] S.J. Brodsky et al., Nucl. Phys. **B369** (1992) 519.

[41] G. Ingelman, L. Jönsson and M. Nyberg, DESY 92-178 (1992).

[42] J. Wiss (E687), in *Proc. of XIII Internat. Conf. on Physics in Collision, Heidelberg, Germany, June 16-18, 1993.*

[43] J.Blümlein et al., Z. Phys. **C45** (1990) 501.

[44] M. Drees and K. Grassie, Z. Phys. **C28** (1985) 451.

[45] H. Abramowicz, K. Charchula and A. Levy, Phys. Lett. **B269** (1991) 458; see also H. Abramowicz et al., DESY 91-057 (1991).

[46] M. Glück, E. Reya and A. Vogt, Phys. Rev. **D45** (1992) 3986; *ibid* **D46** (1992) 1973.

[47] L.E. Gordon and J.K. Storrow, Z. Phys. **C56** (1992) 307.

[48] V.N. Gribov and L.N. Lipatov, Sov. Jour. Nucl. Phys. **15** (1972) 438 and 675.

[49] G. Altarelli and G. Parisi, Nucl. Phys. **126** (1977) 297.

[50] M. Diemoz et al., Z. Phys. **C39** (1988) 21;
J.V. Allaby et al., Phys. Lett. **B197** (1987) 281.

[51] C. Peterson et al., Phys. Rev **D27** (1983) 105.

[52] M. Sokoloff (E791); talk presented at DESY, June 28, 1993, and private communication.

[53] L.N. Lipatov, Sov. J. Nucl. Phys. 23 (1976) 338; E.A. Kuraev, L.N. Lipatov and V.S. Fadin, Sov. Phys. JETP 45 (1977) 199; Ya. Balitskii and L.N. Lipatov, Sov. J. Nucl. Phys. 28 (1978) 822.

[54] A.H. Mueller, Nucl. Phys. **B307** (1988) 34; *ibid* **B317** (1989) 573; *ibid* **B335** (1991) 115.

[55] J. Bartels, A. De Roeck, M. Loewe, *Z.Phys.* **C54** (1992) 635.

[56] J. Bartels, M. Besancon, A. De Roeck, J. Kurzhoefer, in *Proceedings of the HERA Workshop*, DESY, Hamburg, W.Buchmüller and G.Ingelman eds. (1992).

[57] J. Kwiecinski, A.D. Martin, P.J. Sutton, Phys. Lett. **B287** (1992) 254; Phys. Rev. **D46** (1992) 921.

[58] W.-K. Tang, Phys. Lett. **B278** (1991) 363.

[59] E. Eichten, C. Quigg, I. Hinchliffe and K. Lane, Rev. Mod. Phys. **56** (1984) 579.

[60] M. Drees and R.M. Godbole, Phys. Rev. Lett. **61** (1988) 682.

[61] J.C. Collins and R.K. Ellis, Nucl. Phys. **B360** (1991) 3.

[62] E.M. Levin et al., Sov. J. Nucl. Phys. **53** (1991) 657.

[63] E.A. Kuraev, L.N. Lipatov and V.S. Fadin, Sov. Phys. JETP **45** (1977) 199.

[64] B.R. Webber, Cavendish-HEP-91/18 (1991).

[65] E. B. Zijlstra and W.L. van Neerven, Nucl. Phys. **B283** (1992) 525.

[66] J.G. Morfin and Wu-Ki Tung, Z. Phys. **C52** (1991) 13.

[67] E.L. Berger and D. Jones, Phys. Rev. **D23** (1981) 1521.

[68] R. Baier and R. Rückl, Nucl. Phys. **B218** (1983) 289.

[69] S.P. Baranov and H. Jung, DESY Report 93-048 (1993).

[70] B.H. Denby et al. (FTPS), Phys. Rev. Lett. **52** (1984) 795.

[71] R. Barate et al. (NA14), Z. Phys. **C33** (1987) 505.

[72] J.J. Aubert et al. (EMC), Nucl. Phys. **B213** (1983) 1.

[73] D. Allasia et al. (NMC), Phys. Lett. **B258** (1991) 493.

[74] K.J. Abraham, Phys. Lett. **B240** (1991) 224.

[75] A. Donnachie and P.V. Landshoff, Nucl. Phys. **B244** (1984) 322.

[76] A. Donnachie and P.V. Landshoff, Phys. Lett. **B296** (1992) 227.

[77] G.A. Schuler and T. Sjöstrand, Phys. Lett. **B300** (1993) 169.

[78] H. Abramowicz et al., Phys. Lett. **B269** (1991) 465.

[79] G. Wolf, DESY 92-190 (1992).

[80] G. Schuler and S. Terrón, DESY Report 92-017 (1992).

[81] E.L. Berger et al., Nucl. Phys. **B286** (1987) 704.

[82] S. Aoki et al. (WA75), CERN-PPE/92-157 (to appear in Prog. Theor. Phys.).

[83] F. Muheim (CLEO), talk at the *5th. International Symposium on Heavy Flavour Physics*, Montreal, Canada, July 6-10 (1993).

[84] See, P. Burchat, Review talk at the *5th. International Symposium on Heavy Flavour Physics*, Montreal, Canada, July 6-10 (1993).

[85] M. Aguilar-Benitez et al. (Particle Data Group), Phys. Lett. **B239** (1990) 1.

[86] G. Martinelli, in *Proceedings of the ECFA Workshop on a European B-Meson Factory*, ECFA 93/151, DESY 93-053, R. Aleksan and A. Ali eds. (1993).

[87] J.F. Donoghue et al., Phys. Rev. **D33** (1986) 179.

[88] L. Wolfenstein, Phys. Lett. **B164** (1985) 170.

[89] H. Georgi, Phys. Lett. **B297** (1992) 353.

[90] T. Ohl, G. Ricciardi, and E. Simmons, Harvard Univ. Preprint #HUTP-92/A053 (1992).

[91] H. Yamamoto (CLEO), talk at the *5th. International Symposium on Heavy Flavour Physics*, Montreal, Canada, July 6-10 (1993).

[92] M. Leurer, Weizmann Institute Preprint WIS-93/26 (1993).

[93] An Experiment to Study CP Violation in the B System Using an Internal Target at the HERA Proton Ring (Letter of Intent), H. Albrecht et al., DESY-PRC 92/04 (1992). See also, W. Hofmann, DESY Report 93-026 (1993).

[94] F. Schrempp, DESY Report 93-096 (1993).

[95] W. Kwong and J.L. Rosner, Phys. Rev. **D44** 212 (1991) 212; A. Martin and J.M. Richard, Phys. Lett. **B185** (1987) 426; D.B. Lichtenberg et al., Z. Phys. **C47** (1990) 83.

[96] R.A. Bertlman and A. Martin, Nucl. Phys. **B168** (1980) 111.

[97] A. Martin, in *Heavy Flavors and High Energy Collisions in the 1-100 TeV Range*, A. Ali and L. Cifarelli, eds. p. 141 (Plenum, New York, 1989).

[98] For a recent review see, F. Pierre, talk at the *5th. International Symposium on Heavy Flavour Physics*, Montreal, Canada, July 6-10 (1993).

[99] P.D. Acton et al. (OPAL), Phys. Lett. **B295** (1992) 357.

[100] D. Buskulic et al. (ALEPH), CERN-PPE/93-97 (1993).

[101] M. Lusignoli, M. Masetti, and S. Petrarca, Phys. Lett. **B266** (1991) 142.

[102] C. Albajar et al. (UA1), Phys. Lett. **B273** (1991) 540.

[103] B. Decamp et al. (ALEPH), Phys. Lett. **B278** (1992) 209.

[104] P.D. Acton et al. (OPAL), Phys. Lett. **B281** (1992) 394.

[105] P. Abreu et al. (DELPHI), CERN-PPE-93-32 (1993).

[106] A. Ali and Z.Z. Aydin, Nucl. Phys. **B148** (1979) 165.

[107] A. Ali and D. London, DESY Report 92-075 (1992) (to appear in J. Phys. G. (UK)); and in *Proceedings of the ECFA Workshop on a European B-Meson Factory*, ECFA 93/151, DESY 93-053, R. Aleksan and A. Ali eds. (1993).

[108] For a recent review of rare B decays and further references see A. Ali, C. Greub and T. Mannel DESY Report 93-016 (1993) and in *Proceedings of the ECFA Workshop on a European B-Meson Factory*, ECFA 93/151, DESY 93-053, R. Aleksan and A. Ali eds. (1993).

[109] H. G. Moser, Review talk at the *5th. International Symposium on Heavy Flavour Physics*, Montreal, Canada, July 6-10 (1993).

[110] E.H. Thorndike (CLEO), Talk at the American Physical Society Meeting, Washington (D.C.), April 12 (1993).

[111] R. Ammar et al. (CLEO), CLNS-93-1212 (1993).

[112] C. Albajar et al. (UA1), Phys. Lett. **B262** (1991) 163.

[113] A. Ali and C. Greub , Z. Phys. **C49** (1991) 431; Phys. Lett. **B259** (1991) 182; *ibid* **B287** (1992) 191; *ibid* **B293** (1992) 226; DESY Report 93-065 (1993) (to appear in Z. Physik C).

[114] A.K. Likhoded et al., Theory Working Group, UNK B Factory Workshop, Liblice Castle (1993); Univ. of Bari Report BARI-TH/93-137 (1993).

Figure 5: *The $p_\perp^2$-distribution of D-mesons produced in the charm photoproduction process a)-off Be target by the E691 collaboartion fitted with the curve described in the text (from [32]); b)-off Si target by the NA14/2 collaboration. The curves shown in fig. b correspond to the assumed values for m_c and an exponential form* $\exp(-6m_T)$ *(from [33]).*

Figure 6: *Total photoproduction cross section results from low energy fixed-target experiments and HERA. The curves labelled DL and ALLM are predictions from Donnachie and Landshoff [75] and Abramowicz et al. [78], respectively. The top two curves are the LAC1 predictions [45], with the $p_{\perp min}$ cut of 1.4 (dotted) and 2.0 GeV (dashed). The dash-dotted curves are the Drees-Grassie parametrization [44] with 1.4 (upper) and 2.0 GeV (lower).*

Figure 7: The x-dependence of the ratio $K_2^{(2)}(x, Q^2, m_c^2)$ for $Q^2 = 10$ (GeV)^2 (lower set of curves) and for $Q^2 = 100$ (GeV)^2 (upper set of curves) and three mass factorization scales, i.e. $M = (Q^2 + m_c^2)^{1/2}/2$ (the long-dashed curve), $M = (Q^2 + m_c^2)^{1/2}$ (the solid curve) and $M = 2(Q^2 + m_c^2)^{1/2}$ (the short-dashed curve) (from ref. [24]).

Figure 8: The x-dependence of the ratio $K_2^{(2)}(x, Q^2, m_b^2)$ for $Q^2 = 10$ (GeV)^2 (lower set of curves) and for $Q^2 = 100$ (GeV)^2 (upper set of curves) and three mass factorization scales, i.e. $M = (Q^2 + m_b^2)^{1/2}/2$ (the long-dashed curve), $M = (Q^2 + m_b^2)^{1/2}$ (the solid curve) and $M = 2(Q^2 + m_b^2)^{1/2}$ (the short-dashed curve) (from ref. [24]).

Figure 9: Resummed and 1-loop coefficient functions for beauty quark production in the small ρ region ($m = 5$ GeV, $\Lambda = 210$ MeV) (γg channel).

Figure 10: Resummed coefficient function for beauty quark production ($m = 5$ GeV, $\Lambda = 210$ MeV) after performing the matching with the one-loop result (γg channel) (from ref. [25].)

Figure 11: Resummed coefficient function for beauty quark production ($m = 5$ GeV, $\Lambda = 210$ MeV) after performing the matching with the one-loop result (γq channel) (from ref. [25].)

Figure 12: The gluon density $x_g G(x_g)$ reconstructed from the monte carlo simulation of the process $ep \to D^{*+}X$ at HERA, using the Morfin-Tung parametrizations. The statistical error corresponds to an integrated luminosity of 6 pb^{-1} (from ref. [17]).

Figure 13: The differential distribution $d^2\sigma/dQ^2dx$ in DIS events as a function of Q^2 for the two indicated x-ranges, measured by the ZEUS collaboration. Comparison with the extrapolation of three parametrizations for the proton structure functions is also shown. (from ref. [5]).

Figure 14: J/ψ-photoproduction mechanisms (basic processes) and possible models (from ref. [18]).

Figure 15: The distribution $d\sigma/dz$ for the J/ψ-photoproduction calculated in the CS model compared to the data from the FTPS ($\diamond$), NA14 ($\circ$) at $E_\gamma = 100\ GeV$, and NMC ($\Box$) at $E_\gamma = 150\ GeV$. The CS-model is multiplied by $K = 5.6$ in order to fit the NA14 data (from ref. [18]).

Figure 16: The distribution $d\sigma/dz$ for the J/ψ-photoproduction calculated in the CS model with and without the relativistic corrections. The three curves shown correspond to the following values of the parameter ϵ/m_c: Solid curve($\epsilon/m_c = 0$), dashed curve ($\epsilon/m_c = 0.16$), dotted curve ($\epsilon/m_c = 0.30$). A K-factor of 2.0 has been used . Also shown are the data from the EMC and NMC (from ref. [20]).

Figure 17: Slope of the elastic ρ^0 production measured by the H1 collaboration and by lower energy fixed target experiments. For the H1 data only statistical errors are shown: a) $W_{\gamma p} < 40\ GeV$, b) $W_{\gamma p} > 40\ GeV$, c) the slope versus E_γ^{lab} with the curve from ref. [77]. (from ref. [10]).

Figure 18: The gluon density $x_g G(x_g, \mu^2)$ reconstructed from the monte carlo simulation of the inelastic J/ψ electroproduction at HERA, using the Morfin-Tung parametrizations. The statistical error corresponds to an integrated luminosity of 50 pb^{-1} for the set B_1 and 20 pb^{-1} for the set B_2. The $\diamond$ represent the recent NMC data (from ref. [18]).

Table 4: Estimates of the branching fractions for FCNC B-decays in the Standard Model for $m_t = 150\ GeV$, $\mu = 5\ GeV$ and $f_{B_d} = 200\ MeV$. Note that the CKM-suppressed decays depend quadratically on $|V_{td}|$, and the numbers correspond to $|V_{td}| = 9.18 \times 10^{-3}$. Experimental status is also given; upper limits are mostly 90 % C.L. except for the decays $(B_d, B_u) \to X_s + \gamma$, where the 95 % C.L. upper bound from CLEO is quoted.

Decay Modes	Br	Experimental Status
$(B_d, B_u) \to X_s\gamma$	3.5×10^{-4}	$< 5.4 \times 10^{-4}$ [CLEO][110]
$(B_d, B_u) \to K^*\gamma$	$(4.0 - 6.8) \times 10^{-5}$	$5.4 \pm 1.9 \times 10^{-5}$ [CLEO][111]
$(B_d, B_u) \to X_d\gamma$	1.5×10^{-5}	–
$(B_d, B_u) \to \rho + \gamma$	$(2.8 - 4.5) \times 10^{-6}$	–
$(B_d, B_u) \to X_s e^+e^-$	1.5×10^{-5}	–
$(B_d, B_u) \to X_d e^+e^-$	6.5×10^{-7}	–
$(B_d, B_u) \to X_s \mu^+\mu^-$	8.5×10^{-6}	$< 5.0 \times 10^{-5}$ [UA1][112]
$(B_d, B_u) \to X_d \mu^+\mu^-$	3.7×10^{-7}	–
$(B_d, B_u) \to K e^+e^-$	6.0×10^{-7}	$< 5.0 \times 10^{-5}$ [PDG][85]
$(B_d, B_u) \to K \mu^+\mu^-$	6.0×10^{-7}	$< 1.5 \times 10^{-4}$ [PDG][85]
$(B_d, B_u) \to K^* e^+e^-$	5.6×10^{-6}	–
$(B_d, B_u) \to K^* \mu^+\mu^-$	2.9×10^{-6}	$< 2.3 \times 10^{-5}$ [UA1][112]
$(B_d, B_u) \to X_s\ \nu\bar{\nu}$	8.0×10^{-5}	–
$(B_d, B_u) \to X_d\ \nu\bar{\nu}$	3.5×10^{-6}	–
$(B_d, B_u) \to K\ \nu\bar{\nu}$	6.4×10^{-6}	–
$(B_d, B_u) \to K^*\ \nu\bar{\nu}$	2.3×10^{-5}	–
$B_s \to \gamma\gamma$	1.5×10^{-8}	–
$B_d \to \gamma\gamma$	6.5×10^{-10}	–
$B_s \to \tau^+\tau^-$	3.8×10^{-7}	–
$B_d \to \tau^+\tau^-$	1.7×10^{-8}	–
$B_s \to \mu^+\mu^-$	1.8×10^{-9}	–
$B_d \to \mu^+\mu^-$	8.0×10^{-11}	–
$B_s \to e^+e^-$	4.2×10^{-14}	–
$B_d \to e^+e^-$	1.9×10^{-15}	–