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B+L violating rates in the symmetric phase of the standard model

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Abstract

The rate of fermion number violating processes in the symmetric phase of the standard model is a quantity of cosmological importance but difficult to compute due to the nonperturbative infrared behavior of Yang-Mills theories. Here some recent lattice as well as analytical calculations to estimate this rate are reviewed.

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1. Introduction

in the past few years a lot of effort has been spent on the problem of baryon and lepton number violation in the standard model. If fermion number violating processes occur at very high rates in the framework of standard model physics any preexisting B+L asymmetry of the early universe created during some earlier GUT epoch is washed out. In the case of vanishing B-L asymmetry the baryon asymmetry as observed today would then have to be generated at the electroweak phase transition. Several suggestions to realize this interesting possibility have been made, for recent reviews see [1,2]. If the rates are not quite as high but still cosmologically significant (i.e. larger than the expansion rate of the universe) another option is to generate an asymmetry during some GUT epoch and subsequently dilute it to its present day value by electroweak baryon number dissipation. Clearly, in order to decide which of these scenarios was realized by nature we need to know the rate of baryon number dissipation in the symmetric phase of the standard model. It is this quantity that tells us wether or not there is some asymmetry left over when the universe undergoes the electroweak phase transition. Furthermore, it also plays an important role in generating the asymmetry at the bubble walls during the phase transition. Unfortunately, calculating B+L violating rates in the symmetric phase is a very difficult problem, as will be explained below, and a definite answer to these questions can not yet be given. In this article I will review some recent attempts to make progress on this issue. After a brief introduction to the mechanism of fermion number violation in the standard electroweak theory the approximations will be discussed which allow to treat an essentially nonequilibrium problem by equilibrium calculational methods. Section 2 reports on how these methods have been implemented in some lattice simulations while section 3 summarizes an anlytical model calculation. These different approaches are compared in the discussion of section 4 which also tries to extract what can be concluded from those studies and what has to be subjected to further investigations.

Since the work of t'Hooft [3] it is well known that fermion number in the standard electroweak theory is not strictly conserved. Rather, because of the Adler-Bell-Jackiw anomaly, baryon and lepton number have nonvanishing time derivatives,

$$\frac{d}{dt}N_B = \frac{d}{dt}N_L = -n_f\frac{d}{dt}N_{CS}(t)\ , \tag{1}$$

where the Chern-Simons number $N_{CS}(t)$ is a functional of the gauge field and in the temporal gauge, $W_0 = 0$, is given by

$$N_{CS}[W] = \frac{g^2}{16\pi^2}\int d^3x\ \varepsilon_{ijk}\ \mathrm{Tr}\left[W_iF_{jk} + \mathrm{i}g\frac{2}{3}W_iW_jW_k\right]\ . \tag{2}$$

The Chern-Simons number is intimately connected with the nontrivial vacuum structure of the electroweak theory. Fig. 1 shows the minimal potential energy of the gauge fields as a periodic function of the Chern-Simons number. For pure gauge vacuum configurations $W_i = -\mathrm{i}/g\,(\partial_iU)U^{-1}$ the Chern-Simons number takes integer values corresponding to the winding number of the gauge transformation matrix U. All vacuum configurations are related by large gauge transformations and separated by a potential barrier. The top of the potential barrier, which is called sphaleron configuration, is an unstable, static solution of the three dimensional field equations and represents a saddle point between two vacuum sectors [4]. At high temperatures it is possible for thermally excited modes to classically overcome the potential barrier and to get into a neighbouring vacuum sector. Because of the ABJ anomaly (1) each such crossing implies a change in baryon and lepton number. In 1985 Kuzmin, Rubakov and Shaposhnikov [5] suggested that the rate at which these transitions occur is governed by the Boltzmann factor $\Gamma \sim \exp[-E_S/T]$, and hence at very high temperatures, such as in the early universe, B+L violating processes are unsuppressed. Since there is equal probability for the thermal fluctuations to cross to the left or to the right one may think of the system to perform a random walk in the topological charge $Q(t)$,

$$Q(t) = \frac{1}{32\pi^2}\int_0^t dt' \int d^3x F^a_{\mu\nu}\tilde{F}^a_{\mu\nu} = N_{CS}(t) - N_{CS}(0) . \tag{3}$$

The rate at which the crossings occur is then given by the diffusion rate of the random walk problem which for very long times $t \to \infty$ is defined as

$$< Q^2(t) >= 2\Gamma t . \tag{4}$$

Up to this point the discussion was one of a purely bosonic system and its vacuum structure. However, in reality the situation is different. What we are really interested in is that we start out with an initial B+L excess and then study how this initial asymmetry gets dissipated. To supplement the previous scenario with the presence of fermions one has to impose an initial condition

$$< N_{B,L}(t=0) > \neq 0 \tag{5}$$

on the fermionic operators and to provide them with chemical potentials $\mu_{B,L}$. Inclusion of the fermion free energy then modifies the potential of the system. For a sufficiently dilute fermion gas which can be treated as noninteracting a quadratic envelope is obtained, as shown in Fig. 2. If the system now starts out with nonzero baryon/lepton number, there is a preferred direction and it will eventually relax to the absolute minimum of the potential at zero baryon and lepton number. What we whish to calculate is the relaxation rate Γ_B of this dissipative process, defined by a linear kinetic equation

$$\frac{d}{dt} < N_{B,L} >= -\Gamma_B < N_{B,L} > . \tag{6}$$

Clearly, this defines a nonequilibrium problem. It has been shown by Khlebnikov and Shaposhnikov [6] that an appropriate treatment is possible if two assumptions are made. First, there has to be a hierarchy of relaxation times, i.e. all collisions in the plasma involving quarks and leptons have to be fast compared to the B+L violating processes. In other words, the system of quarks and leptons at a given baryon number is in thermal equilibrium while collective excitations of the gauge-Higgs system lead to baryon number changing nonequilibrium

transitions. Second, the initial asymmetry has to be small, such that it is possible to expand in small chemical potentials, $\frac{\mu_{B,L}}{T} \ll 1$. For a free fermion gas this is equivalent to the statement that $\frac{N_{B,L}}{VT^3} \ll 1$. Both conditions are certainly met in the early universe [7]. It is then possible to calculate the statistical average

$$< \dot{N}_{B,L} >= \frac{\mathrm{Tr}[\rho(t)\dot{N}_{B,L}]}{Z} , \quad Z = \mathrm{Tr}\rho , \tag{7}$$

using a time dependent nonequilibrium statistical operator as defined by Zubarev [8],

$$\begin{aligned}\rho(t) &= \exp\left\{-\beta H + \epsilon\beta\int_{-\infty}^t dt' e^{\epsilon(t'-t)}\left[\mu_B(t')N_B(t') + \mu_L(t')N_L(t')\right]\right\} \\ &\equiv \exp\left\{-\beta\left[H + h(t)\right]\right\} .\end{aligned} \tag{8}$$

This operator satisfies the quantum Liouville equation

$$\dot{\rho}(t) = \mathrm{i}\,[H, \rho(t)] \tag{9}$$

in the limit $\epsilon \to 0^+$ and should be a good approximation if only baryon and lepton number are out of equilibrium. With the assumption $\frac{\mu_{B,L}}{T} \ll 1$ the statistical operator can now be expanded to linear order in small deviations from the equilibrium operator,

$$\frac{\rho}{Z} = \left[1 + \beta\langle\int_0^1 d\lambda\, e^{-\beta H\lambda} h\, e^{\beta H\lambda}\rangle_0 - \beta\int_0^1 d\lambda e^{-\beta H\lambda} h\, e^{\beta H\lambda}\right]\frac{\rho_0}{Z_0} + O(h^2) , \tag{10}$$

where the subscript zero indicates equilibrium operators with $h = 0$. Calculating the statistical averages $< \dot{N}_{B,L} >$, $< N_{B,L} >$ in the above linear approximation one can derive the desired kinetic equation [6,11]

$$\frac{d}{dt} < N_{B,L} >= -K\left[\frac{< N_B >}{< N_B^2(0) >_0} + \frac{< N_L >}{< N_L^2(0) >_0}\right] \tag{11}$$

with

$$K = \int_{-\infty}^t dt'\, e^{\epsilon(t'-t)} \int_0^1 d\lambda \langle \dot{N}_B(t) e^{-\beta H\lambda} \dot{N}_B(t') e^{\beta H\lambda}\rangle_0 .$$

Note that the kinetic coefficient K now is related to an equilibrium correlation function which may be evaluated by euclidean technique. This is an application of the more general fluctuation-dissipation theorem (see e.g. [12]). In [6] equation (11) was rewritten into a defining equation for the rate of topological charge changing processes in the presence of fermions,

$$< \dot{N}_B >= -n_f \frac{\Gamma(N_B)}{T}\frac{\partial F(N_B)}{\partial N_B} . \tag{12}$$

Here $F(N_B)$ is the free energy of the system at a given baryon number N_B. To obtain the kinetic equation in the form of (11) $F(N_B)$ has to be evaluated. In [9,10,11] this was done for a massless free fermion gas, while in [13] interactions with static background gauge fields of arbitrary topological charge were taken into account. It is remarkable that both calculations yield the same result, namely

$$< \dot{N}_{B,L} >= -n_f \frac{\Gamma(0)}{VT^3} \left(\frac{9}{2} < N_B > +2 < N_L > \right) + O\left(\frac{< N_{B,L} >^2}{V^2 T^6} \right) , \tag{13}$$

where $\Gamma(0)$ is the rate for a system without chemical potentials as in (4). This result tells us that to leading order in $\frac{N_{B,L}}{VT^3}$, i.e. in the limit of a small initial deviation from equilibrium, it is possible to calculate the rate of B+L violating processes in the absence of fermions using equilibrium thermal averages. Remembering that calculations of the partition function at very high temperatures are done with a dimensionally reduced, purely bosonic theory (nonstatic modes as well as fermions acquire thermal masses $\sim T$ and decouple as $T \to \infty$ [14,15,16]) we may thus turn our attention back to a theory without fermions.

Consider the SU(2)-Higgs theory which approximates the bosonic part of the standard electroweak model in the limit of vanishing Weinberg angle. Neglecting the U(1) sector of the standard model does not change any of the conclusions because it is abelian, has trivial vacuum structure and is therefore not asossiated with fermion number violation. The only influence of the U(1) sector is to modify the sphaleron solution of the theory and its corresponding energy, but these effects are $\sim 1\%$ corrections only [17,18]. The sphaleron solution for the SU(2)-Higgs theory in the phase of broken symmetry with Higgs vacuum expectation value v was found by Klinkhamer and Manton [4]. Its energy which determines the barrier height is

$$E_S = \frac{m_W(T)}{\alpha_W} B(\lambda/\alpha_W) \ , \ m_W(T) = \frac{gv}{2} \left(1 - \frac{T}{T_c} \right)^{1/2} , \tag{14}$$

where $B(\lambda/\alpha_W)$ is a slowly varying function of the weak coupling and the Higgs coupling. As was mentioned before, the sphaleron solution represents a saddle point in functional space. It is therefore possible to calculate the rate of transitions over the barrier using the Langer-Affleck formula [19,20]

$$\Gamma = \frac{| \omega_- |}{\pi T} \mathrm{Im} F , \tag{15}$$

where ω_- denotes the negative eigenvalue of the unstable mode and F is the free energy of the system. It can be evaluated by expanding in Gaussian fluctuations around the sphaleron solution to obtain [9,21–23]

$$\frac{\Gamma}{V} = 0.007 (\alpha_W T)^4 \left(\frac{3 m_W}{T \alpha_W} \right)^7 \mathrm{e}^{-3 m_W / T \alpha_W} . \tag{16}$$

The exponential is just the expected Boltzmann factor, while its prefactor represents the determinant of the small fluctuations around the saddle point. Unfortunately, this calculation is only valid in a narrow temperature range, $m_W \ll T \ll m_W/\alpha_W$. Since the high temperature effective coupling of the three dimensional theory is given by $\alpha_3 = \alpha_W T/(2 m_W(T))$ and the mass of the W-boson (14) goes to zero as $T \to T_c$, the one loop approximation breaks down, and furthermore the potential barrier vanishes for temperatures approaching the critical

temperature of the phase transition. In the symmetric phase the Higgs vacuum expectation value remains zero and there exists no saddle point to expand around. Finally, the well known infrared divergencies of nonabelian gauge theories with massless gauge bosons [14] prohibit a loop expansion in the symmetric phase. Because of all these difficulties a quantitative analytical treatment of the problem in the symmetric phase starting from the full SU(2)-Higgs action is yet lacking. It is however possible to extract the qualitative behavior of the rate at high temperatures by scaling arguments. In the symmetric phase temperature is expected to replace the gauge boson mass as the only dimensionful quantitiy to set the scale of the problem. Substituting $m_W \to \alpha_W T$ in (16) one finds that the equation for the rate assumes the form

$$\frac{\Gamma}{V} = \kappa (\alpha_W T)^4 \tag{17}$$

with κ an unknown purely numerical coefficient (for more refined arguments see [6,21]). With the semiclassical analysis failing here it is natural to turn to nonperturbative methods in order to determine κ.

2. Results of numerical simulations for $T > T_c$

A possible way out of the various problems encountered in the symmetric phase is to do Monte Carlo lattice simulations. Ambjørn et al. attempted to determine κ by setting up a microcanonical real-time simulation of topological charge changing processes [24,25]. These authors start out with the following argument: The configurations which are relevant for topological charge changing processes in the symmetric phase are expected to have energy $E \sim T$ and spatial extension $r \sim (\alpha_W T)^{-1}$, just as the sphaleron configuration in the broken phase has $E \sim m_W/\alpha_W$ and extension $r \sim m_W^{-1}$ (a configuration of the size $r \sim T^{-1}$ would be suppressed due to the Boltzmann factor $e^{-\beta E} \sim e^{-1/\alpha_W}$). However, the generic quantum fluctuations of the hot plasma have momenta of the order $p \sim T$. It is therefore expected that the quantum fluctuations decouple from the topological charge changing processes. If this is correct a description entirely in terms of classical statistical mechanics is suitable. A microcanonical ensemble can then be generated in the following way. First a field configuration specified by its canonical coordinates and conjugate momenta $\{W_i^a, \Phi^a; E_i^a, \Pi^a\}$ is picked according to the Gibbs distribution $\exp(-\beta H)$ built with the classical SU(2)-Higgs Hamiltonian in temporal gauge, $W_0 = 0$,

$$H = \int d^3x \left[\frac{1}{2} E_i^a E_i^a + \frac{1}{4} F_{ij}^a F_{ij}^a + | \Pi |^2 + | D_i \Phi |^2 + M^2 | \Phi |^2 + \lambda | \Phi |^4 \right] , \tag{18}$$

supplemented with the Gauss constraint

$$D_i^{ab} E_i^b = ig(\Phi^\dagger \tau^a \Pi - \Pi^\dagger \tau^a \Phi) . \tag{19}$$

(This Hamiltonian is derived from the finite temperature euclidean theory in the limit of very high temperatures employing dimensional reduction [26].) Then one lets the system evolve according to the classical equations of motion

$$\frac{dW_i^a}{dt} = \frac{\delta H}{\delta E_i^a} = E_i^a \ , \ \frac{dE_i^a}{dt} = -\frac{\delta H}{\delta W_i^a} \ . \tag{20}$$

During the evolution the time development of the topological charge can be measured,

$$Q(t) = \frac{1}{32\pi^2}\int_0^t dt' \int d^3x F^a_{\mu\nu}\tilde{F}^a_{\mu\nu} = N_{CS}(t) - N_{CS}(0) \ . \tag{21}$$

The result of such a measurement is shown in Fig. 3a. It shows how the topological charge thermally fluctuates around a plateau of certain Chern-Simons number before jumping to another plateau. Fig. 3b displays the same picture with "cooled" configurations, i.e. the short wavelength thermal fluctuations have been stripped off in order to check if the transitions between the plateaus survive. The picture is largely unchanged if only the energy sitting in the gauge field is relaxed. This indicates that the Higgs field decouples to a large extent from the gauge field in the symmetric phase. By use of the lattice parameters the rate equation $\Gamma/V = \kappa(\alpha_W T)^4$ can be converted to an equation for the number of topological charge changing transitions on a given lattice,

$$N(t) = \kappa \; const(lattice) \; t \ . \tag{22}$$

Counting the number of transitions Ambjørn et al. then obtain $\kappa \sim 0.1 - 1$ for the prefactor implying that fermion number violation is unsuppressed in the symmetric phase.

However, no temperature range is given in [25], for which this result is valid. The T^4-behavior was derived by a scaling argument, so one can expect it to hold in the asymptotic region of very high temperatures. But what happens at temperatures just slightly above the critical temperature? In order to answer this question Karsch et al. [27] studied the temperature dependence of the probability distribution of the Chern-Simons number,

$$P(N_{CS}) = \int [dW][d\Phi]\, e^{-S}\, \delta\left(N_{CS}[W] - \frac{g^2}{16\pi^2}\int d^3x\, \epsilon_{ijk}\, \mathrm{Tr}\left[W_i F_{jk} + ig\frac{2}{3}W_i W_j W_k\right]\right) \ , \tag{23}$$

by means of lattice simulations. Here S is the full four dimensional euclidean SU(2)-Higgs action at finite temperatures. Fig. 4 shows a sequence of distributions for temperatures $T = 1.5\,T_c$, $T = 2.0\,T_c$ and $T = 2.5\,T_c$ respectively. Due to the periodicity of the Chern-Simons potential it is sufficient to consider only the gauge invariant non-integer part of N_{CS}, which can be mapped into the interval $N_{CS} \in [-1/2, 1/2]$. One observes a rapid broadening of the distributions with increasing temperature indicating that more and more configurations pass half-integer values of Chern-Simons number to get into a neighbouring vacuum sector. In order to study the temperature dependence of the rate of topological charge changing transitions Karsch et al. define $F(\epsilon, T)$ to be the fraction of configurations with Chern-Simons number in some neighbourhood of one half, $1/2- \mid N_{CS} \mid < \epsilon$. They then assume $\Gamma(T) \sim F(\epsilon, T)$ and look at the ratios $\Gamma(T_1)/\Gamma(T_2)$ to get rid of the proportionality constant. In Table 1 the numerical results for these ratios are compared with the ratios of the semiclassical result (16) in two

limiting cases: (a) gives the values for $m_W = m_W(T=0)$ while (b) gives those for $m_W \sim T$. Even though the numerical uncertainty is still rather high one can see that the T^4-behavior is not reproduced but the lattice data seem to favor a only weakly temperature dependent mass scale for the temperature range investigated. If this is correct it implies that the T^4-behavior sets in at much higher temperatures only. Another question concerns the validity of the static approximation as used in the dimensionally reduced effective theories employed in the semiclassical calculations as well as in the previous lattice simulations. Karsch et al. tested the validity of this approximation by measuring correlation functions $< N_{CS}(t) N_{CS}(0) >$. If the static appproximation is correct one should expect strong correlations. Fig. 5 shows the Chern-Simons numbers on neighbouring time slices plotted against each other. Strong correlations with increasing temperature are clearly visible.

3. Sphalerons in the symmetric phase

As we have seen in the previous section, the lattice results are not precise yet and, in the case of the determination of κ in [24,25], they use an effective Hamiltonian approach describing entirely classical statistical mechanics. Clearly, it would be desirable to have an independent analytical estimate for the rate to see if it is possible to make contact to the lattice results. Thus one has to search for a high temperature effective theory that describes the electroweak model in the phase of restored symmetry. As with the effective theories used before (e.g. [21,25]) we are interested in the static, magnetic sector of the theory, i.e. we work in the gauge $W_0 = 0$. However, now we would like to include complicated, nonperturbative plasma effects. One candidate for such a phenomenological model, which in this context was first suggested by Cornwall [30], is a massive pure gauge theory where the gauge boson mass is dynamically generated. The corresponding effective Lagrangian is a gauged nonlinear σ-model,

$$\mathcal{L}_{eff} = -\frac{1}{4}F^a_{ij}F^a_{ij} + \frac{m_W^2}{g^2}\mathrm{Tr}\left[\left(U^{-1}D_iU\right)\left(U^{-1}D_iU\right)\right] \ , \tag{24}$$

with U a unitary matrix transforming as $U' = VU$ such that the theory is invariant under static SU(2) gauge transformations $V(\mathbf{x})$. Why should this be a good description of the electroweak theory at very high temperatures?

First, it is well known [14,15] that higher order contributions to the gauge boson self-energy in high temperature Yang-Mills theories develop directional singularities which can generate a magnetic mass $\sim g^2T$ for the spatial components of the gauge field. The numerical value of this mass cannot be calculated perturbatively, but there are analytic calculations [31–33] as well as lattice simulations [35–38] which indicate that indeed $m_W \neq 0$ in the symmetric phase. While these investigations are not entirely conclusive yet it is interesting to assume the magnetic mass to be nonzero and investigate its consequences on B+L violation. Since the precise value of the magnetic mass is unknown it is expedient to give it a coefficient m_0, i.e. $m_W = m_0 g^2 T$, which can then be treated as a parameter of the theory. Second, the lattice calculations of the previous section have provided evidence for a large degree of decoupling of the Higgs and gauge field as well as for the validity of the static approximation if one is deeply in the symmetric phase. These observations are in agreement with the conclusions drawn from the analytical procedure of dimensional reduction [28,29], where the Higgs field and the time component of the gauge field acquire thermal masses $\sim gT$ such that they may be treated as decoupling heavy particles leaving us with a pure gauge theory. Thus the suggested model (24) exhibits all qualitative features of the high temperature behavior of the full electroweak theory. In addition, this theory has a saddle point solution again and a semiclassical calculation of the rate of topological charge changing transitions is possible in a certain parameter range.

Such a calculation proceeds exactly along the lines of those for the broken phase [21–23] and has been done in [39]. To begin with, one has to identify the correct saddle point solution. The field equations

$$(D_i F_{ij})_a = \mathrm{i}\frac{m_W^2}{g}\left[(D_j U)\,U^{-1}\right]_a \tag{25}$$

can be solved by a spherically symmetric ansatz

$$W_i^a(\mathbf{x}) = \frac{1}{g}\left[\frac{f_A(r)-1}{r^2}\epsilon_{iam}x_m + \frac{f_B(r)}{r^3}(r^2\delta_{ia} - x_i x_a) + \frac{f_C(r)}{r^2}x_i x_a\right] ,$$

$$U(\mathbf{x}) = \exp\left[\frac{\mathrm{i}}{2}\theta(r)\frac{\mathbf{x}\cdot\tau}{r}\right] , \tag{26}$$

turning them into a set of ordinary differential equations for the parameter functions which is then evaluated numerically. The nonlinear σ-model (24) is mathematically equivalent to the one obtained from SU(2)-Higgs theory by taking the limit of infinite Higgs mass and coupling [40,41,42] (it is emphasized that the analogy is only mathematical, (24) is intended to describe a pure gauge theory). From these works it is known that there is an infinite set of solutions to the field equations. In particular, there is one solution with energy $E_S = 5.41 m_W/\alpha_W$ and Chern-Simons number $N_{CS}[W_S] = 0.5$, commonly referred to as the sphaleron solution. However, this solution is not the lowest energy solution and furthermore has an infinite number of instabilities. It has been shown in [43] that only saddle points with a single unstable mode are related to the decay of a metastable state. There are further solutions termed deformed sphalerons because they have winding numbers different from 0.5. In contrast to the sphaleron these configurations are not CP invariant, so they always come in pairs connected by CP conjugation. For the lowest energy solution having only a single negative eigenmode one finds

$$E_S = 5.07 m_W(T)/\alpha_W \;, N_{CS}[W_S] = \pm 0.375 \; . \tag{27}$$

The precise shape of the minimal energy path from one vacuum sector to another has not been calculated, but schematically it should look as indicated in Fig. 6. Now there are two distinct, asymmetric paths connecting neighbouring vacuum sectors. Note that the energy of the sphaleron solutions and therefore the barrier height rises linearly with $m_W \sim T$, so one does not lower the Boltzmann suppression by raising the temperature. To calculate the

rate in Gaussian approximation the Hamiltonian is expanded up to quadratic order in small fluctuations of the fields, $W_i^a = W_{Si}^a + \chi_i^a$, $\theta = \theta_S + \beta$,

$$H = E_S + \frac{1}{2}\int d^3x \int d^3y \,\frac{\delta^2 H_{fl}}{\delta\psi_n^\alpha(\mathbf{x})\delta\psi_m^\beta(\mathbf{y})}\psi_m^\beta(\mathbf{y})\psi_n^\alpha(\mathbf{x}) , \tag{28}$$

where $\psi^{\alpha_n} = (\chi_{in}^a, \beta_n)$ are eigenfunctions of the quadratic fluctuation Hamiltonian H_{fl}

$$\int d^3y \frac{\delta^2 H_{fl}}{\delta\psi_n^\alpha(\mathbf{x})\delta\psi_m^\beta(\mathbf{y})}\psi_m^\beta(\mathbf{y}) = \omega_n^2\psi_n^\alpha(\mathbf{x}) \, . \tag{29}$$

This eigenvalue problem can be solved numerically. For the negative eigenvalue one finds $\omega_-^2 = -3.95 m_W^2$. As a byproduct of the numerical computation one may also get an estimate of the radially symmetric part (i.e. the j=0 partial wave) of the fluctuation determinant $\Delta(m_0)$ which turns out to be of $O(10)$ and only weakly m_0-dependent (see Table 2).

The final result for the rate can be written as

$$\frac{\Gamma}{V} = \gamma(m_0)\Delta(m_0)\,\tilde{\kappa}\,T^4 , \tag{30}$$

where the various factors have the following meaning: $\gamma(m_0)$ contains the contributions from the saddle point, the negative mode as well as the rotational and translational zero modes. $\Delta(m_0)$ is the radially symmetric part of the fluctuation determinant and $\tilde{\kappa}$ parametrizes the higher partial waves that are still unknown. Fig. 7 shows the dominant factor $\gamma(m_0)$ comprising only quantities that have been computed exactly. Remember that the magnetic mass is sitting in the exponential of the Boltzmann factor which accounts for the strong m_0-dependence. The maximal rate that can be achieved in this model is $\Gamma/V|_{max} \approx O(10^{-8} - 10^{-7})\;\tilde{\kappa}\;T^4$ or $\kappa \sim 10^{-2} - 10^{-1}$ for $\tilde{\kappa} \sim O(1)$. Hence only the maximally possible rate at $m_0 \approx 0.1$ is comparable to the lattice result, $\Gamma/V|_{lattice} \approx O(10^{-7} - 10^{-6})\;T^4$ or $\kappa \sim 10^{-1} - 1$, while for other values of the magnetic mass the rates are considerably smaller than those predicted by the numerical simulations. Unfortunately, for the most interesting parameter range with $m_0 < 0.2$ the Gaussian approximation is not a good one anymore with the prefactor being already $\sim 30\%$ of the saddle point contribution.

4. Discussion

When comparing the results of the previous two sections one has to be aware of the fact that they are rather different in the sense that they are generated by different models and configurations. To see this, let us remember the "ingredients" of the calculations. The lattice simulations of Ambjørn et al. started with an effective three dimensional Hamiltonian describing entirely classical statistical mechanics. Thus, there is no barrier in the potenial of the gauge-Higgs sector and one can not identify the configurations responsible for the topological charge changing processes. From this underlying picture ist is clear that these configurations have a much more random nature than the sphaleron configurations in the broken phase, for instance they need not be spherically symmetric. This is to be contrasted with the analytical calculation of the previous section. There, again, the potential has a barrier structure with saddle point solutions of a definite symmetry and the semiclassical analysis implicitly assumes these configurations to dominate the rate of fermion number violating processes. Note also, that for a theory with asymmetric potential barriers as in Fig. 6 the rate would not be proportional to the number of configurations with winding number close to one half, as was assumed in [27]. The approaches of [24,25] and [30,39] have in common that they work with an effective three dimensional theory. However, while the Hamiltonian used in [24,25] is just a classical SU(2)-Higgs theory, the idea behind the model of the last section is to incorporate nonperturbative plasma effects into an effective theory. In view of this fact the difference between the results for the rate is surprisingly small. The work of [27] has provided us with evidence that dimensional reduction works well if one is far enough in the symmetric phase, but it does unfortunately not give us any hint as to what is the correct effective dimensionally reduced theory. Furthermore, it indicates that the T^4-behavior of the rate is only valid far above the critical temperature and thus we do not have any knowledge of what it is like in the neighbourhood of the phase transition.

Another caveat concerns the role of the fermions. In the process of getting a linear kinetic equation for the relaxation of fermion numbers (13) the free energy of the system was computed in a weak interaction approximation that left the fermions massless. However, if it is correct to assign plasma masses to the bosons within an effective high temperature theory one should expect the same thing to hold for the fermions. It is well known [44,45] that in a plasma with fermions and gauge bosons at very high temperatures the spectrum of collective fermion excitations exhibits a mass gap $m_f \sim gT$ which breaks Lorentz covariance but preserves chirality. In a B+L violating process the plasma fluctuations create a sphaleron configuration which then decays again into a multi boson and fermion state. But the final state has a different number of fermions or antifermions than the initial state. If the fermion spectrum has a mass gap then energy of the sphaleron is used to overcome this gap in order to produce additional fermion excitations. This may possibly lead to an additional suppression.

Considering all these open questions it is fair to say that the problem of B+L violating rates in the symmetric phase is far from being solved. Lattice simulations have provided us with evidence for the random nature of the topological charge changing processes, the decoupling of the Higgs field in the symmetric phase and the validity of the static approximation. Even though the lattice and analytical calculations mentioned here use different effective theories, or the full four dimensional action as in [27], as a starting point, they all yield significant rates of B+L violating processes in the symmetric phase. We may therefore be confident in the qualitative picture we have. However, on a quantitative level the problem of course depends on the model that one works with. In order to get better precision for this kind of calculation we first of all need to know the correct effective theory (including nonperturbative effects) for both bosonic and fermionic part of the standard model in the symmetric phase.

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Tables

T_1/T_2	$\epsilon = 0.1$	$\epsilon = 0.05$	(a)	(b)
2.0/1.8	3.1 ± 1.3	3.4 ± 2.0	3.9	1.5
2.5/2.0	6.5 ± 1.4	7.4 ± 2.0	10.0	2.4

Table 1: The first column gives the ratio of temperatures, the second and third the fraction $F(\epsilon, T_1)/F(\epsilon, T_2)$ for different ϵ-values and the fourth and fifth the ratio $\Gamma(T_1)/\Gamma(T_2)$ from the semiclassical formula (16) with (a) $m_W = m_W(T=0)$ and (b) $m_W \sim \alpha_W T$. From [27]

m_0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
$\Delta(m_0)$	42.3	27.1	19.5	14.8	11.8	9.5	8.1	6.9

Table 2: Determinant of the spherically symmetric fluctuations. From [39]

Figure captions

Fig. 1: Minimal free energy or "effective potential" of a nonabelian gauge theory

Fig. 2: Effective potential including the fermion free energy. Here N_B stands for $N_B(0) + N_{CS}[W] - N_{CS}[0]$.

Fig. 3: Measurement of the Chern-Simons number as a function of time. Fig. 3(a) shows the time evolution with thermal fluctuations included, while Fig. 3(b) shows the same time evolution with the thermal flutuations partly stripped off. From [25]

Fig. 4: Chern-Simons number distribution for $\xi = T/T_c = 1.5, 2$ and 2.5. From [27]

Fig. 5: Scattering of Chern-Simons number on two timeslices for increasing $\xi = T/T_c$. From [27]

Fig. 6: Schematic plot of the effective potential for the model (24)

Fig. 7: The contributions of the single negative mode and the zero modes to the prefactor in the transition rate. From [39]

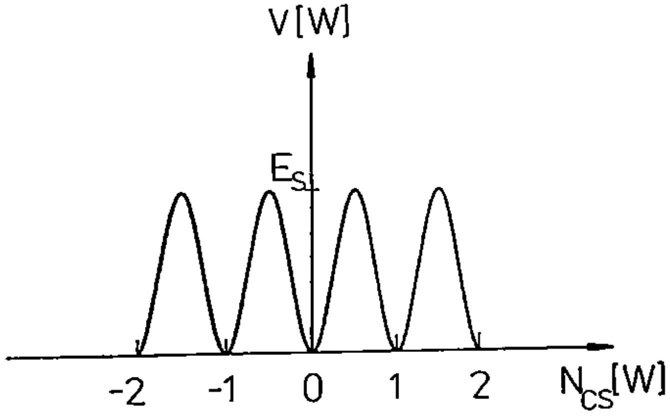

Fig.1

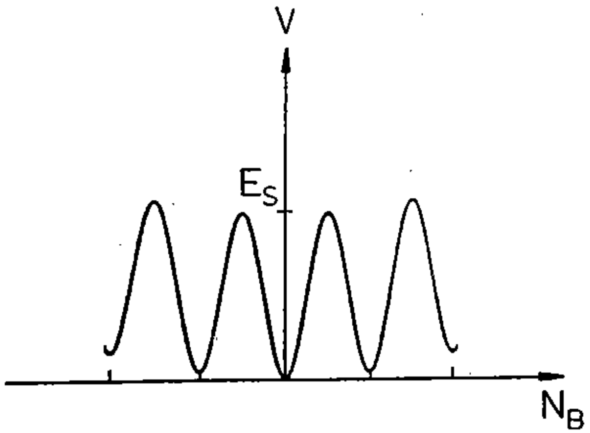

Fig.2

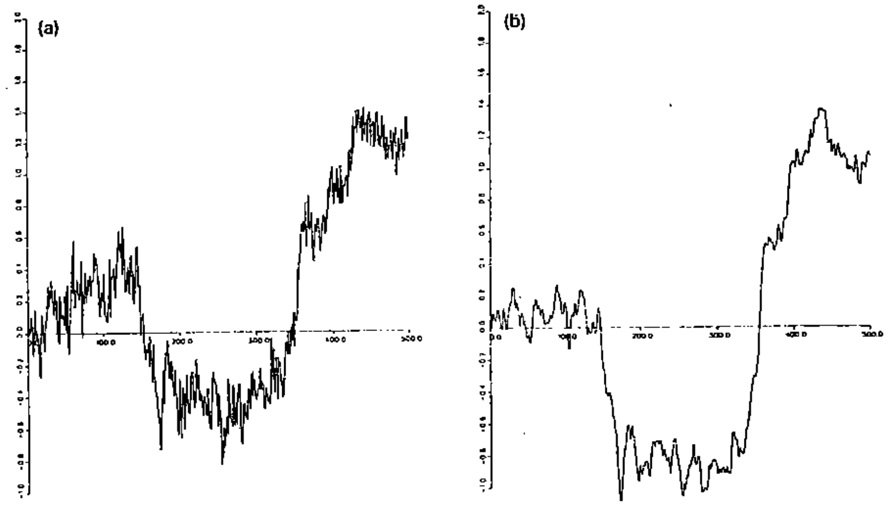

Fig.3

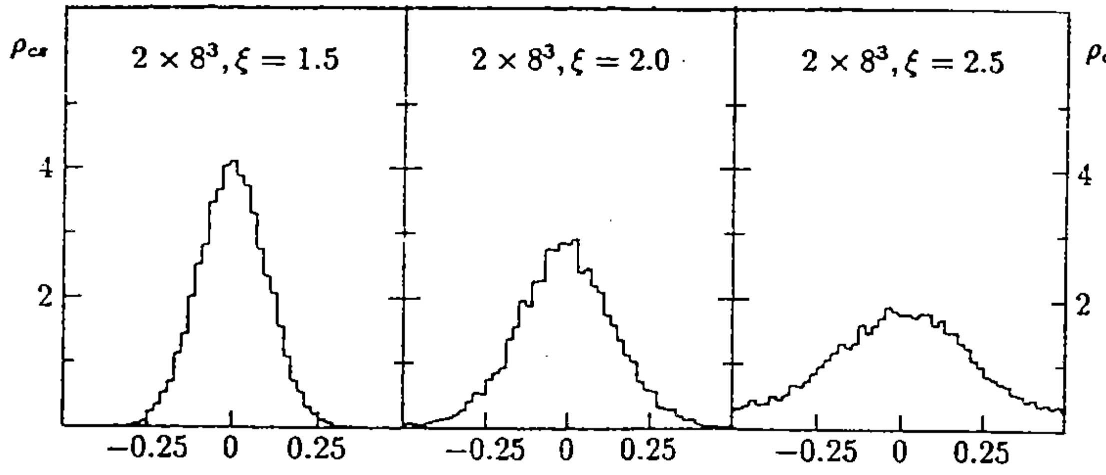

Fig.4

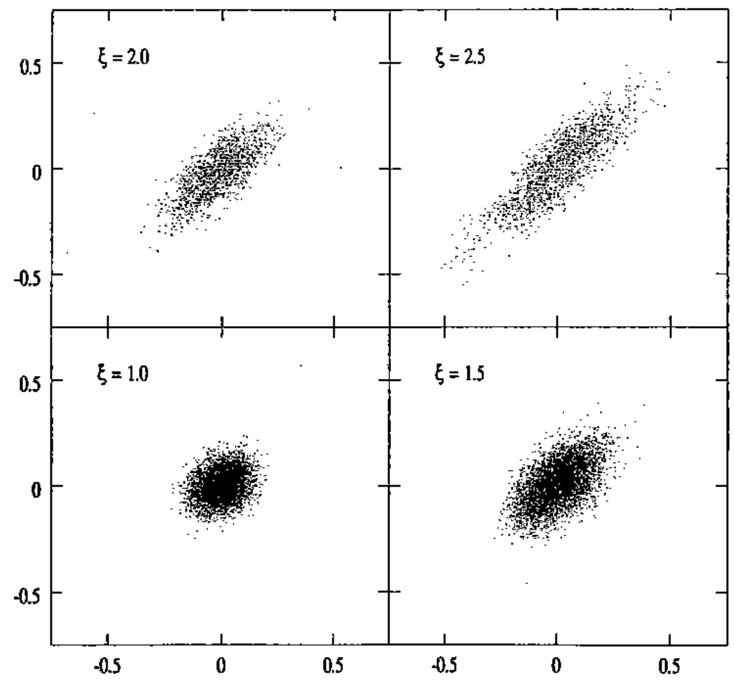

Fig.5

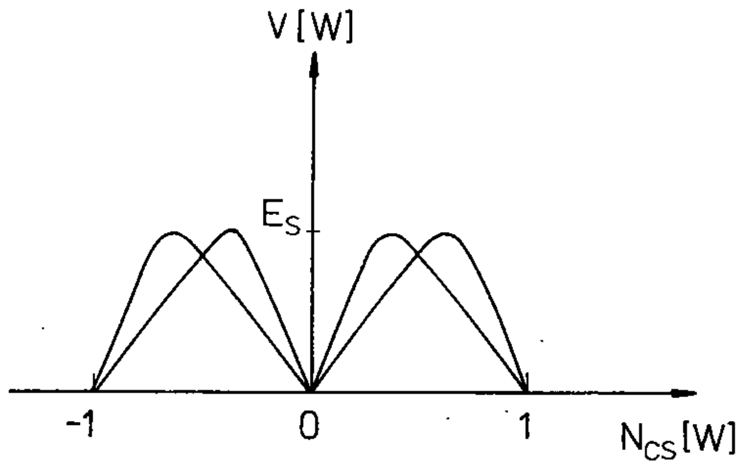

Fig.6

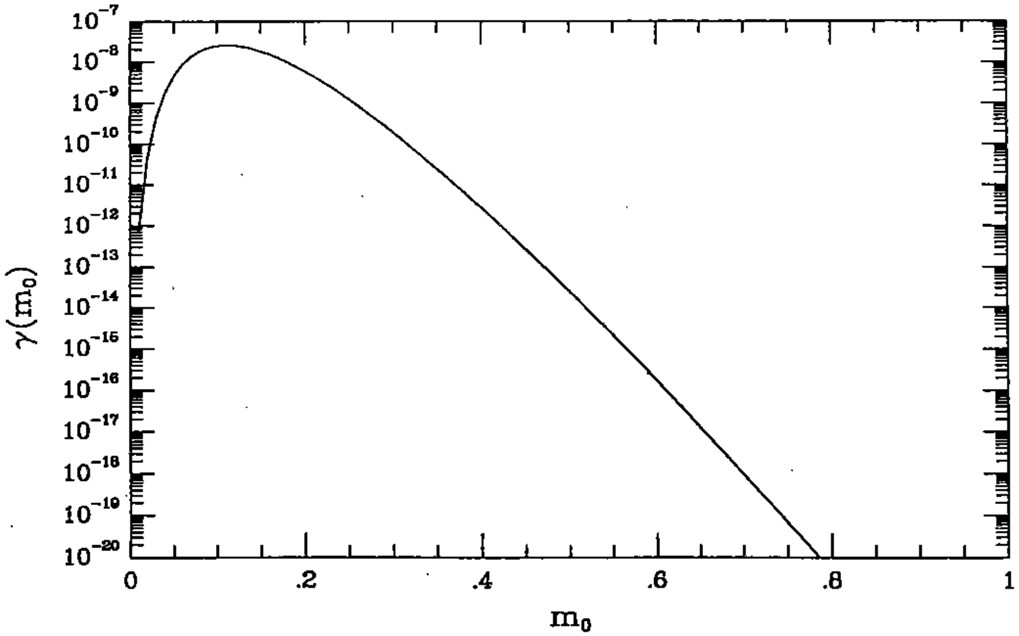

Fig.7