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Abstract

QED and QCD radiative corrections are calculated for the pair production cross sections of leptoquarks in e^+e^- annihilation. Large corrections are found for the production process near threshold. For $\beta = 0.1$ the QED corrections are of $\mathcal{O}(-40\%$ to $-50\%)$. QCD corrections may reach $\mathcal{O}(+100\ \%)$

[1]Contribution to the Proceedings of the Workshop 'e^+e^- Collisions at 500 GeV', - Munich, Annecy, Hamburg, 1993.

1 Introduction

Recently leptoquark pair production in e^+e^- annihilation at $\sqrt{s} = M_Z$, in the energy range of LEP200, and at future high energy linear colliders has been studied in a widely model independent way [1, 2] calculating the contributions at Born level. For the leptoquarks considered rather general assumptions were made demanding that their couplings both to fermions and gauge bosons are dimensionless, baryon and lepton number conserving, family diagonal, and respect $SU(3)_c \times SU(2)_L \times U(1)_Y$ symmetry. In the case of the s–channel contributions both for scalar and vector leptoquarks all couplings are predicted within the Standard Model. Only a minor uncertainty exists for the prediction of lower bounds on the respective production cross sections due to the unknown leptoquark–fermion couplings, which are, however, limited by experimental constraints [3]. The calculation of the production cross sections showed [2] that almost all leptoquark states which are compatible with the above conditions may be produced at sufficient rates to be discovered or definitely excluded in the energy range up to 1 TeV at future linear colliders.

As known from other processes [4], the QED radiative corrections for pair production at threshold – a typical situation in the search for new particles – may be rather large. Furthermore, sizeable QCD final state corrections are expected. In the present paper we will calculate the dominant corrections for leptoquark pair production extending the calculations done in [2].

The largest contributions are due to initial state QED corrections and QCD final state corrections. The QED final state corrections turn out to be of a few per cent only except of the close vicinity of the Coulomb singularity, a range, however dominated by the QCD correction. Since $\mathcal{O}(\alpha^2)$–terms turn out to be still of the size of $\sim 10\%$ of the Born cross section we included them as well as the exponentiation of the soft terms to all orders in the case of initial state corrections.

Because in the threshold range also beamstrahlung effects are sizeable, we have included also this effect. Finally, we investigated also the prospects of leptoquark production in the $\gamma\gamma$ collision mode[2]. In section 2 basic relations and the cross section formulae at the Born level are summarized. The QED corrections and the effect of beamstrahlung are delt with in section 3. For the case of scalar leptoquarks the $\mathcal{O}(\alpha_s)$ corrections are given in section 4. Leptoquark pair production in $\gamma\gamma$ fusion is discussed in section 5, and section 6 contains the conclusions.

2 The Born Cross Section

Leptoquarks carrying $SU(3)_c \times SU(2)_L \times U(1)_Y$ invariant couplings, which are B and L conserving and family diagonal, have been classified in [7]. There exist nine scalar and nine vector states with these properties. The effective Lagrangian which describes their interaction with fermions and neutral gauge bosons in the low energy range ($M_\Phi \lesssim 1\,\mathrm{TeV}$) is given by

$$\mathcal{L} = \mathcal{L}^f_{|F|=2} + \mathcal{L}^f_{F=0} + \mathcal{L}^{\gamma,Z,g} \tag{1}$$

[2]The possibility to produce leptoquarks in $e\gamma$ [5] and e^-e^- [6] collisions has been also studied recently.

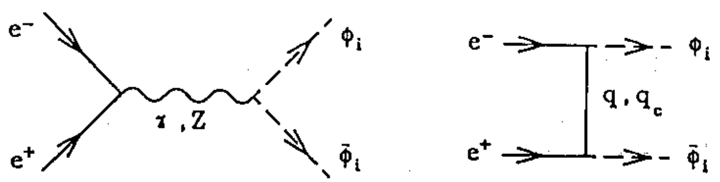


Figure 1: Feynman–diagrams contributing to leptoquark pair–production in e^+e^- annihilation

with

$$\begin{aligned}\mathcal{L}^f_{|F|=2} &= (g_{1L}\bar{q}^c_L i\tau_2 l_L + g_{1R}\bar{u}^c_R e_R)S_1 \\ &+ \tilde{g}_{1R}\bar{d}^c_R e_R \tilde{S}_1 + g_{3L}\bar{q}^c_L i\tau_2\vec{\tau} l_L \vec{S}_3 \\ &+ (g_{2L}\bar{d}^c_R\gamma^\mu l_L + g_{2R}\bar{q}^c_L\gamma^\mu e_R)V_{2\mu} \\ &+ \tilde{g}_{2L}\bar{u}^c_R\gamma^\mu l_L \tilde{V}_{2\mu} + h.c.,\end{aligned} \tag{2}$$

$$\begin{aligned}\mathcal{L}^f_{F=0} &= (h_{2L}\bar{u}_R l_L + h_{2R}\bar{q}_L i\tau_2 e_R)R_2 + \tilde{h}_{2L}\bar{d}_R l_L \tilde{R}_2 \\ &+ (h_{1L}\bar{q}_L\gamma^\mu l_L + h_{1R}\bar{d}_R\gamma^\mu e_R)U_{1\mu} \\ &+ \tilde{h}_{1R}\bar{u}_R\gamma^\mu e_R \tilde{U}_{1\mu} + h_{3L}\bar{q}_L\vec{\tau}\gamma^\mu l_L \vec{U}_{3\mu} + h.c.\end{aligned} \tag{3}$$

where $F = 3B + L$ and

$$\mathcal{L}^{\gamma,Z,g} = \sum_{scalars}\left[(D^\mu\Phi)^\dagger(D_\mu\Phi) - M^2\Phi^\dagger\Phi\right] + \sum_{vectors}\left[-\frac{1}{2}G^\dagger_{\mu\nu}G^{\mu\nu} + M^2\Phi^{\mu\dagger}\Phi_\mu\right] \tag{4}$$

The notation for the leptoquarks follows [7]. The covariant derivative is

$$D_\mu = \partial_\mu - ieQ^\gamma A_\mu - ieQ^Z Z_\mu - ig_s\frac{\lambda_a}{2}\mathcal{A}^a_\mu \tag{5}$$

and A_μ, Z_μ and $\mathcal{A}_\mu$ denote the photon–, Z–boson, and gluon fields, respectively, and the field strength tensor is $G_{\mu\nu} = D_\mu\,\Phi_\nu - D_\nu\,\Phi_\mu$. $Q^\gamma = Q_{em}$ denotes the electromagnetic charge of a given leptoquark, $Q^Z = (T_3 - Q_{em}\sin^2\theta_w)/\cos\theta_w\,\sin\theta_w$, with T_3 the third component of the weak isospin and θ_w the weak mixing angle, g_s is the strong coupling constant and λ_a are the Gell–Mann matrices. The quantum numbers of the different leptoquark species are summarized in table 1 of ref. [2]. Note, that the triple boson couplings appearing in (4) for vector leptoquarks are *not* gauge couplings[3].

The lowest order Feynman diagrams for leptoquark production in e^+e^- annihilation are shown in figure 1. The production cross sections for the scalar and vector leptoquark species defined above are summarized as follows (cf. [2])[4]:

[3]Here, the *minimal* coupling is choosen. More complicated structures similar to the case of non–standard couplings of the weak bosons are also possible (cf. e.g. [8]).

[4]For scalar leptoquarks the production cross sections were given in [9, 10] also.

The differential distributions are

$$\frac{d\sigma_{scalar}}{d\cos\theta} = \frac{3\pi\alpha^2}{8s}\beta^3\sin^2\theta\sum_{a=L,R}\left\{|\kappa_a(s)|^2 + \left(\frac{\lambda_a}{e}\right)^2\frac{4Re[\kappa_a(s)]}{t(\beta,\cos\theta)} + \left(\frac{\lambda_a}{e}\right)^4\frac{4}{t^2(\beta,\cos\theta)}\right\} \tag{6}$$

$$\frac{d\sigma_{vector}}{d\cos\theta} = \frac{3\pi\alpha^2}{8M_\Phi^2}\beta\sum_{a=L,R}\left\{|\kappa_a(s)|^2\tilde{F}_1(\theta,\beta) + \left(\frac{\lambda_a}{e}\right)^2 Re[\kappa_a(s)]\tilde{F}_2(\theta,\beta) + \left(\frac{\lambda_a}{e}\right)^4\tilde{F}_3(\theta,\beta)\right\} \tag{7}$$

The cross sections are the same for $F = 0$ and $|F| = 2$ type leptoquarks since the fermions entering the process (figure 1) are treated as massless. In the above,

$$\kappa_a(s) = \sum_{V=\gamma,Z} Q^V_a(e)\frac{s}{s - M_V^2 + iM_V\Gamma_V}Q^V(\Phi), \tag{8}$$

where $Q^\gamma_{L,R}(e) = -1$, $Q^Z_L(e) = (-1/2 + \sin^2\theta_w)/\cos\theta_w\sin\theta_w$, and $Q^Z_R(e) = \tan\theta_w$ are the relevant charges for the electron. M_V and Γ_V denote the mass and the width of the neutral current gauge bosons. Furthermore, $t(\beta,\cos\theta) = 1 + \beta^2 - 2\beta\cos\theta$ with $\beta = \sqrt{1 - 4M_\Phi^2/s}$ and the functions $\tilde{F}_i(\theta,\beta)$ are

$$\tilde{F}_1(\theta,\beta) = \beta^2\left[1 + \frac{1}{4}(1 - 3\beta^2)\sin^2\theta\right] \tag{9}$$

$$\tilde{F}_2(\theta,\beta) = 2\left[1 - \frac{1-\beta^2}{t(\beta,\cos\theta)}\right](1-\beta^2) + 4\beta^2 - \beta^2\left[1 - 2\frac{1-\beta^2}{t(\beta,\cos\theta)}\right]\sin^2\theta \tag{10}$$

$$\tilde{F}_3(\theta,\beta) = 4 + \frac{\beta^2}{4}\left\{(1-\beta^2)\left[\frac{4}{t(\beta,\cos\theta)}\right]^2 + \frac{s}{M_\Phi^2}\right\}\sin^2\theta \tag{11}$$

The integrated cross sections are given by

$$\begin{aligned}\sigma_{scalar}(s) &= \frac{\pi\alpha^2\beta^3}{2s}\sum_{a=L,R}\left\{|\kappa_a(s)|^2 + \left(\frac{\lambda_a}{e}\right)^2 Re[\kappa_a(s)]F_1(\beta) + \left(\frac{\lambda_a}{e}\right)^4 F_2(\beta)\right\} \\ \sigma_{vector}(s) &= \frac{\pi\alpha^2\beta}{2M_\Phi^2}\sum_{a=L,R}\left\{|\kappa_a(s)|^2\tilde{F}_1(\beta) + \left(\frac{\lambda_a}{e}\right)^2 Re[\kappa_a(s)]\tilde{F}_2(\beta) + \left(\frac{\lambda_a}{e}\right)^4\tilde{F}_3(\beta)\right\}\end{aligned} \tag{12}$$

where

$$F_1(\beta) = \frac{3}{2}\left(\frac{1+\beta^2}{\beta^2} - \frac{(1-\beta^2)^2}{2\beta^3}\ln\frac{1+\beta}{1-\beta}\right) \tag{13}$$

$$F_2(\beta) = 3\left(-\frac{1}{\beta^2} + \frac{1+\beta^2}{2\beta^3}\ln\frac{1+\beta}{1-\beta}\right) \tag{14}$$

$$\tilde{F}_1(\beta) = \beta^2\left(\frac{7-3\beta^2}{4}\right) \tag{15}$$

$$\tilde{F}_2(\beta) = \frac{15}{4} + 2\beta^2 - \frac{3}{4}\beta^4 - \frac{3}{8\beta}(1-\beta^2)^2(5-\beta^2)\ln\frac{1+\beta}{1-\beta} \tag{16}$$

$$\tilde{F}_3(\beta) = 3(1+\beta^2) + \frac{\beta^2}{4}\frac{s}{M_\Phi^2} + \frac{3}{2\beta}(1-\beta^4)\ln\frac{1+\beta}{1-\beta}. \tag{17}$$

Asymptotically, for $s \gg M_Z^2, M_\Phi^2$, the cross sections for scalar leptoquark production behave like $\sim \ln(s/M_\Phi^2)/s$. In the case of vector leptoquarks with *minimal* coupling to the photon and Z–boson the pure s–channel contribution and the interference of the s– and t–channel terms approach a finite value *contrary* to the case of gauge couplings. The pure t–channel contribution grows proportional to s. Note, that there is no genuine relation between the fermion and gauge boson couplings for leptoquarks in general. The effective Lagrangian (1) is assumed to describe the interaction for not too large ratios s/M_Φ^2. At huge energies new effects of the underlying theory should restore unitarity.

3 QED Corrections

3.1 Initial State Radiation

Initial state radiation forms a dominant part of the QED radiative corrections for various processes in e^+e^- annihilation [11]. In the following these contributions are calculated using the structure function method which has been successfully used also in other applications [12, 13]. We will restrict us on the leading log approximation for the terms up to $\mathcal{O}(\alpha^2)$. Only the exponentiation of soft photons is treated in higher order. The Feynman diagrams of the different contributions up to $\mathcal{O}(\alpha^2)$ are depicted in figure 2. For the photon bremsstrahlung and the part of e^+e^- pair creation not associated with photon–polarization the radiator functions Γ_{ij} are described by

$$\begin{aligned}\Gamma_{ij}(z, L_m) &= \delta_{ij}\delta(1-z) + \frac{\alpha}{2\pi}P_{ij}^{(0)}(z)L_m \\ &+ \frac{1}{2}\left(\frac{\alpha}{2\pi}\right)^2\left[\left(P_{ik}^{(0)} \otimes P_{kj}^{(0)}\right)(z)L_m^2 + P_{ij}^{(1)}(z)L_m\right] + \text{h.o.} \qquad (18)\end{aligned}$$

in general. Here, we will consider the leading terms of $\mathcal{O}((\alpha L_m/2\pi)^n)$ only with $L_m = \ln(s/m_e^2)$ and, furthermore, the case $i = j \equiv e$. In $\mathcal{O}(\alpha)$ the splitting function reads

$$P_{ee}^{(0)}(z) = \delta(1-z)\left[\frac{3}{2} + 2\ln\Delta\right] + \theta(1-\Delta-z)\frac{1+z^2}{1-z} \qquad (19)$$

describing single photon bremsstrahlung. The parameter Δ is used here to separate soft and hard contributions. Below we will consider the limit $\Delta \to 0$. The splitting functions $P_{e\gamma}^{(0)}(z)$ and $P_{\gamma e}^{(0)}(z)$

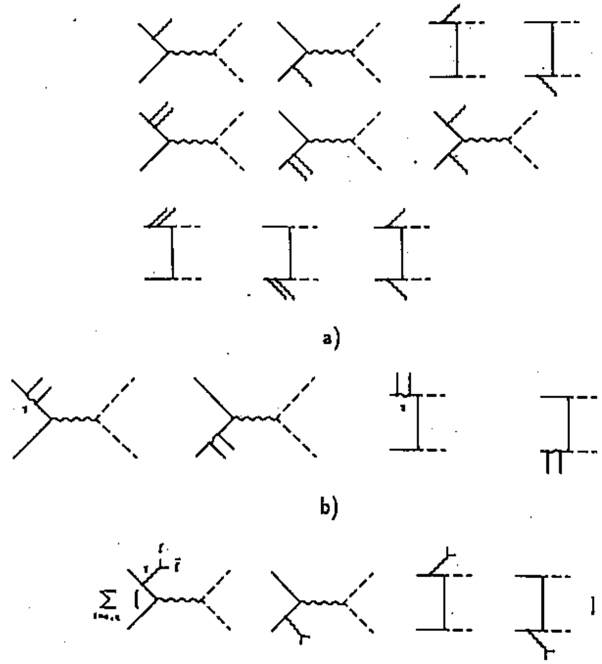
$$P_{e\gamma}^{(0)}(z) = \left[z^2 + (1-z)^2\right] \qquad (20)$$

$$P_{\gamma e}^{(0)}(z) = \frac{1+(1-z)^2}{z} \qquad (21)$$

contribute only with $\mathcal{O}(\alpha^2)$. The convolutions occuring in the 2nd order terms in (18) have the form

$$\begin{aligned}\frac{1}{2}\left[P_{ee}^{(0)} \otimes P_{ee}^{(0)}\right](z) &= \delta(1-z)\left[2\ln^2\Delta + 3\ln\Delta + \frac{9}{8} - 2\zeta(2)\right] \\ &+ \theta(1-\Delta-z)\left\{\frac{1+z^2}{1-z}\left[2\ln(1-z) - \ln z + \frac{3}{2}\right]\right. \\ &+ \left.\frac{1}{2}(1+z)\ln z - (1-z)\right\} \qquad (22)\end{aligned}$$

$$\frac{1}{2}\left[P_{e\gamma}^{(0)} \otimes P_{\gamma e}^{(0)}\right](z) \equiv P_{e\gamma e}^{(1)}(z) = (1+z)\ln z + \frac{1}{2}(1-z) + \frac{2}{3}\frac{1}{z}(1-z^3) \qquad (23)$$



a) b) c)

Figure 2: Feynman diagrams contributing to the leading log radiative QED corrections to leptoquark pair production up to $\mathcal{O}(\alpha^2)$. The bullet denotes the insertion of the respective radiator.

Aside of the above terms also fermion ($f = l^\pm, q(\bar{q})$) pair production (figure 2c) contributes. The associated splitting function is

$$P_{ff}^{(1)}(z) = N_c(f)e_f^2\frac{1}{3}P_{ee}^{(0)}(z)\theta\left(1 - z - \frac{4m_f}{\sqrt{s}}\right) \qquad (24)$$

with $N_c(f) = 3$ for quarks and $N_c(f) = 1$ for leptons, respectively. (24) results from the expansion of $\alpha(s)$ calculating the photon polarization operator in the on–mass–shell scheme for leptons *and* quarks.

$$\alpha(s) = \frac{\alpha}{1 - \frac{\alpha}{3\pi}\sum_f e_f^2 N_c(f)\ln\left(\frac{s}{m_f^2}\right)} \qquad (25)$$

For light quarks this contribution can not be calculated reliably in perturbative QCD. However, one may use the representation (25) to introduce 'effective' quark masses from a fit of

$R(s) = \sigma(e^+e^- \to hadrons)/\sigma(e^+e^- \to \mu^+\mu^-)$. These masses were determined in [14] as

$$m_u = 62 \text{ MeV} \quad m_d = 83 \text{ MeV} \quad m_u = 215 \text{ MeV}$$
$$m_c = 1500 \text{ MeV} \quad m_b = 4500 \text{ MeV}$$

We will use these values in the subsequent analysis.

The total cross section can be represented by six contributions:

$$\sigma_{tot}(s) = \sigma_{tot}^{(0)}(s) + \sigma_{tot}^{(1,\gamma)}(s) + \sigma_{tot}^{(2,2\gamma)}(s) + \sigma_{tot}^{(2,e\gamma e)}(s) + \sigma_{tot}^{(2,f\bar{f})} + \sigma_{tot}^{(>2,soft)}(s) \tag{26}$$

with the individual contributions given by:

$$\sigma_{tot}^{(1)}(s) = \frac{\alpha}{\pi} L_m \int_0^1 dz \frac{1+z^2}{1-z} \left[\sigma_{tot}^{(0)}(zs)\theta\left(z - \frac{4M^2}{s}\right) - \sigma_{tot}^{(0)}(s)\right] \tag{27}$$

$$\sigma_{tot}^{(2,\gamma\gamma)}(s) = \left(\frac{\alpha}{\pi}\right)^2 L_m^2 \int_0^1 dz P_{\gamma\gamma}^{(1)}(z) \left[\sigma_{tot}^{(0)}(zs)\theta\left(z - \frac{4M^2}{s}\right) - \sigma_{tot}^{(0)}(s)\right] \tag{28}$$

with

$$P_{\gamma\gamma}^{(1)}(z) = \frac{1+z^2}{1-z}\left[2\ln(1-z) - \ln z + \frac{3}{2}\right] + \frac{1}{2}(1+z)\ln z - (1-z) \tag{29}$$

$$\sigma_{tot}^{(2,e^+e^-)}(s) = \frac{1}{2}\left(\frac{\alpha}{\pi}\right)^2 L_m^2 \int_0^1 dz P_{e\gamma e}^{(1)}(z)\sigma(zs)\theta\left(z - \frac{4M^2}{s}\right) \tag{30}$$

$$\sigma_{tot}^{(2,f\bar{f})}(s) = \sum_{f=l,q} \frac{1}{2}\left(\frac{\alpha}{\pi}\right)^2 \ln^2\left(\frac{s}{m_f^2}\right) N_c(f) \frac{e_f^2}{3} \int_0^1 dz P_{ee}^{(0)}(z)\sigma(zs)\theta\left(1 - z - \frac{4m_f}{\sqrt{s}}\right)\theta\left(z - \frac{4M^2}{s}\right) \tag{31}$$

and $\sigma_{tot}^{(0)}$ is given by (12), respectively. The soft and virtual parts exponentiated [15] beyond the second order yield

$$\sigma_{tot}^{(3,soft)}(s) = \int_0^1 dz P_{soft}^{(3)}(z) \left[\sigma_{tot}^{(0)}(zs)\theta\left(z - \frac{4M^2}{s}\right) - \sigma_{tot}^{(0)}(s)\right] \tag{32}$$

with

$$P_{soft}^{(3)}(z) = b(1-z)^{b-1}(1+\delta_1+\delta_2) - \frac{b(1+\delta_1) + b^2\ln(1-z)}{1-z} \tag{33}$$

where $b = (2\alpha/\pi)(L_m - 1)$, $\delta_1 = (3\alpha/2\pi)L_m$ and $\delta_2 = (\alpha/\pi)^2[9/8 - 2\zeta(2)]L_m^2$.

In figure 3 the bremsstrahlung corrections to $\mathcal{O}(\alpha^2)$ including soft exponentiation is shown. These corrections grow with $2M_\Phi \to \sqrt{s}$ and reach values in the range of −0.45 to −0.35 for $\beta = 0.1$ for scalar and vector leptoquarks, respectively. The correction varies only weakly with $\sqrt{s}$ in the energy range of future e^+e^- colliders. In figure 4 only the $\mathcal{O}(\alpha^2)$ + soft exponentiation contributions are depicted. Note, that their total contribution is positive and reaches $\mathcal{O}(10\ \%)$ of the Born cross section for $\beta \sim 0.1$.

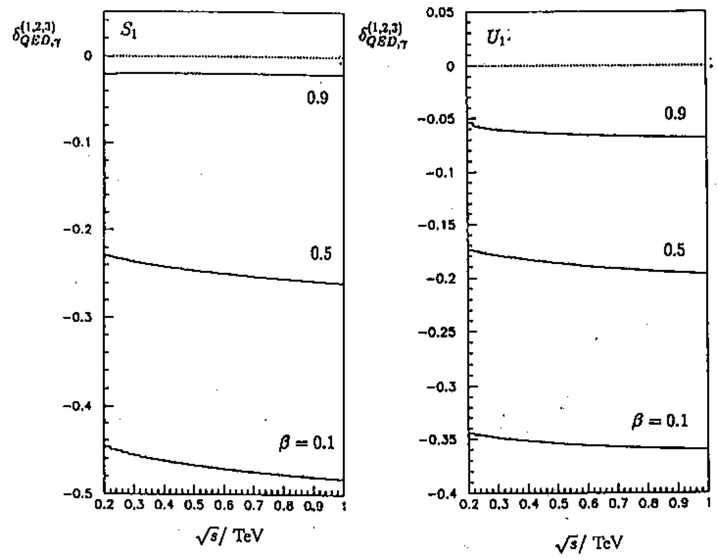


Figure 3: QED initial state Bremsstrahlung contributions (figure 2a) up to $\mathcal{O}(\alpha^2)$ + soft exponentiation for the scalar (S_1) and vector (U_1) leptoquark pair production cross section. Here we assumed $\lambda_L = 0$ and $\lambda_R = e$ for the leptoquark-fermion couplings.

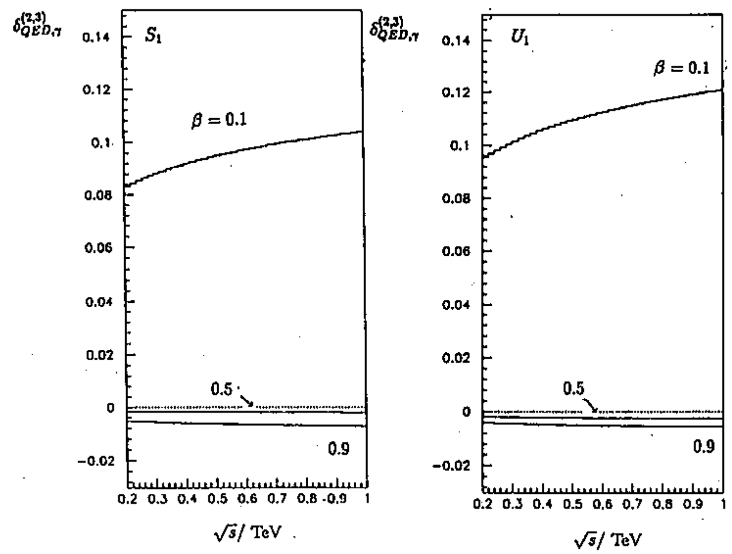


Figure 4: $\mathcal{O}(\alpha^2)$ + soft exponentiation terms of figure 3 only.

Figure 5 shows the e^+e^- and $f\bar{f}$ pair creation contributions in $\mathcal{O}(\alpha^2)$. In the range $\sqrt{s} \leq 1$ TeV they amount to less than 4 % each if $\beta < 0.9$. The results for scalar and vector leptoquarks turn out to be rather similar in the case of these contributions.

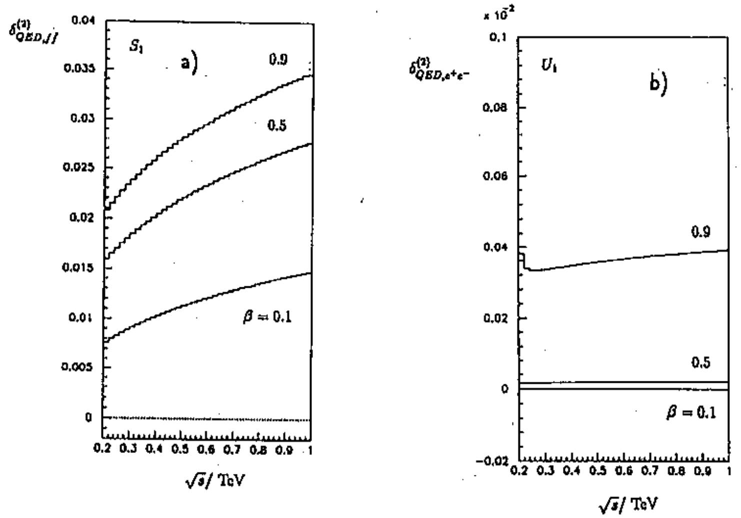


Figure 5: a) $\mathcal{O}(\alpha^2)$ initial state $f\bar{f}$ contribution to scalar (S_1) leptoquark pair production; b) $\mathcal{O}(\alpha^2)$ initial state e^+e^- contribution to vector (U_1) leptoquark pair production.

3.2 Beamstrahlung

At large cms energies $\sqrt{s}$ the effect of beamstrahlung at linear colliders [16] may have a significant impact on the production cross sections. Numerical studies have been performed for a series of accelerators [17]. The beamstrahlung correction is described by the convolution

$$\Delta\sigma_{BS}(s) = \int_0^1 dz \frac{1}{\mathcal{L}}\frac{d\mathcal{L}}{dz}(z)\sigma^{(0)}(zs)\theta\left(z - \frac{4M^2}{s}\right) - \sigma^{(0)}(s). \tag{34}$$

The normalized longitudinal luminosity distribution $(1/\mathcal{L})d\mathcal{L}(z)/dz$ has in general to be determined by Monte Carlo simulations of the accelerator type considered.

As an example in figure 6 the beamstrahlung correction to the production cross section for scalar and vector leptoquarks is shown using a simulated spectrum $(1/\mathcal{L})d\mathcal{L}(z)/dz$ [18] for $\sqrt{s} = 300$ GeV. For $\beta \approx 0.1$ these corrections may reach $\mathcal{O}(-80\ \%)$ although narrow band machines are considered.

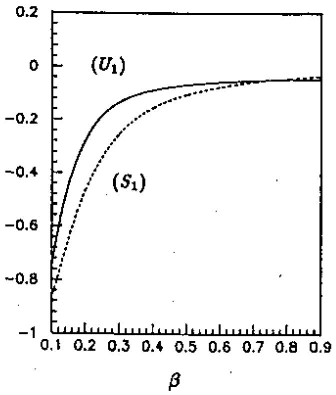


Figure 6: Beamstrahlung correction for a narrow band machine for the pair production cross section of scalar (S_1) and vector (U_1) leptoquarks as a function of β for $\sqrt{s} = 300$ GeV.

4 QCD Corrections

For the case that $\lambda/e \ll 1$ the QCD correction and the final state QED correction for leptoquark pair production is determined by a formfactor $\hat{\mathcal{F}}(\beta, s)$. Here, we will consider this correction for scalar leptoquarks only. The contribution is given by the Feynman diagrams having the same topology as in the case of e.g. the $\gamma q\bar{q}$ vertex, if the Duffin–Kemmer–Petiau representation [19] is used. Otherwise corresponding 'sea-gull' terms have to be added. The form factor was originally calculated by J. Schwinger [20] and reads

$$\begin{aligned}\mathcal{F}_s(\beta) &= \frac{1+\beta^2}{\beta}\left[4Li_2\left(\frac{1-\beta}{1+\beta}\right) + 2Li_2\left(-\frac{1-\beta}{1+\beta}\right) - 3\ln\frac{2}{1+\beta}\ln\frac{1+\beta}{1-\beta} - 2\ln\beta\ln\frac{1+\beta}{1-\beta}\right] \\ &- 3\ln\left(\frac{4}{1-\beta^2}\right) - 4\ln\beta + \frac{1}{\beta^3}\left[\frac{5(1+\beta^2)^2}{4} - 2\right]\ln\frac{1+\beta}{1-\beta} + \frac{3}{2}\frac{1+\beta^2}{\beta^2}\end{aligned} \tag{35}$$

Here, $Li_2(x) = -\int_0^x dt \ln(1-t)/t$ denotes the Spence function. The form factor is dominated by π^2-terms. Thus, the $\mathcal{O}(\alpha_s)$ QCD correction is rather large. One obtains

$$\sigma^{(1,QCD)}_{scalar}(s) = \sigma^{(0)}_{scalar}(s)\left\{1 + \frac{4\alpha_s}{3\pi}\mathcal{F}_s(\beta)\right\} \tag{36}$$

and, correspondingly, the $\mathcal{O}(\alpha)$ QED final state correction reads

$$\sigma^{(1,QED,fs)}_{scalar}(s) = \sigma^{(0)}_{scalar}(s)\left\{1 + \frac{\alpha_s}{\pi}\mathcal{F}_s(\beta)\right\}. \tag{37}$$

Figure 7 shows the relative corrections $\delta_{fs}(\beta) = \sigma^{(1,fs)}/\sigma^{(0)}$.

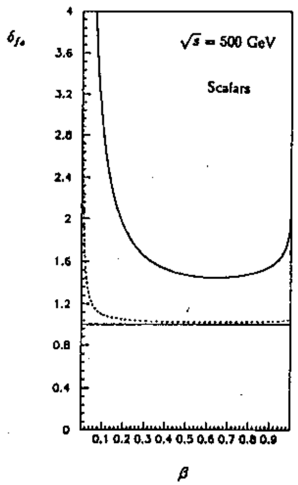


Figure 7: β dependence of the final state correction for scalar leptoquarks $\delta_{f_s} = \sigma^{(1)}/\sigma^{(0)}$ at $\sqrt{s} = 500\,\mathrm{GeV}$; full line: QCD correction, dashed line: QED correction.

For $\beta > 0.2$ the QED correction is smaller than 5 %. However, the QCD correction is always larger than 40 % and exceeds 100 % for $\beta < 0.2$, i.e. $M_\Phi > 245\,\mathrm{GeV}$.

For small values of β $\mathcal{F}_s(\beta)$ grows $\sim \pi^2/(2\beta)$ and both (36,37) become unreliable, indicating bound state formation. In fact, for the related case of scalar quark pair production the formation of squarkonia has been discussed some time ago in [21, 22]. Leptoquarkonia might be created in e^+e^- annihilation for small values of β, if the life–time of the leptoquarks is large enough. The tree–level expression for the decay width of leptoquarks

$$\Gamma_{\Phi_{s,v}} = 0.4\,\mathrm{GeV} f_{s,v} \left(\frac{\lambda_{s,v}}{e}\right)^2 \frac{M_\Phi}{200\,\mathrm{GeV}} \qquad (f_s = 1;\ f_v = 2/3) \tag{38}$$

may be rather small for the values of $\lambda_{s,v}$ still allowed by the current experimental constraints [3]. Otherwise, because leptoquarks are colored particles it is likely that (38) does not yield the dominant contribution to the leptoquark decay width but the leptoquark decay might proceed by leptoquark–antiquark recombination and subsequent decay within typical fragmentation lengths.

Thus, the corresponding widths for $M_\Phi \sim \mathcal{O}(100\,\mathrm{GeV})$ may already be to large to allow bound state formation. Furthermore, the β^3–behaviour of the s–channel contribution complicates the observation of a resonance even if it is formed due to the size of the energy spread of the e^+e^- colliders.

5 $\gamma\gamma$ Fusion

Finally we consider the pair production of scalar leptoquarks via $\gamma\gamma$ fusion. This production process is particularly interesting for possible future photon–photon colliders. In fact, electron beams may be converted into photon beams via laser back scattering [23]. However, a series of technical problems connected with this has still to be solved in future [5].

In the case of $\gamma\gamma$ fusion the total production cross section is obtained convoluting the point cross section with the photon spectra:

$$\sigma = \int_0^{z_{max}} dz_1 \int_0^{z_{max}} dz_2 \Phi_\gamma(z_1)\Phi_\gamma(z_2)\hat{\sigma}(\hat{s})\theta\left(\hat{s} - 4M_\Phi^2\right) \tag{39}$$

with $\hat{s} = z_1 z_2 s$. For the process $\gamma\gamma \to \Phi_s\overline{\Phi}_s$ the cross section reads [24]

$$\sigma_{scalar}(s) = \frac{\pi\alpha^2}{s} Q_\Phi^4 \left\{2(2-\beta^2)\beta - (1-\beta^4)\ln\left|\frac{1+\beta}{1-\beta}\right|\right\} \tag{40}$$

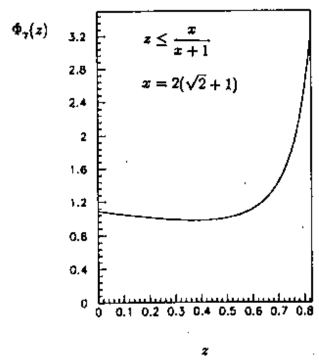
The photon spectra are [23]

$$\Phi_\gamma(z) = \frac{1}{N(x)}\left[1 - z + \frac{1}{1-z} - \frac{4z}{x(1-z)} + \frac{4z^2}{x^2(1-z)^2}\right] \tag{41}$$

with the normalization

$$N(x) = \frac{16 + 32x + 18x^2 + x^3}{2x(1+x)^2} + \frac{x^2 - 4x - 8}{x^2}\ln(1+x) \tag{42}$$

with $z \le z_{max} = x/(x+1)$ and $x = 2(\sqrt{2}+1)$.



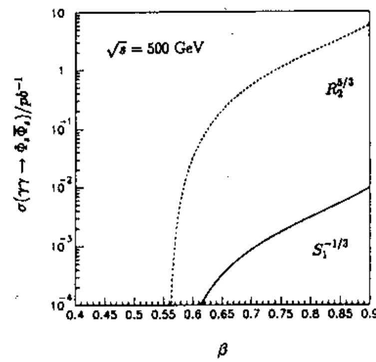


Figure 8: a) Compton spectrum as function of z; b) pair production cross section for scalar leptoquarks for $\gamma\gamma$ fusion as a function of β.

In figure 8a the photon spectrum (41) is shown. It peaks at the upper end. The upper boundary of z marking the onset of pair–production causes a lower bound in β, $\beta_{min} =$

[5] It appears to be unlikely, that one may convert beams from linac–facilities with 0° beam crossing. A geometrical form of the collision area has to be found which prevents the focussing system of the electron beams from irradiation of the high intensity photon beams after the beam crossing.

$\sqrt{4\sqrt{2}+5}/(2\sqrt{2}+3) \approx 0.560$. For an assumed integrated luminosity of $\mathcal{L} = 10 fb^{-1}$ the observation of $S_1^{-1/3}$ leptoquarks is possible at a rate of 20 events for $\beta > 0.75$ at $\sqrt{s} = 500\,\mathrm{GeV}$, i.e. $M_\Phi < 165\,\mathrm{GeV}$ (see figure 8b). The cross section of other leptoquarks carrying higher electric charge scales as $\times\, Q_\Phi^4$. For the case of $R_2^{5/3}$ leptoquarks the same event rate is obtained for $\beta > 0.57$ already, i.e. $M_\Phi \leq 205\,\mathrm{GeV}$. An advantage of the photoproduction cross section of scalar leptoquarks is the proportionality to β for small cms velocities rather than to β^3 as observed in the case of e^+e^- annihilation.

6 Conclusions

QED and QCD corrections to leptoquark pair production cross sections in e^+e^- annihilation have been calculated. These corrections are found to be large in the threshold range. In the case of QED corrections the initial state contributions dominate yielding a negative correction of $\mathcal{O}(-40$ to $-50\%)$. It is imortant to consider $\mathcal{O}(\alpha^2)$ terms as well, since the bremsstrahlung contributions in this order yield about $+10$ % of the Born term. A small contribution is due to fermion pair production and final state photon radiation (in the range $\beta \gtrsim 0.1$),

Large beamstrahlung corrections (also for narrow band machines) are found in the range of small β degrading the event numbers by about 80 %.

The $\mathcal{O}(\alpha_s)$ final state QCD corrections for scalar leptoquarks are always larger than $+40$ % and grow beyond 100 % for $\beta \leq 0.2$ due to the Coulomb singularity which indicates resonance formation. The size of the QCD correction is dominantly due to the emergence of π^2 terms.

Production cross sections for scalar leptoquarks via photon–photon fusion at $\gamma\gamma$–colliders in the range of 0.1 to $10^3 fb$ for $\sqrt{s} = 500\,\mathrm{GeV}$ are obtained using Compton spectra. In this process dominantly the leptoquarks with large charges ($Q_\Phi \leq 5/3$) are produced.

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