hep-th/9511023 3 NOV 95

On the Manifestations of Particles*

Detlev Buchholz
II. Institut für Theoretische Physik, Universität Hamburg
D-22761 Hamburg, Federal Republic of Germany

Abstract

The mathematical description of stable particle-like systems appearing in relativistic quantum field theory at large, respectively small scales or non-zero temperatures is discussed.

1 Introduction

Stable particle-like systems are met in high energy physics in very different settings: elementary particles, as well as their bound states, are observed at asymptotic times in collision processes. At the other end of the scale, i.e. at short distances, particle-like structures confined in hadronic matter (partons) appear in deep inelastic processes. Proceeding to thermal states, they may also exist as individual constituents of a hot plasma (quarks, gluons). It is a somewhat embarassing fact that for most of these fundamental elementary systems a fully satisfactory mathematical description is not at hand.

In the conventional treatment, originating from E. Wigner's pioneering analysis [1], particles are identified with vector states in irreducible representations of the Poincaré group, resp. its covering group. This characterization of particles has proved to be extremely useful: it led to an understanding of the role of mass and spin, it is a key ingredient in the construction of field theoretic models, entering in the renormalization procedure, and it is the basis for the physical interpretation of the theory in terms of collision states. These applications reveal the value of a sound *a priori* concept of particle within the theoretical setting.

Wigner's particle concept applies only to a restricted class of elementary systems, however, a well known counter-example being particles carrying electric charge. The states of such particles always contain infinitely many low energy photons and therefore cannot be described by vectors in some Poincaré invariant superselection sector of the physical Hilbert space; in fact, neither mass nor spin can sharply be defined for these states [2, 3]. The inappropriate Wigner concept of particle is at the origin of the infrared problems in the early treatments of

*Talk given at the "Workshop on Mathematical Physics Towards the 21st Century", Beer Sheva, 14-19 March 1993

quantum electrodynamics and led B. Schroer to the notion of infraparticle [4]. Although one has learned in the meantime how to handle these difficulties, a clear-cut mathematical characterization of electrically charged particles has not yet emerged. Summarizing the folklore, these particles are regarded as certain limit cases of Wigner type particles, which are accompanied by photon clouds (cf. [5, 6] for a rigorous formulation of this idea). This picture suffices in many applications for a consistent interpretation of theoretical calculations, e.g. of collision cross sections. But it somewhat obscures the genuine nature of the underlying physical systems.

In the case of partons the situation is even less satisfactory. These entities made their first appearance in an ingenious interpretation of experimental data [7], relying on Wigner's particle concept. Their theoretical description has later been formalized in the perturbative treatment of quantum chromodynamics, which also relies on that concept through the use of Feynman rules. In this way the existence of partons has now been firmly established, but it is also clear that the particle concept used has only a tentative meaning since partons are (expected to be) confined and physical states of these entities do not exist. Again one pays for the inappropriate particle concept by tantalizing infrared problems in the perturbative setting and the need to introduce in an *ad hoc* way phenomenological fragmentation functions which are tailored to convert the perturbative results into physically significant statements. These difficulties may in principle be avoided by considering only physical states, as e.g. in lattice gauge theory. But then one loses contact with the simple parton picture, and the problem of a description of the parton content of states reappears.

Problems of still another kind arise if one wants to describe the particle features of thermal states. Thinking of the deconfinement transition in quantum chromodynamics, for example, it would be of interest to characterize the constituents of such states. Of similar physical interest are the quasiparticles [8], i.e. collective excitations of the constituents which frequently exhibit particle properties as well. The constituents of a thermal state may be expected to have properties which do not deviate too much from those of an elementary particle, at least in the case of low densities. Nevertheless, their mathematical description causes problems because of the influence of the thermal background. In fact, neither the constituents, nor the quasiparticles can obey a sharp dispersion law if there is interaction [9]. This fact makes it difficult to distinguish them from other modes, and again one runs into inconsistencies if one imposes from the outset a wrong particle concept [10].

It is the aim of this article to report on attempts to overcome these conceptual difficulties. The results obtained so far are not yet complete, but they shed new light on the problems and seem to be a solid basis for further studies. Thus, in accord with the idea of this workshop, this contribution intends to be both, a status report and a bit of science fiction.

The article is organized as follows: Sect. 2 contains an outline of algebraic quantum field theory, which is the natural framework for particle analysis. The particle structures which appear in this setting at asymptotic times are described

in Sect. 3. These results are fairly complete and lead to a unified treatment of elementary particles and infraparticles in terms of weights [11]. Moreover, they provide a general method by which the asymptotic particle content can be extracted from any state.

The particle structures appearing in the short distance limit are the theme of Sect. 4. For their characterization the concept of a scaling algebra is introduced which provides an algebraic version of the renormalization group and allows a systematic discussion of the scaling limit of any quantum field theory [12]. It turns out that this limit can always be interpreted in terms of vacuum states on the scaling algebra. The particle content of the theory in the short distance limit can thus be determined by a particle analysis of the local excitations of these vacua, as indicated above, and one may therefore hope for a complete classification of the pertinent structures.

Section 5 is dedicated to the particle aspects of thermal equilibrium states. For the investigation of the particle structures in the scaling limit one can apply the method of the scaling algebra, yielding the same results as before. But if one proceeds to larger scales, the analysis becomes more difficult since the specific dynamical properties of the thermal background come into play. Nevertheless it appears to be possible to distinguish the constituents of thermal states in precise mathematical terms [13]. A clear-cut mathematical characterization of quasiparticles remains, however, as an intriguing problem.

The article closes with some "concluding ruminations".

2 The framework

Before entering into details, it may be helpful to point to the similarities of our approach and the strategy of Wigner. Wigner took as the starting point of his analysis the principles of special relativity, which are encoded in the properties of the space-time symmetry group, the Poincaré group. Since symmetries act on physical states by automorphisms, it appeared to be natural to assume that particles, being the most elementary systems, give rise to irreducible representations of the Poincaré group, respectively its covering group.[1] This idea of a particle then led naturally to the problem of classifying these representations [1].

In the present approach we start from the principle of locality, which is encoded in algebraic relations between observables. The notion of locality is of vital importance in our characterization of particles, where we appeal to the heuristic picture of a particle as a stable, singly localized entity [14]. This picture will lead us to a comprehensive description of particles in terms of specific states (resp. weights) on the algebra of observables, depending on the respective physical situtation. Thus, like Wigner, we characterize particles by a condition of simplicity,

[1] At the time of its invention this assumption must have seemed to be exceedingly general. But today we see its weak point: symmetries can be spontaneously broken so that their action on states gives rise to incoherent systems, belonging to different superselection sectors. It then makes no sense to classify the states according to the irreducible representations of the symmetry group

and we are then led to the classification of the corresponding representations of the observables, in complete analogy to his approach.

As already indicated, we will make use of the framework of algebraic quantum field theory [15] which allows us to discuss the particle aspects of relativistic quantum field theories in full generality. For the convenience of those readers who are not familiar with this setting, we briefly list the relevant assumptions and add a few comments.

1. *(Locality)* We suppose that the local observables of the underlying theory generate a net of local algebras over d-dimensional Minkowski space $\mathbb{R}^d$, i.e. an inclusion preserving map

$$\mathcal{O} \to \mathcal{A}(\mathcal{O}) \tag{1}$$

from the set of open, bounded regions $\mathcal{O}$ in Minkowski space to unital C*-algebras $\mathcal{A}(\mathcal{O})$. We will think of $\mathcal{A}(\mathcal{O})$ as the algebra generated by observables which can be measured in $\mathcal{O}$. The algebra generated by all local algebras $\mathcal{A}(\mathcal{O})$ (as a C*-inductive limit) will be denoted by $\mathcal{A}$. The net is supposed to satisfy the principle of locality (Einstein causality), i.e.

$$\mathcal{A}(\mathcal{O}_1) \subset \mathcal{A}(\mathcal{O}_2)' \text{ if } \mathcal{O}_1 \subset \mathcal{O}_2', \tag{2}$$

where $\mathcal{O}'$ denotes the spacelike complement of $\mathcal{O}$ and $\mathcal{A}(\mathcal{O})'$ the set of operators in $\mathcal{A}$ which commute with all operators in $\mathcal{A}(\mathcal{O})$. We note that this abstract algebraic setting is quite natural for our purposes since we are interested in physical systems whose Hilbert space description has yet to be determined.

2. *(Covariance)* The Poincaré group $\mathcal{P}_+^\uparrow$ is represented by automorphisms of the net. Thus for each $(\Lambda, x) \in \mathcal{P}_+^\uparrow$ there is an $\alpha_{\Lambda,x} \in \mathrm{Aut}\mathcal{A}$ such that, in an obvious notation,

$$\alpha_{\Lambda,x}(\mathcal{A}(\mathcal{O})) = \mathcal{A}(\Lambda\mathcal{O} + x) \tag{3}$$

for any region $\mathcal{O}$. We amend this fundamental postulate by a continuity condition, i.e. we assume that for any $A \in \mathcal{A}$ the function $(\Lambda, x) \to \alpha_{\Lambda,x}(A)$ is strongly continuous. This is no loss of generality since these observables suffice to separate the states of physical interest.

3. *(States)* Physical states are described by positive, linear and normalized functionals ω on $\mathcal{A}$. By the GNS-construction, any state ω gives rise to a representation π_ω of $\mathcal{A}$ on a Hilbert space $\mathcal{H}_\omega$, and there exists a cyclic vector[2] $\Omega_\omega \in \mathcal{H}_\omega$ such that

$$\omega(A) \;=\; (\Omega_\omega, \pi_\omega(A)\Omega_\omega), \;\; A \in \mathcal{A}. \tag{4}$$

Since the dual space of $\mathcal{A}$ contains an abundance of states corresponding to very different physical situations (few and many body systems, equilibrium and non-equilibrium systems, etc.), one has to select the states of actual interest by additional conditions.

It is a characteristic feature of few body systems that they have finite total energy and momentum. This suggests to characterize the corresponding states ω by the condition [16] that the space-time translations x act on the GNS-space

[2] This means that $\pi_\omega(\mathcal{A})\Omega_\omega$ is dense in $\mathcal{H}_\omega$

$\mathcal{H}_\omega$ by a continuous unitary representation $U_\omega(x)$ which satisfies the relativistic spectrum condition (i.e. the generators have joint spectrum in the closed forward lightcone $\overline{V}_+$) and implements the translations of observables,

$$U_\omega(x)\pi_\omega(A)U_\omega(x)^{-1} \;=\; \pi_\omega(\alpha_x(A)),\; A \in \mathcal{A}. \tag{5}$$

As a matter of fact, under these circumstances there exists a unique choice of the representation $U_\omega(x)$ with Lorentz invariant spectrum, as a consequence of locality [17]. States ω and representations π_ω with these properties are said to have positive energy. If a positive energy state ω is invariant under translations, i.e. $\omega(\alpha_x(A)) = \omega(A)$ for all x and $A \in \mathcal{A}$, its GNS-vector Ω_ω is invariant under the action of $U_\omega(x)$ and may thus be interpreted as vacuum.

In case that also the Lorentz transformations Λ act on $\mathcal{H}_\omega$ by a continuous unitary representation $\Lambda \to U_\omega(\Lambda)$, and $U_\omega(\Lambda)\pi_\omega(A)U_\omega(\Lambda)^{-1} = \pi_\omega(\alpha_\Lambda(A)), A \in \mathcal{A}$, we speak of Poincaré covariant representations. It is reasonable to assume that states ω, representing the vacuum, are invariant under Lorentz transformations, thereby giving rise to Poincaré covariant representations. But, as was pointed out in the Introduction, the condition of full Poincaré covariance for non-vacuum states is in general too restrictive, as it excludes from the outset states carrying electric charge.

We will also consider states ω on $\mathcal{A}$ which describe systems in thermal equilibrium. These states are characterized by the KMS-condition, which is the proper description of Gibbs ensembles in the thermodynamic limit [18, 15]. In the formulation of this condition there enters a distinguished time axis, which is fixed by the rest frame of the heat bath. Denoting by t the translations along this axis, the KMS-condition says that ω is a thermal equilibrium state at inverse temperature $\beta > 0$ if the following holds: for any given $A, B \in \mathcal{A}$ the functions $t \to \omega(A\alpha_t(B))$ and $t \to \omega(\alpha_t(B)A)$ are continuous boundary values of some analytic function in the strip $\{z \in \mathbb{C} : 0 < \mathrm{Im} z < \beta\}$, at its lower and upper rim, respectively.

3 Particles and infraparticles

States ω of positive energy are expected to describe at asymptotic times configurations of a few stable particles, possibly accompanied by clouds of low energy massless particles. Following this idea and the conception of a particle as some well-localized entity, it seems natural to analyze the asymptotic particle aspects of ω by studying the properties of the expectation values $\omega(\alpha_x(A)), A \in \mathcal{A}$, for large timelike x. The dominant contributions should originate from a vacuum state ω_0,

$$\lim_x \omega(\alpha_x(A)) = \omega_0(A),\;\; A \in \mathcal{A}, \tag{6}$$

since the probability for finding some deviation from the vacuum in a finite space-time volume about x (fixed by the localizaton properties of the respective observable A) should tend to 0 in a state of finite energy because of dispersive effects

("spreading of wavepackets").[3] We are here primarily interested in the local deviations from the vacuum, however. Following Araki and Haag [20], we therefore proceed to a subalgebra $\mathcal{C} \subset \mathcal{A}$ of observables which are insensitive to the vacuum.

The algebra $\mathcal{C}$ is constructed as follows: we first consider the vector space $\mathcal{L}_0 \subset \mathcal{A}$ of almost local operators[4] L_0 such that the functions $(\Lambda, x) \to \alpha_{\Lambda,x}(L_0)$ are C^∞ and have, with respect to x, Fourier transforms with support in some arbitrary compact subset Γ in the complement of the closed forward lightcone $\overline{V^+}$ (they "transfer" energy-momentum Γ). Such operators are easily obtained by regularization of (almost) local operators $A \in \mathcal{A}$ with respect to the action of the Poincaré group,

$$L_0 = \int d\mu(\Lambda, x) f(\Lambda, x) \alpha_{\Lambda,x}(A). \tag{7}$$

Here μ is the Haar-measure on $\mathcal{P}_+^\uparrow$ and f is any test function whose Fourier transform with respect to x has support in some Γ. By left multiplication of the elements of $\mathcal{L}_0$ with arbitrary elements from $\mathcal{A}$ and linear combinations we obtain a (non-closed) left ideal $\mathcal{L} \subset \mathcal{A}$ which is invariant under the action of the Poincaré group. We call $\mathcal{L}$ the left ideal of localizing operators for reasons which will become clear later. Because of the specific energy-momentum transfer Γ of the operators in $\mathcal{L}_0$, any operator $L \in \mathcal{L}$ annihilates the vacuum state(s) and, as a matter of fact, all positive energy states of sufficiently small energy (depending on L).

We then define the algebra $\mathcal{C}$ as the linear span of all operators of the form L^*L, $L \in \mathcal{L}$, i.e. $\mathcal{C} = \mathcal{L}^*\mathcal{L}$ in shorthand. The algebra $\mathcal{C}$ is smaller than the "algebra of detectors" introduced in [20]. But it separates all positive energy states which are orthogonal to the vacuum and is therefore big enough for our purposes.

We can now proceed with our asymptotic analysis of positive energy states. Since the operators $C \in \mathcal{C}$ annihilate the vacuum, the at asymptotic times leading contributions to the expectation values $\omega(\alpha_x(C)), C \in \mathcal{C}$, should be due to single stable particles, triggering C. Contributions from multiple excitations of the vacuum may be expected to appear only in next to leading order since the probability for finding more than one particle in a space-time region of fixed size should decay substantially faster with time. As far as the single particle contributions in the functions $x \to \omega(\alpha_x(C))$ are concerned, they will tend to zero at large times because the particles can be anywhere, one loses control of their localization. This picture suggests to integrate these functions over spacelike surfaces and thereby to compensate the effect of dispersion.

At this point there arises a problem: do the integrals $\int d\boldsymbol{x}\, \omega(\alpha_{t,\boldsymbol{x}}(C)), C \in \mathcal{C}$ (where we have introduced proper coordinates $x = (t, \boldsymbol{x})$) exist? An affirmative

[3] This argument is actually not quite correct since the vacuum may be "slowly varying" at infinity, cf. [19]. But one can show within the present general setting that the family of states $\omega \circ \alpha_x$ always has weak *-limit points which are vacuum states

[4] An operator $B \in \mathcal{A}$ is said to be almost local if there exists an approximating sequence $B_r \in \mathcal{A}(\mathcal{O}_r)$, $\mathcal{O}_r$ being a family of concentric balls of radius r, such that $\|B - B_r\|$ tends to 0 for large r faster than any inverse power of r

answer, which may be regarded as a first confirmation of our heuristic picture, follows from the subsequent lemma [21]. Its proof is based on a harmonic analysis of the space-time translations $\alpha_{\mathbb{R}^d}$.[5]

Lemma 3.1: *Let $E(\cdot)$ be the spectral resolution of the generators of space-time translations $U(x)$ in any positive energy representation π of $\mathcal{A}$. Then, for any $C \in \mathcal{C}$ and any compact subset Δ of the spectrum,*

$$\limsup_{r\to\infty} ||E(\Delta) \int_{|\mathbf{x}|<r} d\boldsymbol{x}\, \pi(\alpha_{\boldsymbol{x}}(C))E(\Delta)|| < \infty.$$

Thus if ω is a positive energy state whose spectral support (with respect to the generators of the space-time translations in its GNS-representation) is contained in some compact set Δ, we see from the lemma that the integrals in question are well-defined and uniformly bounded in t (since the spectral support of ω is invariant under time translations). In fact, using the lemma, one can equip $\mathcal{C}$ with a directed set of seminorms, and the positive linear functionals $\rho_t, t \in \mathbb{R}$ on $\mathcal{C}$ given by

$$\rho_t(C) = \int d\boldsymbol{x}\, \omega(\alpha_{t,\boldsymbol{x}}(C)),\ C \in \mathcal{C} \tag{8}$$

form an equicontinuous family with respect to the resulting topology. Thus there exist weak *-limit points if t approaches $\pm\infty$, respectively. Actually, one would expect that in physically reasonable theories the functionals ρ_t converge since the asymptotic particle configurations should stabilize. This question is closely related to the problem of asymptotic completeness, cf. below. In order to deal with possible oscillations of asymptotic particle configurations, we proceed to time averages of the functionals ρ_t,

$$\overline{\rho_t} = \frac{1}{\lambda(t)} \int_t^{t+\lambda(t)} dt' \rho_{t'}, \tag{9}$$

where λ is some positive, slowly increasing function. According to the preceding discussion, we expect that the limit points of the functionals $\overline{\rho_t}$ contain the relevant information about the asymptotic particle content of ω.

Working hypothesis: The limit points $\{\sigma_\pm\}$ of the functionals $\overline{\rho_t}, t \to \pm\infty$ describe the outgoing, respectively incoming stable particle content of the state ω.

The problem of describing particles appearing at asymptotic times has thus turned into a problem of mathematical analysis and interpretation of the structure of the functionals $\sigma_\pm$. Such an analysis has been carried out in cooperation with M. Porrmann [23], cf. also [11], and we give here an account of some of the results. Since in this discission the time direction plays no role, we omit in the following the index $\pm$ in $\sigma_\pm$.

[5] The general harmonic analysis of automorphism groups [22] seems still in its initial stages. It would be useful to have a classification of their possible spectral properties, in analogy to the notion of measure classes in the case of unitary groups

It is apparent from their definition that the functionals σ on $\mathcal{C}$ are positive and linear. (But they are not normalizable; note that $\mathcal{C}$ does not contain a unit.) Thus any σ induces a positive semi-definite scalar product on the left ideal $\mathcal{L}$. Proceeding as in the GNS-construction, one obtains, by taking quotients with respect to elements of zero norm and completion, a Hilbert space $\mathcal{H}$ and a linear map $|\cdot\rangle : \mathcal{L} \to \mathcal{H}$ with dense range, such that

$$\langle L_1|L_2\rangle = \sigma(L_1^* L_2) \;\; L_1, L_2 \in \mathcal{L}. \tag{10}$$

Here $\langle\cdot|$ denotes of course the corresponding antilinear map of $\mathcal{L}$ into the dual space of $\mathcal{H}$. As suggested by the notation, it may be helpful to think of $\langle\cdot|$ and $|\cdot\rangle$ as improper Dirac bra and ket vectors, respectively, which become normalizable after application of the elements of $\mathcal{L}$. Only in special cases, they may also be identified with improper states in a rigged Hilbert space formalism. Some important properties of the maps $|\cdot\rangle$ are summarized in the

Proposition 3.2: *i) The operators $\pi(A), A \in \mathcal{A}$ fixed by*

$$\pi(A)|L\rangle \doteq |AL\rangle, \;\; L \in \mathcal{L}$$

*are well-defined and bounded. The corresponding map $\pi : \mathcal{A} \to \mathcal{B}(\mathcal{H})$ defines a *-representation of $\mathcal{A}$.*
ii) The functions $(\Lambda, x) \to |\alpha_{\Lambda,x}(L)\rangle, \; L \in \mathcal{L}_0$ are C^∞. ($\mathcal{L}_0$ is the generating space of $\mathcal{L}$.)
iii) The operators $U(x)$ fixed by $U(x)|L\rangle \doteq |\alpha_x(L)\rangle, \; L \in \mathcal{L}$ define a continuous unitary representation $x \to U(x)$ of space-time translations on $\mathcal{H}$. The spectrum of the generators of $U(x)$ is contained in some (shifted) light cone $\overline{V}_+ + q$.

We note that the vector q in the latter statement depends on the spectral support of the underlying state ω.

Definition: A linear map $|\cdot\rangle$ of $\mathcal{L}$ into some Hilbert space $\mathcal{H}$ with dense range is called a *particle weight* if it has the properties given in the Proposition.

The representations π of $\mathcal{A}$ induced by the particle weights constructed above will be in general highly reducible. (Note that in the construction of $|\cdot\rangle$ there appeared direct integrals of states.) Since elementary systems, such as particles, should give rise to irreducible representations of $\mathcal{A}$, it is natural to pose the question of whether $|\cdot\rangle$ can be decomposed into pure components. An affirmative answer is given in the following statement.

Proposition 3.3: *Let $|\cdot\rangle$ be a particle weight. There exists a measure space (Σ, μ) and a measurable family of pure particle weights $|\cdot\rangle_\lambda, \lambda \in \Sigma$ (inducing irreducible representations of $\mathcal{A}$) such that there holds the direct integral decomposition*

$$|\cdot\rangle = \int d\mu(\lambda) |\cdot\rangle_\lambda. \tag{11}$$

Remark: In this form the statement holds actually only if $\mathcal{A}$ is separable. We refrain from discussing here the complications arising in the general case.

This result tells us that it is possible to resolve the asymptotic particle structure described by σ into components with definite superselection quantum numbers (charges). More can be said, but before we proceed we have to discuss some technicality.

Definition: A particle weight $|\cdot\rangle$ is said to be regular if $\sum_{i=1}^{n} |L_i^* L_i\rangle = 0$ implies $|L_i\rangle = 0, L_i \in \mathcal{L}, i = 1 \ldots n$. (Note that this implication would trivially hold if $\mathcal{L}$ would contain a unit.)

It seems likely that pure particle weights of physical interest are regular. But we have not been able to establish this property in general.

Proposition 3.4: *Let $|\cdot\rangle_\lambda$ be a pure particle weight. There exists a unique vector $p_\lambda \in \mathbb{R}^d \backslash \overline{V}_-$ such that the continuous unitary representation $x \to U_\lambda(x)$ of space-time translations given by*

$$U_\lambda(x)|L\rangle_\lambda \doteq e^{ip_\lambda x}|\alpha_x(L)\rangle_\lambda, \ L \in \mathcal{L}$$

has Lorentz invariant spectrum in $\overline{V}_+$. If $|\cdot\rangle_\lambda$ is regular, then $p_\lambda \in \overline{V}_+ \backslash \{0\}$.

The interpretation of this result is obvious: pure particle weights $|\cdot\rangle_\lambda$ have sharp energy-momentum p_λ. Appealing to the picture of a plane wave, the role of the operators $L \in \mathcal{L}$ becomes clear then: they localize the wave and thereby produce a normalizable state. Incidentally, this explains the terminology "left ideal of localizing operators". It is not excluded by our results that p_λ is spacelike. The corresponding "tachyonic" particle weights would be non-regular, but become positive energy states after localization.

Let us next discuss the behaviour of particle weights under Lorentz-transformations. Given a pure particle weight $|\cdot\rangle_\lambda$ and any Lorentz transformation Λ, we can define another particle weight $|\cdot\rangle_{\lambda,\Lambda}$, setting

$$|\cdot\rangle_{\lambda,\Lambda} \doteq |\alpha_\Lambda{}^{-1}(L)\rangle_\lambda, \ L \in \mathcal{L}. \tag{12}$$

One easily verifies that $|\cdot\rangle_{\lambda,\Lambda}$ has energy momentum Λp_λ. Restricting Λ to the little (stability) group $\mathcal{R}(p_\lambda)$ of p_λ one can thus explore the internal ("spin") degrees of freedom of $|\cdot\rangle_\lambda$. Here we restrict attention to the physically relevant case, where the number of these degrees of freedom is finite (finite particle multiplets).

Definition: A pure particle weight $|\cdot\rangle_\lambda$ is said to have finite spin if the space of sesquilinear forms on $\mathcal{L} \odot \mathcal{L}$ generated by ${}_{\lambda,\Lambda}\langle\cdot|\cdot\rangle_{\lambda,\Lambda}, \ \Lambda \in \mathcal{R}(p_\lambda)$ is finite dimensional.

Proposition 3.5: *Let $|\cdot\rangle_\lambda$ be a pure particle weight with finite spin and let $(\pi_\lambda, \mathcal{H}_\lambda)$ be the corresponding GNS-representation of $\mathcal{A}$. On $\mathcal{H}_\lambda$ there exists a continuous unitary projective representation $\Lambda \to V_\lambda(\Lambda)$ of the little group $\mathcal{R}(p_\lambda)$ such that*

$$V_\lambda(\Lambda)\pi_\lambda(A)V_\lambda(\Lambda)^{-1} = \pi_\lambda(\alpha_\Lambda(A)), \ A \in \mathcal{A}.$$

Moreover, the linear space of particle weights generated by $V_\lambda(\Lambda)^{-1}|\cdot\rangle_{\lambda,\Lambda}, \Lambda \in \mathcal{R}(p_\lambda)$ is finite dimensional.

Remark: We recall that there may not exist a representation of the full Lorentz group (infraparticle problem).

Let us discuss the consequences of this result for regular particle weights in physical space-time $d = 4$. There we have to distinguish two cases: if the mass-square p_λ^2 is positive, the little group of p_λ is isomorphic to O(3). Hence the projective representation V_λ can be lifted to a true representation of its covering group SU(2), and the particle weights can be decomposed into components of fixed (half) integer spin. So in this case we arrive at the same conclusions as Wigner. If $p_\lambda^2 = 0$ the situation is different, however. In this case the little group of p_λ is isomorphic to E(2), and it may well be that V_λ cannot be lifted to a true representation of its two-fold covering, if Lorentz boosts are spontaneously broken in the representation $(\pi_\lambda, \mathcal{H}_\lambda)$. The spin (helicity) of particle weights can still be defined in these cases, but it does not have to be (half) integer. Thinking of the spin-statistics theorem, this leaves open the possibility of massless infraparticles in physical space-time with unusual statistics.

There are further results which substantiate the heuristic expectation that particle weights describe the timelike asymptotic particle content of states (such as specific localization properties of the systems described by particle weights, local normality properties etc. [23]). But the results quoted so far may suffice for the motivation of the following.

Postulate: Particles appearing at asymptotic times in few body systems are described by (pure) particle weights $|\cdot\rangle$, mapping the left ideal $\mathcal{L}$ of localizing operators into some Hilbert space $\mathcal{H}$. The ensembles which can experimentally be prepared by the process of localizing particles are described by the range of $|\cdot\rangle$.

What have we gained by this point of view? On one hand a formalism which allows a unified treatment of particles, infraparticles, tachyons (should they exist), etc. Having solved this conceptual problem, it amounts to a well defined mathematical problem to determine the specific properties of these systems (Poincaré covariance versus spontaneous breaking of Lorentz boosts, negative mass squares, etc.). On the other hand, our point of view is the basis for a simple and general formula by which the asymptotic particle content can be extracted from any state ω,

$$\langle L|L\rangle = \text{"}\lim_t\text{"} \int d\boldsymbol{x}\, \omega(\alpha_{t,\boldsymbol{x}}(L^*L)),\ L \in \mathcal{L}. \tag{13}$$

(Here the quotation marks indicate the possible need for time averages and the transition to subnets.) It is a striking fact, first stated implicitly in [20] in a more restricted setting, that in this way the (charged) particle content of a theory can be uncovered by using only the local observables in the vacuum sector.

We conclude this discussion with a list of open problems and remarks.

i) When does a theory have a non-trivial asymptotic particle structure, i.e. under which conditions are the limits $\langle\cdot|\cdot\rangle$ in (13) non-zero? It would be of great interest to formulate these conditions in general terms (using the underlying net structure) since it would clarify the question of how the particle aspects are encoded in the local properties of a theory. It seems likely that the following two

conditions are the key ingredients: the condition of primitive causality [24], being the abstract expression of the existence of a dynamical law, and a condition of causal independence [25], now called split-property [26]. A heuristic argument to this effect has been given in [27], but it has not yet matured into a proof.

ii) When do the asymptotic particle configurations stabilize, i.e. under which conditions can the quotation marks in (13) be removed (existence of limits)? As is clear from quantum mechanical scattering theory [28], a positive answer will depend on decent "phase space properties" of the theory. In the present setting, this requirement can be expressed in terms of compactness or nuclearity conditions (cf. [29] for a comprehensive account and references).

iii) Is there a general collision theory based on particle weights? Some interesting progress on this problem has been made in collaboration with U. Stein. We have developed a novel algorithm which seems suitable for the construction of collision states of (infra-) particles of prescribed charge, spin, mass and momentum (possibly accompanied by an unspecified cloud of massless particles with arbitrarily small total energy), cf. [11]. It is based on the algebra $\mathcal{C} = \mathcal{L}^*\mathcal{L}$ and does not require the knowledge of charge carrying fields. It thereby fills the remaining gap in [20] in the direct construction of collision cross sections from observables in the vacuum sector. A proof that the method works has so far only been given in massive, asymptotically complete theories [30]. But the method can be directly applied (and tested) in any model.

iv) What is the statistics of particle weights? An approach to this problem could be based on an extension of the DHR-analysis [31] to superselection sectors describing non-localizable charges (such as the electric charge), and there are some promising ideas to this effect [32].

Another approach could look as follows: it seems possible to construct with the help of the operators $\mathcal{L} \in \mathcal{L}$ loops in the space of collision states, describing the exchange of two (or more) particles of the same kind. In order to determine the statistics of the particles one has to lift these loops to the space of collision state vectors. The appropriate lift seems to be the Berry lift, respectively its generalizations (cf. [33] and references quoted there), and the statistics would then be given by the corresponding monodromy operator (Berry phase). Some preliminary calculations indicate that this definition of statistics gives, in the case of localizable charges, the same answers as the definition of statistics in [31]. Another question, closely related to this problem is: When do there exist antiparticle weights?

v) When does a theory have a complete particle (weight) interpretation? A definite mathematical formulation of this question has not yet been accomplished. (Note that an unambiguous scattering matrix does not exist in the case of infraparticles.) But there is some closely related problem which is of interest in its own right: in high energy collision experiments only some very limited aspect of completeness is really tested, which primarily relies on the existence of conservation laws. One prepares, for example, (initial) states ω with relatively sharp energy momentum and checks whether the energy-momentum carried by the outgoing particles adds up to the same result. So there are two quantities

involved: the energy-momentum pumped into the state ω in the process of its preparation, which may be identified with $\omega(P)$, P being the energy-momentum operator. And the energy-momentum carried by the asymptotic particles, which according to Proposition 3.4 can be identified with the values p_λ corresponding to the particle weights $|\cdot\rangle_\lambda$ appearing in the asymptotic decomposition (11) of $|\cdot\rangle$. Hence, in accord with the experimental facts, there should hold

$$\omega(P) = \int d\mu(\lambda)\, p_\lambda, \tag{14}$$

where μ is the measure appearing in the decomposition (11),[6] and a similar relation should hold for the other conserved quantities (charge, spin, etc.) which can be attributed to particles. It seems that such a weak form of asymptotic completeness suffices for a consistent interpretation of most collision experiments. We conjecture that relation (14) holds in all theories admitting a local stress energy tensor.

4 Ultraparticles

First some word about terminology: the term infraparticle was coined by B. Schroer [4] as a catch phrase alluding to the infrared problems which arise in models with long range forces. In the preceding section we have seen how infraparticles (like particles) appear at large distances in states of moderate energy. Thus in this respect the term infraparticle also fits well. In the present section we are interested in the particles appearing at short distances in the limt of arbitrarily high energies. We call these particles *ultraparticles*. Some of them may be infraparticles as well, e.g. leptons, others do not show up at large scales, e.g. partons. The latter phenomenon (confinement) may be understood as an infrared problem. But the term ultraparticle seems more appropriate also in these cases since it specifies the domain where these particles are observed.

Within the perturbative treatment of field theory, a powerful tool for the analysis of the ultraviolet properties of models is based on scaling transformations of fields (renormalization group transformations [34]). They allow an interpretation of the theory at small scales and have led to fundamental concepts such as the notion of asymptotic freedom. Although well known, let us recall that the scaled fields are regarded as elements of another theory at the original scale with e.g. different (running) coupling constants. Phrased differently: the interpretation of the original theory at small scales is accomplished in terms of other theories, but at fixed scale. We emphasize this fact in order to make clear from the outset that the particle structures which are of interest here will (and can) not be described by states or weights on the given algebra of observables $\mathcal{A}$.

The utility of scaling transformations for the structural analysis in algebraic quantum field theory has first been realized by Roberts [35] in the special case

[6] An unambiguous definition of μ requires a normalization of the particle weights $|\cdot\rangle_\lambda$, a problem not touched upon here

of scale-invariant theories. The methods were later refined in [36] (cf. also [37] and [38]) to cover also the case of non-scale-invariant theories. These proposals contain, however, a conceptually unsatisfactory element since they depend on the existence of pointlike quantum fields. From the algebraic point of view such fields (if they exist) are usually interpreted as a kind of coordinatization of the local algebras. This raises then the questions: does the "principle of local stability", proposed in [36], depend on a specific choice of such coordinates? Do the results on the structure of local algebras in [37] depend on the existence of such coordinates? Moreover, such interesting questions as to when there exist quantum fields describing the ultraviolet properties of a net of observables, or what is the possible ultraviolet structure if there are no such fields are put aside from the outset.

In joint work with R. Verch, we have set up a framework which allows the systematic discussion of these problems [12], and we are presently investigating the ultraviolet properties of local nets with emphasis on the ultraparticle aspects. In the following an outline of the basic ideas in our approach is given, and some pertinent results with regard to the ultraparticle problem are discussed.

What is required in order to express the ideas of the renormalization group in the algebraic setting? One first needs a method to proceed from the local algebras $\mathcal{A}(\mathcal{O})$ at scale $\lambda = 1$, say, to algebras $\mathcal{A}_\lambda(\mathcal{O})$ describing the observations made at smaller scales $\lambda > 0$ in the regions $\lambda\mathcal{O}$, i.e. $\mathcal{A}_\lambda(\mathcal{O}) = \mathcal{A}(\lambda\mathcal{O})$.[7] There are many ways of introducing such maps: conditional expectations, canonical endomorphisms based on the Tomita-Takesaki theory, scaling transformations of underlying quantum fields, etc. Apart from an important physical constraint which will be discussed below, it should not really matter which map one takes since they yield the same nets (they merely reshuffle the operators in the local algebras in different ways). So why not consider them all?

A convenient way of expressing this idea is provided by the concept of a scaling algebra $\underline{\mathcal{A}}$ associated with any given local net $\mathcal{A}$. The elements $\underline{A}$ of $\underline{\mathcal{A}}$ are bounded functions of the scaling parameter $\lambda \in (0, \infty)$ with values in $\mathcal{A}$, i.e. $\underline{A}_\lambda \in \mathcal{A}, 0 < \lambda < \infty$. The algebraic operations in $\underline{\mathcal{A}}$ are pointwise defined by the corresponding operations in $\mathcal{A}$, and there is a C*-norm on $\underline{\mathcal{A}}$ given by

$$||\underline{A}|| = \sup_\lambda ||\underline{A}_\lambda||. \tag{15}$$

The local structure of $\mathcal{A}$ is lifted to $\underline{\mathcal{A}}$ by setting

$$\underline{\mathcal{A}}(\mathcal{O}) = \{\underline{A} \in \underline{\mathcal{A}} : \underline{A}_\lambda \in \mathcal{A}(\lambda\mathcal{O}), 0 < \lambda < \infty\}. \tag{16}$$

Hence $\mathcal{O} \to \underline{\mathcal{A}}(\mathcal{O})$ defines a net over Minkowski space, and it suffices for our purposes to assume that $\underline{\mathcal{A}}$ is the C*-inductive limit of the local algebras $\underline{\mathcal{A}}(\mathcal{O})$. It is easily verified that the net is local, i.e. $\underline{\mathcal{A}}(\mathcal{O}_1) \subset \underline{\mathcal{A}}(\mathcal{O}_2)'$ if $\mathcal{O}_1 \subset \mathcal{O}_2{}'$. Moreover, the action of the Poincaré group on $\mathcal{A}$ can be lifted to a covariant

[7] Note that the map $\mathcal{O} \to \mathcal{A}_\lambda(\mathcal{O})$ defines again a local net over Minkowski space with Poincaré-transformations given by $\alpha_x^{(\lambda)} \doteq \alpha_{\lambda x}$ and $\alpha_\Lambda^{(\lambda)} \doteq \alpha_\Lambda$

action on the net $\underline{\mathcal{A}}$, induced by automorphisms $\underline{\alpha}_x, \underline{\alpha}_\Lambda \in \mathrm{Aut}\underline{\mathcal{A}}$ given by

$$(\underline{\alpha}_x(\underline{A}))_\lambda \doteq \alpha_{\lambda x}(\underline{A}_\lambda),\ (\underline{\alpha}_\Lambda(\underline{A}))_\lambda \doteq \alpha_\Lambda(\underline{A}_\lambda). \tag{17}$$

Not quite unexpectedly, there appears an additional symmetry in this setting: dilations. This group acts by automorphisms $\underline{\delta}_\mu, 0 < \mu < \infty$, on the net, given by

$$(\underline{\delta}_\mu(\underline{A}))_\lambda \doteq \underline{A}_{\mu\lambda}. \tag{18}$$

This formalism nicely expresses the geometrical aspects of scaling transformations. But it is probably of little use as it stands since there is some important physical constraint missing: we want to change the length scale, but the other physical units should stay constant. The velocity of light, $c = 1$, has not been changed by our procedure since we have scaled space and time by the same factor λ. But what about Planck's constant? Many of the (only abstractly characterized) functions $\underline{A}$ will describe a situation, where simultaneously with length also Planck's constant is scaled in some uncontrolled manner. Thus we are not yet in a setting corresponding to the framework of the renormalization group, unless we manage to get rid of these unwanted elements.

In a first attempt to solve this problem, we have tried to isolate in $\underline{\mathcal{A}}$ some "Planck-ideal" which, by taking quotients, fixes the value of Planck's constant at each scale. But this idea did not work. The final solution of the problem turned out to be amazingly simple and consists of a direct characterization of the quotient of $\underline{\mathcal{A}}$, based on the follwing idea: since the product of the dimensions of length and momentum is equal to the dimension of action, we have to scale momentum by λ^{-1} if we scale length by λ and want to keep Planck's constant fixed. The energy-momentum scale can be set by determining the energy-momentum transferred by "standard operators" to physical states. Hence if $\underline{A}_1 \in \mathcal{A}$ is such an operator which at scale $\lambda = 1$ transfers energy momentum Γ, where Γ is some compact subset of $\mathbb{R}^d$, then its counterpart $\underline{A}_\lambda \in \mathcal{A}_\lambda$ at scale λ should transfer energy-momentum $\lambda^{-1}\Gamma$. Now the local operators $A \in \mathcal{A}$ can transfer any energy-momentum. But they are "quasi-local in momentum space" as a consequence of their continuity properties with respect to space-time translations. In fact, given $A \in \mathcal{A}$ and $\varepsilon > 0$ one can find some operator A_Γ with compact energy-momentum transfer Γ such that $||A - A_\Gamma|| < \varepsilon$.

After these remarks it is straightforward now to express the requirement that Planck's constant is kept fixed under scaling transformations: we have to restrict attention to functions $\underline{A}$ which, at every scale λ, can be approximated with uniform accuracy $\varepsilon > 0$ by operators with energy-momentum transfer $\lambda^{-1}\Gamma$, where Γ is some fixed compact set. Phrased in formula form: $\sup_\lambda ||\underline{A}_\lambda - \underline{A}_{\lambda,\lambda^{-1}\Gamma}|| < \varepsilon$. After a moments reflection one sees that this condition is equivalent to the requirement that $\sup_\lambda ||\alpha_{\lambda x}(\underline{A}_\lambda) - \underline{A}_\lambda|| \to 0$ as $x \to 0$, which simply means that the function $x \to \underline{\alpha}_x(\underline{A})$ is to be strongly continuous. Thus the condition "$\hbar = 1$ at all scales" amounts, in the framework of the scaling algebra $\underline{\mathcal{A}}$, to the condition of strong continuity under space-time translations.

The same considerations, as indicated above for the case of energy-momentum, apply of course also to angular momentum and lead to the requirement that

also the functions $\Lambda \to \underline{\alpha}_\Lambda(\underline{A})$ have to be strongly continuous. Hence we are led to the

Definition: Let $\mathcal{A}$ be any local net complying with the assumptions given in Sect. 2. The canonically associated scaling algebra $\underline{\mathcal{A}}$ is the C*-inductive limit of the local net $\mathcal{O} \to \underline{\mathcal{A}}(\mathcal{O})$ of functions $\underline{A}$ which are strongly continuous with respect to the action of the Poincaré group, cf. relations (16) and (17).
(We note that the concept of scaling algebra can also be applied to local nets on curved space-time [12].)

Remark: Within this setting the significance of the automorphisms $\underline{\delta}_\mu, 0 < \mu < \infty$ becomes clear: they induce scaling (renormalization group) transformations on $\underline{\mathcal{A}}$. In view of this fact it may seem natural to assume that $\underline{\delta}_\mu$ acts strongly continuously on $\underline{\mathcal{A}}$ (which amounts to further restrictions on the elements of $\underline{\mathcal{A}}$). It would, however, be too much to require continuity of the functions $\mu \to \underline{\delta}_\mu(\underline{A})$ at $\mu = 0$, as this would imply that n-$\lim_{\mu\to 0} \pi(\underline{A}_\mu) = const \cdot 1$ in all positive energy representations π of $\mathcal{A}$. The corresponding functions $\underline{A}$ would therefore not be suitable to explore the ultraviolet properties of a theory. In the following discussion we do not anticipate any continuity properties of the scaling transformations.

We next discuss how the structure of the states ω at small scales can be analyzed with the help of the scaling algebra. To this end we have to lift these states to $\underline{\mathcal{A}}$.

Definition: Let ω be any state on $\mathcal{A}$. Its canonical lift $\underline{\omega}$ to the scaling algebra $\underline{\mathcal{A}}$ is given by

$$\underline{\omega}(\underline{A}) \doteq \omega(\underline{A}_{\lambda=1}), \ \underline{A} \in \underline{\mathcal{A}}. \tag{19}$$

Before we proceed let us clarify the physical interpretation of the states $\underline{\omega}$ on $\underline{\mathcal{A}}$. In order to recover this interpretation, one has to regard $\underline{\omega}$ as a (projected) state on the quotient $\underline{\mathcal{A}}/ker\,\underline{\pi}$, where $\underline{\pi}$ is the GNS-representation induced by $\underline{\omega}$ and $ker\,\underline{\pi}$ the kernel of $\underline{\pi}$. To see this, note that if π is the GNS-representation induced by ω, the nets $\underline{\mathcal{A}}/ker\,\underline{\pi}$ and $\mathcal{A}/ker\,\pi$ ($= \mathcal{A}$ if π is faithful) are isomorphic[8], an isomorphism ϕ being given by $\phi(\underline{\pi}(\underline{A})) = \pi(\underline{A}_{\lambda=1}), \underline{A} \in \underline{\mathcal{A}}$. This isomorphism also intertwines the projected actions of the Poincaré group on the respective nets if the kernel of π, and hence that of $\underline{\pi}$, is Poincaré invariant. These properties characterize ϕ up to internal symmetries. Moreover, if $proj\,\underline{\omega}$ denotes the state obtained by projecting $\underline{\omega}$ to the quotient $\underline{\mathcal{A}}/ker\,\underline{\pi}$, there holds $proj\,\underline{\omega} \circ \phi^{-1} = \omega$. Thus the state $proj\,\underline{\omega}$ on the net $\underline{\mathcal{A}}/ker\,\underline{\pi}$ contains the same physical information as ω, provided there exist no internal symmetries of the algebra of observables $\mathcal{A}$. If there are such symmetries, one still knows that $proj\,\underline{\omega}$ describes some state on $\mathcal{A}$ which lies on the orbit of ω under the action of the internal symmetry group. Hence if ω is a vacuum state, then $proj\,\underline{\omega}$ also describes a vacuum state which coincides with ω, unless the internal symmetry group is spontaneously broken.

[8]Two nets are said to be isomorphic if there exists an isomorphism between the respective global algebras of observables, which maps the local algebras corresponding to the same regions $\mathcal{O}$ onto each other. Two such nets describe "the same physics"

This information suffices for our purposes.

In order to analyze the state ω at scale λ, we have to apply the scaling transformation $\underline{\delta}_\lambda$ to its canonical lift $\underline{\omega}$, giving the state $\underline{\omega} \circ \underline{\delta}_\lambda$ on $\underline{\mathcal{A}}$. If we proceed as above and consider the projected state $proj\, \underline{\omega} \circ \underline{\delta}_\lambda$ on the net $\underline{\mathcal{A}}/ker\, \underline{\pi}_\lambda$, where $\underline{\pi}_\lambda$ is the GNS-representation induced by $\underline{\omega}$, we recover the state ω on the net $\mathcal{A}_\lambda$, up to internal symmetry transformations. (The internal symmetry group is of course the same at each scale $\lambda > 0$.)

We are here interested in the structure of the physical states ω at scales $\lambda \to 0$, so we have to consider the corresponding sequence of states $\underline{\omega} \circ \underline{\delta}_\lambda$ in this limit. Here some technical nuisance appears since these sequences do not converge. To explain the simple origin of this phenomenon, let us consider the following toy model: we take as our algebra of observables the complex numbers, $\mathcal{A} = \mathbb{C}$. The corresponding scaling algebra $\underline{\mathcal{A}}$ is then the algebra of all bounded c-number functions $\underline{A} : (0, \infty) \to \mathbb{C}$ with norm $||\underline{A}|| = \sup_\lambda |\underline{A}_\lambda| < \infty$. The canonical lift of the (unique) state ω on $\mathcal{A}$ to the scaling algebra $\underline{\mathcal{A}}$ is given by $\underline{\omega}(\underline{A}) = \underline{A}_1$, and $\underline{\omega}(\underline{\delta}_\lambda(\underline{A})) = \underline{A}_\lambda$.

Since we may not assume from the outset that all functions $\underline{A}$ converge for $\lambda \to 0$ (cf. the previous remark) it is clear that the sequence $\underline{\omega} \circ \underline{\delta}_\lambda$ will not converge in this limit. The problem is that the scaling algebra provides us with an abundance of mappings from the algebra $\mathcal{A}$ at scale $\lambda = 1$ to the corresponding algebras $\mathcal{A}_\lambda$ ($= \mathcal{A}$) at other scales λ. (This was, after all, our initial idea.) To proceed, we make use of the fact that the set of states $\underline{\omega} \circ \underline{\delta}_\lambda$ is relatively compact in the weak *-topology induced by $\underline{\mathcal{A}}$. Thus there exist (many) convergent subnets for $\lambda \to 0$. In a sense, each convergent subnet corresponds to some particular choice of the above mentioned mappings.

Let us denote the limits of these subnets by $\underline{\omega}_{0,\iota}, \iota \in \mathbb{I}$ ($\mathbb{I}$ being some index set). Recalling the physical interpretation of states ω at scale λ in terms of $\underline{\omega} \circ \underline{\delta}_\lambda$, we consider the projected states $proj\underline{\omega}_{0,\iota}$ on $\underline{\mathcal{A}}/ker\underline{\pi}_{0,\iota}$, $\underline{\pi}_{0,\iota}$ being the GNS-representation induced by $\underline{\omega}_{0,\iota}$. It is evident that in our model $\underline{\mathcal{A}}/ker\underline{\pi}_{0,\iota}$ is isomorphic to $\mathcal{A}$. Thus the abundance of functionals $\underline{\omega}_{0,\iota}, \iota \in \mathbb{I}$ on $\underline{\mathcal{A}}$ describes a single "physical state", the state ω from which we started. (This is of course no surprise.)

After these preliminary remarks the reader will guess how we proceed: given a state ω on $\mathcal{A}$, we consider the corresponding family of states $\underline{\omega} \circ \underline{\delta}_\lambda$ on $\underline{\mathcal{A}}$ and its weak *-limit points for $\lambda \to 0$: $\underline{\omega}_{0,\iota}, \iota \in \mathbb{I}$. We then propose as

Working hypothesis: Let ω be a state on $\mathcal{A}$. The limit points $\underline{\omega}_{0,\iota}, \iota \in \mathbb{I}$ of $\underline{\omega} \circ \underline{\delta}_\lambda$ for $\lambda \to 0$, projected on the respective nets $\underline{\mathcal{A}}/ker\underline{\pi}_{0,\iota}$, ($\underline{\pi}_{0,\iota}$ being the GNS representation induced by $\underline{\omega}_{0,\iota}$), describe the structure of the state ω in the scaling limit $\lambda \to 0$ (possibly modulo internal symmetry transformations).

If the nets $\underline{\mathcal{A}}/ker\underline{\pi}_{0,\iota}$ in this statement are all isomorphic, one can interpret the asymptotic structure of ω in terms of a single theory. We then say that the original theory has a unique scaling limit. (A justification for this terminology will be given below.) But this does not mean that the states $proj\underline{\omega}_{0,\iota}$ all coincide, there may appear new "degrees of freedom" in the scaling limit. The second

possibility is that not all of the nets $\underline{\mathcal{A}}/ker\underline{\pi}_{0,\iota}$ are isomorphic. It means that the theory has no fixed interpretation in the scaling limit, it looks different at different scales.

After this lengthy preparation of our tools, let us turn now to the actual analysis of the scaling limits. To this end we first have to fix the states which we want to consider. Being interested in states in high-energy physics, we pick the set $\mathcal{S}_0$ of all states which are locally normal [39] with respect to a given pure Poincaré invariant vacuum state ω_0 on $\mathcal{A}$. This set of states is actually quite big, it does not only contain all finite energy excitations of ω_0, but in general also thermal states, etc. Our first result says that it does not matter which state $\omega \in \mathcal{S}_0$ we consider, they all describe the same structure in the scaling limit. In fact there holds a stronger result.

Lemma 4.1: *Let $\omega_1, \omega_2 \in \mathcal{S}_0$. Then*

$$\lim_{\lambda \to 0} (\underline{\omega}_1 \circ \underline{\delta}_\lambda - \underline{\omega}_2 \circ \underline{\delta}_\lambda)(\underline{A}) = 0, \ \underline{A} \in \underline{\mathcal{A}}.$$

In simple terms: all states $\omega \in \mathcal{S}_0$ look alike at small scales. (This justifies our terminology concerning the uniqueness of the scaling limit of a theory.) But how do they look like? Here is the answer for $d > 2$ dimensions:

Proposition 4.2: *Let $\omega \in \mathcal{S}_0$. Then each weak *-limit point $\underline{\omega}_{0,\iota}$ of $\underline{\omega} \circ \underline{\delta}_\lambda$, $\lambda \to 0$, is a pure Poincaré invariant vacuum state on $\underline{\mathcal{A}}$. (Thus it is also a pure, Poincaré invariant vacuum state if projected to the net $\underline{\mathcal{A}}/ker\underline{\pi}_{0,\iota}$, since the kernel of the GNS-representation $\underline{\pi}_{0,\iota}$, induced by $\underline{\omega}_{0,\iota}$, is Poincaré invariant.)*

This result is the key to a general understanding of the scaling limits of quantum field theories. Let us first indicate how it can be used for their classification. There are the following three possibilities.

1. The theory has a *classical* scaling limit, i.e. $\underline{\mathcal{A}}/ker\underline{\pi}_{0,\iota} \simeq \mathbb{C}$ for $\iota \in \mathbb{I}$. A trivial example of this kind is obtained if one (re)defines the algebras $\mathcal{A}(\mathcal{O}) \subset \mathcal{A}$ to be equal to $\mathbb{C} \cdot 1$ for small regions $\mathcal{O}$. This artificial possibility is not what we here have in mind (and it could be excluded by a condition of additivity on the net $\mathcal{A}$). But our structural result seems to indicate that there may well exist theories which become classical in the scaling limit. Examples of this kind could be some of the models envisaged by J. Klauder, cf. these Proceedings. We note that under these circumstances the "classical ideal" in $\underline{\mathcal{A}}$ (generated by all commutators) is annihilated in the scaling limit, i.e. it is a proper ideal of $\underline{\mathcal{A}}$.

2. The theory has a unique *quantum* scaling limit, i.e. the (isomorphic) nets $\underline{\mathcal{A}}/ker\underline{\pi}_{0,\iota}, \iota \in \mathbb{I}$ are non-abelian. This case corresponds to theories which have (in the language of the theory of the renormalization group) an ultraviolet fixed point. Of particular interest are the asymptotically free theories. According to the folklore, in such theories the underlying fields have almost canonical dimensions (apart from logarithmic corrections) and become free massless fields in the scaling limit. We therefore propose to characterize such theories in the algebraic setting by the condition that (for even dimensions d) the "Huygens-ideal" in $\underline{\mathcal{A}}$ (generated

by all commutators of pairs of operators which are localized in strictly timelike separated regions) is annihilated in this limit.

3. The theory has no scaling limit, i.e. not all of the nets $\underline{\mathcal{A}}/ker\underline{\pi}_{0,\iota}, \iota \in \mathbb{I}$ are isomorphic.[9]

We mention as an aside that, as a consequence of Proposition 4.2 and a result in [37], the weak closures of the local algebras $\mathcal{A}(\mathcal{O})$ (with respect to the folium of states $\mathcal{S}_0$) are of type III_1 for all double cones $\mathcal{O}$, if the underlying theory does not have a classical scaling limit.

Let us turn now to the discussion of ultraparticles, the actual subject of this section. So where are they? Proposition 4.2 provides the tools to answer also this question: the ultraparticles are the particles (in the sense of the previous section) of the theory in the scaling limit. This characterization is meaningful, since in the scaling limit we are still in the framework of positive energy (vacuum) representations of some local net, and the methods of the preceding section can be applied. But even more important: this characterization complies with the physical picture which led to the successful interpretation of experimental data. We emphasize that in our analysis the "ultraparticles" are not put in by hand, they appear naturally as a possible structure within the general framework of local quantum field theory. This fact leads us to propose as a

Postulate: The ultraparticle content of all states in a theory which look locally like few body systems (states of finite energy) is described in terms of the particle content of the vacuum representations of the scaling limit theory, cf. Proposition 4.2 and Sect.3.

We refrain from giving here a survey of our partial results on the structure of ultraparticles. But we would like to point out one interesting aspect of our analysis: although we started from local (gauge-invariant) observables, we expect to recover in the scaling limit the full gauge-group of the underlying theory, including the *local gauge group*. The structure of the full gauge group is expected to be encoded in the global gauge group of the scaling limit theory since there is no difference between global and local symmetry transformations at a fixed space-time point. Since the global gauge group of any theory can be reconstructed from the net of local observables in a systematic manner [42], we have thus found a promising approach to identify gauge quantum field theories, starting from local observables.

We conclude this section with an extract from the long list of intriguing question in this new setting of algebraic quantum field theory.

i) What is the ultraparticle content of a given *state*? So far we have only described the ultraparticle content of a theory (by looking at the particle content of all states in the vacuum sector of the scaling limit theory). For the analysis of

[9] It may very well be that "physics" behaves like this scenario. Then the hope that one may guess a "grand unified theory" which describes physics at all scales would probably be an illusion. But this is not a real problem. After all, high energy experiments can only be carried out up to the scale λ_{budget}, and it should therefore be possible to understand the prevailing data in terms of the simpler cases 2 or 1

the specific ultraparticle structure of a given state ω one can probably proceed as in Sect. 3. One first distinguishes some left ideal $\underline{\tilde{\mathcal{L}}} \subset \underline{\mathcal{A}}$ which annihilates all vacuum states on $\underline{\mathcal{A}}$. But now the operators in $\underline{\tilde{\mathcal{L}}}$ have to localize the ultraparticles in momentum space (they have to cut off large momenta). In complete analogy to the discussion in Sect. 3 one could then try to extract the ultraparticle content of ω from the leading contributions in $\underline{\omega} \circ \underline{\delta}_\lambda(\underline{\tilde{L}}^* \underline{\tilde{L}})$, $\underline{\tilde{L}} \in \underline{\tilde{\mathcal{L}}}$, for small λ. A further step would be the formulation of a collision theory for ultraparticles.

ii) When is a theory asymptotically free? We have characterized asymptotically free theories in an ad hoc way by adopting the folklore and converting its algebraic aspects into a condition which fits naturally into our framework. But it would be desirable to establish this condition in an intrinsic way. A possible approach (in massive theories like QCD) could be based on a study of the assumption that the S-matrices S_λ of the theory at scale λ tend to unity in the scaling limit $\lambda \to 0$.

iii) What is the field content of a theory? The scaling algebra seems to be the adequate instrument to analyze also this question and to explore for example the interesting ideas outlined in [40].

iv) What is the structure of the theory at large scales $\lambda \to \infty$? The general concept of scaling algebra may also be relevant for a study of other "scaling limits" such as $c \to \infty$ or $\hbar \to 0$.

5 Thermoparticles

Let us turn now to the analysis of the particle-like constituents of thermal equilibrium states. As will become clear, these entities have to be distinguished from quasiparticles, describing the collective excitations of thermal states, and also from the other particle structures analyzed so far. We therefore propose to call them *thermoparticles*.[10]

How does one determine the thermoparticle content of a given KMS-state ω on $\mathcal{A}$? A complete answer to this question is not yet known, but for the special case of spatially homogenous KMS-states some promising results have been obtained in an investigation with J. Bros [13], cf. also [41]. In this approach we proceed again from the heuristic idea that thermoparticles, like other particles, are almost point-like objects. It should therefore be meaningful to consider the situation, where such a particle is placed in an equilibrium state into some region, about a space-time point x, whose size is small compared to the mean volume occupied by the other constituents. (Phrased differently: the de Broglie wavelength of the particle should be small compared to its mean free path.) This particle will either collide with constituents of the state and thereby disappear in the thermal background. Or it will manage to propagate to another space-time point y without intermediate collisions. The latter events should give rise to particle-like correlations between operations performed in almost point-like regions due to the

[10] Our terminology does not intend to suggest that for a description of these particles one has to rely on the framework of "thermofield theory"

creation and annihilation of a single particle, respectively hole. The probability for such events may be small because of the effects of the thermal background. Nevertheless, the correlation functions of point-like localized operators may be expected to contain the desired information about thermoparticles in a clear-cut form.

Let us translate now these heuristic ideas into our mathematical setting. In order to simplify the discussion we assume, for the time being, that the constituents of the underlying equilibrium state ω are "neutral" in the sense that they can be created by local operations described by certain specific operators $A \in \mathcal{A}$. From the preceding discussion it is clear that most operators in $\mathcal{A}$ are not suitable for our purposes, since they describe operations in finite space-time regions $\mathcal{O}$ which in general will contain many constituents. The resulting excitations of ω may have a quasiparticle structure, but it seems hopeless to extract from them some relevant information about the thermoparticles. Since we do not know from the outset how small $\mathcal{O}$ has to be chosen in order to avoid this problem, it seems appropriate to proceed to the idealization, where the localization region of the underlying operators $A \in \mathcal{A}$ is contracted to a point. Thus, as in the preceding section, we are led to consider sequences of local operators $A_\lambda \in \mathcal{A}(\lambda\mathcal{O})$, where λ tends to 0. But now we want to use these operators to create particle-like excitations of the equilibrium state which can be observed at the original scale $\lambda = 1$.

As explained before, we expect that information about the thermoparticles can be extracted from the correlation functions of the operators A_λ. Assuming that ω is invariant under space-time translations, these functions can be represented in the form

$$\omega(\alpha_y(A_\lambda)^* \alpha_x(A_\lambda)) = \int d\mu_\lambda(p)\, e^{ip(x-y)}, \tag{20}$$

where μ_λ is some positive measure. The dominant contribution to this measure will in general be a discrete part at $p = 0$, corresponding to those events where the operation described by A_λ has no effect on the state ω. This uninteresting part has to be removed from μ_λ (e.g. by subtracting from A_λ its expectation value $\omega(A_\lambda)$ if ω describes a pure phase), and we assume in the following that this has been done. The remaining part of the measure describes the correlations between well-localized excitations of ω.

In order to extract from μ_λ the contributions due to the propagation of a single thermoparticle, we have to proceed to the limit $\lambda \to 0$. Here some further preparations are necessary. On one hand, the energy-momentum content of the excitations produced by A_λ will become infinite in the limit $\lambda \to 0$. As in the case of quantum fields, this effect can be controlled by taking suitable space-time averages of the operators $\alpha_x(A_\lambda)$, i.e. the respective limit has to be understood in the sense of distributions. On the other hand, the probability that the operators A_λ create some excitation of ω will tend to 0 for $\lambda \to 0$ if the norms of these operators are uniformly bounded. In order to compensate the latter effect, the measure μ_λ has to be renormalized. This is accomplished by picking any $N \in \mathbb{N}$

and fixing a corresponding normalization constant

$$Z_\lambda = \int d\mu_\lambda(p)(1+|p|)^{-N},\ \lambda > 0. \tag{21}$$

The resulting renormalized correlation functions

$$W_\lambda(x) = {Z_\lambda}^{-1} \int d\mu_\lambda(p)\, e^{ipx},\ \lambda > 0, \tag{22}$$

regarded as elements of the space of tempered distributions, form an equicontinuous family. Thus there exist subsequences $\lambda_\iota \to 0$ such that the corresponding functions W_{λ_ι} converge in the sense of distributions,

$$\lim_\iota W_{\lambda_\iota}(x) = W(x). \tag{23}$$

(In order not to overburden the notation, we do not indicate the possible dependence of the limit W on the choice of the subsequence or the number N in the normalization constant.) Making use of the KMS-condition, one finds that also the permuted functions $W^\pi_\lambda(-x) = {Z_\lambda}^{-1}\omega(\alpha_x(A_\lambda)A^*_\lambda)$ converge for the respective subsequences λ_ι,

$$\lim_\iota W^\pi_{\lambda_\iota}(-x) = W^\pi(-x). \tag{24}$$

By performing this procedure for all possible sequences of operators A_λ and numbers N one arrives at a collection of tempered distributions W, W^π which describe the correlations between arbitrary point-like operations in the underlying KMS-state ω. These distributions can be shown to have the following general properties:
i) *Locality:* $W(x) = W^\pi(-x)$ for spacelike x.
ii) *KMS-condition:* Denoting by t the translations along the time axis fixed by the rest system of the underlying heat bath at temperature $\beta^{-1} > 0$, the distribution-valued function $t \to W(x+t)$ has an analytic continuation into the strip $0 < \mathrm{Im}t < \beta$, and its boundary value at the upper rim of this strip is given by $\lim_{t \nearrow \beta} W(x+it) = W^\pi(-x)$.
iii) *Cluster-property:* $\lim_{t\to\pm\infty} W(x+t) = 0$.

It turns out that there exists a universal integral representation for these distributiones [41]. We give here the result for the case where the commutator function $C(x) = W(x) - W^\pi(-x)$ is antisymmetric in the time variable. (The general case can be reduced to this one.)

Proposition 5.1: *Let W be a tempered distribution with properties specified above. There holds*

$$W(x) = \int_0^\infty dm\, D(\boldsymbol{x}; m) W^{(0)}(x; m), \tag{25}$$

where $D(\boldsymbol{x}; m)$ is a distribution with respect to $\boldsymbol{x}, m$ and $W^{(0)}(x; m)$ is the correlation function of a non-interacting particle of mass m in a thermal equilibrium state at temperature β^{-1},

$$W^{(0)}(x; m) = const \int dp\, \varepsilon(p_0)\delta(p^2 - m^2)\,(1 - e^{-\beta p_0})^{-1} e^{ipx}.$$

This result provides us with the desired tool to uncover the thermoparticle content of equilibrium states: the individual terms $D(\boldsymbol{x};m)W^{(0)}(x;m)$ in (25) describe the propagation of an excitation of mass m which has been created at the origin. Here the free correlation function $W^{(0)}(x;m)$ corresponds to the unperturbed motion; it is in general modified by a "damping factor" $D(\boldsymbol{x};m)$ which subsumes the loss of probability of finding the excitation at other space-time points due to intermediate collisions. In the case of the stable constituents of thermal equilibrium states, one would expect that such collisions are the only mechanism by which these probabilities are reduced, in contrast to excitations such as metastable bound states, pairs and other composite systems, which disintegrate also without external impact. We therefore conjecture that thermoparticles contribute to the discrete (atomic) part of the integral representations (25), whereas all other excitations contribute to the continuous part. The contributions due to thermoparticles can thus be uncovered by inverting relation (25), cf. [13].

If the underlying thermoparticles are not "neutral", i.e. cannot be created and annihilated by operators $A \in \mathcal{A}$, one has to proceed from the algebra of observables to a corresponding field algebra $\mathcal{F}$ of charge-carrying fields. In the case of localizable charges, this field-algebra is still generated by a net $\mathcal{O} \to \mathcal{F}(\mathcal{O})$ of local field algebras [42], and the thermal equilibrium states can be extended to this net [43]. One can then proceed in the same way as in the preceding discussion and construct a family of correlation functions W, describing the effects of point-like localized operations induced by fields in $\mathcal{F}$. Again there hold integral representations for the distributions W of the kind given in relation (25), and the thermoparticles should manifest themselves by discrete contributions in these representations. This leads us to propose as a

Working hypothesis: Thermoparticles, appearing as constituents of a thermal equilibrium state and carrying a localizable charge, manifest themselves by discrete contributions in the integral representations of the respective correlation functions of point-like observables, respectively fields, cf. relations (23) and (25).

In contrast to other proposals [10], our approach provides a characterization of thermoparticles in unambiguous mathematical terms. But there remain several open problems, and we conclude this section by listing some of them.

i) What are the properties of thermoparticles? We have identified thermoparticles by the discrete contributions appearing in the integral decomposition of the correlation functions W. In this way we are able to specify their mass m and the corresponding "damping factors" $D(\boldsymbol{x};m)$ which describe the dissipative effects of the thermal background. Since these factors are responsible for the modification of the dispersion law of thermoparticles, it would be desirable to gather information about their specific properties. For example, if the correlation functions satisfy the relativistic KMS-condition proposed in [44], the damping factors $D(\boldsymbol{x};m)$ are locally analytic in $\boldsymbol{x}$, which in turn implies that the energy-momentum content of the corresponding thermoparticles is concentrated about the mass shell $p^2 = m^2$. Another problem of interest is the determination of the internal degrees

of freedom of thermoparticles.

ii) How does one identify thermoparticles carrying non-localizable charges? A solution of this problem does not seem possible along the lines indicated in this section, since such particles cannot be created by almost point-like operations in view of the long-range fields which they carry along. Instead one should try to localize the theromoparticles already present in the state with the help of suitable localizing operators, in analogy to the strategy outlined in Sect. 3. By proceeding to point-like localizing operators, one may then hope to extract the thermoparticle content from the resulting correlation functions.

iii) How does one determine the thermoparticle content of spatially non-homogenous states? The basic strategy should be the same as for homogenous states, but there arise additional mathematical complications in the construction and analysis of the correlation functions W.

iv) When does a theory describe thermoparticles? A general analysis of this problem would require the following steps: first one has to understand under which conditions the limits W of the correlation functions are non-trivial. This question is closely related to the existence problem of point-like localized quantum fields. In a second step one would then have to clarify when there appear discrete contributions in the respective integral representations of the distributions W. Here the conditions of primitive causality and of causal independence, mentioned in Sect. 3 in a similar context, should again be of relevance.

6 Concluding ruminations

The topic of the present article has been a conceptual problem in quantum field theory: the notion of particle. We have seen that, depending on the physical situation under consideration, the description of particles leads to specific idealizations which call for their own mathematical framework (particle weights, scaling algebras, etc.).

The clarification of physical concepts requires a model independent setting, and algebraic quantum field theory has proven to be a very useful tool in this respect. As the motto of this workshop is an invitation to contemplate about the achievements and perspectives of our subjects, we conclude this article with some further remarks and views about the value of model independent investigations for the development and understanding of relativistic quantum field theory.

The idea that one may gain some insight into physics by studying a mathematical setting (in contrast to a specific model) is sometimes regarded with suspicion. It is argued that a general framework, such as that of algebraic quantum field theory outlined in Sect. 2, contains too little specific information and can therefore only yield "soft" results which are applicable to all possible theories, no matter how weird.

The point missed here is the fact that a general framework is the appropriate arena for the *classification* of theories, and detailed information can frequently be obtained for the resulting special cases. Let us mention as a prominent example

the analysis of Doplicher, Haag and Roberts, which led to a complete classification of the possible statistics of states carrying a localizable charge and the clarification of the relation between the type of statistics and the structure of the underlying global gauge group.

The problem of classifying theories often leads also to new and powerful *concepts*, an example being the "split property", first conceived by Borchers, which proved to be useful not only in the structural analysis but also in the investigation of concrete models.

A model independent analysis is furthermore the basis for the development of general *algorithms*, well known examples being the Haag-Ruelle collision theory and the Lehmann-Symanzik-Zimmermann reduction formulas for the calculation of S-matrix elements. It also leads to the formulation of *criteria* which characterize physically significant properties of a theory. For example, the analysis of the possible range of the local degrees of freedom of quantum field theories, initiated by Haag and Swieca, led to compactness and nuclearity conditions which turned out to be a key e.g. to the understanding of thermodynamical properties of quantum field theories.

Finally, the results of a model independent analysis may point to the existence of interesting *new structures* which are compatible with the general principles, but remained undiscovered in the realm of models. An example of this kind are theories of massive particles carrying a quantum topological charge. Their specific properties were uncovered by Fredenhagen and the present author in a general analysis, which subsequently triggered a (successful) search for models. (Cf. [15] for more detailed information on the various results mentioned.)

As may be seen from these remarks, it is one of the principle and most interesting aims of algebraic quantum field theory to amend the basic postulates by further conditions, resulting either from a structural analysis or from physical considerations. The strategy is thus to tighten the framework and thereby to obtain more detailed information about the properties of physically significant theories. One may even hope that this strategy leads to new methods for the actual construction of models. Some interesting results in this direction have recently been obtained by Wiesbrock [45], who proved that conformal theories on the light ray can be constructed from special inclusions of two von Neumann algebras.

Another objection raised against the general framework of (algebraic) quantum field theory is the view that it may be too restrictive: all attempts to construct a mathematically consistent and physically significant quantum field theory fitting into this framework have failed so far, and it may be necessary to include "gravitation" in order to cure the agonizing ultraviolet problems. (The reader may recognize this view in some of the contributions to these proceedings.)

It is difficult to argue against this kind of critique if one cannot exhibit a counter example. But we dare to recall that the gravitational constant is small and that the development of physics has shown that "orders of magnitude" provide a natural grading of phenomena. It would therefore be a surprise if the theoretical understanding of present day high energy physics (and may be that

of the next century) could not be based in a consistent way on the idealizations of special relativity. At least, there is no indication that there exists some basic obstruction to this effect. The present lack of relevant examples seems to have a much simpler explanation: it is a difficult task to construct them.

In order to get an idea about the degree of difficulties it may be instructive to compare the progress in quantum field theory with the development of quantum mechanics. It took 30 years from Planck's first observations till von Neumann's final rigorous formulation of the theory. Vital intermediate steps were taken by Einstein, Bohr, de Broglie and, finally, by Heisenberg, respectively Schrödinger. So, roughly speaking, the time scale for progress in quantum mechanics was 6 years.

The corresponding scale in quantum field theory seems to be about 20 years: quantum field theory was already conceived by the fathers of quantum mechanics. The next vital step (covariant perturbation theory and renormalization) was taken 20 years later by Tomonaga, Schwinger, Feynman and Dyson and led to the consolidation of quantum electrodynamics. It then took more than 20 years until the systematic study of the ultraviolet properties of quantum field theories led Gross and Wilczek, Politzer, Symanzik and others to the fundamental concept of asymptotic freedom, which is now a basic ingredient in the treatment of gauge theories, such as quantum chromodynamics. According to this time scale some further important progress in quantum field theory seems to be due, but one should not expect that it will lead to its ultimate consolidation. It may take another 20 or 40 years until quantum field theory reaches a form which also pleases mathematical physicists. So many of us may not be able to appreciate it.

It is perhaps this unpleasant perspective which makes us susceptible to all kinds of unconventional ideas which offer an apparently quicker solution. But the solution of a difficult problem needs time, sometimes much time, and continuous efforts (compare mathematics). It would therefore be regrettable if the beauties of physics in 2 and 10 dimensions let us forget the inherent possibilities of the more conventional approach and distract courageous people from pushing this problem towards its solution.

But what if the lord is malicious and has enticed us to explore for more than six decades some cul-de-sac? Probably the only way to find him out would be a structural analysis which either reveals the internal inconsistency of our present physical ideas or an unavoidable clash between experimental facts and theoretical predictions. Thus the model independent analysis of the "basic principles" is a vital part of the development of the theory and one may therefore expect that algebraic quantum field theory will continue to play its part in the consolidation of relativistic quantum field theory also in the 21st century.

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