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W. Bernreuther

Deutsches Elektronen-Synchrotron DESY, Hamburg

T. Schröder

Institut für Theoretische Physik, Universität Heidelberg

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Strongly Interacting Higgs Sector and W-Pair Production in e^+e^- Collisions

Werner Bernreuther[a] and Thomas Schröder[b1]

[a]Deutsches Elektronen-Synchrotron DESY, Hamburg
[b]Institut für Theoretische Physik, Universität Heidelberg

Abstract:
Within the strongly-interacting electroweak symmetry breaking scenario (SIH) we investigate a few observables which are sensitive to SIH effects in $e^+e^- \to W^+W^-$ at TeV collision energies.

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1 Introduction

The origin of electroweak symmetry breaking (SSB) constitutes one of the biggest mysteries of elementary particle physics. More specifically the question whether the scalar $SU(2)$ field Φ which is used in the standard electroweak theory (SM) to describe SSB is associated with an elementary spin-zero particle or whether Φ is a phenomenological order parameter (similar to the one of the Ginzburg-Landau model of superconductivity) may eventually be resolved only by experiment.

Another dichotomy, not necessarily the same as the one above, is the "light versus heavy Higgs boson" scenario. Investigations which explored the "triviality" of the Φ^4–type symmetry-breaking Lagrangean of the SM [1] suggest a weakly coupled Higgs boson with mass well below 1 TeV. Superymmetric extensions of the SM also predict weakly coupled spin-zero Higgs particles with masses in the hundred GeV range. Nevertheless, the alternative with no light Higgs boson (i.e., with a mass of, say, below 1 TeV) and, as a consequence, strongly coupled longitudinal gauge bosons – strongly interacting Higgs sector (SIH) for short – is also conceivable and has been extensively discussed for many years[2]. A prominent suggestion [2] of reactions where SIH effects may be detected is weak gauge boson fusion in multi-TeV proton-proton collisions. In the sequel, gauge boson fusion in multi-TeV e^+e^- collisions was also considered [3]. Another possiblity to search for signatures of an SIH sector is final-state rescattering in $e^+e^- \to W^+W^-$ in the TeV range [4, 5, 6].

Motivated by recent studies of the physics possibilities of high luminosity and high energetic e^+e^- linear colliders [7, 8] we add to previous investigations [4, 5, 6, 9, 10] in this paper a study of some observables which are sensitive to SIH effects in $e^+e^- \to W^+W^-$ at TeV c. m. energies. In sect.2 we outline how to model the amplitude for $e^+e^- \to W_L^+W_L^-$ in the case of a strongly-interacting symmetry breaking sector. In sect.3 we collect the results of several unitarization schemes [11, 12, 13] for the elastic longitudinal gauge boson scattering amplitude assuming that the symmetry-breaking Lagrangean has a global $SU(2) \times SU(2)$ symmetry broken down to a custodial $SU_V(2)$ [14]. In sect.4 we present our results for observables which are sensitive to SIH effects on the real and/or absorptive of the $e^+e^- \to W_L^+W_L^-$ scattering amplitude.

2 SIH effects in $e^+e^- \to W_L^+W_L^-$

In the following we consider the reaction

$$e^+(\boldsymbol{p}_+)\, e^-(\boldsymbol{p}_-) \to W^+(\boldsymbol{k}_+)\, W^-(\boldsymbol{k}_-) \tag{1}$$

far above threshold. The variables in bold face denote the three-momenta of the particles in the c.m. frame. Within the standard electroweak theory the scattering amplitude for (1) is given

[2]A Lagrangean model which is phenomenologically satisfactory is however lacking so far.

to Born approximation by the s-channel photon and Z-boson and t-channel neutrino exchange diagrams. It is useful to define helicity amplitudes,

$$\mathcal{M}(\omega_-,\omega_+;\eta_-,\eta_+) \;=\; \langle W^-(\omega_-)W^+(\omega_+)|\mathcal{T}|e^-(\eta_-)e^+(\eta_+)\rangle, \tag{2}$$

where $\omega_\pm = -1, 0, +1$ and $\eta_\pm = -\frac{1}{2}, +\frac{1}{2}$ are the helicities of $W^\pm$ and $e^\pm$, respectively.

In the high energy limit $s \gg m_W^2$ the dominant amplitudes correspond to transversely $(W_T^+W_T^-)$ and longitudinally $(W_L^+W_L^-)$ polarized final states. To Born approximation these amplitudes are given in the kinematical region $s, |t|, |u| >> m_W^2$, up to terms of order $m_W/\sqrt{s}$, by

$$\begin{aligned}
\mathcal{M}_0\left(-1,+1;-\frac{1}{2},+\frac{1}{2}\right) &= -\frac{e^2}{2\sin^2\theta_W}\frac{1+\cos\vartheta_W}{1-\cos\vartheta_W}\sin\vartheta_W e^{-i\varphi_W}\\
\mathcal{M}_0\left(+1,-1;-\frac{1}{2},+\frac{1}{2}\right) &= \frac{e^2}{2\sin^2\theta_W}\sin\vartheta_W e^{-i\varphi_W}\\
\mathcal{M}_0\left(0,0;-\frac{1}{2},+\frac{1}{2}\right) &= \frac{e^2}{4\sin^2\theta_W\cos^2\theta_W}\sin\vartheta_W e^{-i\varphi_W}\\
\mathcal{M}_0\left(0,0;+\frac{1}{2},-\frac{1}{2}\right) &= -\frac{e^2}{2\cos^2\theta_W}\sin\vartheta_W e^{+i\varphi_W}.
\end{aligned} \tag{3}$$

Here $\cos\vartheta_W = \hat{\boldsymbol{p}}_-\cdot\hat{\boldsymbol{k}}_-$ is the cosine of the scattering angle whereas θ_W denotes the weak mixing angle. The factors $\sin\vartheta_W\exp(\pm i\varphi_W)$ are associated with the angular momentum projections $m = \pm 1$ onto the e^- beam direction. Note that the phase φ_W is physically irrelevant if one considers unpolarized e^+e^- beams.
The first amplitude in (3) which corresponds to the production of transversely polarized $W^\pm$ pairs has a strong forward peak due to the t channel neutrino exchange. Therefore cuts around the beam axis mainly reduce the number of produced transverse $W^\pm$ bosons.

Only the amplitudes for $W_L^+W_L^-$ production will be significantly modified if a strongly interacting Higgs sector exists. Within this scenario a model for these amplitudes may be obtained as follows (cf. also [5, 4]): First we decompose the helicity amplitudes for $e^+e^- \to W_L^+W_L^-$ into partial waves:

$$\mathcal{M}(0,0;\pm 1/2,\mp 1/2) = \sum_{l\geq 1} f_{l,\pm 1}Y_{l,\pm 1}(\vartheta_W,\varphi_W) \tag{4}$$

where $\mathcal{M}$ are the full amplitudes[3] of (2).
Then we explore the consequences of unitarity, $\mathcal{T}-\mathcal{T}^+ = i\mathcal{T}^+\mathcal{T}$. Assuming the particle content of the SM (but no Higgs boson with mass below, say, 800 GeV) with the top quark being the

[3]Since helicity is conserved in the high energy limit and the e^+ and e^- couplings are chirality conserving the angular momentum projection onto the beam axis is restricted to $m = \pm 1$.

heaviest fermion, only longitudinal W and Z bosons are relevant in the intermediate state. As we are interested in c.m. energies up to 2 TeV we can restrict ourselves to two-particle states since the contribution of the four[4] (or more) gauge boson intermediate state to the unitarity equation is phase-space suppressed.

We shall argue below that only the p-wave contribution matters for which Z_LZ_L is forbidden by Bose symmetry. Therefore we keep only the $W_L^+W_L^-$ intermediate state (see Fig.1). The unitarity equation then reads in terms of partial wave amplitudes

$$\mathrm{Im}[f_{l,\pm 1}(s)] \;=\; f_{l,\pm 1}(s)\cdot a_l^*(s) \;\; (l\geq 1). \tag{5}$$

where $a_l(s)$ are the partial wave amplitudes of elastic $W_L^+W_L^-$ scattering

$$\mathcal{T}(W_L^+W_L^- \to W_L^+W_L^-) = 16\pi\sum_l (2l+1)a_l(s)P_l(\cos\vartheta), \tag{6}$$

Here P_l denote the Legendre polynomials.

The partial wave amplitudes $f_{l,\pm 1}(s)$ satisfy a once-subtracted dispersion relation:

$$f_{l,\pm 1}(s) \;=\; f_{l,\pm 1}(\mu^2) + \frac{s-\mu^2}{\pi}\cdot\int_0^\infty \mathrm{d}s'\frac{\mathrm{Im}[f_{l,\pm 1}(s')]}{(s'-s-i\epsilon)(s'-\mu^2-i\epsilon)}. \tag{7}$$

Here μ^2 is the subtraction point and $\epsilon > 0$. Using the unitarity relation (5) and elastic unitarity in $W_L^\pm$ rescattering,[5] i.e.,

$$a_l(s) = \exp(i\delta_l(s))\sin\delta_l(s) \tag{8}$$

one obtains the integral equation

$$f_{l,\pm 1}(s) \;=\; f_{l,\pm 1}(\mu^2) + \frac{s-\mu^2}{\pi}\cdot\int_0^\infty \mathrm{d}s'\frac{f_{l,\pm 1}(s')\cdot\exp(-i\delta_l(s'))\sin\delta_l(s')}{(s'-s-i\epsilon)(s'-\mu^2-i\epsilon)}. \tag{9}$$

whose solution is [15]

$$f_{l,\pm 1}(s) \;=\; f_{l,\pm 1}(\mu^2)\cdot\exp\left[\frac{s-\mu^2}{\pi}\cdot\int_0^\infty \mathrm{d}s'\frac{\delta_l(s')}{(s'-s-i\epsilon)(s'-\mu^2-i\epsilon)}\right], \tag{10}$$

This formula is reasonable up to energies $\sqrt{s}$ of order 2 TeV, because for these energies the contributions to the dispersion integral from the region $s' >2$ TeV, where two-body unitarity eventually becomes a bad approximation, is still suppressed. The subtraction point μ^2 can be

[4]Assuming parity invariance of the SIH sector.

[5]We neglect the masses of the W bosons in the following.

chosen at low energies where $f_{l,m}(\mu^2)$ can be substituted by the SM Born amplitudes $f^{(0)}_{l,m}(\mu^2)$. On the other hand, in the chiral limit $m_W/\sqrt{s} \ll 1$, which we use, only $f^{(0)}_{1,\pm1}$ are non-zero, and these amplitudes can be read off from (3). In other words, only the modification of the p-wave amplitude matters for the above reaction. The full p-wave amplitude reads:

$$f_{l,\pm1}(s) \;=\; f^{(0)}_{l,\pm1} \cdot H(s), \tag{11}$$

where H is given by (P denotes the principle value integral)

$$H(s) \;=\; [\cos\delta_1(s) + i\sin\delta_1(s)] \cdot \exp\left(\frac{s}{\pi} P \int_0^\infty \mathrm{d}s' \frac{\delta_1(s')}{(s'-s)s'}\right), \tag{12}$$

Models for the p-wave phase shift $\delta_1(s)$ will be discussed in the next section.

3 The elastic $\mathrm{W}_L^+\mathrm{W}_L^-$ scattering amplitude

Let us now compare various approaches in modeling the p-wave phase shift, respectively the amplitude for elastic $W_L^+W_L^-$ scattering within the SIH scenario. As we are interested in this reaction only for $s >> m_W^2$ we can use the equivalence theorem [16, 2] in order to relate the $W_L^+W_L^-$ scattering amplitude to the one which involves the associated Goldstone bosons $w^\pm$:

$$\mathcal{T}(W_L^+W_L^- \to W_L^+W_L^-) = \mathcal{T}(w^+w^- \to w^+w^-) + \mathcal{O}\left(\frac{m_W}{\sqrt{s}}\right). \tag{13}$$

A definite Lagrangean model for the SIH sector has not been developed yet. Nevertheless, empirical knowledge about the global symmetries of the symmetry-breaking Lagrangean $\mathcal{L}_{SSB}$ and general principles of S matrix theory allow to model the Goldstone boson scattering amplitudes[6] — in analogy to pion scattering. Experimentally the ρ parameter ($\rho = m_W^2/m_Z^2\cos^2\theta_W$) is very close to one. In order to protect $\rho = 1$ against strong radiative corrections from $\mathcal{L}_{SSB}$ it is sufficient to require that $\mathcal{L}_{SSB}$ possesses a global $SU(2)_L \times SU(2)_R$ symmetry — which is spontaneously broken to $SU(2)_V$ [14]. Neglecting effects from electroweak gauge and Yukawa interactions, which are small, the amplitudes for $W_L^\pm, Z_L$ scattering are then constrained by the unbroken $SU(2)_V$ group. This "isospin" symmetry and crossing symmetry lead to the result (well-known from pion physics) that the elastic w_aw_b $(a,b = 1,2,3)$ scattering amplitudes are determined by a single analytic function. We may decompose these amplitudes into isospin amplitudes $\mathcal{T}_I$ $(I = 0,1,2)$ with respect to s channel isospin eigenstates. Their partial wave expansion reads

$$\mathcal{T}_I \;=\; 32\pi \sum_l (2l+1) a_{Il}(s) P_l(\cos\vartheta). \tag{14}$$

[6]For a recent review see [6].

The "threshold" behaviour of these amplitudes is fixed by the chiral $SU(2)_L \times SU(2)_R$ symmetry of $\mathcal{L}_{SSB}$. More precisely this refers to the kinematic range $m_W^2 \ll s \ll m_{SSB}^2$, where m_{SSB} is the characteristic mass scale of $\mathcal{L}_{SSB}$. (Typically, m_{SSB} may be estimated by $4\pi v$, where $v = 246$ GeV) [2, 17]. For instance for the elastic $W_L^+W_L^-$ channel one gets for the corresponding Goldstone boson amplitude, up to terms of order s^2:

$$\mathcal{T}(w^+w^- \to w^+w^-) = \frac{s}{v^2}\,\frac{1+\cos\vartheta}{2}. \tag{15}$$

For the partial wave amplitudes chiral symmetry implies that the only amplitudes which are nonzero to order s are

$$a_{00}^{(0)} = \frac{s}{16\pi v^2}, \qquad a_{11}^{(0)} = \frac{s}{96\pi v^2}, \qquad a_{20}^{(0)} = -\frac{s}{32\pi v^2}, \tag{16}$$

where the upper index denotes the order in s. As $a_{00}^{(0)}$ and $a_{11}^{(0)}$ are attractive resonances may appear in the $I = l = 0$ and $I = l = 1$ channels. In view of the discussion in the previous section we discuss only the p wave amplitude in the following.

The corrections of order s^2 are known from chiral perturbation theory at one loop. For the p-wave amplitude the one-loop correction is [18]

$$a_{11}^{(1)}(s) \;=\; \frac{s^2}{96\pi v^4}\left[const + \frac{1}{96\pi^2}\left(-\frac{1}{3} + \ln(s/\nu^2) - \ln(-s/\nu^2)\right)\right], \tag{17}$$

We need this correction in the physical region. The unknown constant depends on the renormalization scale ν. The logarithms determine the analytic structure of the amplitude.

In the TeV region the result from chiral perturbation theory, $a_{11} \approx a_{11}^{(0)} + a_{11}^{(1)}$, cannot be trusted since it violates partial wave unitarity. As discussed above we shall impose the requirement of elastic unitarity on the partial wave amplitudes which means that, using the normalization (14), the partial waves have the form $a_{Il} = \exp(i\delta_{Il})\sin\delta_{Il}$. For c.m. energies up to about 2 TeV elastic unitarity should be a good approximation.

Several extrapolation schemes, well-known for a long time from low-energy hadron physics, which incorporate the tree level and one-loop results from chiral perturbation theory and satisfy elastic unitarity were discussed in the literature: notably the K matrix, Padé, and N/D methods (cf. [6] for a review and [11] for a discussion of the p wave amplitude). If one uses the tree-level and one-loop amplitudes from chiral perturbation theory the K matrix method yields a p-wave amplitude which, as shown in [11], violates analyticity. However, the zeroth-order K matrix amplitude [7]

$$a^{(K)}(s) := \frac{a^{(0)}(s)}{1 - i a^{(0)}(s)}. \tag{18}$$

[7]In the following we frequently drop the indices $l = I = 1$.

is acceptable in this respect; it may serve as a model for a non-resonant $l = I = 1$ partial wave.

In the Padé method a unitarized partial wave is constructed from $a^{(0)}, a^{(1)}$ by

$$a^{(Pade)}(s) \ := \ \frac{a^{(0)}(s)}{1 - a^{(1)}(s)/a^{(0)}(s)}. \tag{19}$$

Using (17) and putting

$$m_V^2 = \frac{288\pi^2 v^2}{288\pi^2 v^2 \cdot const - 1} \tag{20}$$

the Padé approximant for the partial wave $a_{11}(s)$ reads in the physical region $s > 0$:

$$a_{11}^{(Pade)}(s) = -\frac{s m_V^2}{96\pi v^2} \cdot \frac{1}{s - m_V^2 + i m_V \Gamma_V(s)} \tag{21}$$

with

$$\Gamma_V(s) = \frac{s m_V}{96\pi v^2}. \tag{22}$$

If the r.h.s. in eq. 20 is positive a vector resonance with mass m_V occurs whose width is

$$\Gamma_V = \Gamma_V(m_V^2) = \frac{m_V^3}{96\pi v^2}. \tag{23}$$

This is consistent with the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation [20]. From eq. 21 we get the scattering phase

$$\delta_{11}(s) = -\arctan\left(\frac{s}{96\pi v^2}\frac{m_V^2}{s - m_V^2}\right). \tag{24}$$

The resulting w^+w^- scattering amplitude suffers, however, from the lack of crossing symmetry. We will address this problem below.

A third often used unitarization method is the N/D method [12]. A partial wave is written as

$$a(s) = \frac{N(s)}{D(s)} \tag{25}$$

where $D(s)$ contains the right-hand cut and $N(s)$ the left-hand cut in the complex s plane. The condition of elastic unitarity is then for $s > 0$

$$\mathrm{Im}[D(s)] = -N(s). \tag{26}$$

Analyticity is guaranteed if $D(s)$, $N(s)$ fulfill the once-subtracted dispersion relation

$$D(s) = D(0) - \frac{s}{\pi}\int_0^\infty ds' \frac{N(s')}{s'(s'-s)}. \tag{27}$$

One may obtain $a(s)$ iteratively from (27) by starting with the Born result $N^{(0)}(s) = a^{(0)}(s)$.

Applying this procedure to the p wave also leads to the appearance of a vector resonance whose width satisfies to good accuracy the KSFR relation. A more elaborate analysis was made in [13] in order to obtain amplitudes which are approximately crossing-symmetric. For the case where $W_L W_L$ scattering is dominated by a resonance in the vector channel Ref. [13] obtains for its width $\Gamma_V \approx m_V^3/144\pi v^2$ which is smaller by a factor of 2/3 than the one obtained with the Padé procedure. Comparison with (21) shows that the Padé unitarized p-wave amplitude has, for $\sqrt{s} \leq 2$ TeV, a behaviour which is very similar to the N/D amplitude. In order to get a feeling whether or not crossing symmetry is badly violated by the Padé amplitude we employ the following simple procedure using vector dominance: Assuming C invariance of $\mathcal{L}_{\rm SSB}$ the coupling of the vector boson V to $w^\pm$ is

$$\langle V^\mu | T | w^+(p_+) w^-(p_-) \rangle = -G(p_- - p_+)^\mu \tag{28}$$

where the momenta $p_\pm$ are incoming and the form factor G is taken to be constant. A crossing symmetric w^+w^- scattering amplitude is then obtained from V exchange in the s and t channel:

$$\langle w^+w^- | T | w^+w^- \rangle \ = \ G^2 \cdot \frac{-s\cos\vartheta}{s - m_V^2 + i m_V \Gamma_V} + G^2 \cdot \frac{-\frac{s}{2}(3 + \cos\vartheta)}{\frac{s}{2}(1 - \cos\vartheta) - m_V^2 + i m_V \Gamma_V}. \tag{29}$$

The coupling G is fixed to be $G = \sqrt{m_V^2/3v^2}$ by requiring that (29) agrees for $s \ll m_V^2$ with $a_{11}^{(0)}(s)$, as dictated by chiral symmetry. This value of G yields the vector meson width $\Gamma_V \approx m_V^3/144\pi v^2$ which agrees with the width in [13] in the case of vector dominance. From (29) one obtains a_{11} by projecting onto the $l = I = 1$ channel. Of course, elastic unitarity is not exact for this amplitude. However, $(\mathrm{Im}a_{11} - |a_{11}|^2)$ is very small for $s \ll m_V^2$ and of order $0.1 - 0.2$ in the vicinity of m_V^2. (Only for $s > m_V^2$ the deviations become larger.) Choosing a vector meson mass, say, in the range between 1 and 1.7 TeV, and calculating the real and imaginary parts of this amplitude one finds good agreement with the Padé and N/D results for $s \leq 2$ TeV. Therefore a simple parameterization of SIH effects on $e^+e^- \to W_L^+ W_L^-$ is obtained by using (24) in (12) with various choices of m_V. In the next section we shall use $m_V = 1$ TeV, 1.7 TeV and ∞. The latter case corresponds to the non-resonant K matrix amplitude.

4 Observables and results

If the electroweak symmetry breaking sector leads to the formation of a vector resonance in $e^+e^- \to W^+W^-$ this would of course be easily detectable in the (differential) cross section at sufficiently high c. m. energies. However, since neither the mass nor the very existence of such a resonance can be predicted even if a SIH sector exists one may ask the following question: what is the minimal c. m. energy needed for detecting SIH effects — with or without a spin-one resonance — in W pair production, and which observables are most suited for this purpose. A number of suggestions for such observables were already made ([4, 5, 6]). Here we present a few results for observables which trace SIH effects in the real and/or imaginary parts of the scattering amplitude.

We consider W^+W^- events for which the $W^\pm$ momenta can be reconstructed. This is possible for semihadronic final states, i.e.,

$$e^+e^- \rightarrow W^+W^- \quad \rightarrow \quad \ell^+\nu_\ell \; + 2jets \tag{30}$$

and the charge-conjugated channels, respectively, which allow for charge tagging and momentum reconstruction of the W bosons in a straightforward fashion.

The simplest but efficient way to look for SIH effects is to investigate the cross section with suitable cuts in order to enhance the ratio of $W_L^+W_L^-$ to $W_T^+W_T^-$ events [4]. Using the phase-space cut $|\cos\vartheta_W| < 0.95$ we study, in slight variation of [4], the ratio

$$R \;=\; \frac{N(-0.95 < \cos\vartheta_W < b)}{N(-0.95 < \cos\vartheta_W < 0.95)} \tag{31}$$

where N denotes the number of W pairs in the given range of $\cos\vartheta_W$. Restricting the nominator of (31) to a section of the backward hemisphere turns out to be most favourable. In the following we shall use $\vartheta_W \geq 0.6\pi$, i.e., $b = -0.309$.

In Figs. 2,3 we have plotted the ratio R for a few SIH scenarios and for comparison also R in SM Born approximation. Suffice it to say that this reference function $R_{SM}(s)$ should be calculated with radiative corrections [21] included. However, we do not expect a drastic change of R as compared to its Born values. Fig.2 corresponds to strong rescattering but no spin-one resónance. This leads to a deviation of 5 percent or more from R_{SM} only at c. m. energies above 1.1 TeV. Fig.3 shows the resulting effect on R if there is a vector resonance with mass $m_V = 1$ TeV (1.7 TeV). The value $m_V = 1.7$ TeV may be motivated by using the analogy to hadron physics: $f_\pi \leftrightarrow v$ and $m_\rho \leftrightarrow m_V$. Nevertheless vector meson masses of 1 TeV or even slightly lower are so far not excluded [22], and we consider also this optimistic case. Fig.3 shows that R signals deviations from the SM value significantly below the location of a resonance. In order to make this more precise we calculate the 1σ error δR of R which is given by $\delta R = ((R - R^2)/N_{event})^{1/2}$ where the number of events $N_{event} = \mathcal{L} \cdot \sigma_{WW} \cdot 8/27$. Here $\mathcal{L}$ denotes the integrated luminosity which we take to be 10 $(fb)^{-1}$, σ_{WW} is the W pair production cross section with $|\cos\vartheta_W| < 0.95$, and the last factor is the branching ratio for W pairs to decay into 2 jets plus an electron or muon. We may then define a statistical significance S by

$$S \;=\; (R_{SIH} - R_{SM})/\delta R_{SIH}. \tag{32}$$

In Fig.4 S is plotted as a function of the c. m. energy. With an integrated luminosity of 10 $(fb)^{-1}$ a 1 TeV vector resonance would clearly leave its mark in R as a 6σ effect already at $\sqrt{s}=$ 500 GeV. If $m_V=$ 1.7 TeV a 3σ effect requires a c. m. energy of 750 GeV or larger. Strong $W_L^+W_L^-$ rescattering without a vector resonance can not be detected with R in the considered energy range, given the above integrated luminosity. The trouble in this case is that for energies where $R_{SIH} - R_{SM} \geq 0.05$ the pair production cross section is small.

One may also construct observables $\mathcal{O} = \mathcal{O}(\vartheta_W)$, which enhance the backward hemisphere, according to an optimization procedure [23]. However, we have not found a significant increase in sensitivity. Therefore, R should provide a good tool for searches of SIH effects.

The ratio R is sensitive to the modulus $|H(s)|^2$ of the Omnes function. One may also specifically search for effects of the phase shift, i.e., ImH. This can be done with T-odd observables (which change sign under reflection of momenta and spins). For the reactions at hand these observables must involve the W momentum in the c. m. frame and the $W^\pm$ spins. Since the spins of the W bosons are analyzed by their weak decays, T-odd observables amount to T-odd angular correlations [8] among the $W^\pm$ decay products. It is known (see, e.g., [24]) that SM final state interactions like W, Z, or photon exchange between the gauge bosons induce transverse polarization of the W bosons, i.e., correlations of the form

$$< (\mathbf{s}_+ + \mathbf{s}_-) \cdot (\mathbf{p}_+ \times \mathbf{k}_+) > \tag{33}$$

where $\mathbf{s}_\pm$ denote the spin operators of $W^\pm$. Yet because a strongly intercating Higgs sector affects longitudinally polarized W bosons one has to look for longitudinal-transverse W^+W^- spin-spin correlations in order to be sensitive to SIH effects. A simple and easily measurable quantity [26] which picks out such correlations is the expectation value of

$$U = \hat{\boldsymbol{p}}_+ \cdot [\hat{\boldsymbol{\ell}}_+ \times \hat{\boldsymbol{\ell}}_-]\, \hat{\boldsymbol{p}}_+ \cdot (\hat{\boldsymbol{\ell}}_+ + \hat{\boldsymbol{\ell}}_-) \tag{34}$$

where $\hat{\boldsymbol{\ell}}_\pm$ denote the unit momenta of the charged leptons originating from $W^\pm$ decay in the overall c. m. frame. Yet the sensitivity of $< U >$ to SIH effects turns out to be low: it is about one order of magnitude smaller than the sensitivity for the observable discussed below.

It is apparent that a higher sensitivity can be obtained if the rest frames of both W bosons can be reconstructed. This is possible for semihadronic decays studied above, i.e.,

$$e^+e^- \rightarrow W^+W^- \quad \rightarrow \quad \ell^+(\boldsymbol{\ell}_+)\nu_\ell \; + D(\boldsymbol{q}_-)\bar{U} \tag{35}$$

$D(U)$ denote quarks with electric charge $Q = -\frac{1}{3}(+\frac{2}{3})$. The optimal information is obtained if the charge of the quarks can be tagged. This may be possible for b or c quarks, i.e., the mode

[8] A general classification was given in [25].

$W^+ \to c\bar{s}$ is useful in this respect. (In [27] a tagging method is proposed which uses probability information about the quark flavours.)

In the following we assume "charge tagging" of the jets. Then in addition to the direction of the charged lepton momentum the direction of the D-type jet can also be determined in the respective W rest frame (denoted by an asterisk), i.e.,

$$\begin{aligned} \hat{\boldsymbol{\ell}}^*_- &= \sin\vartheta_- \cos\varphi_- \, \hat{\boldsymbol{m}}_- + \sin\vartheta_- \sin\varphi_- \, \hat{\boldsymbol{n}}_- + \cos\vartheta_- \, \hat{\boldsymbol{k}}_- \\ \hat{\boldsymbol{q}}^*_+ &= \sin\vartheta_+ \cos\varphi_+ \, \hat{\boldsymbol{m}}_+ + \sin\vartheta_+ \sin\varphi_+ \, \hat{\boldsymbol{n}}_+ + \cos\vartheta_+ \, \hat{\boldsymbol{k}}_+ \\ &= -\sin\vartheta_+ \cos\varphi_+ \, \hat{\boldsymbol{m}}_- + \sin\vartheta_+ \sin\varphi_+ \, \hat{\boldsymbol{n}}_- - \cos\vartheta_+ \, \hat{\boldsymbol{k}}_- . \end{aligned} \tag{36}$$

Here $\{\hat{\boldsymbol{m}}_-, \hat{\boldsymbol{n}}_-, \hat{\boldsymbol{k}}_-\}$ is the orthonormal basis

$$\begin{aligned} \hat{\boldsymbol{k}}_- &= (\sin\vartheta_W \cos\varphi_W, \sin\vartheta_W \sin\varphi_W, \cos\vartheta_W) \\ \hat{\boldsymbol{n}}_- &= \frac{\hat{\boldsymbol{p}}_- \times \hat{\boldsymbol{k}}_-}{|\hat{\boldsymbol{p}}_- \times \hat{\boldsymbol{k}}_-|} = (-\sin\varphi_W, \cos\varphi_W, 0) \\ \hat{\boldsymbol{m}}_- &= \hat{\boldsymbol{n}}_- \times \hat{\boldsymbol{k}}_- \\ &= (\cos\vartheta_W \cos\varphi_W, \cos\vartheta_W \sin\varphi_W, -\sin\vartheta_W) \end{aligned} \tag{37}$$

A simple T-odd quantity, which picks out the longitudinal-transverse spin-spin correlations mentioned above, is obtained by correlating the azimuthal angles [6]:

$$\mathcal{O} = \sin(\varphi_- - \varphi_+) \tag{38}$$

(Instead of (38) one may use the CP-even correlation $< \mathcal{O} > + < \bar{\mathcal{O}} >$, where $\bar{\mathcal{O}}$ is the corresponding observable for the channels which are charge conjugated to (35).)
For calculating the expectation value $< \mathcal{O}' >$ of an observable $\mathcal{O}'$ in the SIH scenario we need the (unnormalized) production density matrix defined by

$$R_{\omega_+\omega'_+\omega_-\omega'_-} = \frac{1}{4} \sum_{e^+e^-\,spins} \langle W^+(\omega'_+) W^-(\omega'_-)|T|e^+e^-\rangle^* \langle W^+(\omega_+) W^-(\omega_-)|T|e^+e^-\rangle \tag{39}$$

where the $\omega_\pm, \omega'_\pm$ denote the helicities of the $W^\pm$ bosons, respectively. This matrix is obtained from the first two amplitudes of (3) and those of (4). Further we need the normalized decay matrices of the $W^\pm$ bosons [25] $r^{(+)}_{\omega'_+\omega_+}, r^{(-)}_{\omega'_-\omega_-}$ which are obtained in straightforward fashion from the SM amplitudes for W decay into two massless fermions in the respective $W^\pm$ rest frames.

Correlations $\langle \mathcal{O}' \rangle$ which depend on the momenta of the secondary fermions and the $W^\pm$ bosons are then calculated in the narrow width approximation from

$$\langle \mathcal{O}' \rangle = \frac{1}{\mathcal{N}} \int d\Phi R_{\omega_+\omega'_+\omega_-\omega'_-} \, r^{(+)}_{\omega'_+\omega_+} \, r^{(-)}_{\omega'_-\omega_-} \mathcal{O}' \tag{40}$$

with

$$\mathcal{N} = \int d\Phi \, R_{\omega_+\omega'_+\omega_-\omega'_-} \, r^{(+)}_{\omega'_+\omega_+} \, r^{(-)}_{\omega'_-\omega_-} . \tag{41}$$

Here

$$d\Phi = d\cos\vartheta_W d\cos\vartheta_- d\varphi_- d\cos\vartheta_+ d\varphi_+ . \tag{42}$$

Using the cut $|\cos\vartheta_W| < 0.95$ we have calculated $< \mathcal{O} >$ and the signal-to-noise ratio $< \mathcal{O} > / \Delta\mathcal{O}$ where

$$\Delta\mathcal{O} = \sqrt{\langle \mathcal{O}^2 \rangle - \langle \mathcal{O} \rangle^2} \tag{43}$$

is the width of the distribution of $\mathcal{O}$. The value of this ratio as a function of the c. m. energy is plotted in Fig.5 for a vector resonance with mass 1 TeV and 1.7 TeV, respectively. With the absorptive parts of the one-loop SM amplitudes calculated in [24] we estimate the SM value for $< \mathcal{O} > / \Delta\mathcal{O}$ to be about 0.02. In order to obtain the statistical significance of the signal we use only final states of the form $\ell\nu_\ell + \bar{c}s$, $\ell = e, \mu$ and the charge conjugated channels. Further we assume the efficiency for tagging the c quark to be 0.1. Then the number of available events $N \simeq N_{WW} \cdot 0.015$. With an integrated luminosity of $10 (fb)^{-1}$ one then estimates from Fig.5 that the statistical significance to see an SIH effect by means of $< \mathcal{O} >$ is 1.4σ (3.2σ) at $\sqrt{s}$ = 900 GeV (950 GeV) if there is a vector resonance at 1 TeV, and 2.9σ (3.5σ) at $\sqrt{s}$ = 1600 GeV (1650 GeV) for a resonance at 1.7 TeV. If there is no resonance in the spin-one channel then, for the luminosity above, an SIH effect cannot be traced with $< \mathcal{O} >$ in the energy range considered.
We have also investigated optimized T-odd correlations (in the sense of [23]); however we have found no significant increase in sensitivity as compared to that of $\mathcal{O}$.

In conclusion: a strongly interacting symmetry breaking sector may lead to the formation of a vector resonance in $e^+e^- \to W^+W^-$. If the mass of this resonance is below 1.5 TeV then, as our study shows, one may trace it with the event ratios (31) already at $\sqrt{s}$ = 500 GeV, given an integrated luminosity of 10 $(fb)^{-1}$. The phase shift δ_1 reveals its mark in the T-odd correlation $< \mathcal{O} >$ only in the close vicinity of a resonance. However if the spin-one channel is non-resonant then the prospects of detecting SIH effects in W-pair production are gloomy: even at a c. m. energy of 2 TeV a much higher integrated luminosity , to wit 100 $(fb)^{-1}$, would be required in order to obtain a 3 σ effect in the event ratio R.

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Figure Captions

Figure 1: Strongly rescattering $W_L^\pm$ bosons in the reaction $e^+e^- \to W_L^+W_L^-$. The circles denote full amplitudes.

Figure 2: Ratio R defined in eq. (31) as a function of the c.m. energy in the case of strong $W_L^+W_L^-$ rescattering but no vector resonance. The dashed line depicts the Born value of R.

Figure 3: Ratio R defined in eq. (31) as a function of the c.m. energy in the case of a vector resonance with mass $m_V = 1.7$ TeV (solid line) and $m_V = 1$ TeV (dashed line), respectively. The dash-dotted line depicts the Born value of R.

Figure 4: Statistical significance as a function of the c.m. energy in the case of a vector resonance with mass $m_V = 1.7$ TeV (solid line), $m_V = 1$ TeV (dashed line), and for strong $W_L^\pm$ rescattering without vector resonance (dotted line).

Figure 5: Signal-to-noise ratio $< \mathcal{O} > /\Delta\mathcal{O}$ of the observable (38) as a function of the c.m. energy in the case of a vector resonance with mass $m_V = 1.7$ TeV (solid line) and $m_V = 1$ TeV (dashed line), respectively.

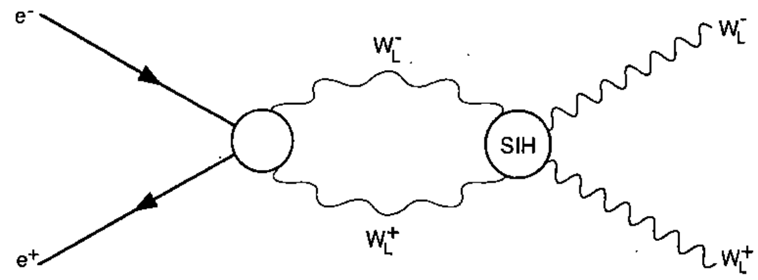
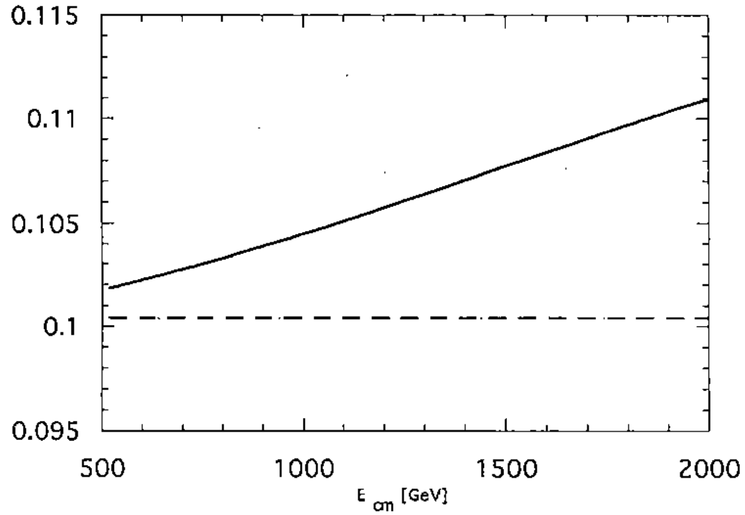


Fig.1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E cm [GeV]

Fig. 2

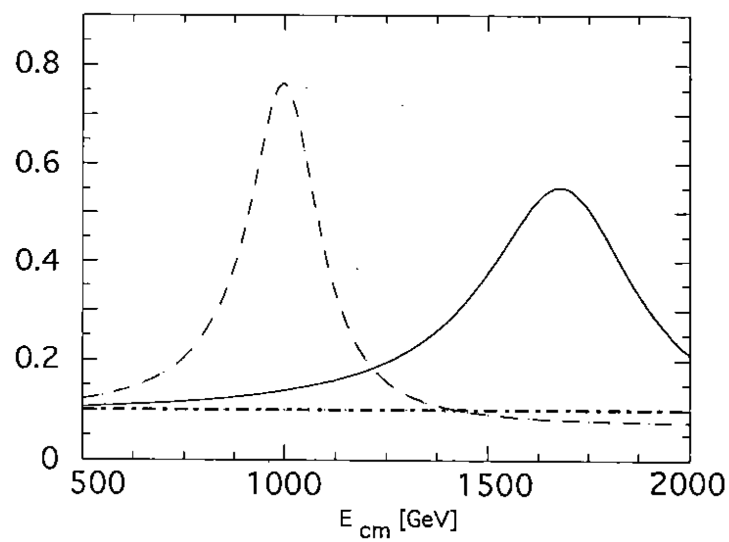
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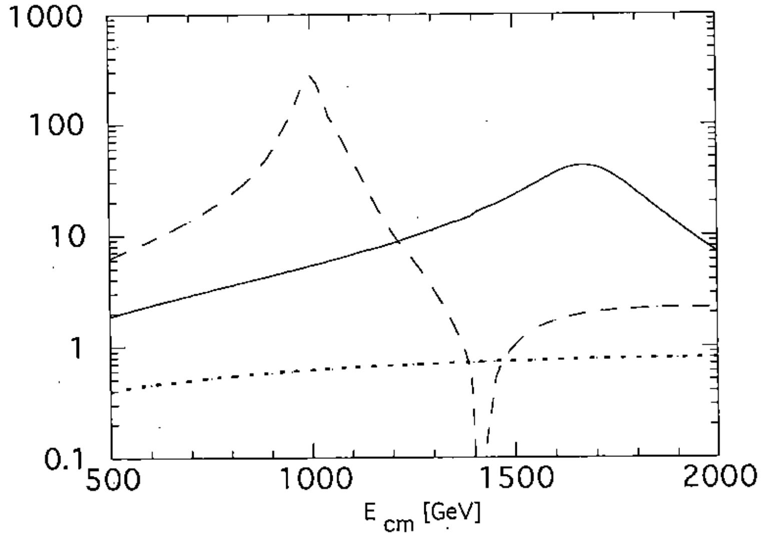
Fig. 3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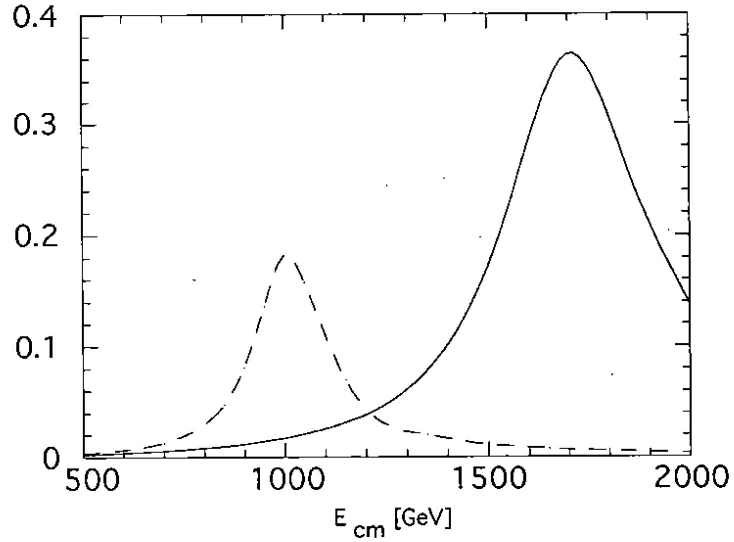
1000
1500
2000
E cm [GeV]

Fig. 4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0.3
0.2
0.1
0
500
1000
1500
2000
E cm [GeV]

Fig. 5