DEUTSCHES ELEKTRONEN-SYNCHROTRON

DESY 93-168
November 1993

Hadronic Tau Decays

H. Kolanoski
Institut für Physik, Universität Dortmund

ISSN 0418-9833

NOTKESTRASSE 85 - 22603 HAMBURG

To be sure that your preprints are promptly included in the
HIGH ENERGY PHYSICS INDEX,
send them to (if possible by air mail):

DESY	DESY-IfH
Bibliothek	Bibliothek
Notkestraße 85	Platanenallee 6
22603 Hamburg	15738 Zeuthen
Germany	Germany

DESY 93-168
November 1993

ISSN 0418-9833

Hadronic Tau Decays

H. Kolanoski*
Universität Dortmund
D-44221 Dortmund, Germany

(Talk presented at the 5th International Symposium on Heavy Flavour Physics, Montreal, Canada, July 6 - 10, 1993)

Abstract

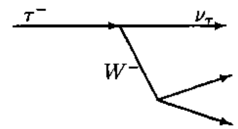
This talk summarizes our current knowledge on the decays of τ leptons into hadrons. The decays are discussed in terms of the hadronic weak currents and compared to theoretical predictions. The presented topics include: the QCD analysis of the hadronic decay width and the determination of the strong coupling constant at the τ mass scale, analyses of the pion form factor and the $a_1(1260)$ resonance parameters, improved limits on second class currents, analyses of the Lorentz structure of the $\tau - \nu_\tau$ vertex and limits on the τ neutrino mass. Also discussed is the (still existing) deficit in the exclusive τ decay branching ratios.

*This work was supported by the German Bundesministerium für Forschung und Technologie under contract number 056DO57P.

1 Introduction

In the Standard Model of electro-weak interactions the τ lepton and its neutrino are repetitions of the first two lepton generations, with the same interaction structure and strength ('universality' of electro-weak couplings). As yet there is not a single experimental evidence against this picture [1, 2].

According to the Standard Model the τ^- lepton decays by transforming into a ν_τ under the emission of a W^- boson* which then couples to a fermion–antifermion pair, either leptons or quarks. The τ mass allows for the coupling to the three lightest quark species, u, d, s.



$$\begin{pmatrix} f \\ \bar{f} \end{pmatrix} = \begin{pmatrix} e^- \\ e^+ \end{pmatrix}, \begin{pmatrix} \mu^- \\ \mu^+ \end{pmatrix}, \begin{pmatrix} d \cos\theta_C + s \sin\theta_C \\ \bar{u} \end{pmatrix}$$

In the Standard Model both W–fermion vertices in the diagram have the same, universal coupling strength given by the Fermi coupling constant G_F (times the corresponding mixing matrix element for quarks). In the limit where the squared momentum, q^2, of the exchanged W is small compared to m_W^2, which is a very good approximation for τ decays ($q^2 < m_\tau^2$), the W propagator becomes constant and the interaction is described by a current-current Lagrangian:

$$\mathcal{L}^{cc} = \frac{G_F}{\sqrt{2}} J^{f\dagger}(x)\, J^\tau(x)$$

For the lepton vertices the currents are simply calculable from the Dirac spinors with a vector and an axial-vector component. Also the hadronic currents have a vector and axial-vector component,

$$J^h_\mu = V_\mu + A_\mu,$$

which are, however, not easily expressed in terms of free-quark spinors. The strong interaction projects out hadronic final states h with well defined quantum numbers.

Constraints on the quantum numbers of the hadronic system:

- Spin-parity: The space-time transformation properties of the V and A currents fix the allowed spin and parity, J^P of the hadronic states:

$$\begin{aligned} V_\mu: \quad J^P &= 1^-,\ 0^+ \\ A_\mu: \quad J^P &= 1^+,\ 0^- \end{aligned}$$

- Isospin: Since the hadronic system in τ decays is charged the isospin is $I \geq 1$ for non-strange system. If the W couples, as expected, initially to $q\bar{q}$ pairs the isospin is further constraint:

$$\begin{aligned} I &= 1 \quad \text{for } u\bar{d} \\ I &= \tfrac{1}{2} \quad \text{for } u\bar{s} \end{aligned}$$

*References to a specific charge state imply also the charge conjugate state

- G-parity: For non-strange $q\overline{q}$ systems isospin and angular momentum fixes also the G-parity ($G(q\overline{q}) = (-1)^{L+S+I}$) leading to multi-pion final states with either an even or odd number of pions:

$$\begin{aligned} GV_\mu G^{-1} &= +V_\mu \rightarrow 2n\,\pi \\ GA_\mu G^{-1} &= -A_\mu \rightarrow (2n-1)\,\pi \end{aligned}$$

Hadronic final states which do not fit into the scheme:

$$\begin{aligned} V_\mu: \; J^{PG} &= 1^{--},\; 0^{+-} \\ A_\mu: \; J^{PG} &= 1^{++},\; 0^{-+} \end{aligned}$$

are called 'second class currents' and are suppressed on the level at which isospin violation is suppressed in strong interactions. No second class currents have been observed yet, in particular not the formation of resonances with the 'wrong' G-parity, e. g.:

$$\begin{aligned} a_0^-(980) &\rightarrow \eta\pi^-(S-wave) \quad J^{PG} = 0^{+-} \\ b_1^-(1235) &\rightarrow \omega\pi^-(S-wave) \quad J^{PG} = 1^{++} \end{aligned}$$

The hadronic τ decay width and spectral functions: The differential decay width of τ leptons into hadrons with a squared mass q^2 $(= m^2(W^*))$ can be decomposed into spectral functions with the allowed quantum numbers for non-strange and strange V and A currents:

$$\begin{aligned} \frac{d\Gamma}{dq^2} = \frac{G_F^2}{32\pi^2 m_\tau^3}(m_\tau^2 - q^2)^2 \Big\{ & \left[(m_\tau^2 + 2q^2)\left(v_1(q^2) + a_1(q^2)\right) + m_\tau^2 a_0(q^2)\right]\cos^2\theta_c \\ & + \left[(m_\tau^2 + 2q^2)\left(v_1^s(q^2) + a_1^s(q^2)\right) + m_\tau^2\left(v_0^s(q^2) + a_0^s(q^2)\right)\right]\sin^2\theta_c \Big\} \end{aligned}$$

There are many theoretical predictions for the spectral functions and for single decay channels:

- The τ decay into a single pion or kaon is related by the universality hypothesis to leptonic pion (kaon) decays.
- The 'conserved vector current' hypothesis (CVC) connects the isovector part of the electro-magnetic current in e^+e^- annihilation with the non-strange vector spectral function. It predicts the scalar vector current (v_0) to vanish.
- Less hard predictions are made for the axial-vector current employing the hypothesis of the 'partially conserved axial-vector current' (PCAC).
- Sum rules connect integrals over spectral functions.
- The ratio of the hadronic to the leptonic decay width has been calculated in the framework of perturbative QCD (very similarly to the corresponding ratio in e^+e^- annihilation).

Experimental measurements of hadronic τ decays: The experimental investigation of the structure of the hadronic current in τ decays has been carried out at e^+e^- colliders. New results will be presented from ARGUS, CLEO, TPC and the LEP experiments.

The LEP experiments have made quite substantial contributions to τ physics in recent years, improving in particular the precision of the branching ratio measurements. Compared to the energies of the ARGUS and CLEO experiments the high energy of LEP offers advantages: the low multiplicity τ decays are well separated from $q\overline{q}$ events with average charged multiplicities around 20 leading to a very small $q\overline{q}$ background and the collimation of τ decay products by the boost increases the detection efficiency (from typically 10 to 20% for ARGUS to about 50% for LEP experiments).

On the other hand particle identification, which is crucial for the reconstruction of exclusive channels, is often easier at lower energies. The larger opening angle of the gammas from π^0 decays (about 4.4° for ARGUS and CLEO compared to about 0.5° at LEP) facilitates π^0 reconstruction.

2 Determination of $\alpha_s(m_\tau^2)$

The QCD calculations

The ratio of the hadronic to the leptonic τ decay width

$$R_\tau = \frac{\Gamma(\tau^- \rightarrow hadrons\,\nu_\tau)}{\Gamma(\tau^- \rightarrow e^-\overline{\nu}_e\nu_\tau)} = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$$

has been calculated perturbatively to $O(\alpha_s^3)$ (separately for the non-strange V, A and the strange part) and it was shown that non-perturbative contributions can be included with few additional parameters (see refs. [3, 4] and references therein). The calculation R_τ is derived from an integral in the complex s plane over a circle at $|s| = m_\tau^2$. At this scale the non-perturbative terms are found to be small. Figure 1 shows the prediction of R_τ as a function of α_s. For the measured leptonic τ branching ratio the QCD prediction lies about 20% above the parton model value R_τ=3 which accounts for one (Cabbibbo-mixed) flavor and 3 color degrees of freedom. This yields an α_s determination on the τ mass scale with a precision of about 2.5% which is surprisingly precise, actually for many people too precise.

It is interesting to note that, since $R_{\tau,V}$ is related via CVC to the e^+e^- cross section, it follows that α_s can be determined from e^+e^- data around 2 GeV! Actually, Narison and Pich [5] have exploited this relation to test the range of validity of their QCD calculation. For that purpose they calculated $R_{\tau,V}(m^2)$ for non-physical τ masses and compared the result to the corresponding measured e^+e^- cross section (Fig.2). The QCD prediction, fixed to $R_{\tau,V}$ at the τ mass, agrees well with the e^+e^- data down to about 1.2 GeV where the calculation is expected to become unreliable because of non-perturbative effects. This demonstration was aimed to disprove critics [6] that the good QCD result may be accidental for $m = m_\tau$.

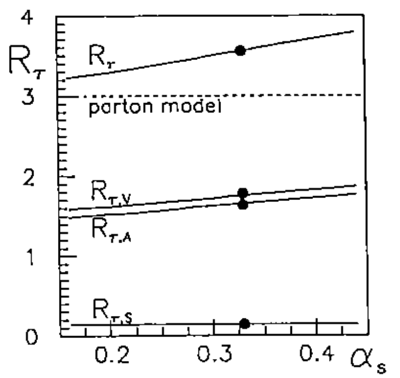


Figure 1: QCD calculations for R_τ as a function of α_s compared to experimental values (taken from [7]).

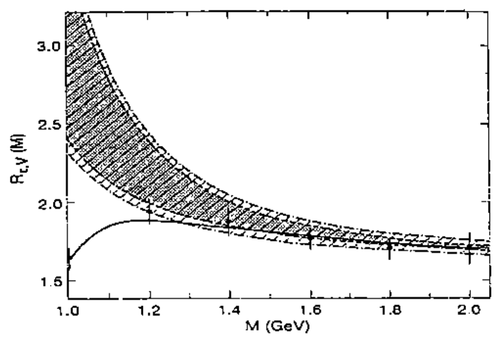


Figure 2: QCD prediction for $\sigma_{ee}^{I=1}$ (band) obtained from $R_\tau(m)$ (fixed to the experimental value at $m = m_\tau$). The points with error bars are experimental e^+e^- data.

Experimental $\alpha_s(m_\tau^2)$ analyses

The collaborations ALEPH and OPAL have presented an analysis of α_s at the τ mass based on the described QCD calculations [4]. In addition to the inclusive hadronic branching ratio the ALEPH group also used moments of the hadronic mass distribution ($s = q^2$):

$$R_\tau^{kl} = \frac{1}{\Gamma_e}\int_0^{m_\tau^2} ds \left(1 - \frac{s}{m_\tau^2}\right)^k \left(\frac{s}{m_\tau^2}\right)^l \frac{d\Gamma_h}{ds}$$

These moments are sensitive to non-perturbative effects and allow for an experimental determination of the parameters describing these effects in the QCD calculation.

The spectrum $\frac{d\Gamma_h}{ds}$ was obtained by unfolding the measured spectra correcting for detector effects. From the measured leptonic branching ratios and the τ lifetime R_τ ($= R_\tau^{00}$) was determined to be:

$$R_\tau^{exp} = 3.579 \pm 0.087$$

A combined fit of the QCD calculation to the experimental results for R_τ, R_τ^{10}, R_τ^{11}, R_τ^{12} and R_τ^{13} yielded α_s and 3 non-perturbative parameters. The value of α_s obtained at the τ mass and the derived value at the Z mass,

$$\alpha_s(m_\tau^2) = 0.330 \pm 0.046$$
$$\Rightarrow \alpha_s(m_Z^2) = 0.118 \pm 0.005,$$

are in remarkable agreement with α_s determinations from hadronic Z decays [8]. With the measurement of α_s at the τ mass the running of α_s can be impressively demonstrated.

3 Vector currents

The (1st class) vector currents have even G-parity: $GVG^{-1} = +V$. That restricts the possible hadronic decay channels:

$$J^{PG} = 1^{-+}: \quad \tau \rightarrow 2n\pi\,\nu,\ \eta\pi\pi\nu,\ \omega\pi\nu,\ K\overline{K}\pi\nu,\ \ldots$$
$$J^{PG} = 0^{++}: \quad \text{forbidden by CVC}$$

The formation of vector states in τ decays and e^+e^- annihilation is related by the CVC hypothesis. CVC forbids the formation of scalars which is related to the fact that $q\bar{q}$ states with this quantum numbers have even C-parity and hence are not produced in e^+e^- annihilations. The occurrance of states with non-$q\bar{q}$ quantum numbers or second class currents at a level higher than expected by isospin violation effects could point to new physics. None of the following states has been observed (L is the final state angular momentum):

state	L	J^{PG}	class
$\pi^-\pi^0$	S-wave	0^{++}	exotic
$\eta\pi^-$	S-wave	0^{+-}	2nd class (e.g. $a^0(980)$)
$\eta\pi^-$	P-wave	1^{--}	2nd class, exotic

3.1 The two-pion final state

Branching ratio: The τ decay

$$\tau^- \rightarrow \pi^-\pi^0\nu_\tau$$

has the largest branching ratio and has been extensively investigated. The two-pion spectrum is dominated by the ρ resonance (Fig.3). The measured branching ratios are shown in Fig.4 and compared to the CVC expectation obtained from the two-pion cross section in e^+e^- annihilation. Here and in most of the following tables we summarize earlier measurements by the 1990 Particle Data Group average [10] and display explicitely only newer, sometimes preliminary, results. The results labelled 'prel.' have been submitted to this conference; the label 'Ohio' indicates results presented at the '92 Ohio τ Workshop [2]. All measurements are corrected such as to contain only the two-pion channel (the $K^-\pi^0$ channel is discussed in Sec.5.2). The average branching ratio of

$$B(\tau^- \rightarrow \pi^-\pi^0\nu_\tau) = (24.2 \pm 0.3)\%$$

agrees well with the CVC [11] expectation. The ALEPH [12] and CLEO [9] values are significantly larger than the other ones.

CVC comparison: Besides the rates CVC relates also the q^2 spectra in τ decays and e^+e^- annihilation. In Fig.3 the two-pion mass spectrum is well fitted by the standard relativistic Breit-Wigner for the ρ resonance except for the high mass tail. Using e^+e^- data parametrized [11] as a pion form factor (in the time-like region), $F_\pi(q^2)$, both the ARGUS

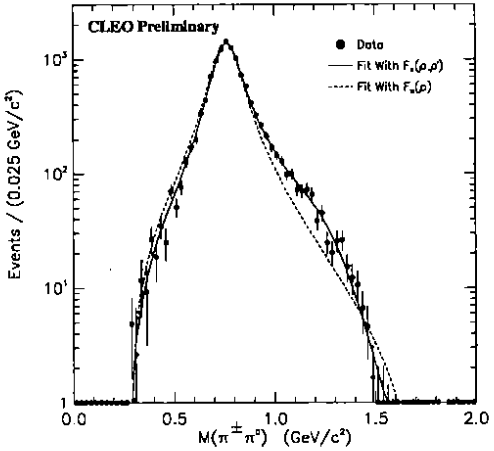


Figure 3: Two-pion mass spectrum of the decay $\tau^- \to \pi^-\pi^0\nu_\tau$ fitted by the ρ resonance (dashed curve) and by the ρ and a ρ' resonance (full curve) [9].

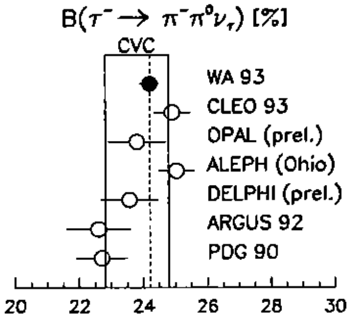


Figure 4: Branching ratios for $\tau^- \to \pi^-\pi^0\nu_\tau$ compared to the CVC expectation. Only results not included in PDG 90 [10] are explicitly shown.

and CLEO (Fig.3) data are fitted much better. The deviation from a Breit-Wigner behaviour has been interpreted by contributions of higher resonances. The $e^+e^- \to \pi^+\pi^-$ data can be fitted by two interfering ρ' resonances (2^3S_1 at $\sim$ 1350 Mev and 1^3D_1 at $\sim$ 1700 MeV) [13]. The τ data support the $\rho'(\sim 1250)$ resonance. There is no sensitivity to the higher resonance because of missing phase space.

3.2 The four-pion final states

The two charge combinations contributing to the four-pion final state have the following average branching ratios [14]:

$$B(\tau^- \to \pi^-\pi^+\pi^+\pi^0\nu_\tau) = (4.8 \pm 0.4)\%$$
$$B(\tau^- \to \pi^-\pi^0\pi^0\pi^0\nu_\tau) = (1.1 \pm 0.2)\%$$

The channel with three charged pions was further analysed for resonant substructures [15] and was found to be dominated by the channels $\rho\pi\pi$ ($\sim$ 64%) and $\omega\pi$ ($\sim$ 33%), no $\rho\rho$ was observed. It is also interesting that in the '3-charged + 1 π^0' topology no $K^\pm$ mesons have been found.

The CVC comparison to the corresponding e^+e^- data in Fig.5 shows agreement within the errors which are already smaller for the τ data. The comparison of the mass spectra is even more limited by statistics.

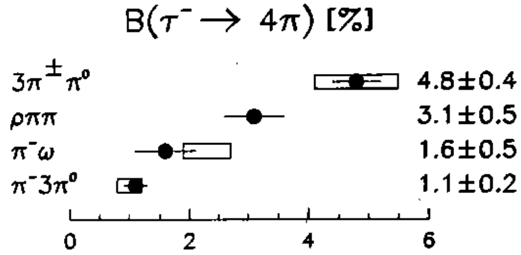


Figure 5: Branching ratios of the four-pion channels compared to the CVC expectation (rectangles). The $\omega\pi$ and $\rho\pi\pi$ channels are part of the $3^\pm\pi^0$ channel.

3.3 Decay modes involving η mesons

Decay modes of the τ involving η mesons are found to have low branching ratios as can be seen from various upper limits quoted in the PDG [16]. Only recently the first observation of the decay mode $\tau^- \to \pi^-\pi^0\eta\nu_\tau$ has been reported by the CLEO group [17]. For this analysis the low energy efficiency and the energy resolution of the CsI calorimeter of the CLEO II detector was essential (see Fig.6). The measured branching ratio

$$B(\tau^- \to \pi^-\pi^0\eta\nu_\tau) = (0.17 \pm 0.02 \pm 0.02)\%$$

is consistent with the CVC expectation $(0.13 \pm 0.02)\%$.

Searching for η's in τ decays had been stimulated some years ago by the unconfirmed observation of the $\eta\pi$ decay channel by one experiment. This hadronic state, independent of its angular momentum, cannot be a first class current (see discussion at the beginning of this section). It should thus be suppressed in τ decays to a level at which isospin violation occurs (here about $1.5 \cdot 10^{-5}$ [18]). The CLEO group published an upper limit on this decay mode which is the most stringent for 2nd class currents [17]:

$$B(\tau^- \to \eta\pi^-\nu_\tau) < 3.4 \cdot 10^{-4} \text{ at } 95\% \text{ c.l.}$$

4 Axial-vector currents

The (1st class) axial-vector currents have odd G-parity: $GAG^{-1} = -A$. It follows that multi-pion final states have an odd number of pions. The non-strange axial-vector currents are nearly saturated by only two states, the π and the $a_1(1260)$:

$$\begin{array}{llll} a_0 \text{ current}, & J^{PG} = 0^{--}: & \tau \to \pi\nu & \\ a_1 \text{ current}, & J^{PG} = 1^{+-}: & \tau \to a_1(1260)\nu & \to 3\pi\nu \\ & & \tau \to 5\pi\nu & \text{only} \approx 0.5\% \end{array}$$

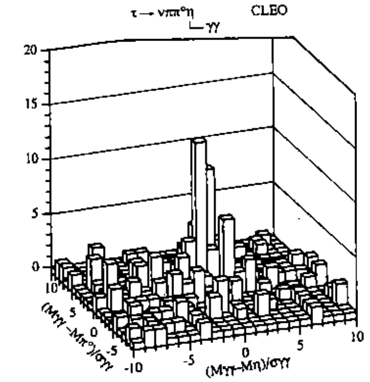


Figure 6: Observation of a correlated $\eta - \pi^0$ signal in the analysis of 4 photons with the CLEO II detector [17].

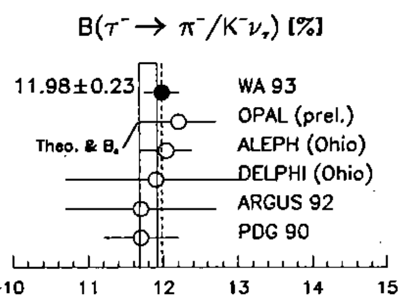


Figure 7: Branching ratios for $\tau^- \to \pi^-/K^-\nu_\tau$ compared to the theoretical expectation from leptonic pion and kaon decays (column).

4.1 The τ decay into a single pion or kaon

In the Standard Model with universal couplings the τ decay into a pion or kaon,

$$\tau^- \to \pi^-(K^-)\nu_\tau,$$

is related to the corresponding leptonic meson decays via the same decay constants f_π and f_K:

$$\Gamma(\tau^- \to \pi^-\nu_\tau) = \frac{G_F^2}{16\pi} f_\pi^2 \cos^2\theta_C \, m_\tau^3 \left(1 - \frac{m_\pi^2}{m_\tau^2}\right)^2 (1 + r_\pi)$$
$$\Gamma(\tau^- \to K^-\nu_\tau) = \frac{G_F^2}{16\pi} f_K^2 \sin^2\theta_C \, m_\tau^3 \left(1 - \frac{m_K^2}{m_\tau^2}\right)^2 (1 + r_K)$$

Predictions have been obtained with $f_\pi = 0.943\, m_\pi$ and $f_K = 0.313\, m_K$ and the radiative corrections from [19] ($r_\pi = r_K = 0.019 \pm 0.01$).

In most experiments pions and kaons have not been separated. In Fig.7 we show the measured branching ratios of the sum of both channels. The world average is

$$B(\tau^- \to \pi^-/K^-\nu_\tau) = (11.98 \pm 0.23)\%$$

which is in very good agreement with the theoretical prediction of $(11.80 \pm 0.12)\%$, obtained from the above expressions for the widths together with world averages for the leptonic branching ratios and the τ lifetime ($B_e = (17.89 \pm 0.12)\%$, $\tau_\tau = (294.7 \pm 3.0)\ fs$).

4.2 The decay into three pions

The two possible three-pion final states $\pi^-\pi^-\pi^+$ and $\pi^-\pi^0\pi^0$ are dominated by the $a_1(1260)$ resonance which itself decays dominantly via an intermediate $\rho\pi$ state as has been observed

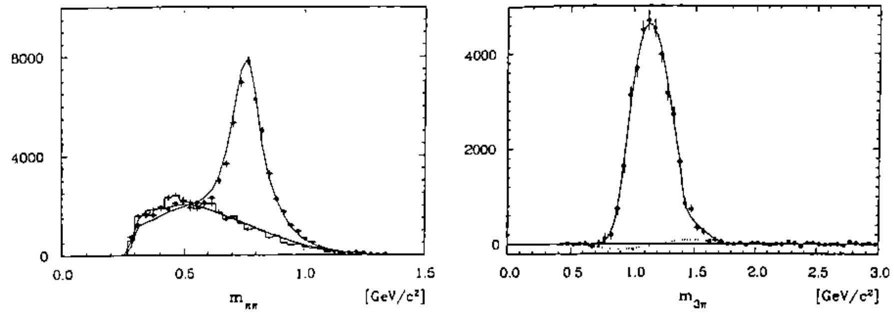


Figure 8: Background corrected two- and three-pion mass spectra of the decay $\tau^- \to \pi^-\pi^-\pi^+\nu_\tau$ [20]. The two-pion plot shows the like-sign (one entry per event) and unlike-sign (two entries per event) combinations (curves: KORALB Monte Carlo). The three-pion mass spectrum is fitted by the model of Isgur et al. [21].

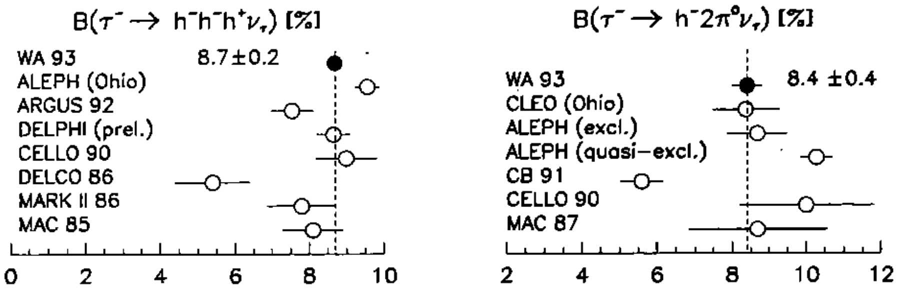


Figure 9: Branching ratios for $\tau^- \to h^-h^-h^+\nu_\tau$ (left) and $\tau^- \to h^-\pi^0\pi^0\nu_\tau$ (right). The two ALEPH results in the right plot are not statistically independent. The average was obtained with the 'excl.' result.

by several experiments (Fig.8). If this is the only decay chain then, by isospin symmetry, both charge modes should have the same branching ratio.

Since most of the experiments do not separate charged pions and kaons we compare in Fig.9 branching ratios for the final states $h^-h^-h^+$ and $h^-\pi^0\pi^0$ where $h^\pm$ means a kaon or pion. The two channels contain kaon modes with branching ratios of about 0.75% and 0.1%, respectively. This correction has not always been taken into account previously. The world averages are:

$$B(\tau^- \to h^-h^-h^+\nu_\tau) = (8.7 \pm 0.2)\%$$
$$B(\tau^- \to h^-\pi^0\pi^0\nu_\tau) = (8.4 \pm 0.4)\%$$

The three-pion decay channel is one of the foremost suspects to cause the deficit in the sum of the measured exclusive branching ratios. Here and in the two-pion channel the largest differences to the 'quasi-exclusive' analysis of the ALEPH group [22] are observed.

Most pronounced, because of the small errors, is the discrepancy between the recent ARGUS and ALEPH measurements of the $h^-h^-h^+$ mode. Both disagree significantly with the average which therefore appears little reliable given a χ^2 of 23.8 for 6 degrees of freedom.

In the $h^-\pi^0\pi^0$ mode the largest discrepancy occurs, surprisingly, for two ALEPH measurements, one called 'exclusive' and the other 'quasi-exclusive' [22]. The other measurements are quite consistent with each other and with the ALEPH 'exclusive' value. Taking the latter for the average a χ^2 of 3.4 for 4 degrees of freedom is obtained. It is interesting to note that the branching ratios of the $h^-\pi^0\pi^0$ mode, accept for the ALEPH 'quasi-exclusive' value, are quite consistent. On the other hand the average branching ratios of both charge modes are about the same as expected from the $a_1(1260)$ dominance. That could be an argument against both the ARGUS and the ALEPH measurement of the $h^-h^-h^+$ mode. This situation will hopefully be clarified by future measurements.

4.3 Determination of the $a_1(1260)$ resonance parameters

The τ decay into three pions is the cleanest known laboratory for studying the $a_1(1260)$ resonance. The most significant determination of resonance parameters has been published by the ARGUS group [20]. Fitting the model of Isgur et al. [21] to the three-pion mass spectrum yields:

$$m_{a1} = (1.211 \pm 0.007) \text{ GeV and } \Gamma_{a1} = (0.446 \pm 0.021) \text{ GeV}$$

The statistical errors have become quite small but the model dependence is large. Using for example the model of Kühn et al. [11], which is implemented into the KORALB and KORALZ Monte Carlo programs and which also describes the data reasonably well, ARGUS obtains:

$$m_{a1} = (1.274 \pm 0.007) \text{ GeV and } \Gamma_{a1} = (0.594 \pm 0.023) \text{ GeV}.$$

A recent preliminary analysis of the DELPHI group [23] using the same model is in good agreement with this result:

$$m_{a1} = (1.270 \pm 0.015) \text{ GeV and } \Gamma_{a1} = (0.604 \pm 0.050) \text{ GeV}.$$

In analyses of the a_1 parameters from diffractive hadronic reactions the width comes out much smaller (see note in [16]). According to Bowler [24] the τ and the hadronic analyses can be made consistent by appropriately treating the Deck amplitude in the hadronic data.

For the interpretation of the observed parity violation in three-pion τ decays [25] it turned out to be crucial to know the three-pion angular momentum decomposition. The ARGUS group has therefore analysed the three-pion final state for intermediate resonances and angular momenta [20]. As is demonstrated in Fig.8 the three pions are consistent with being produced completely via $\rho\pi$. ARGUS finds that less than 6% could come from another intermediate state (like $\epsilon\pi$). The $\rho\pi$ system can be in a S or D wave. An analysis of the three-pion Dalitz-plot in different three-pion mass bins using the model of Isgur et al. yields for the ratio of the amplitudes at the nominal a_1 mass:

$$D/S = -0.11 \pm 0.02$$

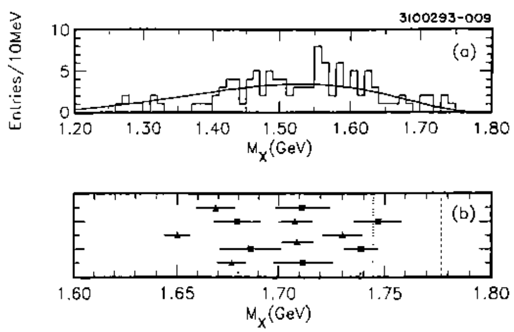


Figure 10: Five-pion mass spectrum used by CLEO to obtain the upper limit for the ν_τ mass (top). The plot below shows the events near the kinematical end point each with error bars corresponding to the mass resolution. The dashed line indicates the kinematical limit for zero neutrino mass, the dotted line the limit for a mass of 32.6 MeV.

The mass dependence of this ratio is build into the model. The fit describes the experimental distributions well, except for the bin between 1.0 and 1.2 GeV. In this mass bin the model seems not to have enough freedom.

4.4 The τ decay into five pions

The τ decay into five charged pions has been analysed by ARGUS and HRS yielding an average branching ratio [16]:

$$B(\tau^- \to \pi^-\pi^-\pi^-\pi^+\pi^+\nu_\tau) = (0.056 \pm 0.016)\%$$

Recently the CLEO collaboration reported also the observation of the five-pion decay with two neutral pions with the surprisingly large branching ratio [26]:

$$B(\tau^- \to \pi^-\pi^-\pi^-\pi^0\pi^0\nu_\tau) = (0.48 \pm 0.04 \pm 0.04)\%$$

The decay proceeds mainly via an intermediate ω meson with the subprocess branching ratio:

$$B(\tau^- \to \pi^-\pi^0\omega\nu_\tau) = (0.39 \pm 0.04 \pm 0.04)\%$$

The ν_τ mass limit: Since the five-pion mass spectrum peaks at high masses near the kinematical limit it is well suited to study effects of a finite mass of the τ neutrino. The ARGUS collaboration found from the spectrum of five charged pions (20 events) based on an integrated luminosity of 390 pb^{-1} the limit [27]:

$$m_{\nu_\tau} < 31.0 \text{ MeV at 95\% c.l.}$$

Table 1: Branching ratios for Cabibbo allowed τ decays involving kaons.

channel	Br.	experiment
K^0K^-	$< 0.26\%$ (95% c.l.)	TPC 87
$K^-K^+\pi^-$	$(0.22^{+0.17}_{-0.11})\%$	DELCO 85
	$(0.14^{+0.09}_{-0.07})\%$	TPC (prel.)
	$(0.15 \pm 0.08)\%$	average
$K^{*0}K^-$	$(0.32 \pm 0.15)\%$	CLEO 90
	$(0.18 \pm 0.07)\%$	ARGUS (prel.)
	$(0.21 \pm 0.07)\%$	average 93

With the much higher luminosity of 1920 pb^{-1} and exploiting both five-pion channels (total of 113 events, Fig.10) the CLEO collaboration obtained [28]:

$$m_{\nu_\tau} \; < \; 32.6 \text{ MeV at } 95\% \text{ c.l.}$$

The fact that ARGUS obtains a better limit with much lower luminosity has been extensively discussed between the groups. It seems that ARGUS has had a lot of luck finding the events in the right kinematical range.

5 Tau decays involving kaons

5.1 Cabibbo allowed decays

Kaon pairs produced in Cabibbo allowed ($\sim \cos^2\theta_C$) decays can have isospin 0 or 1 and thus contribute to both vector and axial-vector currents. As the measurements and QCD predictions of these currents become more precise the knowledge of the kaon contributions to each current becomes increasingly important.

Table 1 summarizes the branching ratio measurements. The whole $K^-K^+\pi^-$ channel is consistent with being produced via $K^{*0}K^-$. In this case all possible K^*K channels together have the branching ratio:

$$B(\tau^- \;\rightarrow\; K^*K\nu_\tau) \;=\; (0.42 \pm 0.14)\% \,.$$

5.2 Cabibbo suppressed decays

As in the non-strange case the strange currents may be dominated by resonances:

spectral fct.	J^P	resonances
a_0^s	0^-	K^-
v_1^s	1^-	$K^*(892) \rightarrow K\pi$
v_0^s	0^+	$K_0^*(1430) \rightarrow K\pi$ (93%)
a_1^s	1^+	$K_1(1270)$, $K_1(1400) \rightarrow K\pi\pi$

Table 2: Branching ratios for Cabibbo suppressed τ decays (top: exclusive, bottom: semi-inclusive).

channel	Br.[%]	experiment
K^-	0.67 ± 0.23	average 92
	0.93 ± 0.20	DELPHI (prel.)
	1.00 ± 0.30	OPAL (prel.)
	0.85 ± 0.14	average 93
K^*	1.42 ± 0.18	average 92
	0.95 ± 0.19	ARGUS (prel.)
	1.71 ± 0.63	DELPHI (prel.)
	1.22 ± 0.13	average 93
$K^-\pi^-\pi^+$	$0.22^{+0.16}_{-0.13}$	DELCO 85
	0.33 ± 0.16	ARGUS (prel.)
	0.37 ± 0.09	average 93
$K_1(1270)$	$0.48^{+0.38}_{-0.32}$	TPC (prel.)
$K_1(1400)$	$0.66^{+0.35}_{-0.29}$	TPC (prel.)
$\sum K_1$	$1.14^{+0.50}_{-0.43}$	TPC (prel.)
$K + K^* + K_1$	3.21 ± 0.53	
$K^- \geq 0n$	1.67 ± 0.24	DELPHI (prel.)
$K^-\pi^-\pi^+ \geq 0n$	0.54 ± 0.15	TPC (prel.)
$K^0 h^- \geq 0n$	1.30 ± 0.30	HRS 88
$(-)\,K^0K^- \geq 0n$	0.23 ± 0.08	(from K^*K)[a]
$\sum$ incl. K's	3.28 ± 0.42	

[a] For strangeness counting (last row) the $K\overline{K} + X$ contribution has to be subtracted.

Table 3: Summary of exclusive τ hadronic branching ratios

channel	B [%]
leptons	35.3 ± 0.25
hadrons	64.7 ± 0.25
π^-/K^-	12.0 ± 0.25
$\pi^-\pi^0$	24.2 ± 0.3
$\pi^-\pi^-\pi^+$	8.0 ± 0.2
$\pi^-\pi^0\pi^0$	8.3 ± 0.4
$\pi^-\pi^-\pi^+\pi^0$	4.8 ± 0.4
$\pi^-\pi^0\pi^0\pi^0$	1.1 ± 0.2
$h^-\pi^0\pi^0\pi^0\pi^0$	0.15 ± 0.07
$\pi^-\pi^-\pi^+\pi^0\pi^0$	0.5 ± 0.1
$5\pi^\pm \geq 0\,n$	0.13 ± 0.03
$7h^\pm \geq 0\,n$	< 0.019
$\eta\pi^0\pi^-$	0.17 ± 0.03
K^0K^-	< 0.26
K^*K	0.42 ± 0.14
K^{*-}	1.22 ± 0.13
$K^-\pi^-\pi^+$	0.3 ± 0.1
total	61.5 ± 0.9
deficit	3.2 ± 1.0

The dominance of the two lowest resonances, K and K^*, in the corresponding currents a_0^s, v_1^s (much like the π and the ρ resonance in the non-strange currents) is well established. The branching ratios are given in Tab.2. The average of the kaon branching ratio

$$B(\tau^- \;\rightarrow\; K^-\nu_\tau) \;=\; (0.85 \pm 0.14)\%$$

is consistent with the theoretical prediction of $(0.664 \pm 0.005)\%$ (exp./theo. $= 1.28 \pm 0.21$). The K^* branching ratio

$$B(\tau^- \;\rightarrow\; K^{*-}\nu_\tau) \;=\; (1.22 \pm 0.13)\%$$

can be related to the non-strange vector current via the Das-Mathur-Okubo sum rule [29]:

$$\int_0^\infty \left[v_1(q^2) - v_1^s(q^2)\right] dq^2 \;=\; 0$$

The evaluation of this integral assuming that the vector currents are saturated by the $\pi\pi$ and $K\pi$ channels fulfils the sum rule:

$$\int_0^{m_\tau^2} \left[v_1(q^2) - v_1^s(q^2)\right] dq^2 \;/\; \int_0^{m_\tau^2} v_1^s(q^2) dq^2 = 0.11 \pm 0.13$$

There is no evidence yet for $K_0^*(1430)$ production via the scalar current (v_0^s) [30]. The $K_1(1270)$ and the $K_1(1400)$ are mixtures of the SU(3) eigenstates of the $^1P_1$ and $^3P_1$ multiplets, corresponding to the $a_1(1260)$ in the non-strange axial-vector current. The TPC group finds [30] that all $K^-\pi^-\pi^+$ final states could come from the two K_1's (see Tab.2).

The sum of the K, K^*, K_1 states in Tab.2 yields the the total strangeness branching ratio:

$$B_s = (3.21 \pm 0.53)\%$$

which compares well with the sum of all inclusively measured kaons (Tab.2) yielding $B_s = (3.28 \pm 0.42)\%$. This is somewhat larger than, but within the errors compatible with, the QCD prediction

$$R_{\tau,s}(\alpha_s) \cdot B_e = (2.6 \pm 0.1)\%.$$

6 Summing the branching ratios

Given the leptonic branching ratios to be $B_l = 35.3\pm0.25\%$ [31] we expect the total hadronic branching ratio to be $B_h = 64.7\pm0.25\%$. This has to be compared with the sum of the world averages of all measured exclusive branching ratios $B_h = 61.5 \pm 0.9\%$ yielding a deficit of $3.2\pm1.0\%$ (Tab.3). The error is obtained assuming uncorrelated errors of the single branching ratios which is probably a wrong assumption. The experimental survey of τ decays suggests that channels which have not been exclusively measured should have branching ratios below 10^{-3}.

The ALEPH group finds from their 'exclusive' analysis a sum of all τ branching ratios of $100.3 \pm 1.3\%$ [22]. With the ALEPH results the chances that the observed deficit points to 'New Physics' have certainly become much smaller. However, I do not believe that we have already reached a good understanding of all exclusive branching ratios. Taking for example the three-hadron final state, which is the most unsettled, even ALEPH alone has two inconsistent results for the $h^{\pm}2\pi^0$ branching ratio using either their 'exclusive' or 'quasi-exclusive' analysis method (see Fig.9). I am optimistic that these problems will be solved in the near future with high statistics results coming from the LEP experiments and CLEO.

From the exclusive branching ratios one finds for the contributions from the non-strange vector and axial-vector currents and from the strange currents:

$$\begin{aligned} B_V &= (30.25 \pm 0.57)\% \\ B_A &= (28.48 \pm 0.55)\% \\ B_s &= (3.21 \pm 0.53)\% \end{aligned}$$

It is interesting to note that the ratio $B_V/B_A = 1.06 \pm 0.04$ is, despite the overall deficit, consistent with the QCD expectation $B_V/B_A = 1.06 \pm 0.02$.

7 Probing the electro-weak coupling by the hadronic final state

The angular momentum of the hadronic final state in τ decays carries information on the Lorentz structure of the leptonic $\tau - \nu_\tau$ vertex. Like in the the Goldhaber experiment, where the measured polarisation of the emitted photon probes the ν_e helicity, the hadron polarisation in τ decays probes the ν_τ helicity.

ρ polarisation: In the decay

$$\tau^- \to \rho^- \nu_\tau$$

the ρ is polarised if the interaction is vector-like (V or A or both). This results in a ρ decay angular distribution $\sim 1 + b\cos^2\theta$. The ARGUS group has determined the coefficient b in the ρ rest system relative to the ρ direction of flight which agrees with the prediction of the KORALB Monte Carlo for a $V - A$ interaction of the $\tau - \nu_\tau$ vertex [32]:

$$\begin{aligned} b_{meas} &= 0.57 \pm 0.12 \\ b_{MC} &= 0.57 \pm 0.01 \end{aligned}$$

This result excludes that the ν_τ spin could be 3/2.

Since the sign of the ρ spin cannot be measured the analysis of the ρ channel is not sensitive to the V, A composition of the interaction. The V, A interference and hence parity violation has been observed in τ decays into three pions and in correlations between hadronically decaying τ pairs.

Parity violation in the three-pion final state: In the decay

$$\tau^- \to \pi^-\pi^-\pi^+\nu_\tau$$

a parity violating asymmetry has been observed [25] in the direction of the normal to the three-pion plane (in their c.m. system) relative to the τ direction. The orientation of the normal is defined by the cross product of the slower and the faster of the two like-sign pions. From this measurement the $V - A$ interference can be determined if one assumes that the three-pion final state is produced via $a_1(1260) \to \rho\pi$. In this case parity violation becomes observable as a result of an interference between the two possible amplitudes to make a ρ from the three pions. The asymmetry depends also on the relative angular momentum of the $\rho - \pi$ system. With the more precise determination of the S and D wave contributions described in section 4.3 the ARGUS collaboration was able to decrease the systematic error on the normalised product of the V and A couplings [20]:

$$\gamma_{AV} = \frac{2g_Ag_V}{g_A^2 + g_V^2} = 1.25 \pm 0.23^{+0.15}_{-0.08}$$

This is consistent with the Standard Model expectation $\gamma_{AV} = +1$ for left-handed τ neutrinos.

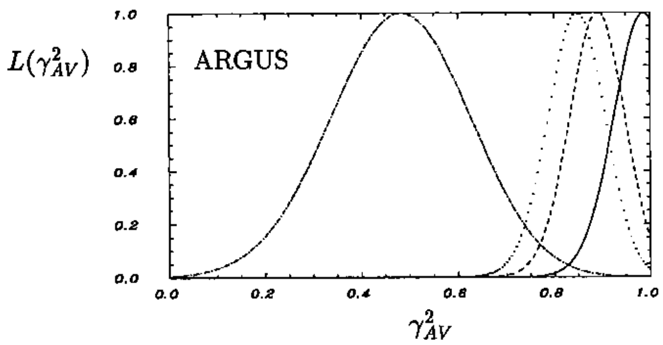


Figure 11: Likelihood as a function of γ^2_{AV} for $\rho^-\rho^+$ events (ARGUS). Shown are the curves obtained directly from the data (dashed), the simulated data (dotted) and simulated background curves (dashed-dotted) and the background corrected data curve (full).

Spin correlations in hadronic decays of τ pairs: In the electro-weak production process

$$e^-e^+ \rightarrow \gamma, Z^0 \rightarrow \tau^-\tau^+$$

the τ pairs have correlated spins, preferring opposite helicities. This leads in general to correlations between the decay products of both τ's.

The ARGUS group has exploited the angular correlations in the final state of the reaction:

$$e^-e^+ \rightarrow \tau^-\tau^+ \rightarrow \nu_\tau\pi^-\pi^0\overline{\nu}_\tau\pi^+\pi^0 \rightarrow \nu_\tau\rho^-\overline{\nu}_\tau\rho^+$$

to determine with high precision $|\gamma_{AV}|$. The matrix element of this reaction depends on 11 variables $\vec{\eta} = (\eta_1, ..., \eta_{11})$. The angles are the two $\pi\pi$ masses, the τ production and decay angles and the ρ decay angles. The matrix element can be expressed as [33]:

$$|\mathcal{M}(\vec{\eta})|^2 = A(\vec{\eta}) + \gamma^2_{AV} \cdot B(\vec{\eta}).$$

The explicite expressions A and B can be found in [33].

Experimentally, however, the kinematics can only be reconstructed up to a two-fold ambiguity because of the undetected neutrinos. The ARGUS group defined a likelihood function using the average matrix element for these two solutions. With a maximum likelihood fit to the data ($\sim$ 2000 events) γ^2_{AV} was determined (Fig.11). The fit result had still to be corrected for background, mainly from the $\pi^-\pi^0\pi^0$ final state where one π^0 was lost. The background has a non-negligible asymmetry since about 50% of the $\pi^-\pi^0$ from the a_1 decay form a ρ resonance with the same spin polarisation as in the direct ρ channel (because of the S-wave decay of the a_1). Monte Carlo studies showed that detector effects and radiative corrections, which are not included in the likelihood function have a very small influence. The preliminary result is:

$$|\gamma_{AV}| = 1.0^{+0.0}_{-0.032-0.029}$$

Together with the sign of γ_{AV} from the three-pion analysis this is a very precise determination of the handedness of the τ neutrino ($\gamma_{AV} = -h_{\nu\tau}$).

Angular correlations at low energies lead to energy - energy correlations when the τ's get high boosts. The ALEPH group has studied energy - energy correlations in decays of τ pairs where each τ decays either to a π or ρ:

$$\tau^-\tau^+ \rightarrow \rightarrow \pi^-\pi^+ (\pi^\pm\rho^\mp, \rho^-\rho^+)\, \nu_\tau\overline{\nu}_\tau$$

Using the energy variables,

$$z_\pi = \frac{2E_\pi - E_\tau}{E_\tau} \qquad z_\rho = \frac{|E_{\pi^\pm} - E_{\pi^0}|}{|\beta_\rho| \cdot E_\tau},$$

a likelihood fit to the two-dimensional distributions

$$\frac{1}{N}\frac{d^2N}{dz_1 dz_2} \sim F(z_1)F(z_2) + \xi_1\xi_2 G(z_1)G(z_2) - P_\tau\left[\xi_1 G(z_1)F(z_2) + \xi_2 G(z_2)F(z_1)\right]$$

was performed. In the Standard Model the parameters ξ_π and ξ_ρ are equal to the ν_τ helicity. Assuming $\xi_\pi = \xi_\rho$ and the Standard Model value for the τ polarisation P_τ one gets [34]:

$$h_{\nu\tau} = -0.96 \pm 0.07 \pm 0.04.$$

With these results from ARGUS and ALEPH we have now reached a remarkable precision in confirming the universality hypothesis also for the leptonic charged currents in τ decays.

8 Summary and conclusion

Hadronic τ decays have been studied in great detail in recent years. Important results are:

- The QCD analysis of the hadronic decay width by the ALEPH group yields a surprisingly precise determination of the strong coupling constant at the τ mass scale:

$$\alpha_s(m^2_\tau) = 0.330 \pm 0.046$$

- The comparison of the vector part of the τ hadronic currents with e^+e^- data confirms the CVC hypothesis.
- The τ data have made important contributions to the light quark spectroscopy, e.g. the measured pion formfactor reveals contributions from excited ρ states, the $a_1(1260)$ resonance parameters have been most precisely determined from τ data.
- No second classs currents and no exotic states have been observed in hadronic τ decays. The best upper limit on 2nd class currents comes now from CLEO:

$$B(\tau^- \rightarrow \eta\pi^-\nu_\tau) < 3.4 \cdot 10^{-4}, \; 95\%\ c.l.$$

- The Lorentz structure of the τ leptonic charged current has been determined using the hadronic states as analysers. From studies of angular correlations of hadronic final states by ARGUS and ALEPH the helicity of the τ neutrino was found to be left-handed with a precision of about 4%. The average of both (preliminary) results is:

$$h_{\nu\tau} = -0.99 \pm 0.04.$$

- From five-pion events near the kinematical limit ARGUS and CLEO obtained upper limits for for the τ neutrino mass:

$$m_{\nu_\tau} < 31.0 \text{ MeV}, \quad 95\%\ c.l. \quad \text{(ARGUS)}$$
$$m_{\nu_\tau} < 32.6 \text{ MeV}, \quad 95\%\ c.l. \quad \text{(CLEO)}$$

- The sum of all known exclusive leptonic and hadronic branching ratios is $(3.2 \pm 1.0)\%$ below 100%. Given the overall precision of the experiments this discrepancy remains puzzling. The quasi-exclusive analysis by ALEPH shows no such deficit. However, from this analysis it seems also not clear which of the exclusive channels are causing the problems (though the two- and three-pion channels are most suspicious).
- Despite the deficit, the ratio of the vector and axial-vector currents is consistent with the QCD prediction as well as the branching ratio into states with strangeness ($B_s = (3.21 \pm 0.53)\%$).

References

[1] M. Perl, Rep. Prog. Phys. 55 (1992) 653;
C. Kiesling, in 'High Energy Electron-Positron Physics', eds. A. Ali, P. Söding, World Scientific (1989) 177;
B.C. Barish and R. Stroynowski, Phys. Rep. 157 (1988) 1.

[2] Proceedings of the 2^{nd} Workshop on Tau Lepton Physics, Columbus (Ohio, USA), ed. K.K. Gan, World Scientific (1992).

[3] F. Le Diberder and A. Pich, Phys. Lett. B289 (1992) 165.

[4] OPAL Collaboration, P.D. Acton et al., Z. Phys. C55 (1992) 1;
ALEPH Collaboration, D. Buskulic et al., Phys. Lett. B307 (1993) 209.

[5] S. Narison and A. Pich, CERN-TH-6769-93 (1993).

[6] Tran N. Truong, Phys. Rev. D47 (1993) 3999.

[7] E. Braaten, S. Narison and A. Pich, Nucl. Phys. B373 (1992) 581.

[8] S. Bethke, Proc. of the 26th International Conference on High Energy Physics, Dallas, (1992) 81.

[9] The $h^-\pi^0$ branching ratio reported by CLEO at the Ohio Workshop [2] was corrected in summer 1993 because of a normalisation error (communicated by D.H. Miller).

[10] Review of particle properties (Particle Data Group), Phys. Lett. B239 (1990) 1-516.

[11] J.H. Kühn and A. Santamaria, Z. Phys. C48 (1990) 445.

[12] S.W. Snow, in [2].

[13] A. Donnachie, Proc. of the HADRON '91 Conf., College Park, MD (USA), 1991.

[14] B. Spaan, in [2].

[15] Argus Collaboration, H. Albrecht et al., Phys. Lett. B260 (1991) 259.

[16] Review of particle properties (Particle Data Group), Phys. Rev. D45 (1992).

[17] CLEO Collaboration, M. Artuso et al., Phys. Rev. Lett. 69 (1992) 3278.

[18] A. Pich, Phys. Lett. B196 (1987) 561.

[19] W.J. Marciano and A.Sirlin, Phys. Rev. Lett. 61 (1988) 1815.

[20] Argus Collaboration, H. Albrecht et al., Z. Phys. C58 (1993) 61.

[21] N. Isgur, C. Morningstar and C. Reader, Phys. Rev. D39 (1989) 1357.

[22] S.W. Snow, in [2]; M. Davier, in [2]; J. Lefrancois, private communication.

[23] F. Matorras, this conference; D. Reid, private communication.

[24] M. Bowler, Phys. Lett. B209 (1988) 99.

[25] Argus Collaboration, H. Albrecht et al., Phys. Lett. B250 (1990) 164.

[26] CLEO Collaboration, D. Bortoletto, et al., CLNS-93-1232 (1993).

[27] Argus Collaboration, H. Albrecht et al., Phys. Lett. B292 (1992) 221.

[28] CLEO Collaboration, D. Cinabro et al., Phys. Rev. Lett. 70 (1993) 3700.

[29] T. Das, V.S. Mathur and S. Okubo, Phys. Rev. Lett. 18 (1967) 761.

[30] M. Ronan, in [2] and private communication.

[31] M. Roney, this conference.

[32] Argus Collaboration, H. Albrecht et al., Z. Phys. C56 (1992) 339.

[33] H. Thurn and H. Kolanoski, Z. Phys. C60 (1993) 277.

[34] M. Schmitt, XXXVIIIth Rencontre de Moriond (1993).