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On the existence of a first order phase transition at small vacuum angle θ in the CP^3 model*

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We examine the phase structure of the CP^3 model as a function of θ in the weak coupling regime. It is shown that the model has a first order phase transition at small θ. We pay special attention to the extrapolation of the data to the infinite volume. It is found that the critical value of θ decreases towards zero as β is taken to infinity.

1. INTRODUCTION

One of the characteristic features of QCD is the existence of an integer valued topological charge Q. This gives rise to an additional term in the action,

$$S_\theta = S + i\theta Q,\ \theta \in [0, 2\pi), \tag{1}$$

where each value of θ results in a different vacuum state. A priori θ is a free parameter. Since the vacuum is an eigenstate of CP only for $\theta = 0, \pi$ and no CP violation is observed in the strong interactions, θ must, however, be very close to zero.

Is there any dynamical reason for that? There are indications that confinement at $\theta = 0$ is caused by the condensation of color magnetic monopoles [1]. In the θ vacuum these monopoles acquire an electric charge $\theta/2\pi$ [2]. One may then argue [3] that a rich phase structure will emerge as a function of θ. In the weak coupling limit it is even conceivable that confinement will be limited to $\theta = 0$ only which would solve the problem rather elegantly.

Since many of the characteristic features of QCD are shared by the CP^{N-1} models, it suggests itself to examine these models first. We have chosen the CP^3 model. For smaller N one may run into the problem of dislocations [4], while for large N the signal one hopes to see will probably die away like $1/N$ [5].

2. LATTICE SIMULATION

We use the action

$$S = -2\beta \sum_{x,\mu} \mathrm{Tr} P(x) P(x+\hat{\mu}), \tag{2}$$

where $P(x) = z(x)z^\dagger(x)$, $\mathrm{Tr}P(x) = |z(x)|^2 = 1$, and we employ periodic boundary conditions. The action can be rewritten in terms of the complex scalar field $z(x)$ and a composite abelian gauge field interacting minimally with each other. The associated $U(1)$ link field has the form

$$U_\mu(x) = \frac{z^\dagger(x) \cdot z(x+\hat{\mu})}{|z^\dagger(x) \cdot z(x+\hat{\mu})|}\ ,\ \mu = 0, 1. \tag{3}$$

The topological charge is obtained from (3) by forming plaquette fields $U_p = \mathrm{e}^{\mathrm{i}F_{01}}$, $-\pi < F_{01} \leq \pi$ and writing

$$Q = \frac{1}{2\pi} \sum_p F_{01}. \tag{4}$$

As can easily be seen, Q is an integer.

In this talk we will restrict ourselves to the calculation of the partition function and a few quantities which derive from it. A full account of our work will be given elsewhere [6]. The partition function reads

$$Z(\theta) = \sum_Q \mathrm{e}^{\mathrm{i}\theta Q} p(Q), \tag{5}$$

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$$p(Q) = \frac{\int [\mathcal{D}z\mathcal{D}z^\dagger]_Q \delta(|z|^2-1)\mathrm{e}^{-S}}{\int \mathcal{D}z\mathcal{D}z^\dagger \delta(|z|^2-1)\mathrm{e}^{-S}}, \qquad (6)$$

where the subscript Q indicates that the path integration is restricted to the given charge sector. In order to compute $p(Q)$ we proceed as follows [7]. We divide the phase space into overlapping sets of five consecutive charges. In each of these sets the z fields are updated by a combination of Metropolis and overrelaxation steps, $z' = \exp(\mathrm{i}\phi_a\lambda_a)z$, where the $U(4)$ generators λ_a are selected randomly [8]. This is supplemented by an additional acceptance criterion: if the new charge is in the same set the configuration is accepted and the new charge recorded; if the new charge is outside the set the change is rejected and the old charge recorded. Furthermore, a trial charge distribution which is approximately equal to the true distribution is incorporated.

So far we have done simulations at two values of β on various lattice volumes. We have chosen $\beta = 2.5$, $V = 28^2, 32^2, 38^2, 48^2$, and $\beta = 2.7$, $V = 46^2, 56^2, 64^2, 72^2$. The correlation lengths at these two β values are [9] $\xi \approx 4.5$ and $\xi \approx 8.8$, respectively. On these lattices we are able to follow $p(Q)$ over more than twenty orders of magnitude.

3. RESULTS

The first quantity we looked at is the free energy $F(\theta)$ which is given by

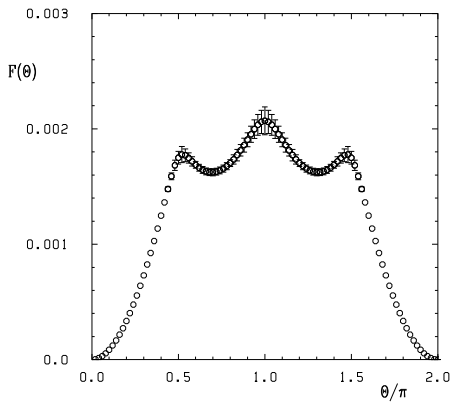

Figure 1. The free energy $F(\theta)$.

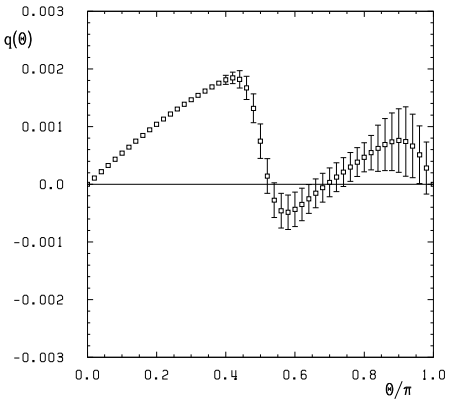

Figure 2. The average topological charge $q(\theta)$. In the second half of the θ interval not shown here $q(\theta) = -q(2\pi - \theta)$.

$$Z(\theta) = \mathrm{e}^{-V F(\theta)}. \qquad (7)$$

A first order phase transition will manifest itself in a kink in $F(\theta)$. In Fig. 1 we show $F(\theta)$ on the $V = 64^2$ lattice at $\beta = 2.7$. The error bars have been computed by a jackknife method. We see two kinks, one at $\theta \approx 0.5\pi$ (and a corresponding one at $\theta \approx 1.5\pi$) and one at $\theta = \pi$. This should be compared with the prediction of the dilute instanton gas approximation, $F(\theta) \propto 1 - \cos\theta$, and the prediction of the $1/N$ expansion [5], $F(\theta) \propto \theta^2$. Clearly, our results do not agree with either of the two. In the following we shall mainly be interested in the phase transition at small θ.

The first derivative of $F(\theta)$ is equal to the average topological charge $q(\theta)$:

$$\frac{\mathrm{d}F(\theta)}{\mathrm{d}\theta} \equiv q(\theta) = -\frac{\mathrm{i}}{V}\frac{1}{Z(\theta)}\sum_Q \mathrm{e}^{\mathrm{i}\theta Q}\, Q\, p(Q). \qquad (8)$$

At a first order phase transition this quantity must be discontinuous. In Fig. 2 we show $q(\theta)$ for the $V = 64^2$ lattice at $\beta = 2.7$. We see that $q(\theta)$ increases almost linearly with θ until it reaches the location of the phase transition where it jumps to a value which is close to zero. According to (4), $q(\theta)$ can be interpreted as a background electric field. In Ref. [6] we shall argue that at the phase transition the charged

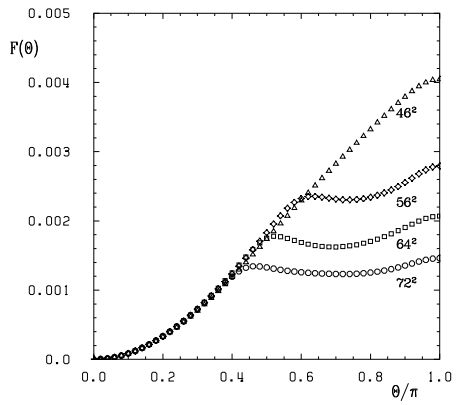

Figure 3. The free energy $F(\theta)$. The error bars are omitted in order to keep the figure legible. They are comparable to the ones in Fig. 1.

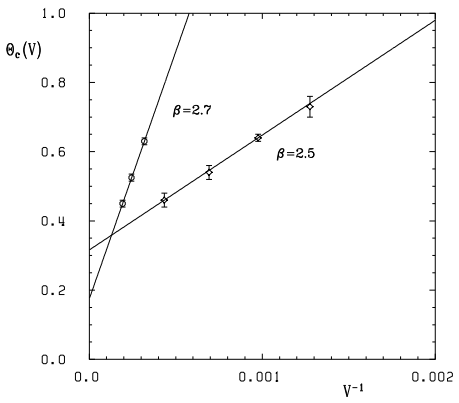

Figure 4. The critical value of θ.

scalars (associated with the field $z(x)$) are liberated which causes the background electric field to collapse.

Let us now look at the volume dependence of the free energy. In Fig. 3 we show $F(\theta)$ for four different volumes at $\beta = 2.7$. We see that the location of the phase transition moves to smaller values of θ as V is increased. Below the phase transition all data points fall on a universal curve. If the lattice volume is too small we see no signal of a phase transition [10].

We denote the critical value of θ by θ_c and identify it with the local minimum of $Z(\theta)$. The volume dependence of θ_c is linked to the order of the phase transition. For a first order transition we expect

$$\theta_c(V) - \theta_c(\infty) \propto V^{-1}. \qquad (9)$$

In Fig. 4 we show θ_c as a function of V^{-1} for $\beta = 2.5$ and 2.7. In both cases our data fall on a straight line in accordance with a first order phase transition. This allows us to extrapolate the data to the infinite volume.

We obtain $\theta_c(\infty) = 0.32(2)$ at $\beta = 2.5$ and $\theta_c(\infty) = 0.18(3)$ at $\beta = 2.7$. It is perhaps not surprising that the critical value of θ depends strongly on β. Note that $\theta_c = \pi$ in the strong coupling limit [11].

4. CONCLUSIONS

We have found a novel, first order phase transition in θ with $\theta_c(\infty)$ decreasing towards zero as we approach the continuum limit. This transition is consistent with a deconfining phase transition [6].

If somebody wants to repeat the calculation he should start with $\beta = 2.7, V = 56^2$.

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