DEUTSCHES ELEKTRONEN-SYNCHROTRON

DESY 93-189
December 1993

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ISSN 0418-9833

NOTKESTRASSE 85 - 22603 HAMBURG

DESY 93-189
December 1993

ISSN 0418-9833

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Abstract
It is shown that synchrotron sidebands of linear betatron coupling resonances which limit the performance of LEP and DORIS III are excited by a vertical dispersion at sextupoles without any linear coupling. Synchrotron sidebands of integer and half integer resonances with a distance of half the synchrotron frequency are excited by a horizontal dispersion at sextupoles.

1. Introduction

Satellite resonances or synchro-betatron resonances appear mostly as sidebands of an integer resonance and are characterized by

$$Q_{x,y} + mQ_s = n \qquad (1)$$

where Q_x, Q_y, and Q_s are the horizontal and vertical betatron frequency and the synchrotron frequency in units of the revolution frequency, respectively. m and n are integers. These synchro-betatron resonances are excited by a horizontal or vertical dispersion in an accelerating cavity [1] or by transverse fields with longitudinal variation [2, 3] which are also produced in accelerating cavities. In the past these resonances have limited the currents and the luminosity in PETRA [4] and have been a serious problem in SPEAR [5] and in LEP [6].

Satellite resonances can also appear as sidebands of coupling resonances of horizontal and vertical betatron oscillations and satisfy the relation

$$Q_x \pm Q_y + mQ_s = n. \qquad (2)$$

Sidebands of coupling resonances have been observed in DORIS III [7] and in LEP [8] and have limited the operation of these machines. It has already been shown that these resonances cannot be excited by a dispersion in a cavity together with linear coupling [9] and there is no investigation showing that they can be excited by transverse fields with longitudinal variation.

In this note we will prove that satellites of the betatron coupling resonances can be excited by a vertical dispersion in sextupoles without any linear coupling element. Since DORIS III has a large vertical dispersion due to the vertical bending near the interaction points and LEP has a relatively large vertical dispersion due to the large number of quadrupoles and sextupoles, which can be a source of spurious vertical dispersion, it is understandable that these two machines suffer from these synchrotron sidebands.

It will also be shown that horizontal and vertical satellites of integer and half integer resonances with a distance of half the synchrotron frequency are excited by a horizontal dispersion at sextupoles. These resonances have already been observed with tracking studies [10] but they are not of practical importance since an operating point so close to an integer or half integer has several other disadvantages.

A third type of satellite resonances excited by sextupoles has a distance of twice the synchrotron frequency from integer resonances. These satellites are excited by a combination of horizontal and vertical dispersions and coincide with those caused by other mechanisms as mentioned above.

2. A Single Sextupole

The changes of the horizontal and vertical betatron angles at a sextupole are given by

$$\begin{aligned}\Delta x'_\beta &= k_s(x^2-y^2)\\ &= k_s((x_\beta+D_x\delta)^2-(y_\beta+D_y\delta)^2) \qquad (3)\\ \Delta y'_\beta &= -2k_s\,x\,y\\ &= -2k_s(x_\beta+D_x\delta)(y_\beta+D_y\delta) \qquad (4)\end{aligned}$$

where δ is the relative energy deviation $\Delta E/E$, k_s is the integrated sextupole strength, and D_x and D_y are the horizontal and vertical dispersion at the sextupole, respectively. We consider the kicks produced by the sextupoles as a perturbation of the emittances $\epsilon_{x,y}$ which are given by

$$\epsilon_x\beta_x = x_\beta^2+(x'_\beta\beta_x-x_\beta\beta'_x/2)^2 \qquad (5)$$
$$\epsilon_y\beta_y = y_\beta^2+(y'_\beta\beta_y-y_\beta\beta'_y/2)^2 \qquad (6)$$

The changes of the emittances at a sextupole are then given by

$$\Delta\epsilon_x = 2(x'_\beta\beta_x-x_\beta\beta'_x/2)\Delta x'_\beta \qquad (7)$$
$$\Delta\epsilon_y = 2(y'_\beta\beta_y-y_\beta\beta'_y/2)\Delta y'_\beta \qquad (8)$$

With

$$x_\beta = \sqrt{\epsilon_x\beta_x}\sin\phi_x\,, \qquad x'_\beta\beta_x-x_\beta\beta'_x/2 = \sqrt{\epsilon_x\beta_x}\cos\phi_x\,.$$
$$y_\beta = \sqrt{\epsilon_y\beta_y}\sin\phi_y\,, \qquad y'_\beta\beta_y-y_\beta\beta'_y/2 = \sqrt{\epsilon_y\beta_y}\cos\phi_y$$
$$\delta = \hat\delta\sin\phi_s$$

one obtains

$$\begin{aligned}\Delta\epsilon_x &= 2k_s\sqrt{\epsilon_x\beta_x}\cos\phi_x\big(\epsilon_x\beta_x\sin^2\phi_x+2\sqrt{\epsilon_x\beta_x}D_x\hat\delta\sin\phi_x\sin\phi_s\\ &\quad -\epsilon_y\beta_y\sin^2\phi_y-2\sqrt{\epsilon_y\beta_y}D_y\hat\delta\sin\phi_y\sin\phi_s+(D_x^2-D_y^2)\hat\delta^2\sin^2\phi_s\big) \qquad (9)\end{aligned}$$

and

$$\begin{aligned}\Delta\epsilon_y &= -4k_s\sqrt{\epsilon_y\beta_y}\cos\phi_y\big(\sqrt{\epsilon_x\epsilon_y\beta_x\beta_y}\sin\phi_x\sin\phi_y\\ &\quad +(\sqrt{\epsilon_x\beta_x}D_y\sin\phi_x+\sqrt{\epsilon_y\beta_y}D_x\sin\phi_y)\,\hat\delta\sin\phi_s+D_xD_y\hat\delta^2\sin^2\phi_s\big) \qquad (10)\end{aligned}$$

The phases $\phi_{x,y,s}$ vary with time as $2\pi f_o Q_{x,y,s}t+\alpha_{x,y,s}$ where f_o is the revolution frequency and $\alpha_{x,y,s}$ are constant phases. The average values of $\Delta\epsilon_x$ and $\Delta\epsilon_y$ for many revolution are then given by

$$\begin{aligned}\langle\Delta\epsilon_x\rangle &= k_s\sqrt{\epsilon_x\beta_x}\Big(\epsilon_y\beta_y\big(\cos(2\phi_y+\phi_x)+\cos(2\phi_y-\phi_x)\big)/2-\epsilon_x\beta_x\cos(3\phi_x)/2\\ &\quad +\sqrt{\epsilon_x\beta_x}D_x\hat\delta\big(\cos(2\phi_x-\phi_s)-\cos(2\phi_x+\phi_s)\big)\\ &\quad +(D_y^2-D_x^2)\hat\delta^2\big(\cos(\phi_x+2\phi_s)+\cos(\phi_x-2\phi_s)\big)/2\\ &\quad +\sqrt{\epsilon_y\beta_y}D_y\hat\delta\big(\cos(\phi_x+\phi_y+\phi_s)+\cos(\phi_x-\phi_y-\phi_s)-\cos(\phi_x+\phi_y-\phi_s)\\ &\quad -\cos(\phi_x-\phi_y+\phi_s)\big)\Big) \qquad (11)\end{aligned}$$

and

$$\begin{aligned}\langle\Delta\epsilon_y\rangle &= k_s\sqrt{\epsilon_y\beta_y}\Big(\sqrt{\epsilon_x\epsilon_y\beta_x\beta_y}\big(\cos(\phi_x+2\phi_y)-\cos(\phi_x-2\phi_y)\big)\\ &\quad +\sqrt{\epsilon_y\beta_y}D_x\hat\delta\big(\cos(2\phi_y+\phi_s)-\cos(2\phi_y-\phi_s)\big)\\ &\quad +D_xD_y\hat\delta^2\big(\cos(\phi_y+2\phi_s)+\cos(\phi_y-2\phi_s)\big)\\ &\quad +\sqrt{\epsilon_x\beta_x}D_y\hat\delta\big(\cos(\phi_x+\phi_y+\phi_s)-\cos(\phi_x-\phi_y-\phi_s)-\cos(\phi_x+\phi_y-\phi_s)\\ &\quad +\cos(\phi_x-\phi_y+\phi_s)\big)\Big) \qquad (12)\end{aligned}$$

Eqs.(11) and (12) show that the third order resonance $3Q_x = n$, the two betatron coupling resonances $Q_x \pm 2Q_y = n$, and the following satellite resonances can be excited

horizontal satellites

$2Q_x \pm Q_s = n$	excited by D_x
$Q_x \pm 2Q_s = n$	excited by $D_x^2-D_y^2$

vertical satellites

$2Q_y \pm Q_s = n$	excited by D_x
$Q_y \pm 2Q_s = n$	excited by D_xD_y

satellites of the betatron coupling resonances

$Q_x \pm Q_y \pm Q_s = n$	excited by D_y

Eqs.(11) and (12) show also that both emittances increase or decrease at the same time on both satellites of the sum resonance, i. e. on $Q_x+Q_y\pm Q_s = n$, whereas one emittance increases and the other decreases on both satellites of the difference resonance, i. e. on $Q_x-Q_y\pm Q_s = n$. There is no difference between the two satellites of one coupling resonance since the synchrotron oscillation is considered to be constant.

In this investigation we assume that the synchrotron oscillation energy is much larger than the betatron oscillation energies, so that the synchrotron coordinates are not changed by the betatron coordinates. For a more precise calculation one has to take into account the path lengthening due to the betatron oscillation [4] which may be neglected here.

The rise times for a satellite of a coupling resonance are

$$\frac{1}{\tau_{1x,y}} = \frac{1}{\epsilon_{x,y}}\frac{d\epsilon_{x,y}}{dt} = f_o|k_sD_y|\sqrt{\beta_x\beta_y}\sqrt{\frac{\epsilon_{y,x}}{\epsilon_{x,y}}}\,\hat\delta\,. \qquad (13)$$

The rise times for the satellites $2Q_{x,y}\pm Q_s = n$ are given by

$$\frac{1}{\tau_{2x,y}} = f_o|k_sD_x|\beta_{x,y}\,\hat\delta \qquad (14)$$

and the rise times for $Q_{x,y} \pm 2Q_s = n$ are

$$\frac{1}{\tau_{3x}} = f_o|k_s(D_x^2 - D_y^2)|\sqrt{\beta_x/\epsilon_x}\;\hat{\delta}^2/2 \tag{15}$$

$$\frac{1}{\tau_{3y}} = f_o|k_s D_x D_y|\sqrt{\beta_y/\epsilon_y}\;\hat{\delta}^2 \tag{16}$$

3. Arbitrary sextupole distribution

In order to calculate the rise times for N sextupoles one has to add the kicks (Eqs.(9) and (10)) for one revolution taking into account the phase advance between the sextupoles. Since the sum depends on the initial phase and we are interested in the maximum of the sum, which occurs for one special phase, we take the cosine and the sine with the same but arbitrary initial phase. The rise times for the satellites of the betatron coupling resonances are then given by

$$\frac{1}{\tau_{1x,y}} = f_o\sqrt{C_1^2 + S_1^2}\sqrt{\frac{\epsilon_{y,x}}{\epsilon_{x,y}}}\;\hat{\delta} \tag{17}$$

with

$$C_1 = \sum_{n=1}^{N} k_{sn} D_{yn}\sqrt{\beta_{xn}\beta_{yn}}\cos(\phi_{xn} \pm \phi_{yn})$$

$$S_1 = \sum_{n=1}^{N} k_{sn} D_{yn}\sqrt{\beta_{xn}\beta_{yn}}\sin(\phi_{xn} \pm \phi_{yn})$$

In a similar way one obtains, for the satellites with a distance of $Q_s/2$ from integer and half integer resonances,

$$\frac{1}{\tau_{2x,y}} = f_o\sqrt{C_{2x,y}^2 + S_{2x,y}^2}\;\hat{\delta} \tag{18}$$

with

$$C_{2x,y} = \sum_{n=1}^{N} k_{sn} D_{xn}\beta_{x,yn}\cos(2\phi_{x,yn})$$

$$S_{2x,y} = \sum_{n=1}^{N} k_{sn} D_{xn}\beta_{x,yn}\sin(2\phi_{x,yn})$$

and for satellites with a distance of $2Q_s$ from integer resonances

$$\frac{1}{\tau_{3x,y}} = f_o\sqrt{(C_{3x,y}^2 + S_{3x,y}^2)/\epsilon_{x,y}}\;\hat{\delta}^2 \tag{19}$$

with

$$C_{3x} = \sum_{n=1}^{N} k_{sn}(D_{xn}^2 - D_{yn}^2)\sqrt{\beta_{xn}}\cos(\phi_{xn})/2$$

$$C_{3y} = \sum_{n=1}^{N} k_{sn} D_{xn} D_{yn}\sqrt{\beta_{yn}}\cos(\phi_{yn})$$

$$S_{3x} = \sum_{n=1}^{N} k_{sn}(D_{xn}^2 - D_{yn}^2)\sqrt{\beta_{xn}}\sin(\phi_{xn})/2$$

$$S_{3y} = \sum_{n=1}^{N} k_{sn} D_{xn} D_{yn}\sqrt{\beta_{yn}}\sin(\phi_{yn})$$

As an example the 24 sextupoles in DORIS III give, with a 4 cm optic and for $\epsilon_y = \epsilon_x/10$, the following rise times for the satellites in the region around the working point

$Q_x + Q_y \pm Q_s = 11$:	$\tau_x = 1.34$ msec
	$\tau_y = 0.13$ msec
$2Q_x - Q_s = 14$:	$\tau_x = 0.033$ msec
$2Q_y + Q_s = 8$:	$\tau_y = 0.0068$ msec
$Q_x - 2Q_s = 7$:	$\tau_x = 0.11$ msec
$Q_y + 2Q_s = 4$:	$\tau_y = 0.12$ msec

All rise times are shorter than the damping time of 2 msec at 5.3 GeV. The rms-values of the horizontal and vertical dispersion at the sextupoles are 1.27 m and 0.14 m, respectively.

The rise times were calculated for amplitudes of one standard deviation for all three directions. Here one has to take into account that only the rise times on $2Q_{x,y} \mp Q_s = n$ give an exponential increase, but the other rise times depend on the amplitudes and change with increasing amplitude according to Eqs.(17) and (19).

If the vertical dispersion is produced by machine errors it is difficult to calculate the rise time with Eq.(14) since it is usually not possible to measure the dispersion at all sextupoles. In this case one can estimate the rise times by assuming that the vertical spurious dispersion is distributed randomly at the sextupoles with a given rms value which can be estimated by measurements at other positions, and one obtains approximately.

$$\frac{1}{\tau_{1x,y}} = f_o\sqrt{N\langle k_s^2\beta_x\beta_y\rangle\langle D_y^2\rangle}\sqrt{\frac{\epsilon_{y,x}}{\epsilon_{x,y}}}\;\hat{\delta} \tag{20}$$

In a similar way one can estimate the rise times of the other resonances.

4. Computer Simulations

The simulations were done for a single sextupole per revolution. At the sextupole the changes of the betatron coordinates are given by Eqs.(3) and (4), whereas the synchrotron coordinates remain constant. For the whole revolution the linear transformation of the four betatron coordinates $z_1 = x_\beta$, $z_2 = x'_\beta\beta_x - x_\beta\beta'_x/2$, $z_3 = y_\beta$, and $z_4 = y'_\beta\beta_y - y_\beta\beta'_y/2$ is given by the matrices

$$M_{x,y} = \begin{pmatrix} \cos(2\pi Q_{x,y}) & \sin(2\pi Q_{x,y}) \\ -\sin(2\pi Q_{x,y}) & \cos(2\pi Q_{x,y}) \end{pmatrix} \qquad (21)$$

The synchrotron oscillation can be described in terms of the two coordinates $z_5 = s/\sigma_s$ and $z_6 = \delta/\sigma_\delta$ where σ_s and σ_δ are the standard deviations of the longitudinal particle distribution and the relative energy distribution, respectively. The change of the synchrotron coordinates per revolution is then given by

$$\Delta z_5 = -2\pi Q_s z_6 \qquad (22)$$

$$\Delta z_6 = \frac{Q_s\lambda}{\cos\psi_s\sigma_s}(\sin(2\pi z_5\sigma_s/\lambda + \psi_s) - \sin\psi_s) \qquad (23)$$

where λ is the wave length of the rf frequency, and ψ_s is equilibrium phase. If nonlinearities of the synchrotron oscillation are taken into account, as the sine like synchrotron potential or distortions of the potential due to longitudinal wake fields, resonances with $m > 1$ in Eq.(2) can be excited. For small $z_5\sigma_s/\lambda$ one gets linear synchrotron oscillations and Eq.(23) simplifies to

$$\Delta z_6 = 2\pi Q_s z_5 \qquad (24)$$

The motion of 12 particles with different initial phases were simulated for 10000 revolutions with the following parameters: $k_s = 4 \times 10^{-6}\text{m}^{-2}$, $\sigma_s/\lambda = 0.1$ and $\psi_s = 0.5$. The horizontal and vertical dispersion are assumed to give equal synchrotron and betatron beam sizes at the sextupoles.

Fig. 1 shows for the case of a horizontal dispersion the satellites with a distance of half the synchrotron frequency from the integer and half integer resonance and the satellite with a distance of twice the synchrotron frequency from the integer resonance. Also the satellites with a distance of the simple synchrotron frequency from the integer and half integer resonance and the satellite with a distance of three times the synchrotron frequency from the integer resonance can be seen, which are caused by the nonlinearity of the synchrotron oscillation. They vanish if Eq.(25) is replaced by Eq.(24).

Fig. 2 shows for the case of a vertical dispersion the satellites of the betatron sum resonance $Q_x + Q_y = 1$ in the region $0 \le Q_x \le 0.5$ and $0.5 \le Q_y \le 1$ of the tune diagram. The satellites with a distance of $2Q_s$ from the coupling resonance are relatively weak since only the nonlinearities of the natural synchrotron potential have been applied. They will be stronger with wake fields produced by the bunch itself. The strongest resonance is $Q_x + 2Q_y = 0$, whereas the difference resonance $Q_x - 2Q_y = 0$ is very weak.

Fig. 3 shows, again for a vertical dispersion, the satellites of the betatron difference resonance $Q_x - Q_y = 0$ in the region $0 \le Q_x \le 0.5$ and $0 \le Q_y \le 0.5$ of the tune diagram. These satellites are considerably weaker than the satellites of the sum resonance. Thus the strength of the satellites behaves similarly to the strength of the betatron coupling resonances as already mentioned with respect to Eq.(11) and (12).

All satellites which are caused by the nonlinearity of the synchrotron oscillation are current dependent, since the nonlinearity can increase considerably with longitudinal wake fields produced by the bunch.

References

[1] A. Piwinski, A. Wrulich, DESY 76/07 (1976)

[2] N. A. Vinokurov et al., Intern. Conf. on High Energy Accel., II, 254 (1977)

[3] R. M. Sundelin, 1979 Particle Accel. Conf., IEEE Trans. V.NS-26, No.3, 3604 (1979)

[4] A. Piwinski, 11th Intern. Conf. on High-Energy Accel., Geneva, p. 636 (1980)

[5] SPEAR Group, 1975 Particle Accel. Conf., IEEE Trans. NS-22,1366 (1975)

[6] J. M. Jowett et al., Proc. of the 2nd Europ. Acc. Conf., Nice, France, p. 1612(1990)

[7] W. Decking, 6th Advanced ICFA Beam Dynamics Workshop, Madeira, 1993

[8] S. Meyers, 6th Advanced ICFA Beam Dynamics Workshop, Madeira, 1993

[9] A.W. Chao, A. Piwinski, Int. Report DESY PET-77/09 (1977)

[10] W. Mooshammer, 6th Advanced ICFA Beam Dynamics Workshop, Madeira, 1993

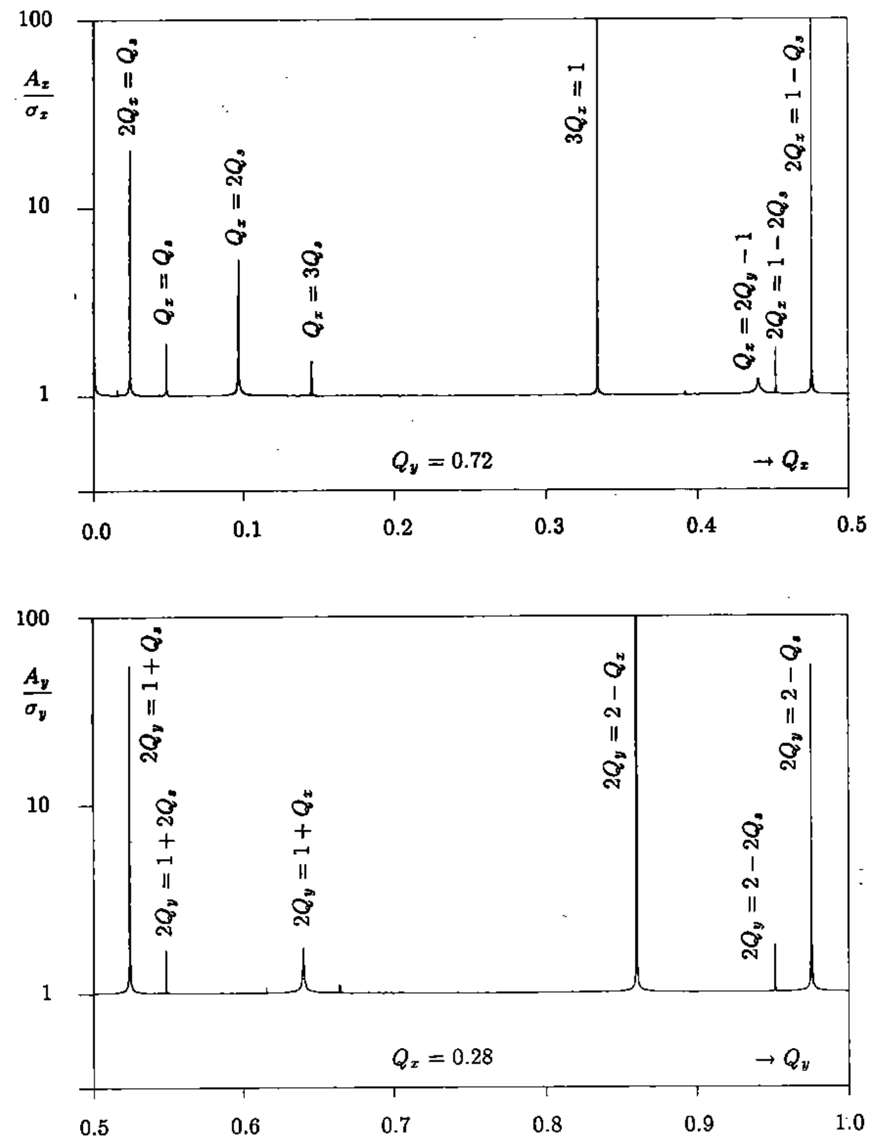

Fig. 1 Maximum horizontal and vertical betatron amplitudes as a function of the tune for $D_x > 0$ and $D_y = 0$.

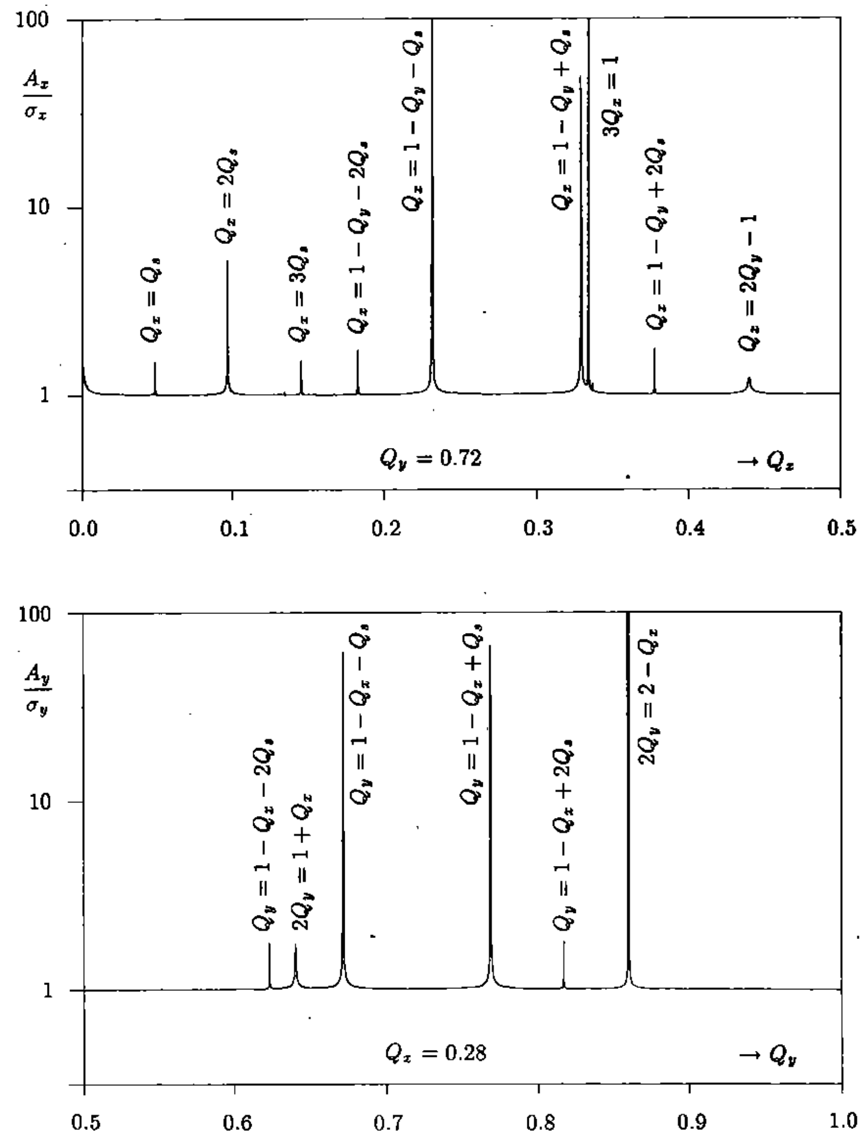

Fig. 2 Maximum horizontal and vertical betatron amplitudes as a function of the tune for $D_x = 0$ and $D_y > 0$.

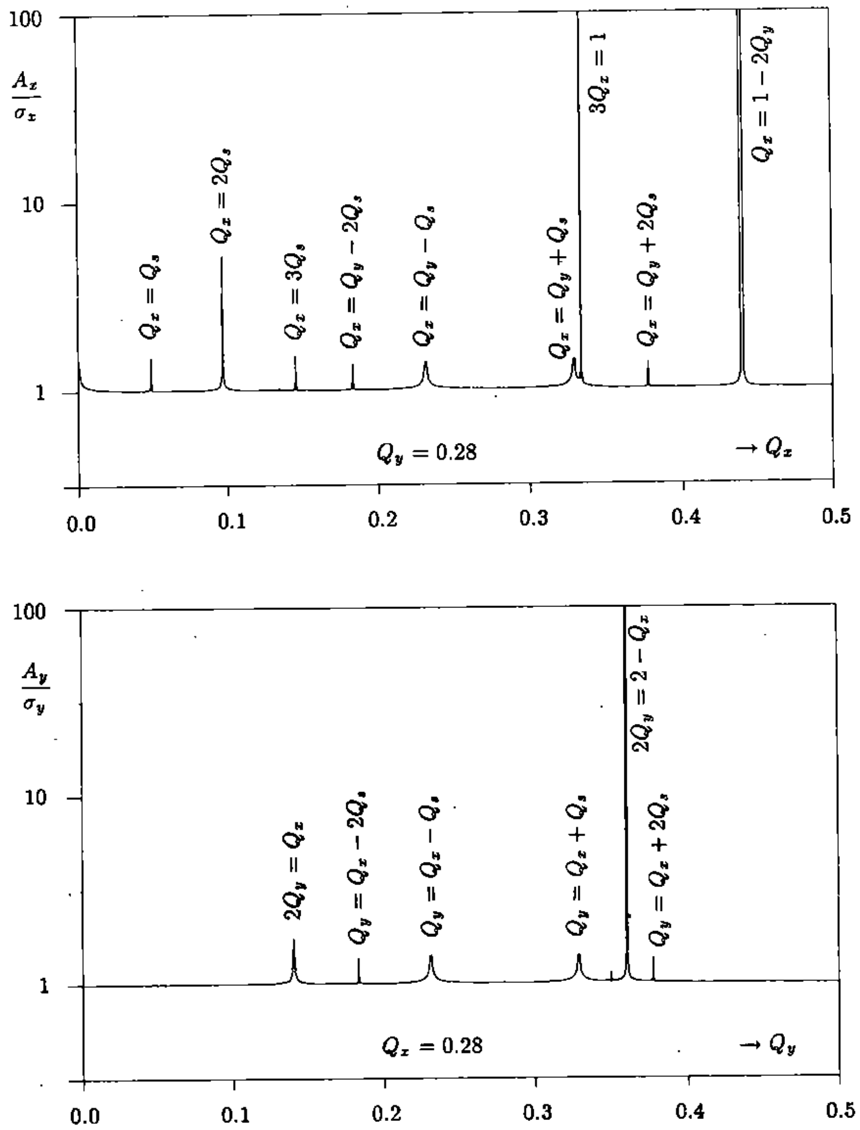

Fig. 3 Maximum horizontal and vertical betatron amplitudes as a function of the tune for $D_x = 0$ and $D_y > 0$.