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Gauge Invariant Treatment of the Electroweak Phase Transition

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Abstract

We evaluate the gauge invariant effective potential for the composite field $\sigma = 2\Phi^\dagger\Phi$ in the SU(2)-Higgs model at finite temperature. Symmetric and broken phases correspond to the domains $\sigma \leq T^2/3$ and $\sigma > T^2/3$, respectively. The effective potential increases very steeply at small values of σ. Predictions for several observables, derived from the ordinary and the gauge invariant effective potential, are compared. Good agreement is found for the critical temperature and the jump in the order parameter. The results for the latent heat differ significantly for large Higgs masses.

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Detailed recent studies of the electroweak phase transition are all based on the effective action for the Higgs field Φ in Landau gauge [1–4]. Although the effective action is gauge dependent, physical observables, derived from this action, must be gauge independent. To verify this explicitly is an important and non-trivial task, especially for quantities like the surface tension, which involve not only the effective potential but also derivative terms.

Hence, it appears desirable to use a manifestly gauge invariant approach as far as possible. This is of particular importance for comparison with lattice Monte Carlo simulations which are usually carried out without gauge fixing [5]. It is known that the expectation value of the operator $\Phi^\dagger\Phi$ is well suited to characterize the broken or Higgs phase in lattice simulations [6], and the corresponding effective potential has been evaluated [7].

In the following we will calculate the effective potential for the composite field $\sigma = 2\Phi^\dagger\Phi$ in continuum perturbation theory. It turns out that this potential, which is gauge invariant by definition, has two qualitatively new features. First, the symmetric phase, which in the conventional framework corresponds to a single point, $\Phi = 0$, is now related to the half-axis $\sigma < T^2/3$. The local minimum in this domain is rather narrow, and at small values of σ the potential increases very steeply. Second, the new potential is valid at temperatures above and below the critical temperature T_c and can in fact be smoothly extrapolated down to $T = 0$.

For simplicity, we shall restrict our discussion in this paper to the Higgs model in three dimensions. With proper identification of parameters we will then obtain the finite-temperature result for small values of the Higgs field. We have also performed the analogous calculation at finite temperature, which will be discussed, together with two-loop results, in a forthcoming paper [8].

The SU(2)-Higgs model in three dimensions is described by the lagrangian

$$\mathcal{L} = \frac{\infty}{\triangle}\mathcal{W}^{\dashv}_{\mu\nu}\mathcal{W}^{\dashv}_{\mu\nu} + (\mathcal{D}_\mu\oplus)^\dagger\mathcal{D}_\mu\oplus + \mathcal{V}_\prime(\varphi^\epsilon) \quad , \tag{1}$$

where

$$V_0(\varphi^2) = \frac{1}{2}m^2\varphi^2 + \frac{1}{4}\mu\lambda\varphi^4 \ , \ \varphi^2 = 2\Phi^\dagger\Phi \ . \tag{2}$$

Here $W^a_{\mu\nu}$ is the ordinary field strength tensor and $D_\mu = \partial_\mu - i\mu^{1/2}\frac{g}{2}W^a_\mu\tau^a$ is the covariant derivative; g and λ are gauge and scalar self-coupling, respectively, and μ is the mass scale used to define dimensionless couplings in three dimensions.

In order to obtain the effective potential for the field $\sigma = 2\Phi^\dagger\Phi$, one has to evaluate

first the "free energy" in the presence of an external source J,

$$e^{-\Omega W(J)} = \int DW D\Phi D\Phi^\dagger e^{-S(\Phi,\Phi^\dagger,W) - \int d^3x 2\Phi^\dagger\Phi J} \ , \tag{3}$$

where Ω is the total volume. For constant J the effective potential is obtained as the Legendre transform,

$$\begin{aligned} \frac{\partial}{\partial J} W(J) &= \sigma \ , \\ V(\sigma) &= W(J(\sigma)) - \sigma J \ . \end{aligned} \tag{4}$$

The free energy $W(J)$ can be calculated in the standard semiclassical or loopwise expansion [1]. The equation for a spatially constant stationary point,

$$(m^2 + 2\mu\lambda\Phi_c^\dagger\Phi_c + 2J)\Phi_c = 0 \ , \tag{5}$$

has two solutions, $\Phi_c = \Phi_s$ and $\Phi_c = \Phi_b$, which correspond to the symmetric and the broken phase, respectively,

$$\Phi_s = 0 \ , \tag{6}$$

$$\Phi_b = \left(-\frac{1}{2\mu\lambda}(m^2 + 2J)\right)^{1/2} \Phi_0 \ , \tag{7}$$

where $|\Phi_0| = 1$. The determinants of fluctuations around the two stationary points depend on the masses of vector bosons, Higgs (φ) and Goldstone (χ) bosons. In the broken phase ($\Phi_c = \Phi_b$) one has, in any covariant gauge,

$$m_W^2 = -\frac{g^2(m^2 + 2J)}{4\lambda} \ , \ m_\varphi^2 = -2(m^2 + 2J) \ , \ m_\chi^2 = 0 \ , \tag{8}$$

whereas in the symmetric phase ($\Phi_c = \Phi_s$) the masses are given by

$$m_W^2 = 0 \ , \ m_\varphi^2 = m_\chi^2 = m^2 + 2J \ . \tag{9}$$

Φ_s and Φ_b correspond to the global minima of the classical action for $m^2 + 2J > 0$ and $m^2 + 2J < 0$, respectively.

The one-loop contribution to the free energy in covariant gauge (cf. [10]) is given by

$$\begin{aligned} W_1(J) = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\Big(& 6\ln(k^2 + m_W^2) + \ln(k^2 + m_\varphi^2) \\ & + 3\ln(k^4 + k^2 m_\chi^2 + \alpha m_W^2 m_\chi^2) - 6\ln k^2 \Big) \ , \end{aligned} \tag{10}$$

[1] See, e.g., ref. [9], chapter 5.3

where α is the gauge parameter. This expression is gauge independent since the product $m_W m_\chi$ vanishes in the symmetric and broken phase. One easily verifies that the same result for $W_1(J)$ is obtained in R_ξ-gauge.

Subtracting linear divergencies by means of dimensional regularization one obtains for the finite part,

$$W(J) = W_b(J)\Theta(-m^2 - 2J) + W_s(J)\Theta(m^2 + 2J) \ , \tag{11}$$

where

$$\begin{aligned} W_b(J) &= -\frac{1}{4\mu\lambda}(m^2+2J)^2 - \frac{1}{2\pi}\left(-\frac{g^2}{4\lambda}(m^2+2J)\right)^{3/2} && (12)\\ &\quad -\frac{1}{12\pi}\left(-2(m^2+2J)\right)^{3/2} \ , && (13)\\ W_s(J) &= -\frac{1}{3\pi}\left(m^2+2J\right)^{3/2} \ . && (14) \end{aligned}$$

Note, that in the broken phase only vector bosons and the Higgs boson contribute. In the symmetric phase all four scalar degrees of freedom contribute equally, whereas the gauge boson contribution vanishes.

The one-loop effective potential can now be obtained by performing the Legendre transformation according to eq. (4). In the broken phase, to order $\mathcal{O}(\}^\ni, \lambda^{\ni/\in})$, it is sufficient to determine $\sigma(J)$ from the tree-level expression for $W(J)$. The resulting potential, which is not convex, represents the energy of an appropriately defined, homogeneous state [11]. A straightforward calculation yields

$$V(\sigma) = V_b(\sigma)\Theta(\sigma) + V_s(\sigma)\Theta(-\sigma) \ , \tag{15}$$

where

$$\begin{aligned} V_b(\sigma) &= \frac{1}{2}m^2\sigma + \frac{1}{4}\mu\lambda\sigma^2 - \frac{1}{2\pi}\left(\frac{1}{4}\mu g^2\sigma\right)^{3/2} - \frac{1}{12\pi}\left(2\mu\lambda\sigma\right)^{3/2} \ , && (16)\\ V_s(\sigma) &= \frac{1}{2}m^2\sigma - \frac{\pi^2}{6}\sigma^3 \ . && (17) \end{aligned}$$

Here the couplings depend on the renormalization scale, i.e., $g = g(\mu)$, $\lambda = \lambda(\mu)$, $m^2 = m^2(\mu)$.

It is very instructive to compare this result with the familiar effective potential for the field Φ in Landau gauge. The one-loop correction is given by the integral in eq. (10). Shifting the scalar field Φ in the usual way by the background field $\varphi/\sqrt{2}$ yields the masses,

$$m_W^2 = \frac{1}{4}\mu g^2\varphi^2 \ , \ m_\varphi^2 = m^2 + 3\mu\lambda\varphi^2 \ , \ m_\chi^2 = m^2 + \mu\lambda\varphi^2 \ . \tag{18}$$

For the potential in Landau gauge one then obtains

$$V_{LG}(\varphi^2) = \frac{1}{2}m^2\varphi^2 + \frac{1}{4}\mu\lambda\varphi^4 - \frac{1}{12\pi}\left(6m_W^3 + m_\varphi^3 + 3m_\chi^3\right) \ . \tag{19}$$

Comparing the two potentials (15) and (19) the first striking difference is the range of the fields. For the potential (19) one has $0 \leq \varphi^2 < \infty$, whereas for the potential (15) the field varies in the range $-\infty < \sigma < \infty$. In the first case the symmetric phase is represented by the point $\varphi = 0$, whereas in the second case it corresponds to the half-axis $\sigma \leq 0$. This difference is a consequence of the different source terms for which the "free energy" W is calculated. Note, that for the gauge invariant potential at one-loop order only the four scalar degrees of freedom contribute in the symmetric phase. At small values of σ the potential increases very steeply.

The second important difference between the potentials (15) and (19) concerns the contribution of scalar loops in the broken phase. Contrary to the ordinary potential, the non-analytic terms of the gauge invariant potential do not depend on m^2. Hence, this potential can also be used for $m^2 < 0$, where the symmetric phase is unstable.

We are interested in the SU(2)-Higgs model at finite temperature, for which the ordinary, ring-improved one-loop potential is given by (cf. [2])

$$\begin{aligned} V_{ring}(\varphi^2, T) &= \frac{1}{2}\left(\frac{3}{16}g^2 + \frac{1}{2}\lambda\right)\left(T^2 - T_b^2\right)\varphi^2 + \frac{1}{4}\lambda\varphi^4 & (20) \\ &\quad -\frac{T}{12\pi}\left(3m_L^3 + 6m_T^3 + m_\varphi^3 + 3m_\chi^3\right) + \mathcal{O}(\}^\triangle, \lambda^\in). & (21) \end{aligned}$$

The masses of longitudinal and transverse vector bosons, Higgs and Goldstone bosons are

$$\begin{aligned} m_L^2 &= \frac{5}{6}g^2T^2 + \frac{1}{4}g^2\varphi^2 \ , \ m_T^2 = \frac{1}{4}g^2\varphi^2 \ , & (22) \\ m_\varphi^2 &= \left(\frac{3}{16}g^2 + \frac{1}{2}\lambda\right)(T^2 - T_b^2) + 3\lambda\varphi^2 \ , & (23) \\ m_\chi^2 &= \left(\frac{3}{16}g^2 + \frac{1}{2}\lambda\right)(T^2 - T_b^2) + \lambda\varphi^2 \ . & (24) \end{aligned}$$

Here we have neglected the effect of the top quark, which is not of interest for our discussion. For values of the Higgs field φ small compared to the temperature T, one can expand m_L in powers of φ^2/T^2. Up to terms of order $\mathcal{O}(\}^\ni\varphi/\mathcal{T}^\in)$ the result is then identical with the effective potential of the three-dimensional theory in Landau gauge (cf. [12]),

$$V_{ring}(\varphi^2, T) = TV_{LG}\left(\frac{\varphi^2}{T}\right) \ , \tag{25}$$

if parameters are identified as follows,

$$\mu = T \ , \ \lambda(\mu) = \lambda - \frac{3}{128\pi}\sqrt{\frac{6}{5}}g^3 \ , \ g(\mu) = g \ , \tag{26}$$

$$m^2(\mu) = \left(\frac{3}{16}g^2 + \frac{1}{2}\lambda - \frac{3}{16\pi}\sqrt{\frac{5}{6}}g^3\right)(T^2 - \bar{T}_b^2) \ , \ \bar{T}_b^2 = \frac{3g^2 + 8\lambda}{3g^2 + 8\lambda - \frac{3}{\pi}\sqrt{\frac{5}{6}}g^3} \ . \tag{27}$$

Inserting these parameters into eqs. (15) - (17) we finally obtain for the gauge invariant finite-temperature potential,

$$V(\sigma, T) = V_s(\sigma, T)\Theta(\sigma) + V_b(\sigma, T)\Theta(-\sigma) \ , \tag{28}$$

where

$$V_b(\sigma, T) = \frac{1}{2}m^2(T)\sigma + \frac{1}{4}\lambda\sigma^2 - \frac{T}{12\pi}\left(6\left(\frac{1}{4}g^2\sigma\right)^{3/2} + (2\lambda\sigma)^{3/2}\right) \ , \tag{29}$$

$$V_s(\sigma, T) = \frac{1}{2}m^2(T)\sigma - \frac{\pi^2}{6}\frac{\sigma^3}{T^2} \ . \tag{30}$$

Contrary to the conventional potential $V_{ring}(\varphi^2, T)$, the gauge invariant potential $V(\sigma, T)$ is valid at temperatures above and below the barrier temperature $\bar{T}_b$.

We have also evaluated the gauge invariant effective potential directly in the finite-temperature theory [8]. In the high temperature expansion the result is essentially the same, it can be obtained from eq. (28) by a shift in the field σ,

$$\sigma \to \sigma - \frac{1}{3}T^2 \ . \tag{31}$$

This shift is obtained by subtracting divergencies using dimensional regularization. Note, that in general the shift is arbitrary and fixed by a renormalization condition. Only its temperature dependence has physical significance.

In figs. (1) and (2) the ordinary potential $V_{ring}(\varphi^2, T)$ and the gauge invariant potential $V(\sigma, T)$ are shown for a Higgs mass of $m_H = 70$ GeV. Each potential is shown at its critical temperature, defined by the degeneracy of the two minima. In the broken phase the two potentials are very similar. The main difference concerns the symmetric phase where the gauge invariant potential shows a strong increase at small values of σ. Note, that the barrier is higher by about a factor of two for the gauge invariant potential.

We have evaluated several observables for the two potentials. The critical temperatures are different, but very similar. For Higgs masses between 30 GeV and 120 GeV the ratio $(T_c - \bar{T}_b)/\bar{T}_b$ differs by at most 40%. Fig. (3) shows that also the predictions for φ_c, the shift in the Higgs field at the critical temperature, are in good agreement. The

latent heat, shown in fig. (4), differs by about 70% at $m_H = 120$ GeV. We expect similar discrepancies for the surface tension.

In summary, the main differences between the ordinary effective potential in Landau gauge and the new gauge invariant potential concern the symmetric phase and the effect of scalar loops in the broken phase. The new potential can be used at temperatures above and below the barrier temperature. The gauge invariance of the new potential should be an advantageous feature also with respect to non-perturbative effects which are expected to be important in the symmetric phase of the non-abelian, and possibly also the abelian, Higgs model.

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Figure captions

Fig.1 Effective potential in Landau gauge, $m_H = 70$ GeV.

Fig.2 The gauge invariant effective potential, $m_H = 70$ GeV.

Fig.3 Shift of the Higgs field as function of the Higgs mass, as predicted by the potential in Landau gauge (dashed line) and the gauge invariant potential (full line).

Fig.4 Latent heat as function of the Higgs mass, as predicted by the potential in Landau gauge (dashed line) and the gauge invariant potential (full line).