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Branching ratio and direct CP-violating rate asymmetry of the rare decays B$\to K^*\gamma$ and B$\to \rho\gamma$ [1]

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Abstract

We calculate CP-violating rate asymmetries in the rare radiative decays $B^\pm \to K^{*\pm}\gamma$ and $B^\pm \to \rho^\pm\gamma$. They arise because of the interference between leading-order penguin amplitudes and one-gluon corrections with absorptive phases, and provide unambiguous evidence for direct CP violation. Complementing earlier studies, we also investigate gluon exchange with the 'spectator' quark. The bound state effects in the exclusive matrix elements are taken into account by a covariant model, which yields a branching ratio $BR(B \to K^*\gamma) = (4-5)\times 10^{-5}$ in good agreement with the observed value. The bound state effects increase the CP asymmetry, which is of order 1% in the channel $B \to K^*\gamma$ and 15% for $B \to \rho\gamma$.

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1. Introduction

There is much hope to detect CP violation in B-meson physics [1, 2, 3]. It would complement existing results in Kaon decays and help solving the origin of this phenomenon. In fact, the large variety of decay modes of the B-meson makes it possible to investigate CP violation from many angles.

A few decay modes, such as $B^0 \to K_S J/\Psi$, allow for a simple, essentially model-free prediction of the CP-violating rate asymmetries [4]. These originate mainly from the mixing of B^0 and $\overline{B^0}$, and are analogous to the ϵ-parameter in K-decay. However, one would also like to test direct CP violation in the decay amplitudes, such as manifested by the ϵ' parameter (for a recent review of direct CP violation, see [5]). Theoretical investigations are restricted to inclusive decay modes, which are difficult to relate to experimental data or require a calculation of an exclusive decay within a phenomenological model.

It is well known that a non-vanishing asymmetry due to direct CP-violation requires two interfering amplitudes with different weak (CKM) and strong (rescattering) phases. The main difficulty is to calculate the latter. Within the present understanding of the basic CP-violating mechanism, consisting in the simultaneous contribution of several generations of particles to the physical processes in question, it is natural to locate the strong phases in (penguin) diagrams on the quark level, involving virtual heavy and on-shell light particles of all these generations. This picture was introduced a long time ago by Bander, Silverman and Soni [6] and underlies most treatments of direct CP violation in charged B-decays; we will essentially follow these lines as well.

A prominent (penguin) decay is $B \to K^*\gamma$, recently observed by the CLEO group with a branching ratio of $(4.5 \pm 1.0 \pm 0.9) \times 10^{-5}$ [7]. With only one hadron in the final state it is an interesting candidate: One may hope that the theoretical uncertainties of the calculated rate asymmetry

$$a_{CP} \equiv \frac{\Gamma(B^- \to K^{*-}\gamma) - \Gamma(B^+ \to K^{*+}\gamma)}{\Gamma(B^- \to K^{*-}\gamma) + \Gamma(B^+ \to K^{*+}\gamma)} \tag{1}$$

are minimal and that the clear signature should make experimental detection feasible.

While the rate asymmetry for the underlying quark process, $b \to s\gamma$, vanishes up to the one-loop level (as the penguin diagram generates an absorptive phase only when the photon is off shell), Soares [8] showed that there is a non-zero effect at two loops. When true hadronic transitions are considered, also bound state effects must be included. They can give rise to additional contributions due to absorptive parts of diagrams where additional gluons are emitted from inside of the penguin-loop and couple to the other constituents of the hadrons. In this paper we include these effects and estimate the rate asymmetry in charged B-decays using a simple model for the mesons.

2. Perturbative contributions

We write the effective Hamiltonian for the decay $B \to K^*\gamma$ as

$$\begin{aligned} \mathcal{H}_{eff} &= \frac{4G_F}{\sqrt{2}}\, v_c \left(C_2(\mu) O_2^{(c)} + C_7(\mu) O_7 + ... \right) \\ &+ (c \to u)\,. \end{aligned} \tag{2}$$

where

$$O_2^{(q)} = (\bar{q}_\alpha \gamma^\mu L b_\alpha) \cdot (\bar{s}_\beta \gamma_\mu L q_\beta) \tag{3}$$

is the familiar four-Fermi operator and

$$O_7 = \frac{e}{16\pi^2}\, \bar{s}\, \sigma^{\mu\nu}\, (m_b R + m_s L)\, b\; F_{\mu\nu} \tag{4}$$

is the magnetic dipole operator. L and R are the left and right projectors. The operator O_7 arises from diagrams with an internal top-quark, and hence its coefficient is proportional to $v_t \equiv V_{tb}V_{ts}^*$. It is convenient to express v_t in terms of the two independent CKM combinations $v_i \equiv V_{ib}V_{is}^*$ $(i = u, c)$ by making use of the unitarity relation $v_t = -v_c - v_u$. This enables us to write the amplitudes [2] for our process in the convenient form

$$A = v_u A_u + v_c A_c\ . \tag{5}$$

For the decay B$\to \rho\gamma$ one simply replaces the s-quark in the above expressions for the operators and in the CKM factors by the d-quark. The ellipses in eq. (3) denote other operators; in particular, four-Fermi operators whose one-loop matrix elements in general contribute to the rate. As pointed out in [9], their effects can, at least in leading logarithmic approximation, be absorbed into an effective coefficient $C_7^{eff}(\mu)$. For a top-quark mass of 174 GeV, the corresponding values of the Wilson coefficients are given by $C_2(5\ GeV) = 1.096$ and $C_7^{eff}(5\ GeV) = -0.305$ [3] [9, 10]; these values include some next-to-leading log (NLL) corrections. Of course, a systematic calculation at the NLL level not only requires the knowledge of the coefficients to this precision, but also two-loop QCD-corrections to the matrix elements (real and imaginary parts) have to be calculated.

The CP violating asymmetry a_{CP} has the form [11]

$$a_{CP} = \frac{-4\mathrm{Im}[v_u v_c^*]\,\mathrm{Im}[A_u A_c^*]}{|v_u A_u + v_c A_c|^2 + |v_u^* A_u + v_c^* A_c|^2} \tag{6}$$

which implies the well known fact that the absorptive parts of A_u and A_c must be evaluated. These are subleading, i.e. vanish in the leading log approximation

[2] Of course, there are several amplitudes corresponding to the different helicity states of K^* and γ. However, as long as not both the polarization of the photon and the angular distribution of the decay products of the K^* are observed, there are no other CP-violating observables besides the rate asymmetry and we ignore the helicity structure throughout.

[3] The relative sign between C_2 and C_7 depends on the definition of the covariant derivative; we use $D = \partial + ig_s T^a A^a + ieQA$

and when the matrix element are treated only at tree level. In fact, the imaginary parts do not arise from the coefficients C_i, but only from the matrix elements of the various operators when evaluated to sufficient order in α_s. Therefore, when we consider an expansion of the amplitude in α_s, it will be consistent to neglect subleading terms, except in the imaginary parts in the numerator in eq.(6). In particular, these absorptive parts are renormalization scheme independent, which is of course not generally true for the real parts of the one-loop matrix elements. The leading contribution to the decay rate comes from the tree-level matrix element of O_7 in Fig. 1 which does not involve the spectator anti-quark.

At next order in α_s there are many terms. In particular, when considering the bound state, one would also include, in a perturbative calculation, gluon exchange between the b-quark or its decay products and the spectator line. In fact, in a particular treatment of high momentum transfer processes [12], the gluon exchange plays the major role. In this scheme the constituents have essentially no transverse momentum which must be provided by the gluon. However, in B-meson decays this approach tends to yield too small branching ratios [12] and is likely to serve only as corrections to the decay width. To estimate their importance, we have analyzed some higher order graphs in the framework of our model (see below) and found that they only give corrections of up to order 15% in the rate. We therefore expect that an expansion in α_s is meaningful and that very soft gluons can be absorbed in a suitable wave function. Thus we only include the graph of Fig. 1 to calculate the rate.

However, as pointed out, the higher orders are essential for the asymmetry and we must isolate the important ones. At order α_s^0 only the operator O_7 contributes to the decay (after absorbing the contributions of four-quark operators into C_7^{eff}); thus, A_c and A_u are equal and real at lowest order. Including order α_s corrections, we write $A_c = A^0 + \alpha_s A_c^1$ and $A_u = A^0 + \alpha_s A_u^1$. Then, $\mathrm{Im}[A_u A_c^*]$ in eq.(6) reads

$$\mathrm{Im}[A_u A_c^*] = \alpha_s A^0 \left(\mathrm{Im} A_u^1 - \mathrm{Im} A_c^1 \right) \quad . \tag{7}$$

This shows explicitly that the internal particles, which generate the absorptive phase, must have different masses for a nonzero asymmetry (i.e. the $i = u, c$ quarks from $O_2^{(i)}$ must be involved in $\mathrm{Im}A_i$). All operators omitted in eq. (3) enter with a coefficient proportional to $v_u + v_c$ and therefore do not yield an absorptive phase which contributes to the asymmetry at order α_s. For the same reason, absorptive parts of matrix elements of the operators O_7 and O_8 (the analog to O_7, but with a gluon instead of the photon) do not contribute; only the loop-level matrix elements of O_2 with a gluon emitted from the loop are relevant.

There are basically two classes of diagrams depending on whether the gluon from the internal quark loop is attached to the final s-quark (Fig. 2) or to the spectator (Fig. 3).

The first class has already been studied in ref. [8] and corresponds to the finite part of those QCD radiative corrections which are also responsible for the considerable operator mixing of O_2 into O_7 [10, 9]. For completeness, and ease of use, we

will recast the result of Soares in the framework of effective operators which also gives rather compact expressions. Applying the language of Cutkosky rules, the absorptive parts arise from "cuts" which contain the $u\bar{u}$ or $c\bar{c}$ pair of O_2, respectively. Other cut diagrams either vanish or would generate an "universal" phase, i.e. one multiplied by $v_u + v_c$, which contributes only at higher order to the rate asymmetry.

The relevant cut diagrams are shown in Fig. 2. They yield local contributions proportional to the tree-level matrix element of O_7 (i.e. proportional to the contribution of Fig. 1)

$$\mathrm{Im}\langle s\gamma|O_2^{(i)}|b\rangle_{Fig.2a,b} = g_s^2\, Q_{a,b}\, C_N\, \rho_{a,b}(m_i, m_b)\, \langle s\gamma|O_7|b\rangle_{tree}\,, \tag{8}$$

where $C_N = (N^2-1)/(2N)$ is a color factor ($C_N = 4/3$ for $N = 3$ colors). $Q_a = -1/3$ and $Q_b = 2/3$ are the charges of the quarks to which the photon couples in Fig. 2a,b, respectively. The masses of the internal and of the b-quark are denoted by m_i and m_b, respectively. To evaluate the factors $\rho_{a,b}$ in eq. (8), we apply the Cutkosky rules in the form

$$\mathrm{Im}A(b \to f) = \frac{(2\pi)^4}{2} \sum_i \int \prod_{n=1}^{N_i} \frac{d^3p_n}{(2\pi)^3 2E_n}\, \delta^4\Big(p_b - \sum_{n=1}^{N_i} p_n\Big)\, \hat{A}(b \to i)\, \hat{A}(i \to f) \quad , \tag{9}$$

where $\hat{A}$ are the (real) amplitudes of the corresponding subprocesses and the summation runs over all possible intermediate states i.

The diagrams with only the gluon coupled to the quark loop (Fig. 2a) contain penguin subdiagrams with insertion of $O_2^{(i)}$. They can be represented by an effective $\bar{s}bg$ vertex

$$I_\mu^a = \frac{g_s}{16\pi^2}\, V\Big(\frac{q^2}{m_i^2}\Big)\, \Big(q^2\gamma_\mu - q_\mu \not{q}\Big)\, L\, \frac{\lambda^a}{2} \quad , \tag{10}$$

where q is the momentum of the gluon, and V is proportional to the vacuum polarization with internal $i = u$ or c quarks. With this building block the sum of the diagrams in Fig. 2a yields

$$\rho_a(m_i, m_b) = \frac{1}{16\pi^2}\, \frac{1}{m_b^6} \int_{4m_i^2}^{m_b^2} \mathrm{Im}V\Big(\frac{q^2}{m_i^2}\Big)\, (m_b^2 - q^2)^2\, dq^2 \quad , \tag{11}$$

where

$$\mathrm{Im}V(x) = \frac{2\pi}{3}\, \frac{x+2}{x} \sqrt{\frac{x-4}{x}}\, \Theta(x-4) \quad . \tag{12}$$

The sum of the diagrams with the gluon *and* the photon emitted from the quark loop (Fig. 2b) involves an effective $\bar{s}bg\gamma$ vertex which can be derived from ref. [13]

$$\begin{aligned} I_{\mu\nu}^a \;=\; & -\frac{g_s e Q_u}{8\pi^2}\, \frac{\lambda^a}{2} \Big\{ i\epsilon_{\beta\mu\nu\alpha}\, (q^\beta \Delta i_5 + p_\gamma^\beta \Delta i_6) \\ & + i\frac{\epsilon_{\rho\sigma\mu\alpha}}{p_\gamma q}\, q^\rho p_\gamma^\sigma q_\nu \Delta i_{23} + i\frac{\epsilon_{\rho\sigma\nu\alpha}}{p_\gamma q}\, q^\rho p_\gamma^\sigma p_{\gamma\mu} \Delta i_{26} \Big\}\, \gamma^\alpha\, L \quad . \end{aligned} \tag{13}$$

In general, for the gluon and the photon being off shell, the quantities $\Delta i_n \equiv \Delta i_n(z_0, z_1, z_2)$ are functions of the three variables $z_0 = s/m_i^2$, $z_1 = q^2/m_i^2$ and $z_2 = p_\gamma^2/m_i^2$, where q and p_γ denote the four-momenta of the gluon and the photon, respectively, and s is the invariant mass squared of the internal quark pair. In the present situation, where the photon is on shell, the form factors Δi_n can be written as

$$\begin{aligned}
\Delta i_5(z_0, z_1, 0) &= -1 + \frac{z_1}{z_0 - z_1}(Q_0(z_0) - Q_0(z_1)) - \frac{2}{z_0 - z_1}(Q_-(z_0) - Q_-(z_1)) \\
\Delta i_6(z_0, z_1, 0) &= +1 + \frac{z_1}{z_0 - z_1}(Q_0(z_0) - Q_0(z_1)) + \frac{2}{z_0 - z_1}(Q_-(z_0) - Q_-(z_1)) \\
\Delta i_{23}(z_0, z_1, 0) &= \Delta i_5(z_0, z_1, 0) = -\Delta i_{26}(z_0, z_1, 0) \quad . \qquad (14)
\end{aligned}$$

The functions Q_0 and Q_- are defined by the integrals

$$Q_-(x) = \int_0^1 \frac{du}{u} \log\left(1 - xu(1-u)\right) \quad , \quad Q_0(x) = \int_0^1 du \log\left(1 - xu(1-u)\right) \quad , \qquad (15)$$

and have imaginary parts for $x \geq 4$. On the sheet $\mathrm{Im}x \geq 0$ they are given by

$$\mathrm{Im}Q_-(x) = -2\pi \log\left(\frac{\sqrt{x} + \sqrt{x-4}}{2}\right) \quad , \quad \mathrm{Im}Q_0(x) = -\pi\sqrt{\frac{x-4}{x}} \; . \qquad (16)$$

In terms of these functions, we obtain for the total contribution of the diagrams in Fig. 2b

$$\begin{aligned}
\rho_b(m_i, m_b) &= \frac{1}{8\pi^2} \int d\hat{E}_s' \, dz \, \frac{\hat{E}_s'}{[1 - \hat{E}_s'(1-z)]^2} \\
&\quad \times \Bigg\{ \Big(2 - 4\hat{E}_s'(1-z)\Big) \, \frac{m_i^2}{m_b^2} \, \mathrm{Im}Q_-\Big(\frac{s}{m_i^2}\Big) \\
&\quad + \Big(3\hat{E}_s' - \hat{E}_s' z - 4\hat{E}_s'^2(1-z)\Big) \, \mathrm{Im}Q_0\Big(\frac{s}{m_i^2}\Big) \Bigg\} \quad , \qquad (17)
\end{aligned}$$

where $\hat{E}_s' m_b$ is the energy of the intermediate (cut) s-quark in the rest frame of the decaying b-quark and z is the cosine of the angle between the intermediate s-quark and the photon. In terms of the integration variables the invariant mass squared of the internal quark pair reads $s = m_b^2(1 - 2\hat{E}_s')$.

At $z = -1$ the integral in eq. (17) is divergent. In an inclusive (free) b-quark decay this divergence would have to be cancelled by including the radiation of additional gluons or $s\bar{s}$-pairs. In the present exclusive situation this logarithmic divergence is cut off by requiring that the momentum squared q^2 of the virtual gluon is sufficiently off shell in order to avoid double counting with contributions from the wave functions. The condition $q^2 = -m_b^2 \hat{E}_s'(1+z) \leq -\Lambda^2$ yields the integration

intervals

$$\begin{aligned} \frac{\Lambda^2}{2m_b^2} \leq \quad \hat{E}'_s \quad &\leq \frac{1}{2} \\ -1 + \frac{\Lambda^2}{m_b^2 \hat{E}'_s} \leq \quad z \quad &\leq 1 \quad . \end{aligned} \tag{18}$$

We close this section with a physical picture of the diagrams involved. For instance, when the internal quark loop contains a $c\bar{c}$ pair, the above diagrams can be viewed as perturbative contributions to the decay of the B-meson into some $(s\bar{c})$ and $(c\bar{u})$ states, e.g. a $D_s^{(*)}$ and a $D^{(*)}$ meson. These intermediate hadrons rescatter then into the final state $K^* + \gamma$ by radiating the photon and exchanging gluons in all possible ways. This implies the presence of the diagrams (see Fig. 3) of the second class, involving the spectator line. They yield non-local contributions and their explicit form depends on the model by which the meson states are described; the one we use will be presented in the next section.

3. A model for the exclusive matrix elements

To describe exclusive decays of B mesons involving a light meson in the final state, like $B \to \pi e\nu$ and $B \to \rho e\nu$, we have developed a covariant model [14] which yields good results for the branching ratio and energy spectrum in these decays; we apply it here to the rare decay $B \to V\gamma$, where $V = K^*$ or ρ. In fact, since most models coincide when the hadronic momentum transfer squared is approximately zero, we expect that the results are relatively model independent. We assume that both B and K^* can be described by two (effective) constituents only: $B = (b, \bar{q})$ and $K^* = (s, \bar{q})$. To guarantee covariance of the results we require that the four-momenta of the constituents add up to the four-momentum of the bound state. Due to binding effects, this condition can only be fulfilled if at least one of the two constituents has a variable mass [4]. We take the mass m_q of the spectator anti-quark to be fixed; more specifically we put $m_q = 0$ (for reasons which are related to our prescription of the light mesons, see below). Consequently, the b-quark mass becomes momentum dependent

$$m_b = \sqrt{m_B^2 - 2p_B p_q} \quad , \tag{19}$$

where p_B and p_q are the four-momenta of the B meson and the spectator, respectively, and m_B is the mass of the B meson. Eq. (19) only makes sense if the momentum $|\vec{p}_q|$ in the B rest-frame is restricted to $|\vec{p}_q| \leq m_B/2$. As derived in detail in ref. [14], the B meson can be represented by a matrix Ψ_B (in spinor and color space)

$$\Psi_B = C_B \int \frac{d^3p_q}{2E_q(2\pi)^3} \sqrt{\frac{m_B}{2(m_B^2 - p_B p_q)}}\, \phi_B(|p_B p_q / m_B|)\, \Sigma_B \otimes \frac{1}{\sqrt{N}} 1_N \quad , \tag{20}$$

[4] This is quite similar in the model of Altarelli et al. [15]

with

$$\Sigma_B = -(\not{p}_b + m_b)\,\gamma_5\ \not{p}_q \quad . \tag{21}$$

For the wave function $\phi_B(p)$ we use a harmonic Ansatz

$$\phi_B(p) = \exp\left\{-p^2/(2p_F^2)\right\} \quad . \tag{22}$$

The normalization factor C_B is chosen such that $\int d^3p_q |C_B \Phi_B(|\vec{p}_q|)|^2 = (2\pi)^3$ when the integration region is restricted to $|\vec{p}_q| \leq m_B/2$. Numerically, $C_B \approx \sqrt{8}\,\pi^{3/4}\,p_F^{-3/2}$ to a very high precision. The only parameter of the wave function, p_F, is fixed in such a way that our model yields the right value for the meson decay constant f_B. Numerical values of p_F for different choices of f_B are given in table 1.

While the above construction is adequate for the B meson, it turns out to be impossible to describe the light (vector) meson along these lines because whatever one chooses for the wave function, it is impossible to get a value for the decay constant which is compatible with experimental data. (Here we refer to the analogous statement given in detail for the pion in ref. [14]). Therefore, we use a different picture for the light meson, assuming that the meson is represented by two constituents with parallel momenta $p_s = y p_V$ and $p_q = (1-y)p_V$, respectively. As the vector mesons considered here are much lighter than the B meson, we neglect their masses and consequently also those of the constituents. The final state vector meson is represented by the matrix

$$\Psi_V = C_V \int_0^1 dy\ \phi_V(y)\,\Sigma_V \otimes \frac{1}{\sqrt{N}} 1_N \quad , \tag{23}$$

with

$$\Sigma_V = \lim_{m_V \to 0} \not{\epsilon}_V^*(\not{p}_V + m_V)\ , \tag{24}$$

where p_V and ϵ_V are the momentum and polarization vector of the meson, respectively. $\phi_V(y)$ is the quark distribution amplitude in the vector meson, and the normalization factor C_V is determined by the value of the decay constant f_V, defined as $\langle 0|\bar{u}\gamma_\mu s|V\rangle = m_V\ f_V\ \epsilon_V^\mu$ ($f_{K^*} \approx f_\rho \approx 216$ MeV). Explicitly, we take

$$\phi_V(y) = 6y(1-y)\,(1+\cdots) \quad ; \quad C_V = f_V/(4\sqrt{N}) \tag{25}$$

where the ellipses denote deviations from the asymptotic form of the wave function and $SU(3)_{Flavour}$ breaking effects (see e.g. ref. [16]).

The exchange of extra gluons may lead to ambiguities (double counting and IR divergences) in two situations: In the first, there is a quark line, like the b-line in Fig. 4a, which is off shell because of the exchange of a gluon which can be absorbed into one of the meson wave functions. In order to avoid counting this configuration a second time in the perturbative treatment, we should impose a condition that the quark line is sufficiently off shell.

In the second, gluons which couple directly to a constituent quark of a meson should not be included when they are almost on shell: This case would be degenerate

with the contribution from the adequate wave function for a three-particle Fock state (e.g. $\bar{u}gs$ in Fig. 2); since the latter are not included in our simple model treatment, we should not include such almost on-shell gluons as well.

Both conditions will be built into our model by requiring that quarks or gluons in the above situations are off shell by at least an amount Λ. (Of course, the cutoff parameter may a priori be different in the two situations; however, for simplicity we take the same value Λ in both cases.) This parameter is part of the definition of our model and a value of $\Lambda = 0.2 - 1$ GeV should be physically reasonable. This automatically cuts off possible soft and collinear divergencies.

In the following, we denote by p_γ (p_V) and ϵ_γ (ϵ_V) the momentum and polarization of the photon (vector meson), respectively. The transition matrix elements of the effective operators in eq. (3) have the general form (for an on-shell photon)

$$\langle V\gamma|O_i|B\rangle \equiv e\, m_B\, \epsilon_\gamma^\mu \left\{ i\epsilon_{\mu\nu\alpha\beta} p_\gamma^\nu \epsilon_V^\alpha p_V^\beta\, F_1[O_i] + \Big((p_\gamma \epsilon_V)\, p_{V\mu} - (p_\gamma p_V)\, \epsilon_{V\mu} \Big)\, F_5[O_i] \right\} \tag{26}$$

which serves as a definition for the 'form factors' $F_{1,5}[O_i]$. In the following, when the mass of the final-state meson is neglected, we have always $F_1 = F_5$ due to the $\sigma_{\mu\nu}(1+\gamma_5)$ structure of O_7; hence, we shall drop the subscript of F [5].

In the framework of our model for the bound states, the hadronic matix elements are determined by

$$\begin{aligned} \langle V\gamma|O_i|B\rangle &= C_B\, C_V \int \frac{d^3p_q\, dy}{(2\pi)^3 2E_q} \sqrt{\frac{m_B}{2(m_B^2 - p_B p_q)}} \\ &\quad \times \phi_B\Big(\frac{p_B p_q}{m_B}\Big)\, \phi_V(y)\, \frac{1}{N} \mathrm{Tr}\left[\Sigma_V\, M_{sb}\, \Sigma_B\, M_{qq'}\right] \quad , \end{aligned} \tag{27}$$

where the trace is in Dirac and color space. The spin projectors Σ_B and Σ_V in eq. (27) are given in eqs. (21) and (24), respectively, and the matrices M_{sb} and $M_{q'q}$ (acting in spinor and color space) are related to the quark-level matrix elements by

$$\langle s\bar{q}'\gamma|O_i|b\bar{q}\rangle = \bar{u}_s\, M_{sb} u_b \cdot \bar{v}_q M_{qq'} v_{q'} \quad . \tag{28}$$

Note, that in the case of a transition where the spectator is not directly involved, $M_{qq'}$ can be written as [14]

$$M_{qq'} = 2\gamma_0\, (2\pi)^3\, \delta^3(p_q - p_{q'}) \otimes 1_N \quad . \tag{29}$$

From the decomposition in eq. (26) one readily derives the decay width

$$\Gamma(B \to V\gamma) = \frac{\alpha_{em}}{4}\, m_B^5\, |F[\mathcal{H}_{eff}]|^2 \quad . \tag{30}$$

In the (leading) approximation when only the matrix element of O_7 is taken into account, one obtains

$$\Gamma(B \to V\gamma) \approx 2\, G_F^2 m_B^5 \alpha_{em}\, |v_c + v_u|^2\, (C_7^{eff})^2\, |F[O_7]|^2 \quad . \tag{31}$$

[5] For the operator O_7, our definition of F coincides with the form factor $2F_1$ commonly used in the literature

4. Evaluation of $F[O_7]$ and branching ratios

As mentioned in section 2, only $F[O_7]$ contributes to $\langle V\gamma|\mathcal{H}_{eff}|B\rangle$ in a leading-log calculation of the decay rate. Evaluating the trace in eq. (27) and working out the d^3p_q integration by making use of the 3-dimensional delta function (which directly relates $m_b = m_B\sqrt{y}$ to the momentum fraction y) we get

$$F[O_7] = -\frac{\sqrt{m_B}\,C_B\,C_V}{4\pi^2}\int dy\,\frac{y}{\sqrt{1+y}}\,\phi_V(y)\,\phi_B\Big(\frac{m_B(1-y)}{2}\Big) + \mathcal{O}(\alpha_s) \quad . \tag{32}$$

In order to get a rough idea of the effect of higher order contributions to the amplitude for $B \to K^*\gamma$, we consider those order α_s corrections to the matix element of O_7 which involve an extra gluon exchange with the spectator. The corresponding diagrams, with the gluon radiated either off the b- or s-quark, are shown in Fig. 4. The comparison of the lowest order contributions in our model with these diagrams is of particular interest, because the latter would be the leading contributions in the approach of ref. [12], where the exchange of hard gluons is essential.

While the second diagram of Fig. 4 vanishes for $m_s = 0$, the first diagram yields the following contribution to $F[O_7]$

$$\begin{aligned} F[O_7]_{Fig.4} &= \frac{g_s^2}{\pi^2}\,m_B\,C_N\,C_B\,C_V \\ &\times \int dy\,dE_q\,dz\;\phi_V(y)\,\phi_B(E_q)\;\frac{E_q^2(m_B-2E_q)\,(2-y(1-z))}{N_b\,q^2\,\sqrt{2(m_B-E_q)}} \end{aligned} \tag{33}$$

where E_q is the energy of the spectator in the rest frame [6] of the B-meson, and z is the cosine of the angle between the spectator and the K^*-meson. N_b is the denominator of the b-quark propagator and q^2 is the invariant mass squared of the exchanged gluon. In terms of the integration variables we have

$$N_b = -(1-y)m_B^2 + 2m_B E_q \quad , \quad q^2 = -m_B E_q(1-y)(1-z) \quad . \tag{34}$$

From pure kinematics, both can become zero and generate divergencies in eq. (33). However, this is avoided by the cutoffs included in our model (see section 3): When the b-quark propagator, N_b, goes to zero the corresponding gluon exchange can be viewed as already been taken into account in an adequate wave function of the B meson. We may therefore exclude these contributions in the perturbative treatment and impose the cutoff $|N_b| \geq \Lambda^2$.

Similarly in q^2, a vanishing factor $(1-z)$ (the factor $(1-y)E_q$ is already cancelled by the wave function $\phi_V(y)$ and by E_q in the numerator) corresponds to the kinematical situation where the gluon is on shell and parallel to the spectator. Such kinematical configurations would also enter when we consider higher fock-components of the mesons, e.g. $\bar{u}gs$. Since we do not go beyond 2-Fock states here,

[6] Since the model is covariant, the result is of course frame independent.

we should also cut off this kinematical region by imposing the (covariant) condition $|q^2| \geq \Lambda^2$. Of course, this leads to a weak (logarithmic) dependence of the results on the value of Λ. We find that for $\Lambda = 0.2$ GeV (1.0 GeV) the non-leading corrections of eq. (33) contribute at most 23% (6%) to the rate.

In order to estimate the branching ratio we normalize the rate, evaluated at leading order, by the theoretical prediction for the inclusive semileptonic decay width. To evaluate the latter, we apply our bound-state model to the decaying B-meson and obtain

$$\Gamma_{sl} = \frac{G_F^2}{192\pi^3} |V_{cb}|^2 C_B^2 \int \frac{d^3p_q}{(2\pi)^3} |\phi_B(|\vec{p}_q|)|^2 \frac{m_b^6(p_q)}{m_B - E_q} g\Big(\frac{m_c}{m_b(p_q)}\Big) \quad , \tag{35}$$

where $g(r) = 1 - 8r^2 + 8r^6 - r^8 - 24r^4 \log(r)$ is the usual phase space function. As we work in the leading-log approximation we have neglected QCD corrections in Γ_{sl}.

The branching ratio is then obtained by multiplying with the measured value for $BR(B \to X\ell\nu_\ell) = 0.11$:

$$BR(B \to V\gamma) \equiv \frac{\Gamma(B \to K^*\gamma)}{\Gamma_{sl}} \times 11\% \ . \tag{36}$$

Finally, it is straigthforward to apply our description of the B-meson to the inclusive decays $B \to X_s\gamma$. For the ratio of exclusive to inclusive decay rates, which is the more convenient quantity for phenomenological investigations, we obtain

$$R_{K^*} \equiv \frac{\Gamma(B \to K^* + \gamma)}{\Gamma(B \to X_s + \gamma)} = \frac{m_B^5}{m_{b,eff}^5} 64\pi^4 |F[O_7]|^2 \ , \tag{37}$$

where $m_{b,eff}$ is given by

$$m_{b,eff}^5 \equiv \int \frac{d^3p_q}{(2\pi)^3} C_B^2 |\phi_B(|\vec{p}_q|)|^2 \frac{m_b(p_q)}{m_B - E_q} m_b^5(p_q) \quad . \tag{38}$$

Neglecting $SU(3)_{Flavour}$ breaking effects, R is the same for $B \to K^*\gamma$ and $B \to \rho\gamma$.

5. Absorptive parts from $ImF[O_2]$ and rate asymmetries

For the calculation of the CP asymmetry the imaginary part of $F[O_2]$ is needed. Corresponding to the contributions from the different classes of diagrams (see Figures 2 and 3), we split it into

$$F[O_2] = F[O_2]_{Fig.2a} + F[O_2]_{Fig.2b} + F[O_2]_{Fig.3a} + F[O_2]_{Fig.3b} \quad . \tag{39}$$

The diagrams in Fig. 2, where *no* gluon is exchanged with the spectator, are readily incorporated into our model. As the effective structure of the relevant subdiagram is proportional to $\langle s\gamma|O_7|b\rangle$ [see eq. (8)], the expression is similar to $F[O_7]$

from eq. (32)

$$\begin{aligned}\mathrm{Im}F[O_2^{(i)}]_{Fig2a,b} &= -\sqrt{m_B}\frac{g_s^2}{4\pi^2}\,Q_{a,b}\,C_N\,C_B\,C_V \\ &\times \int dy\,\frac{y}{\sqrt{1+y}}\,\phi_V(y)\,\phi_B\Big(\frac{m_B(1-y)}{2}\Big)\,\rho_{a,b}(m_i,m_b) \quad , \end{aligned} \tag{40}$$

where $Q_a = -1/3$, $Q_b = 2/3$, and $\rho_{a,b}$ have been defined in section 2 [eqs. (11) and (17)]. Note that $m_b = m_B\sqrt{y}$ depends on y according to our bound-state model; and therefore [see eq. (18)], ρ_b is only non-zero for

$$\frac{\Lambda^2}{m_B^2} \le y \le 1 \quad . \tag{41}$$

In the diagrams *with* gluon exchange to the spectator, the photon can be emitted either from one of the external fermion legs (Fig. 3a) or from the $i = u, c$ quark in the fermion loop (Fig. 3b). Diagrams in which the photon is emitted from the b- or s-quark line do not develop imaginary parts, because the corresponding momentum squared of the gluon, q^2, is negative. The diagrams in Fig. 3a where the photon is emitted from the spectator quark involve the effective $\bar{s}bg$ vertex of eq. (10) and we obtain for their sum

$$\begin{aligned}\mathrm{Im}F[O_2^{(i)}]_{Fig.3a} &= \frac{g_s^2}{16\pi^4}\,\frac{1}{m_B^2}\,Q_u\,C_N\,C_B\,C_V \int dz\,dE_q\,dy \\ &\times \frac{E_q^2}{\sqrt{2(m_B-E_q)}}\,\phi_B(E_q)\,\frac{\phi_V(y)}{1-y}\,\mathrm{Im}V\Big(\frac{q^2}{m_i^2}\Big) \quad , \end{aligned} \tag{42}$$

where $\mathrm{Im}V$ is given in eq. (12) and the momentum of the gluon is related to the integration variables according to

$$q^2 = m_B^2(1-y) - 2m_B E_q + m_B E_q y(1-z) \quad . \tag{43}$$

In the diagrams of Fig. 3b the photon *and* the gluon is emitted from the fermion loop, and the sum of the corresponding two one-particle irreducible subdiagrams is represented by the effective $\bar{s}bg\gamma$ vertex of eq. (13). After implementing it in the bound-state model, we obtain in terms of the form factors Δi_n of eq. (14)

$$\begin{aligned}\mathrm{Im}F[O_2^{(i)}]_{Fig.3b} &= \frac{g_s^2 Q_u C_N C_B C_V}{64\pi^4} \int dz\,dE_q\,dy\,\frac{E_q^2}{\sqrt{2(m_B-E_q)}}\,\phi_B(E_q)\,\phi_V(y) \\ &\times \Big\{\frac{4(1-z)E_q}{m_B}\,\mathrm{Im}\Delta i_5 - 4\mathrm{Im}\Delta i_6 + \frac{2(1-z^2)E_q^2}{p_\gamma q}\mathrm{Im}\Delta i_{23} \\ &+ \frac{[m_B(1-y)-2E_q]\,(1+z)m_B}{p_\gamma q}\mathrm{Im}\Delta i_{26}\Big\}\,\frac{1}{q^2} \quad , \end{aligned} \tag{44}$$

where

$$\begin{aligned} q^2 &= -m_B E_q (1-y)(1-z) \quad , \\ p_\gamma q &= \frac{m_B}{2}[(1-y)m_B - E_q(1+z)] \quad . \end{aligned} \tag{45}$$

The arguments of the functions $\Delta i_n(z_0, z_1, 0)$ are given by $z_0 = (p_\gamma + q)^2/m_i^2$ and $z_1 = q^2/m_i^2$. The cut on the virtuality of the gluon ($q^2 \leq -\Lambda^2$) leads to the restricted integration intervals

$$\begin{aligned} 0 \leq \quad y \quad &\leq 1 - \frac{\Lambda^2}{m_B^2} \\ \frac{\Lambda^2}{2m_B(1-y)} \leq \quad E_q \quad &\leq \frac{m_B}{2} \\ -1 \leq \quad z \quad &\leq 1 - \frac{\Lambda^2}{m_B E_q (1-y)} \quad . \end{aligned} \tag{46}$$

We can now evaluate the rate asymmetry a_{CP} defined in eq. (1). In terms of the above form factors $F[O_k]$, the amplitudes A_u and A_c entering in eq. (6) are proportional to

$$A_i \sim C_7^{eff}(m_B)\, F[O_7] + C_2(m_B) F[O_2^{(i)}] \quad . \tag{47}$$

Retaining only the leading (order α_s) terms we get

$$a_{CP} = -2 \frac{\mathrm{Im}[v_u v_c^*]}{|v_u + v_c|^2} \frac{C_2(m_B)}{C_7^{eff}(m_B)} \frac{\mathrm{Im}F[O_2^{(u)}] - \mathrm{Im}F[O_2^{(c)}]}{F[O_7]} \quad . \tag{48}$$

To render the dependence on the CKM matrix more transparent it is convenient to use the Wolfenstein parametrization [17] which yields

$$\frac{\mathrm{Im}[v_u v_c^*]}{|v_u + v_c|^2} = \begin{cases} -\eta\lambda^2 & (for\ b \to s\ transitions) \\ \frac{\eta}{(1-\rho)^2+\eta^2} & (for\ b \to d\ transitions) \end{cases} \quad . \tag{49}$$

Our numerical calculation of the asymmetries is based on a CKM matrix with $\lambda = 0.2205$ [see eq. (49)] and $\rho = -0.3$, 0.0, and +0.3 for values of $f_B = 150$ MeV, 200 MeV and 250 MeV, respectively. We vary η in the allowed range 0.15 ...0.5 (for $m_t = 174$ GeV). For the mass values we use $m_B = 5.28$ GeV and $m_c = 1.5$ GeV, while $m_{b,eff}$ from eq. (38) is given in table 1 for different values of f_B. The corresponding values of the CP asymmetries, together with the branching ratio and R, are given in table 2 for the two choices of the cutoff parameter $\Lambda = 1$ GeV and $\Lambda = 0.2$ GeV.

In our approximation, α_s which we choose as $\alpha_s = 0.2$, enters linearly and the asymmetry is indirect proportional to $C_7^{eff}(\mu)$ which suffers from a considerable uncertainty related to the choice of μ [18, 9]. We have also estimated $SU(3)_{Flavour}$ breaking effects in the wave functions eq. (25). Using a parametrization as in ref. [19], the branching ratio for $B \to K^*\gamma$ ($B \to \rho\gamma$) increases (decreases) by up to 30%,

compared to the values in table 2, while $|a_{CP}|$ decreases (increases) less than 4% (7%), respectively. We also note that for the contributions of the diagrams in Fig. 2 alone, the rate asymmetry is essentially independent of uncertainties from the details of the hadronic matrix elements[7]; this is not the case for the diagrams involving the spectator (see Fig. 3).

6. Discussion

In this paper we have calculated the rate and the CP-violating asymmetry for B$\to K^*\gamma$. The branching ratio of $4-5\times 10^{-5}$ is in good agreement with recent CLEO measurements [7] and other theoretical approaches [18, 19]. With a typical asymmetry near 1%, about 10^{10} B-mesons are required to resolve a_{CP} experimentally. A simple rescaling yields for B$\to \rho\gamma$ an asymmetry of above 10% and a branching ratio of 10^{-6}, requiring about 10^9 B-mesons. Similar numbers of B-mesons are needed for all such rate asymmetries which test *direct* CP violation. One expects such samples of B-mesons at the planned hadronic B-facilities [20].

The asymmetry receives contributions from pure spectator-type decays (considered first in ref. [8]) *and* from transitions involving the non-decaying meson constituents. The two add constructively, a fact stressing the importance of bound state effects and possible enhancements of the observables.

The occurence of both mechanisms is expected if one views the rate asymmetry as arising from the rescattering of intermediate DD_s or $K\pi$ meson pairs. In a quark picture, there is no reason to neglect one of the possible gluon exchanges between the intermediate c or u quark and the other constituents; taking into account both contributions just reflects the symmetry of the problem. If one neglects soft rescattering of the mesons, the asymmetry is essentially a phase space effect (generated by the difference of the u and c quark masses) and probably well represented by the quark diagrams.

The asymmetry depends on a cutoff parameter for the momentum of the exchanged gluon. Choosing it between 200 MeV and 1 GeV leads to a variation of the asymmetry by about 20%. We feel that this procedure gives reasonable estimates; since a good understanding of the infrared region is still lacking, it is difficult to go beyond.

Within our simple model which takes into account the internal (transverse) motion of the quarks, corrections to the branching ratio coming from hard gluon exchange between the quark constituents are small. This confirms the fact that such gluons are not sufficient to account for the magnitude of large momentum transfer processes (for a discussion of these problems see [21]).

The rate of $B\to K^*\gamma$ has recently been recalculated, using various QCD sum rule techniques [19, 22, 23]. Our approach yields results most similar to the light-cone sum rules of ref. [19]; in particular, the form factor $F[O_7]$ scales as $m_B^{-3/2}$. In

[7] In our model, the effect of the bound-state wave functions does not completely cancel because the argument m_b of ρ in eq. (40) depends on the quark momenta.

contrast, it goes as $m_B^{+1/2}$ at maximum recoil $p_\gamma^2 = (m_B - m_V)^2$ in ref. [22]. We believe that such a scaling is not correct. Using the Gordon decomposition and the known result $f_B \sim m_B^{-1/2}$, one obtains on general grounds that $F[O_7]$ scales as $m_B^{-3/2}$ at maximum recoil.

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Figure captions

Fig. 1: Leading order contribution from O_7, which we use to determine the rate.

Fig. 2: Order α_s matrix elements of O_2 with an additional gluon exchange between the internal quark loop and the final state s-quark. A cross ($\times$) indicates from where the photon can be emitted. The dashed line denotes the cut generating the absorptive phase.

Fig. 3: Order α_s matrix elements of O_2 with an additional gluon exchange between the internal quark loop and the spectator quark. A cross ($\times$) indicates from where the photon can be emitted. The dashed line denotes the cut generating the absorptive phase.

Fig. 4: Order α_s corrections to the matrix element of O_7 with an additional gluon exchange between the b- or s-quark and the spectator.

Tables

f_B [MeV]	p_F [MeV]	$m_{b,eff}$ [GeV]
150	516	4.68
200	643	4.53
250	778	4.37

Table 1: Wave function parameter p_F and effective b-quark mass $m_{b,eff}$

f_B [MeV]	ρ	R [%]	BR $[10^{-5}]$	a_{CP} for $\Lambda = 1.0$ GeV [%]	a_{CP} for $\Lambda = 0.2$ GeV [%]
$B \to K^*\gamma$					
150	—	12	3.6	0.23 ... 0.76 (0.18 ... 0.60)	0.30 ... 1.0 (0.23 ... 0.76)
200	—	14	4.3	0.24 ... 0.81 (0.18 ... 0.60)	0.32 ... 1.1 (0.23 ... 0.77)
250	—	15	4.8	0.25 ... 0.85 (0.18 ... 0.60)	0.34 ... 1.1 (0.23 ... 0.77)
$B \to \rho\gamma$					
150	−0.3	12	0.30	−2.7 ... −8.1 (−2.2 ... −6.3)	−3.6 ... −11 (−2.7 ... −8.1)
200	0.0	14	0.21	−4.9 ... −13 (−3.6 ... −9.8)	−6.4 ... −18 (−4.6 ... −13)
250	+0.3	15	0.12	−10 ... −24 (−7.2 ... −17)	−14. ... −31 (−9.3 ... −22)

Table 2: Ratio of exclusive to inclusive decays R, branching ratio BR, and CP-asymmetry a_{CP} for the decays $B \to K^*\gamma$ and $B \to \rho\gamma$ and for two different cut-off parameters Λ. The numerical values correspond to η varying in the range $0.15 \ldots 0.5$ and $\alpha_s = 0.2$. The weak η-dependence of the BR($B \to \rho\gamma$) is not shown. The numbers in parentheses correspond to taking into account only the contributions of Fig. 2 in $F[O_2]$.