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Instanton-induced production of QCD jets

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Abstract:

We consider the instanton contributions for the production of a gluon jet with large transverse momentum in QCD. We find that Mueller's corrections corresponding to the rescattering of hard quanta are likely to remove contributions of large instantons, making this cross section well defined. This observation generalizes the previous discussion of instanton effects in the deep inelastic scattering and suggests that all hard processes in the QCD receive hard non-perturbative corrections from instantons of small size, of order $\rho \sim 1/(Q\alpha_s)$.

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1. Gauge theories such as the QCD are known to possess a rich structure of the scattering matrix which reflects the disappearance of quark and gluon singularities and appearance of numerous poles and cuts corresponding to hadrons. It is a common belief that this complex structure reveals itself in full at small energies, while at large energies (in hard processes) all the influence of the rich texture of QCD reduces to a few phenomenological characteristics such as parton densities. Given these densities all properties of a given hard process can be calculated within perturbative QCD. Even at small distances, however, there is a remnant of the nontrivial QCD vacuum structure which can lead to non-perturbative contributions to hard amplitudes — the small-size instantons.

In recent years there is a revival of interest to instanton effects in gauge theories, inspired by the conjection [1] that in high-energy collisions such effects are enhanced enough to produce an observable violation of the baryon number. Although present theoretical arguments rather seem to disfavour such a strong enhancement [2], so that the instanton effects in the electroweak theory presumably remain far below the level at which they might become observable at future colliders, these studies have trigged an increasing interest to semiclassical effects in gauge theories at high energies in a more general context.

In [3, 4] we have suggested to study the instanton contributions in high-energy collisions in the QCD. In this case the coupling is not so small as in the electroweak theory, and the instanton contributions might be observable even if they remain exponentially supressed. The Ringwald's phenomenon — enhancement of the cross section through the dominance of final states with many gluons — allows one to hope that these cross sections may reach observable values, and similtaneously provides a good trigger for their observation, since a fireball of $\sim 2\pi/\alpha_s$ gluons is likely to produce an event with a very high density of particles in the final state.

The major difficulty for the identification of instanton effects in QCD is that in the generic situation they are not infra-red (IR) stable, typically involving a IR-divergent integral over the instanton sizes. There are speculations that in the true QCD vacuum this integration is effectively cut off at sizes of order $\rho \sim 1/600$ MeV, but this assertion is difficult to justify theoretically. To be on a safer side, one should take special care to select contributions of small instantons, which implies going over to a certain hard process. This notion is applied to reactions in which hard scale is related either to a large virtuality of the external particle (photon or W,Z boson), or to a large momentum transfer. In perturbation theory this distinction is subtle: a large momentum transfer necessarily involves an exchange by a highly-virtual gluon (quark). Thus, perturbative description of these processes is similar, in both cases dynamics of small distances can be factorized from the large distance effects. It is this way that most of the QCD predictions arise.

This distinction is crucial, however, for the discussion of instanton effects. In the deep inelastic lepton-hadron scattering the hard scale is brought in by the virtuality Q^2

of the photon. In this case [3, 4] the contribution of small instantons is distinguished by a non-trivial power dependence on Q, corresponding to a fractional twist, and can be disentangled from IR divergent contributions of large instantons, included in parton distributions. This is largely due to the fact that the instanton field at large virtualities contains a gauge-independent piece proportional to $\exp(-\rho|Q|)$ [5]. Thus, coefficient functions in front of parton distributions receive well-defined non-perturbative corrections

$$C(x,Q^2) \sim \exp\left\{ -\frac{4\pi}{\alpha_s(\rho)} \left[1 - \frac{3}{2}\left(\frac{1-x}{1+x}\right)^2 + \ldots \right] \right\}, \tag{1}$$

coming from instantons with the size of order $\rho \sim \pi/\alpha_s \cdot 1/Q$. These corrections have been calculated in [4] and turn out to be of order $10^{-2} - 10^{-5}$ in the region of sufficiently large $Q^2 > 100$ GeV^2 and Bjorken variable $x > 0.3$, where the derivation of (1) is justified.

The situation proves to be essentially different in large momentum transfer reactions, from which we consider production of a gluon jet with large $q_\perp$ as a representative example. On physical grounds it is obvious that this cross section cannot be affected by large instantons. However, a semiclassical calculation fails in this case: to the accuracy to which (1) has been derived in [4], the instanton contribution is given by a power-like divergent integral, and contributions of small instantons $\rho \sim 1/q_\perp$ do not produce any non-trivial dependence on $q_\perp$. Indeed, to the semiclassical accuracy the effect of small instantons is to introduce new *point-like* multi-particle vertices, which do not involve any momentum transfer dependence. Thus, instanton-induced amplitudes do not decrease at large momentum transfers.

Conceptually, it is easy to realize what is missing: to obtain a sensible result one must take into account an (exponentially small) overlap between the initial state, which involves a few hard quanta, with the semiclassical final state [6]. This necessarily involves taking into account quantum corrections to semiclassical amplitudes in the instanton background, the study of which has been pioneered by Mueller [7], see [2] for a review and further references. We demonstrate that the "Mueller's corrections" indeed remove contributions of large instantons to the jet production with large $q_\perp$, making the non-perturbative contribution to this cross section well defined.

2. Mueller finds [7] that to the one-loop accuracy the asymptotics of the gluon propagator in the instanton background takes the factorized form

$$G(p,q) \simeq A_I(p) A_I(q) \frac{\alpha_s}{8\pi} (pq) \ln(pq)\,, \tag{2}$$

assuming $pq \gg 1/\rho^2$, $p^2 = q^2 = 0$. From this expression, it is possible to derive that the corresponding quantum correction to the instanton-induced amplitude acquires the multiplicative factor

$$\exp\left\{ -\frac{\alpha_s}{8\pi} M(p_i) \right\}\,, \tag{3}$$

where

$$M(p_i) = \sum_{i<j} (p_i \cdot p_j) \ln(p_i \cdot p_j)\,. \tag{4}$$

The summation goes over all the ingoing and outgoing particles. To $O(\alpha_s)$ accuracy this formula follows directly from (2), provided $(p_i \cdot p_j) \gg 1/\rho^2$, while the exponentiation of this result is a plausible conjecture beyond, which has created a lot of discussions, see [2]. Our strategy in this work is to take the quantum correction in (3) for granted, and try to understand its qualitative effect on the jet production.† We note in passing that in the deep inelastic scattering the quantum correction (3) is necessary to cancel the ambiguity in the $\bar{I}I$ interaction. In this case the instanton size determined from the saddle-point equations is of order $\rho \sim (4\pi)/\alpha_s \cdot 1/Q \cdot 1/\xi^2$ where $\xi \simeq (R/\rho)^2$ [3], and the $\bar{I}I$ separation R is fixed by the Bjorken x alias by the initial energy. At large $\xi \gg 1$ one has, generally, $M \sim Q^2$ and thus the expression under the exponent in (3) is of order $\pi/\alpha_s \cdot 1/\xi^4$, same as ambiguities in $U_{int}^{\bar{I}I}$.

3. The Mueller's factor M (4) involves momenta of hard particles (a few), which we denote by p_i, and soft particles ($n \sim (4\pi)/\alpha_s U_{int}^{\bar{I}I}$), denoted by k_i. Hard particles are the two colliding partons (gluons) and the final state gluons with large momentum, which are resolved as jets. We consider the inclusive cross section, summing over all soft particles and integrating over their phase space. As it is well known, an explicit integration over the final phase space can be rewritten in terms of the instanton-antiinstanton interaction, which is exemplified by writing

$$\begin{aligned} e^{-(4\pi)/\alpha_s(1-6/\xi^2)} &= e^{-2S_0} \exp\Big\{\frac{24\pi}{\alpha_s}\frac{\rho_1^2\rho_2^2}{R^4}\Big\} \\ &= e^{-2S_0} \exp\Big\{-\frac{16\pi^3}{\alpha_s}\rho_1^2\rho_2^2 \int \frac{dk}{(2\pi)^4}\, e^{-ikR}\frac{(k\cdot E)^2}{E^2k^2}\Big\}, \end{aligned} \tag{5}$$

where $S_0 = 2\pi/\alpha_s$ is the instanton action, E is the total momentum of soft particles, and the maximal attractive $\bar{I}I$ orientation is assumed. For further use we have introduced the notation $\xi = R^2/(\rho_1\rho_2)$. Here ρ_1 and ρ_2 are the sizes of the instanton and antiinstanton, and R is their separation which we take to be parallel to E. The restriction to the special cases of maximal attractive $I\bar{I}$ orientation in color space and $R \parallel E$ orientation in normal space are justified *a posteriori* when it turns out that the final integral over the collective coordinates of instantons is determined by the saddle point where these two properties are valid (see [3] for details).

Eq.(5) corresponds to the approximation when all interactions between the soft particles in the final state are neglected. Taking them into account corresponds to the substitution of the first two terms in the expansion $S(\xi) = 1 - 6/\xi^2 + \ldots$ by the full expression $S(\xi)$ for the QCD action on the $\bar{I}I$ valley configuration [8].

The Mueller's correction generally does not allow to eliminate the explicit dependence on momenta of soft particles, because of the terms corresponding to "soft-hard" interactions, which involve particles with both the hard (initial and final) and soft momenta.

†In what follows we neglect imaginary parts of the logarithms in (4) which produce a phase factor and to our accuracy cancel in the answer for the cross section.

We are going to demonstrate, however, that at large values of the conformal parameter ξ the soft momenta appearing in M can be substituted by

$$k_i \to \frac{E = \sum k_i}{n} \,. \tag{6}$$

where n is the number of soft particles which eventually disappears in the final expressions. The total energy of soft particles E is determined by the energy conservation.[‡] Thus, to this accuracy M is expressed entirely in terms of hard momenta.

The idea of the derivation is to use a freedom to introduce an arbitrary common factor under the logarithms in (4), see [7], to choose this factor in such a way that the terms quadratic in soft momenta become small and can be neglected. To this end, we write down

$$M(p_i) = \sum_{i<j}(p_i \cdot p_j)\ln(\rho^2 p_i \cdot p_j) - \sum_{i,j}(p_i \cdot k_j)\ln(\rho^2 p_i \cdot k_j) + \sum_{i<j}(k_i \cdot k_j)\ln(\rho^2 k_i \cdot k_j)\,, \tag{7}$$

separating contributions of hard and soft momenta explicitly and introducing the scale ρ under the logarithms. To our accuracy it is enough to take $\rho_1 \sim \rho_2 \sim \rho$. The three terms in (7) correspond to "hard-hard", "soft-hard" and "soft-soft" corrections, respectively. The derivation of (4) is justified for the first two terms, involving large relative momenta, while the third piece should be absorbed in the full "soft-soft" correction. The crucial observation, which will be justified *a posteriori*, is that in the form in (7) (that is after the proper scale is included under the logarithms) the third term is small, of order $\sim 1/\xi^3$ compared to the first two terms, and can be neglected.

Restricting to the "soft-hard" terms only, the instanton-induced cross section can be written, apart from the ρ-integration, as

$$\begin{aligned}\sigma_I \;\sim\; & \int dR\, e^{iER} \exp\Bigg\{ -\frac{16\pi^3}{\alpha_s}\rho_1^2\rho_2^2 \int \frac{dk}{(2\pi)^4}\frac{(k\cdot E)^2}{E^2k^2} \\ & \times \exp\Big[-ikR - \frac{\alpha_s}{8\pi}(\rho_1^2+\rho_2^2)\sum_{i,j}(p_i\cdot k_j)\ln(\rho^2 p_i\cdot k_j)\Big]\Bigg\}\,. \end{aligned} \tag{8}$$

As we shall soon demonstrate, large values of ξ correspond to a small fraction of the total energy transferred to the instanton, of order $E^2/s \sim \xi^{-3}$, (see eq. (16) below). Since by assumption $R \parallel E$, we find that

$$\frac{(p_i\cdot p_j)R^2}{(p_i\cdot R)(p_j\cdot R)} \sim \xi^{-3}\,,$$

[‡]It is convenient to use the same notation E for the four-vector of the sum of the soft particle momenta and for their energy in the c.m. frame, which, hopefully, cannot produce a confusion.

because typically $(p_i \cdot E) \sim (p_i \cdot p_j) \sim s$ whereas $E^2 \sim s/\xi^3$. Using this small parameter, the internal integral over k can be taken and gives:

$$\int \frac{dk}{(2\pi)^4} \frac{(k\cdot E)^2}{E^2 k^2} \exp\Big[-ikR - \frac{\alpha_s}{8\pi}(\rho_1^2+\rho_2^2)\sum_{i,j}(p_i\cdot k_j)\ln(\rho^2 p_i\cdot k_j)\Big] =$$
$$= \quad -\frac{3}{4\pi^2}\Big[R - i\frac{\alpha_s}{8\pi}\rho^2\sum_i p_i \ln(-2p_i\cdot R)\Big]^{-4} \qquad (9)$$

Making the shift of the integration variable in the remaining integral over the $\bar{I}I$ separation

$$R \to R' = R - i\frac{\alpha_s}{8\pi}\rho^2\sum_j p_j \ln(-2p_j\cdot R)$$

we obtain

$$\sigma_I \sim \int dR\, \exp\Big[iER - \frac{\alpha_s}{8\pi}(\rho_1^2+\rho_2^2)\sum_j (p_j\cdot E)\ln(p_j\cdot E) - \frac{4\pi}{\alpha_s}S(\xi)\Big]\,. \qquad (10)$$

It is easy to see that the remnant of the "soft-hard" interaction is exactly of the form corresponding to the substitution in (6). Indeed, inserting (6) in (7) and assuming $n \gg 1$ one gets

$$M(p_i) =$$
$$= \sum_{i<j}(p_i\cdot p_j)\ln(\rho^2 p_i\cdot p_j) - \sum_i (p_i\cdot E)\ln(\rho^2 p_i\cdot E/n) + \frac{n(n-1)}{2}(E/n)^2\ln(\rho^2(E/n)^2)$$
$$= \sum_{i<j}(p_i\cdot p_j)\ln(\rho^2 p_i\cdot p_j) - \sum_i (p_i\cdot E)\ln(\rho^2 p_i\cdot E) + E^2\ln(\rho^2 E^2)\,, \qquad (11)$$

where we have used the momentum conservation implying $\sum_i p_i = E$. The second term in (11) reproduces (10), and the last term can now be neglected, since it is of the order $\sim 1/\xi^3$, see the second of eqs.(16) below.

3. In this approximation, it is straighforward to calculate the Mueller's factor for particular processes. For the back-to-back production of a pair of gluon jets $g + g \to g + g + X$ we find

$$M = 2\ln 2 (p-q)^2 + 4pq\ln 2\, G(\theta)\,, \qquad (12)$$

$$\ln 2\, G(\theta) = -\sin^2\frac{\theta}{2}\ln\sin^2\frac{\theta}{2} - \cos^2\frac{\theta}{2}\ln\cos^2\frac{\theta}{2}\,, \qquad (13)$$

where p, q are the momenta of ingoing and outgoing gluons in c.m. frame, respectively, and θ is the angle between them. The instanton-induced cross section to the exponential accuracy reads

$$\sigma_I \sim \int dR\, d\rho\, \exp\left\{ER - \frac{4\pi}{\alpha_s}S(\xi) - \frac{\alpha_s}{4\pi}M(p,q)\rho^2\right\}. \qquad (14)$$

Here $E = 2(p-q)$ is the energy transferred to the instanton (and released in soft particle emission).

The integral is taken by the saddle-point method. The saddle-point values for ρ and R are determined from the equations

$$\begin{aligned} E\rho &= \frac{8\pi}{\alpha_s}\sqrt{\xi-2}S'(\xi)\,, \\ M\rho^2\left[\frac{\alpha_s}{\pi}+\frac{b}{4}\left(\frac{\alpha_s}{\pi}\right)^2\right] &= \frac{16\pi}{\alpha_s}(\xi-2)S'(\xi)+4bS(\xi)\,, \end{aligned} \tag{15}$$

where $S'(\xi) = (d/d\xi)S(\xi)$ and $b = 11-(2/3)n_f$. Neglecting for simplicity the running of the QCD coupling and taking into account the dipole interaction term in the expansion of the action $S(\xi) = 1 - 6/\xi^2 + \ldots$, one gets

$$\frac{\alpha_s}{\pi}\rho = \frac{2E}{M}\sqrt{\xi}\,, \quad \frac{E^2}{M} = \frac{48}{\xi^3}\,, \tag{16}$$

justifying the assumptions which we have made to calculate the "soft-hard" corrections.

Now comes the central point. The function $G(\theta)$ varies between 0 and 1, with a minimum value $G = 0$ at $\theta = 0$ and $\theta = \pi$, and a maximum $G = 1$ at $\theta = \pi/2$. Consider first the collinear jet production, $\theta = 0, \pi$. Hence $M = 2\ln 2(p-q)^2 = E^2/2\cdot\ln 2$ is of order of the energy transferred to the instanton. From the second of the saddle-point equations in (16) one finds then $\xi^3 = 24\ln 2$, independent on the external momenta. Thus, in this case the cross section is defined by the region of $R \sim \rho$ where instantons interact strongly and the calculation is not justified (parametrically). On the other hand, consider jets with large transverse momentum, $\theta = \pi/2$. Then $M = 2\ln 2(p^2+q^2) \simeq s\ln 2$, where $s = 4p^2$ is the total energy, and substituting this to the saddle-point equation we find $\xi^3 = 48\ln 2\cdot s/E^2$. Keeping $E^2 \ll s$ (which means that momenta of gluon jets are close to momenta of coliding gluons) we get $R \gg \rho$ and the calculation is under control. Note that we get $\alpha_s/4\pi\cdot\rho^2 M \sim \pi/\alpha_s\cdot 1/\xi^2$, which indicates that the Mueller's correction now contributes on the leading $1/\xi^2$ level, same as the dipole interaction.

The same effect is observed in the instanton-induced production of monojets $g+g\to g+X$, in which case for $\theta = \pi/2$ in the c.m. frame we obtain

$$\begin{aligned} M &= s\ln 2 - (s/2)\ln(1+E^2/s) + (E^2/2)\ln 2 \\ &\quad - (E^2/2)\ln(1+s/E^2)\,. \end{aligned} \tag{17}$$

The cross section is given again by the integral in (14), and the saddle-point values in the limit $E^2/s \ll 1$ are

$$\xi^3 = 48\ln 2\, s/E^2\,, \quad \sqrt{s}\,\rho = \frac{4\pi}{\alpha_s}\cdot\frac{\sqrt{3\ln 2}}{\xi}\,. \tag{18}$$

Thus, at least in this academic limit, the calculation is under control. This example can be interesting phenomenologically, since it has a clear signature and smaller perturbative

background. Typical numbers are as follows: in gluon-gluon collisions with $\sqrt{s} = 400$ GeV, one could look for production of a monojet with $q_\perp > 180$ GeV, balanced by $n_g \sim 10 - 15$ gluons and $2n_f$ quarks with transverse momenta of order $\rho^{-1} \sim 10$ GeV each. The cross section is difficult to estimate, but is expected to be of the same order or larger than in the deep inelastic scattering [4].

To summarise, we have shown that Mueller's corrections are likely to cut off the IR divergent integrals over the instanton size in the process of gluon jet production with large transverse momentum, indicating that their role is more important than usually believed. In general, one may thus conjecture that in *any* hard process there is a well-defined nonperturbative contribution due to small instantons with the size of order $\rho \sim 1/(Q\alpha_s(Q))$, where Q is the corresponding hard scale. A search for instanton-induced effects in large $q_\perp$ reactions may be most fruitful because of larger rates and smaller backgrounds.

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References

[1] A. Ringwald, Nucl. Phys. B330 (1990) 1.

[2] M.P. Mattis, Phys. Rep. 214 (1992) 159;
P.G. Tinyakov, Int. J. Mod. Phys. A8 (1993) 1993.

[3] I.I. Balitsky and V.M. Braun, Phys. Rev. D47 (1993) 1879.

[4] I.I. Balitsky and V.M. Braun, Phys. Lett. B314 (1993) 237.

[5] V.V. Khoze and A. Ringwald, Phys. Lett. B259 (1991) 106.

[6] T. Banks et al., Nucl. Phys. B347 (1990) 581.

[7] A.H. Mueller, Nucl. Phys. B348 (1991) 310; Nucl. Phys. B353 (1991) 44.

[8] I.I.Balitsky and A.V.Yung, Phys. Lett. 168B (1986) 113; A.V.Yung, Nucl. Phys. B297 (1988) 47.