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Topological structures around the thermal phase transition of pure $SU(3)$ gauge theory studied on a Quadrics Q16

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The temperature dependence of the topological susceptibility around and above the deconfining phase transition is studied both by cooling and the field theoretic method. The susceptibility has a peak at $T \simeq 0.83T_c$ and drops steeply at T_c. In the cooled configurations a characteristic anisotropy of the correlation function of topological charge is detected in the range $T \approx (1..2)T_c$.

1. INTRODUCTION

Instantons are probably the most prominent gauge field degrees of freedom for a working description of the low-lying hadrons [1,2], but cannot contribute to a confining quark-antiquark potential. Therefore their only phenomenological purpose within *pure Yang-Mills theory* seems to provide the solution of the $U_A(1)$ problem, giving a mass to the η' in accordance to the Witten-Veneziano [3,4] argument. The loss of confinement of quenched QCD is probably caused by the disappearance of *other gauge field structures*. If the density of instantons changes rapidly at T_c, this would be merely another indicator of the deconfining phase transition.

In contrast to this, instantons in non-quenched QCD seem to play a more active role in the chiral symmetry restoring phase transition [5]. This mechanism is related to their quark-induced interactions. Whether *purely gluonic interactions* in pure gauge theory also lead to nontrivial topological density correlations has not been checked on the lattice so far. The quenched QCD vacuum at $T = 0$ is reported [6] to be describable as an *uncorrelated instanton ensemble*, but this cannot be true at higher temperature[7].

The instanton density and topological susceptibility is theoretically described by Debye-type thermal screening in a free quark-gluon gas [8] only at very high temperature. This picture holds only at $T > 3T_c$ and is completely unjustified in the confinement phase [7]. For purposes of hadron phenomenology an unambiguous lattice measurement of the topological susceptibility *vs.* temperature would be highly desirable. For $SU(2)$ pure gauge theory this has been studied in Ref.[9] but, due to a discrepancy between the cooling and the field theoretic [10] methods *above deconfinement*, there was no final conclusion. One of our objectives was to redo this analysis for $SU(3)$.

2. MONTE CARLO AND COOLING

During the last months, the Bielefeld group has run a high statistics project of pure $SU(3)$ thermodynamics [11] on the Quadrics Q16 (sponsored by the DFG). Temperatures $T = (0.8...3.8)T_c$ were investigated on a lattice of size $32^3 \times 6$. For a subset of β-values we had the opportunity to analyse every 50-th configuration. Typically, we could use 400 configurations for each β in this pilot study, which was a preparation for the non-quenched case. Due to the I/O bottleneck of the present Quadrics machines it was not possible to store the configurations for an off-line analysis. On-line it would not have been possible to use ge-

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ometric or integral methods to measure the topological charge within an acceptable time. Therefore we used the naive topological charge density on the lattice[13] ($\Pi_{\mu\nu}(x)$ denotes a plaquette)

$$q^L(x) = -\frac{1}{2^4 32\pi^2}\sum_{\pm 1}^{\pm 4}\epsilon_{\mu\nu\sigma\rho}Tr(\Pi_{\mu\nu}(x)\Pi_{\sigma\rho}(x)). \quad (1)$$

The naive charge has been measured for the Monte Carlo configurations. After that these were subject to the following cooling schedule: 20 steps of slow cooling (with maximal step size $\delta = 0.05$ in all three $SU(2)$ subgroups), followed by 50 fast cooling steps (without step size control) in order to find out whether the corresponding topological charge (and susceptibility) stays constant. During the slow cooling the correlation function of topological density has been measured for spacial and temporal distances.

3. DATA ANALYSIS

3.1. Topological susceptibility: cooling

After 20 steps of slow cooling the naive topological charge $\sum q^L(x)$ clusters around multiples of some $Q_0(\beta) < 1$. This *unit of charge* takes care of the roughnesss of the charge distribution (which depends on β !). Although the configurations are neither selfdual or antiselfdual, they can be selected in topological sectors. Using $Q_0(\beta)$ we build charge multiplicity distributions for all cooling steps t, $N(Q,t,\beta)$, by binning around integer Q's, from which cooling histories of the topological susceptibility $\chi^L_{top}(t) = \langle Q^2\rangle(t)/V$ are constructed for each temperature. The histories look very different above and below T_c. Within 15...20 slow cooling steps a plateau is approached from below *in the confinement phase and just at the transition* at $\beta = 5.9$ ($T/T_c = 1.01$). It extends throughout the following fast cooling regime. For $T \geq 1.6T_c$ ($\beta \geq 6.2$) the susceptibility drops to the plateau within the first 5...10 slow cooling steps and stays also constant during the following fast cooling.

In general, we can determine the cooling value χ^{Lc}_{top} through $\chi^L_{top}(t = 20)$ for definiteness. In Fig.1 we show χ^{Lc}_{top}/T_c^4 as function of T/T_c. The cooling history for $\beta = 6.0$ ($T/T_c = 1.2$), how-

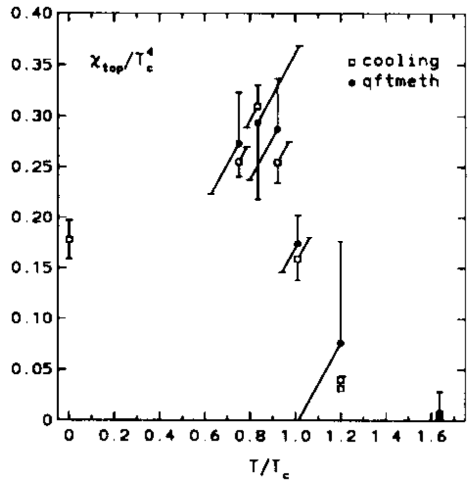

Figure 1. Topological susceptibility around T_c

ever, poses the ambiguity shown in Fig.1. At this temperature $\chi^L_{top}(t)$ does not have a plateau. After a steep minimum it goes through a narrow maximum around $t \approx 12$ (both shown in the figure) before it decays to the lower value over all the fast cooling steps. The value at $T = 0$ is taken from Ref.[12], and is based on the field theoretic method. Notice the peak of the topological susceptibility preceding the deconfinement transition.

3.2. Topological susceptibility: field theoretic method

We want to compare this temperature dependence with the results of the field theoretical analysis. In principle we do not need to use cooled data together with an $Z(\beta_{eff}(t))$. This possibility had been proposed in Ref.[10] in order to use more data and to have control over the renormalization of charge. The lattice topological density is related to the continuum one through a Z-factor, $q^L(x) = a^4 Z(\beta)q(x) + O(a^6)$ with $Z(\beta) = 1 + z_1/\beta + z_2/\beta^2 + \ldots$, while the susceptibility contains perturbative mixing terms

$$\chi^L_{top}(\beta) = Z(\beta)^2\chi_{top} + A(\beta)\langle T\rangle + P(\beta)/a^4 \quad (2)$$

with the energy momentum tensor (known to negligible) and the unit operator (which has to be subtracted). The polynomials $Z(\beta)$ and $A(\beta)$

are perturbatively known up to the leading coefficients ($z_1 = -5.4508$ and a_2), $P(\beta) = c_3/\beta^3 + c_4/\beta^4 + \ldots$ up to the second one ($c_3 = 3.575\ 10^{-3}$, $c_4 = 8.423\ 10^{-4}$) (all numbers correspond to the $32^3 \times 6$ lattice). The next coefficient $c_5 = (2. \pm 0.75)10^{-2}$ has been obtained from a fit to supplementary data taken by us at $\beta = 8., 9., 10.$ and 11., where the topological susceptibility can be safely neglected. Finally, $z_2 = 4.3 \pm 0.6$ has been determined from a fit of χ^L_{top} at our lowest $\beta = 5.75$ (also taken after the Bielefeld measurements) and 5.80, where χ_{top} is considered to coincide with χ^{Lc}_{top}.

The comparison is also shown in Fig.1 (with only statistical errors in the field theoretic values). Within much larger errors, the results of the field theoretic analysis are compatible with the cooling results over the range $(0.75...1.6)T_c$, where the susceptibility drops, but not at higher temperature. In this respect the result is clearer than in the $SU(2)$ case[9].

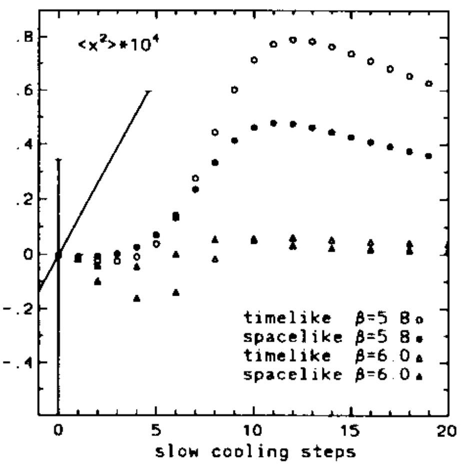

Figure 2. Anisotropy of the moments of $\chi(x, 0)$

3.3. Topological density correlations

The correlation function is dominated by a strong positive core at distances of 0 and 1 lattice spacings. This reflects the spatial size of the instantons themselves. Interesting is a small negative tail which becomes visible in the temporal direction above T_c in an intermediate stage of cooling (around the 5-th slow cooling step). In contrast, for $T < T_c$ both correlation functions behave similarly under the first cooling steps and no negative tail develops.

Lateron the topological charge distribution is washed out by cooling, such that the timelike correlation function becomes more flat and exceeds the spacelike one. In Fig.2 we show how the second moments of the on-axis correlation for spacelike and timelike distances change differently in the process of cooling, both below and above deconfinement.

4. CONCLUSIONS

The results of the cooling method on the temperature dependence of the topological susceptibility around the deconfinement transition are corroborated by the field theoretic method over the range $(0.75..1.6)T_c$. There exists a space-time asymmetry of the topological correlation function, which becomes visible after a few cooling steps for temperatures above T_c.

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