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A Self Consistent Study of the Phase Transition in the Scalar Electroweak Theory at Finite Temperature

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We propose the study of the phase transition in the scalar electroweak theory at finite temperature by a two - step method. It combines i) dimensional reduction to a 3-dimensional *lattice* theory via perturbative blockspin transformation, and ii) either further real space renormalization group transformations, or solution of gap equations, for the 3d lattice theory. A gap equation can be obtained by using the Peierls inequality to find the best quadratic approximation to the 3d action. This method avoids the lack of self consistency of the usual treatments which do not separate infrared and UV-problems by introduction of a lattice cutoff. The effective 3d lattice action could also be used in computer simulations.

1. Scalar Theory

To introduce our method [1] we consider first the scalar $\lambda\Phi^4$-theory on the continuum $\Re^4$ of points z. We define the blockspin, the effective action, the fluctuation field propagator and the interpolation operator. To proceed from temperature $T = 0$ to $T > 0$, we use that the finite temperature propagator v_T can be obtained from the zero-temperature propagator v_0 by periodizing in time: $v_T(r,t) = \sum_{n\in Z} v_0(r, t + n\beta)$. This reproduces the well known finite temperature propagator [2] (sum over Matsubara frequencies $\omega_n = 2\pi n$). We generalize the procedure for the other above mentioned entities.

The continuum is divided into blocks x with extensions L_s in the space and L_t in the time direction respectively. The block centers form a lattice Λ. Following Gawedzki and Kupiainien [3] we associate a block spin $\Phi(x)$ to a scalar field $\phi(z)$ as its block average

$$\Phi(x) = C\ \phi(x) = a v_{z\in x}\ \phi(z)\ .$$

C is called the averaging operator.

Given the continuum action $S[\phi]$, an effective lattice action $S_{eff}[\Phi]$ is defined in Wilson's way

$$\exp(-S_{eff}[\Phi]) = \int \mathcal{D}\phi\, \delta(C\phi - \Phi)\, \exp(-S[\phi])$$

where in place of the δ-function, one may use a Gaussian following [4].

Given a block spin Φ, one determines a background field φ which minimizes the free action $-\frac{1}{2}(\varphi, \Delta\varphi)$ subject to the constraint $C\varphi = \Phi$. This determines the interpolation kernel $\mathcal{A}_T$,

$$\varphi(z) = \int_{z\,\in\,\Lambda} \mathcal{A}_T(z,x)\,\Phi(x), \quad \text{with } C\mathcal{A}_T = 1$$

Following [3] one splits the field ϕ into the low frequency part φ which is determined by the block spin Φ, and a high frequency or fluctuation field ζ which has vanishing block average, $C\zeta = 0$, and propagator $\Gamma_T = P v_T P^*$ with high frequency projector $P = 1 - \mathcal{A}_T C$. The block spin Φ has free propagator $u_T = C v_T C^*$. The effective action and its perturbation expansion to all orders become

$$\begin{aligned} S_{eff}[\Phi] &= \frac{1}{2}\left(\Phi, u_T^{-1}\Phi\right) + V_{eff}[\Phi] + const. \quad (1)\\ V_{eff}[\Phi] &= -\ln\left(\int d\mu_{\Gamma_T}(\zeta)\exp\left(-V_\Phi(\zeta)\right)\right)\\ &= \left\{\exp\left(\frac{1}{2}\frac{\delta}{\delta\zeta}\Gamma_T\frac{\delta}{\delta\zeta}\right)\exp\left(-V_\Phi(\zeta)\right)\right\}_{\zeta=0} \quad (2)\\ V_\Phi(\zeta) &= V(\mathcal{A}_T\Phi + \zeta). \quad (3)\end{aligned}$$

$d\mu_{\Gamma_T}$ is the Gaussian measure with covariance Γ_T. Thus V_{eff} is the free energy of a field theory with free propagator Γ_T and Φ-dependent coupling constants.

At finite temperature, time is periodic with period β. The extension L_t of blocks in time direction must be chosen commensurate with β.

S_{eff} depends on T, but only weakly so as long as $\beta >> L_t$.

A great simplification results if we choose $L_t = \beta$, so that only one block fits in time direction and the lattice Λ becomes 3-dimensional.

The periodic boundary conditions in time direction are now inherited by the blocks. As a consequence one finds that $u_T = \beta^{-1} u_{FT}$ with a *T-independent* 3d lattice propagator u_{FT}, so that the kinetic term in (1) has a factor β, while V_{eff} has only a weak T-dependence. $\Gamma_T(z,z')$ has exponential decay with decay length of order L_s if $L_s \geq O(L_t)$. Therefore the effective action is local modulo exponential tails. It can be expanded,

$$V_{eff}(\Phi) = \int_x \left\{ \frac{1}{2} m^2 \Phi^2 + \frac{\lambda_r}{4!}\Phi^4 + \frac{\lambda_6}{6!}\Phi^6 \right\} + \\ + \frac{\beta}{2} \int_x \int_y \Phi(x)\; u_{FT}^{-1}(x,y)\Phi(y) \left[2\delta_2 + \gamma \Phi(x)^2 \right] + \ldots$$

where all coefficients are (weakly) T-dependent and finite.

Treatment of the effective theory

The effective lattice theory can be analysed in different ways.

1) Further block spin transformations can be performed to increase L_s and ultimately determine the constraint effective potential [7] which determines the probability distribution of the magnetization.

2) Solve a gap equation [8, 9]. Following Feynman and Bogoliubov, one may seek an optimal quadratic approximation $S_0 = \frac{1}{2}(\Phi, J\Phi)$ to S_{eff} around which to expand. The optimal choice is that which maximizes the right hand side of the Peierls inequality [12],

$$\ln Z \geq \ln Z_0 - \langle S - S_0 \rangle_0 \, .$$

The optimal J is determined by the extremality condition

$$\left\langle \frac{\delta^2 S}{\delta\Phi(x)\delta\Phi(y)} \right\rangle_0 = J(x,y) \; .$$

This procedure leads to gap equations whose perturbative solution is the sum of superdaisy diagrams. In principle one can compute corrections, treating $S - S_0$ as a perturbation.

Comparison with other approaches

The preceding formulae are also valid when one averages only over time to obtain a 3-dimensional *continuum* theory [10]. In this case $\mathcal{A}_T = \beta^{-1} 1$ while the fluctuation propagator is translation invariant and given by

$$\Gamma_T(r,t) = \beta^{-1} \sum_{n \neq 0} \int \frac{d^3p}{(2\pi)^3} e^{-ipr - i\omega_n t} [\omega_n^2 + p^2]^{-1}$$

The closest singularity of the integrand is at $|p| = \pm 2\pi i \beta^{-1}$. Therefore Γ_T decays exponentially with r with decay length $\beta/2\pi$. According to the perturbation expansion eq.(2) this leads to nonlocalities in S_{eff} in higher orders. This is a disaster. In contrast with the lattice case, derivative expansions of the nonlocal terms are not possible here because they lead to UV-divergences.

An inconsistency is also encountered in the standard 4-dimensional approach based on a gap equation for the continuum theory whose formal solution is the sum over super daisy diagrams. It is supposed to determine $m^2(T)$. If static and nonstatic contributions are treated on the same footing, one finds a *temperature dependent* logarithmically divergent contribution to the self energy Π:

$$\Pi_{vac} = \frac{1}{16\pi^2} \left[\Lambda^2 - m^2(T) \ln(\frac{\Lambda^2}{m^2(T)}) \right] + O(\frac{1}{\Lambda^2})$$

To cancel the logarithmic UV-divergence a temperature dependent counterterm is needed. This is not acceptable. Our method avoids this by separating UV and IR problems.

2. Scalar Electrodynamics

In this section we outline the extension of the previous sections to scalar electrodynamics. We recall first the Balaban-Jaffe block spin transformation for the abelian gauge field [11]

Given a vector potential $a(z) = a_\mu(z) dz_\mu$ on the continuum, the block spin A is defined through

$$A[b] = A_\mu(x) = av_{\,z \in x} \int_{C_{z,\mu}} d\omega_\nu \; a_\nu(\omega)$$

where b is the link emanating from x in μ-direction, z a point $z \in x$ and $C_{z,\mu}$ the straight path of length one block lattice spacing in μ-direction starting from z.

If $\mu = 4$, $C_{z,\mu}$ connects z with $z + L_t e_4$, and if $\mu \neq 4$ it connects z with $z + L_s e_\mu$ (e_μ = unit vector in μ-direction)

The blocking procedure has to be covariant under gauge transformations. Given a Higgs field $\phi(z)$, we wish to define a covariant block Higgs field $\Phi(x)$. In order to maintain gauge covariance, one must use an averaging kernel which depends on the gauge field a. Our proposal [5] is to define the averaging operator for the Higgs field $C^H(a)$ as lowest eigenmode of the covariant Laplacian Δ_a with Neumann boundary conditions on the block x :

$$-\Delta_a^{N,x}\, C^{H\dagger}(a \mid z, x) = \epsilon_0(a \mid x)\, C^{H\dagger}(a \mid z, x)$$

while $C^H(a \mid z, x) = 0$ for $z \notin x$, and $C^H C^{H\dagger} = 1$. $\epsilon_0(a \mid x)$ is the lowest eigenvalue of $-\Delta_a^{N,x}$ and $C^{H\dagger}(a \mid z, x)$ is the adjoint kernel. C^H admits a perturbation expansion [6].

The choice $L_t = \beta$ leads again to a 3-dimensional lattice. Periodic boundary conditions on blocks in time direction are again imposed. The solution of the eigenvalue equation for the averaging kernel C^H for the Higgs field will depend on T. The effective 3-dimensional lattice action for scalar electrodynamics is

$$S_{eff}(\Phi, A) = -\ln \int \mathcal{D}a \int \mathcal{D}\phi\, \delta_{Ax}(a)\delta(Ca - A)\, \delta(C^H(a)\phi - \Phi)\, e^{-S_M(a) - \frac{1}{2}(\phi, \Delta_a \phi) - V(\phi)}$$

where $V(\phi) = \int dz \left[\frac{1}{2} m_0^2 (\phi\phi^\dagger) + \frac{\lambda}{4!}(\phi\phi^\dagger)^2\right]$, $S_M(a)$ is the Maxwell action, and $\delta_{Ax}(a)$ is the block radial gauge fixing term, which fixes the gauge only locally within each block [11].

After transformation to a block Landau gauge, S_{eff} can be computed by perturbation theory as for the scalar case. The Faddeev Popov transformation to the block Landau gauge as well as the interpolation kernel $\mathcal{A}_T$ and the fluctuation propagator Γ_T were discussed in detail by Balaban and Jaffe [11]. The only new aspect in our situation is the existence of periodic b.c. on blocks in time direction.

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