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QCD CORRECTIONS TO HIGGS BOSON DECAYS*

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ABSTRACT

The two–loop $\mathcal{O}(\alpha_s G_F m_t^2)$ corrections to the $b\bar{b}$ decay rate of the Standard Model Higgs boson as well as its production via $e^+e^- \to ZH$ will be presented. These QCD corrections are obtained by using a low-energy theorem for light Higgs bosons compared to the top quark mass. The results yield strong screening effects of the $\mathcal{O}(G_F m_t^2)$ contributions. After that the two-loop QCD corrections to the $\gamma\gamma$ and gluonic decays of the Higgs bosons of the Standard Model and its minimal supersymmetric extension are discussed. While the corrections to the $\gamma\gamma$ decays remain small of $\mathcal{O}(\alpha_s)$ they are huge $\sim 50 - 70$ % in the case of the gluonic decays.

1. Introduction

1.1. Standard Model [$\mathcal{SM}$]

The $\mathcal{SM}$ contains one Higgs doublet leading to the existence of one elementary scalar [$\mathcal{CP}$-even] Higgs boson H after absorbing the three would-be-Goldstone bosons by the W and Z bosons due to the Higgs mechanism of spontaneous symmetry breaking [1]. The only unknown parameter in the $\mathcal{SM}$ is the Higgs mass. The failure of experiments at LEP1 and SLC to detect the decay $Z \to Hf\bar{f}$ rules out the Higgs mass range $m_H \lesssim 64.3$ GeV [2]. Theoretical analyses of the Higgs sector lead to the consequence that above a cut-off scale Λ the $\mathcal{SM}$ becomes strongly interacting due to the Higgs four point coupling exceeding any limit. Requiring unitarity for the $\mathcal{SM}$ one is left with a consistent formulation of the model up to this cut-off Λ, which leads to an upper bound on the Higgs mass. For a minimal cut-off $\Lambda \sim 1$ TeV this upper bound amounts to about 800 GeV [3], whereas for the $\mathcal{SM}$ being weakly interacting

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up to the GUT scale $\Lambda \sim 10^{15}$ GeV this value comes down to about 200 GeV. On the other hand, requiring the $\mathcal{SM}$ vacuum to be stable, places a lower bound on the Higgs mass depending on the top quark mass $m_t = 176 \pm 13$ GeV [4] and the cut-off Λ. For $\Lambda \sim 1$ TeV the Higgs mass has to exceed about 55 GeV and for $\Lambda \sim 10^{15}$ GeV the value ~ 130 GeV [5]. These bounds decrease dramatically, if the $\mathcal{SM}$ vacuum is required to be metastable [6].

For Higgs masses below about 135 GeV the $\mathcal{SM}$ Higgs boson predominantly decays into $b\bar{b}$ pairs. Consequently this decay mode determines the signature of the Higgs boson in the lower part of the intermediate mass range $m_W \lesssim m_H \lesssim 2m_Z$.

The gluonic decay $H \to gg$ can be detected at future e^+e^- colliders [7]. Its branching ratio amounts to $\lesssim 10^{-1}$ for Higgs masses below ~ 140 GeV. A fourth generation of heavy quarks would increase this branching ratio to a size comparable to the $b\bar{b}$ decay mode. Therefore the precise knowledge of this decay mode within the minimal $\mathcal{SM}$ is mandatory to disentangle novel effects of new physics from the standard profile of the Higgs particle.

The rare decay mode $H \to \gamma\gamma$ with a branching ratio of about 10^{-3} for Higgs masses $m_H \lesssim 150$ GeV yields the main signature for the search of the $\mathcal{SM}$ Higgs particle at the LHC for masses below about 130 GeV [8,9]. Higgs production via photon fusion $\gamma\gamma \to H$ is the relevant mechanism at future high energy photon colliders [10].

1.2. Minimal Supersymmetric Extension of the Standard Model [$\mathcal{MSSM}$]

The $\mathcal{MSSM}$ requires the introduction of two Higgs doublets leading to the existence of five elementary Higgs particles after absorbing the three would-be-Goldstone bosons via the Higgs mechanism of spontaneous symmetry breaking. These consist of two neutral scalar [$\mathcal{CP}$-even] ones h, H, one neutral pseudoscalar [$\mathcal{CP}$-odd] A and two charged ones $H^\pm$. At tree level the Higgs sector can be described by two basic parameters that are usually chosen to be (i) $\mathrm{tg}\beta = v_2/v_1$ with v_1, v_2 being the vacuum expectation values of the neutral scalar Higgs states, and (ii) one of the Higgs masses, usually the pseudoscalar mass m_A. After fixing these two parameters all others are determined due to constraints required by supersymmetry. One of these sets an upper bound on the mass of the lightest neutral scalar Higgs boson h, which must be lighter than the Z boson at tree level. This value increases significantly by the inclusion of radiative corrections, with the leading part increasing as the fourth power of the top mass m_t [11], to about 130 GeV.

The Yukawa couplings of the neutral Higgs bosons to the standard fermions and the couplings to gauge bosons are modified compared to the $\mathcal{SM}$ by additional coefficients fixed by the angle β and the mixing angle α of the neutral scalar Higgs particles h, H. These couplings are shown in Table 1 relative to the $\mathcal{SM}$ couplings. An important feature is the absence of any pseudoscalar A coupling to gauge bosons at tree level.

Table 1. Higgs couplings in the $\mathcal{MSSM}$ to fermions and gauge bosons relative to $\mathcal{SM}$ couplings.

ϕ		t	b	$V = W, Z$
$\mathcal{SM}$	H	1	1	1
$\mathcal{MSSM}$	h	$\cos\alpha/\sin\beta$	$-\sin\alpha/\cos\beta$	$\sin(\beta-\alpha)$
	H	$\sin\alpha/\sin\beta$	$\cos\alpha/\cos\beta$	$\cos(\beta-\alpha)$
	A	$1/\mathrm{tg}\beta$	$\mathrm{tg}\beta$	0

The direct search for the Higgs particles at LEP1 excludes the mass ranges $m_{h,H} \lesssim 45$ GeV for the neutral scalar, $m_A \lesssim 25$ GeV for the neutral pseudoscalar and $m_{H^\pm} \lesssim 45$ GeV for the charged Higgs bosons [12].

The main decay modes of the neutral Higgs particles are in general $b\bar{b}$ decays [~ 90 %] and $\tau^+\tau^-$ decays [~ 10 %]. The gluonic decay mode can reach a branching ratio of a few percent for the light scalar h, with a mass close to its upper end, and the pseudoscalar A as well as the heavy scalar H just below the $t\bar{t}$ threshold for small $\mathrm{tg}\beta$.

Rare $\gamma\gamma$ decays of the neutral scalar Higgs bosons provide the most important signature in the main part of the $\mathcal{MSSM}$ parameter space at the LHC [9].

This paper is organized as follows. In Section 2 we discuss the derivation of the two-loop $\mathcal{O}(\alpha_s G_F m_t^2)$ corrections to the $H \to b\bar{b}$ decay mode of the Standard Higgs boson. Section 3 presents the analogous corrections to Standard Higgs production via $e^+e^- \to ZH$ at LEP. In Section 4 we describe the calculation of the two-loop QCD corrections to the photonic decays $\Phi \to \gamma\gamma$ of the Higgs particles in the $\mathcal{SM}$ and the $\mathcal{MSSM}$ and in Section 5 the corresponding ones to the gluonic decays $\Phi \to gg$.

2. $H \to b\bar{b}$ [$\mathcal{SM}$]

To derive the two-loop $\mathcal{O}(\alpha_s G_F m_t^2)$ corrections to the $H \to b\bar{b}$ decay width we take advantage of a low-energy theorem for light Higgs bosons. This theorem is derived in the limit of vanishing Higgs momentum, where the Higgs field acts as a constant c-number, because $[\mathcal{P}_\mu, H] = i\partial_\mu H = 0$ with $\mathcal{P}_\mu$ denoting the four-momentum operator. Hence the kinetic terms of the Higgs boson in the basic Lagrangian can be neglected in this limit so that the entire interaction with matter particles is generated by the mass substitution $m \to m(1 + H/v)$ for fermions as well as massive gauge bosons [13,14,15]. The parameter $v = 246$ GeV denotes the vacuum expectation value. To extend this theorem to higher orders in perturbation theory all parameters have to be replaced by their bare quantities, marked by the index 0, leading finally to the

low-energy theorem [16,17,24]:

$$\lim_{p_H \to 0} \mathcal{M}(XH) = \sum_{i=f,V} \frac{1}{v_0} \frac{m_i^0 \partial}{\partial m_i^0} \mathcal{M}(X) \tag{1}$$

$\mathcal{M}(X)$ denotes the matrix element of any particle configuration X and $\mathcal{M}(XH)$ the corresponding one with an external Higgs particle added. Using eq.(1) one can build up effective Lagrangians describing the matrix elements $\mathcal{M}(XH)$. One important subtelty for the application of the theorem is that all mass dependent couplings $g_i^0 = m_i^0/v_0$, which are generated by the mass substitution, have to be kept fixed with respect to mass differentiation, so that only dynamical masses in the propagators will be affected by the differentiation in eq.(1).

For the derivation of the two-loop corrections of $\mathcal{O}(G_F m_t^2)$ to the $H \to b\bar{b}$ decay we have to compute the corresponding corrections to the b propagator:

$$\mathcal{M}(b \to b) = m_b^0 \left[1 + \Sigma_S(0)\right] + \not{p} \left[\Sigma_V(0) + \gamma_5 \Sigma_A(0)\right] \tag{2}$$

In the calculation the b mass has to be put equal to zero inside the loops and kept finite only as an overall coefficient. Furthermore we may neglect the W mass and take into account the longitudinal components $w^{\pm}$ only to compute the $\mathcal{O}(\alpha_s G_F m_t^2)$ correction. Applying the low-energy theorem we can derive the effective coupling of the Higgs boson H to b quarks at the same order:

$$\lim_{p_H \to 0} \mathcal{M}(b \to bH) = \frac{1}{v_0} \left(\frac{m_b^0 \partial}{\partial m_b^0} + \frac{m_t^0 \partial}{\partial m_t^0} \right) \mathcal{M}(b \to b) \tag{3}$$

The calculation of the two-loop diagrams yields the following results for the different pieces of the b self-energy in $n = 4 - 2\epsilon$ dimensions [17]:

$$\begin{aligned}
m_b^0 \Sigma_S(0) &= g_b^0 g_t^0 m_t^0 \frac{\Gamma(1+\epsilon)}{(4\pi)^2} \left(\frac{4\pi\mu^2}{(m_t^0)^2} \right)^{\epsilon} \left\{ \frac{2}{\epsilon} + 2 + 2\epsilon \right\} \\
&+ C_F \frac{\alpha_s}{\pi} g_t^0 \frac{\Gamma^2(1+\epsilon)}{(4\pi)^2} \left(\frac{4\pi\mu^2}{(m_t^0)^2} \right)^{2\epsilon} \left\{ \frac{3}{2\epsilon^2} g_b^0 m_t^0 \right. \\
&\qquad \left. + \frac{1}{\epsilon} \left[2 g_b^0 m_t^0 - \frac{3}{4} g_t^0 m_b^0 \right] + \mathcal{O}(1) \right\} \\
\Sigma_V(0) &= (g_t^0)^2 \frac{\Gamma(1+\epsilon)}{(4\pi)^2} \left(\frac{4\pi\mu^2}{(m_t^0)^2} \right)^{\epsilon} \left\{ -\frac{1}{2\epsilon} - \frac{3}{4} - \frac{7}{8}\epsilon \right\} \\
&+ C_F \frac{\alpha_s}{\pi} (g_t^0)^2 \frac{\Gamma^2(1+\epsilon)}{(4\pi)^2} \left(\frac{4\pi\mu^2}{(m_t^0)^2} \right)^{2\epsilon} \left\{ -\frac{3}{8\epsilon^2} - \frac{1}{8\epsilon} + \mathcal{O}(1) \right\}
\end{aligned} \tag{4}$$

with $g_q^0 = m_q^0/v_0$ $(q = t, b)$ and $C_F = 4/3$ denoting the corresponding Yukawa couplings and color factor. After taking the derivative with respect to the top and bottom masses we have to perform the renormalization of the bare couplings, wave functions

and masses. For this purpose we have adopted the on-shell renormalization scheme, which fixes the counter terms as:

$$\begin{aligned}
m_b^0 &= m_b[1-\Sigma_S(0)-\Sigma_V(0)] \\
b_0 &= [1+\frac{1}{2}\Sigma_V(0)]b \\
m_t^0 &= m_t\left(1-\frac{\delta m_t}{m_t}\right) \\
\frac{\delta m_t}{m_t} &= C_F\frac{\alpha_s}{\pi}\left(\frac{4\pi\mu^2}{m_t^2}\right)^{\epsilon}\Gamma(1+\epsilon)\left\{\frac{3}{4\epsilon}+1+2\epsilon\right\} \\
\frac{H_0}{v_0} &= \frac{H}{v}(1+\delta_u)
\end{aligned} \tag{5}$$

with the universal correction [18,19]

$$\delta_u = x_t\left\{\frac{7}{2}-\frac{3}{4}\left[3+2\zeta(2)\right]C_F\frac{\alpha_s}{\pi}\right\} \tag{6}$$

where $x_t = G_F m_t^2/(8\sqrt{2}\pi^2)$. Finally we end up with the effective Lagrangian [17],

$$\begin{aligned}
\mathcal{L}_{eff} &= -m_b\bar{b}b\frac{H}{v}[1+\delta_{nu}][1+\delta_u] \\
\delta_{nu} &= x_t\left(-3+\frac{3}{4}C_F\frac{\alpha_s}{\pi}\right)
\end{aligned} \tag{7}$$

which has to be considered as the basic Lagrangian of the modified theory with the heavy top quark being integrated out so that the perturbative corrections due to the interaction among the light particles have to be added to gain the full correction to the $H \to b\bar{b}$ process. These corrections coincide with the well-known one-loop QCD corrections [20], so that the total correction to the decay width reads [17,19]:

$$\begin{aligned}
\Gamma(H\to b\bar{b}) &= \Gamma_{LO}(H\to b\bar{b})(1+\delta)(1+\delta_{QCD}) \\
\Gamma_{LO}(H\to b\bar{b}) &= \frac{N_c G_F m_H m_b^2}{4\sqrt{2}\pi}\sqrt{1-4\frac{m_b^2}{m_H^2}} \\
\delta &= x_t\left(1-3\left[1+\zeta(2)\right]C_F\frac{\alpha_s}{\pi}\right) \\
\delta_{QCD} &\longrightarrow C_F\frac{\alpha_s}{\pi}\left\{\frac{9}{4}-\frac{3}{2}\log\frac{m_H^2}{m_b^2}\right\} \qquad (m_b \ll m_H)
\end{aligned} \tag{8}$$

The large logarithm of the δ_{QCD} part can be absorbed into the running b mass of the lowest order decay width by changing the scale from the b mass itself to the Higgs mass m_H [20]. The correction δ is numerically given by

$$\delta \approx x_t\left\{1-3.368\alpha_s\right\} \tag{9}$$

yielding a screening effect of about 40% in the leading top mass term, in agreement with the general observation of screening in all known $\mathcal{O}(\alpha_s G_F m_t^2)$ corrections to physical observables in the $\mathcal{SM}$.

3. $e^+e^- \to ZH$ [$\mathcal{SM}$]

To obtain the two-loop $\mathcal{O}(\alpha_s G_F m_t^2)$ corrections to Higgs production via $e^+e^- \to ZH$ the low-energy theorem eq.(1) can again be used. The effective coupling of a light Higgs boson with a mass negligible as compared to the top quark mass m_t can be derived from the corresponding corrections to the on-shell Z self-energy:

$$\mathcal{M}(Z \to Z) = (M_Z^0)^2 + (g_Z^0 v_0)^2 \Pi_{ZZ}(0) \tag{10}$$

Calculating the two-loop corrections to the self-energy Π_{ZZ} for large top masses at vanishing Z mass and Z momentum one arrives at the result [21,22]

$$\begin{aligned} \Pi_{ZZ}(0) &= x_t^0 \left(\frac{4\pi\mu^2}{(m_t^0)^2}\right)^{\epsilon} \Gamma(1+\epsilon)\frac{6}{\epsilon} \\ &+ C_F \frac{\alpha_s}{\pi} x_t^0 \left(\frac{4\pi\mu^2}{(m_t^0)^2}\right)^{2\epsilon} \Gamma^2(1+\epsilon)\left\{\frac{9}{2\epsilon^2} - \frac{21}{4\epsilon} + \frac{3}{8}\right\} \end{aligned} \tag{11}$$

with

$$x_t^0 = \frac{G_F^0 (m_t^0)^2}{8\sqrt{2}\pi^2} \qquad\qquad g_Z^0 = \frac{M_{Z0}}{v_0} \tag{12}$$

The low-energy theorem leads to the following relation to the effective coupling of the Higgs bosons to Z bosons at vanishing Higgs momentum

$$\lim_{p_H \to 0} \mathcal{M}(Z \to ZH) = \frac{1}{v_0}\left(\frac{M_Z^0 \partial}{\partial M_Z^0} + \frac{m_t^0 \partial}{\partial m_t^0}\right)\mathcal{M}(Z \to Z) \tag{13}$$

After taking the differentiation and performing the renormalization of the bare parameters in the on-shell renormalization scheme we end up with the effective Lagrangian

$$\begin{aligned} \mathcal{L}_{HZZ} &= \left(\sqrt{2}G_F\right)^{1/2} M_Z^2 Z^\mu Z_\mu H(1+\delta_{HZZ}) \\ \delta_{HZZ} &= x_t\left\{-\frac{5}{2} + \frac{3}{2}\left[\frac{15}{2} - \zeta(2)\right] C_F \frac{\alpha_s}{\pi}\right\} \end{aligned} \tag{14}$$

This Lagrangian contains only the $\mathcal{O}(G_F m_t^2)$ and $\mathcal{O}(\alpha_s G_F m_t^2)$ corrections to this coupling. All other higher-order corrections are omitted from eq.(14). The correction δ_{HZZ} amounts to $\delta_{HZZ} = -5x_t/2(1 - 4.684\alpha_s/\pi)$ yielding a screening effect of about 20% in the leading top mass contribution to the effective HZZ coupling. In order to derive the correction to the cross section $\sigma(e^+e^- \to ZH)$ an additional term due to

the renormalization of the Ze^+e^- vertex in the on-shell scheme has to be added,

$$\begin{aligned}\sigma(e^+e^- \to ZH) &= \sigma_{LO}(e^+e^- \to ZH)(1+\delta_{HZe^+e^-}) \\ \delta_{HZe^+e^-} &= 2\delta_{HZZ} + \left(1 - 8\frac{c_w^2 Q_e v_e}{v_e^2+a_e^2}\right)\Delta\rho \end{aligned} \tag{15}$$

which is proportional to the correction of the ρ parameter [21,22,23]:

$$\Delta\rho = x_t\left\{3 - \frac{3}{2}\left[1+2\zeta(2)\right]C_F\frac{\alpha_s}{\pi}\right\} \tag{16}$$

The final correction can be cast into the form [24]

$$\delta = -2x_t\left\{1 - \left[\frac{21}{2} - 3\zeta(2)\right]C_F\frac{\alpha_s}{\pi} + 12\frac{c_w^2 Q_e v_e}{v_e^2+a_e^2}\left[1 - \left[\frac{1}{2}+\zeta(2)\right]C_F\frac{\alpha_s}{\pi}\right]\right\} \tag{17}$$

Numerically this amounts to a screening effect of about 20% in the leading top mass term $\mathcal{O}(G_F m_t^2)$.

4. $\Phi \to \gamma\gamma$ [$\mathcal{SM}$, $\mathcal{MSSM}$]

The lowest order $\Phi\gamma\gamma$ coupling [Φ denotes all possible kinds of Higgs bosons within the $\mathcal{SM}$ and the $\mathcal{MSSM}$] is mediated by fermion and W boson loops yielding the following expression for the lowest order decay width [13,14,25]

$$\Gamma(\Phi \to \gamma\gamma) = \frac{G_F\alpha^2 m_\Phi^3}{128\sqrt{2}\pi^3}\left|\sum_f N_c e_f^2 g_f^\Phi A_f(\tau_f) + g_W^\Phi A_W(\tau_W)\right|^2 \tag{18}$$

with g_i^Φ denoting the corresponding couplings of Table 1. The individual amplitudes are given by

$$\begin{aligned} A_f^H(\tau) &= 2\tau[1+(1-\tau)f(\tau)] \\ A_f^A(\tau) &= 2\tau f(\tau) \\ A_W^H(\tau) &= -[2+3\tau+3\tau(2-\tau)f(\tau)] \end{aligned} \tag{19}$$

where the scaling variables are defined as $\tau_i = 4m_i^2/m_\Phi^2\, i$ $(i = f, W)$; the function $f(\tau)$ can be expressed as

$$f(\tau) = \begin{cases} \arcsin^2\dfrac{1}{\sqrt{\tau}} & \tau \geq 1 \\ -\dfrac{1}{4}\left[\log\dfrac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} - i\pi\right]^2 & \tau < 1 \end{cases} \tag{20}$$

Heavy particles provide the dominant contributions to this rare decay mode, so that we restrict ourselves to the W, top and bottom contributions in the following. The

branching ratio amounts to about 10^{-3} in the mass ranges, where this decay process is visible. The cross section for Higgs boson production via photon fusion, the relevant production mechanism at future high energy photon colliders, can be derived from the decay width via

$$\sigma(\gamma\gamma \to \Phi) = \frac{8\pi^2}{m_\Phi^3}\Gamma(\Phi \to \gamma\gamma)\delta\left(1 - \frac{m_\Phi^2}{s}\right) \tag{21}$$

The two-loop QCD corrections to the photonic decay mode can be parameterized as a correction to the quark amplitude

$$A_Q = A_Q^{LO}\left[1 + C_\Phi(\tau_Q)\frac{\alpha_s}{\pi}\right] \tag{22}$$

To evaluate the coefficient $C_\Phi(\tau_Q)$ we reduced the five-dimensional Feynman integrals of the virtual corrections analytically down to one-dimensional ones containing trilogarithms in the integrand. The regularization of ultraviolet singularities is performed in $n = 4 - 2\epsilon$ dimensions. The pseudoscalar γ_5 coupling is defined in the scheme by 't Hooft and Veltman, which has been systematized by Breitenlohner and Maison [26]. This definition of γ_5 in n dimensions reproduces the axial vector anomaly and is consistent up to any order in perturbation theory. The renormalization is performed in the on-shell scheme with the running quark mass defined by the boundary condition $m_Q^{on}(\mu^2 = m_Q^2) = m_Q$, where m_Q denotes the physical mass defined as the pole of the quark propagator. This mass definition does *not* coincide with the usually chosen running $\overline{MS}$ mass, but differs by a finite amount $m_Q^{on}(\mu^2) = m_Q^{\overline{MS}}(\mu^2)\left[1 + 4/3\ \alpha_s(m_Q^2)/\pi + \mathcal{O}(\alpha_s^2(m_Q^2))\right]$. In the limit of large quark masses m_Q compared to the Higgs mass m_Φ the coefficients C_Φ approach the following values [16,27,28]

$$C_H \to -1 \qquad\qquad C_A \to 0 \tag{23}$$

These limits can also be derived by using low-energy theorems:

Scalar Higgs bosons. To derive the QCD correction to the $H\gamma\gamma$ coupling in the limit of small Higgs masses we have to differentiate the vacuum polarization function Π by the heavy quark mass m_Q. The heavy quark part of this function can be expressed in terms of the effective Lagrangian

$$\mathcal{L}_{eff} = -\frac{1}{4}F_0^{\mu\nu}F_{0\mu\nu}\left\{1 + \Pi_Q\left(\frac{\mu^2}{m_Q^2}\right)\right\} \tag{24}$$

where m_Q denotes the physical renormalized heavy quark mass. Rewriting the differentiation by the bare mass m_Q^0 in eq.(1) in terms of the renormalized mass m_Q a correction due to the anomalous mass dimension γ_m is obtained. The differentiation

of the vacuum polarization function by the renormalized heavy quark mass yields the heavy quark contribution to the QED β function,

$$\frac{m_Q^0 \partial}{\partial m_Q^0} \Pi_Q \left(\frac{\mu^2}{m_Q^2} \right) = \frac{1}{1+\gamma_m} \frac{m_Q \partial}{\partial m_Q} \Pi_Q \left(\frac{\mu^2}{m_Q^2} \right) = -\frac{\beta_\alpha^Q / \alpha}{1+\gamma_m} \tag{25}$$

so that finally we end up with the effective Lagrangian for the $H\gamma\gamma$ coupling in the limit of a heavy quark Q compared to the Higgs mass [16,29,30]

$$\mathcal{L}_{H\gamma\gamma} = \frac{1}{4} \frac{\beta_\alpha^Q / \alpha}{1+\gamma_m} F^{\mu\nu} F_{\mu\nu} \frac{H}{v} \tag{26}$$

Expanding the β function and the anomalous mass dimension up to $\mathcal{O}(\alpha_s)$

$$\frac{\beta_\alpha^Q}{\alpha} = 2e_Q^2 \frac{\alpha}{\pi} \left[1 + \frac{\alpha_s}{\pi} \right] \qquad \text{and} \qquad \gamma_m = 2\frac{\alpha_s}{\pi} \tag{27}$$

we arrive at the correction in the limit of light Higgs masses,

$$\frac{m_H^2}{4m_Q^2} \to 0 \; : \qquad 1 + C_H \frac{\alpha_s}{\pi} \to \frac{1+\alpha_s/\pi}{1+2\alpha_s/\pi} = 1 - \frac{\alpha_s}{\pi} \tag{28}$$

in agreement with the explicit expansion of the two-loop diagrams.

Pseudoscalar Higgs bosons. Also for pseudoscalar Higgs bosons a low-energy theorem can be derived based on the ABJ-anomaly of the axial vector current $j_5^\mu = \bar{Q}\gamma^\mu\gamma_5 Q$ [31]

$$\partial_\mu j_\mu^5 = 2m_Q \overline{Q} i\gamma_5 Q + N_c e_Q^2 \frac{\alpha}{4\pi} F_{\mu\nu} \widetilde{F}_{\mu\nu} \tag{29}$$

with $\widetilde{F}_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta} F_{\alpha\beta}$ denoting the dual field strength tensor. A general theorem states that there are *no* radiative corrections modifying eq.(29), which therefore remains valid up to all orders in perturbation theory [32]. The operator $\partial_\mu j_5^\mu$, multiplied with the pseudoscalar field operator A at vanishing momentum, fulfills the low-energy condition [33]

$$\lim_{p_A \to 0} \langle \gamma\gamma | \partial_\mu j_5^\mu A | A \rangle = 0 \tag{30}$$

Using the basic interaction Lagrangian $\mathcal{L}_{int} = -m_Q \bar{Q} i\gamma_5 Q A/v$ for the coupling of the pseudoscalar A to quarks one immediately arrives at the effective Lagrangian [16,27]

$$\mathcal{L}_{eff}(A\gamma\gamma) = N_c e_Q^2 \frac{\alpha}{8\pi} F_{\mu\nu} \widetilde{F}_{\mu\nu} \frac{A}{v} \tag{31}$$

Because of the Adler-Bardeen theorem [the non-renormalization of the ABJ-anomaly] [32] this Lagrangian is valid up to all orders in perturbation theory, so that the QCD corrections are vanishing in the heavy quark limit

$$\frac{m_A^2}{4m_Q^2} \to 0 \; : \qquad C_A \to 0 \tag{32}$$

Fig. 1. Real and imaginary parts of the QCD correction to the scalar two-photon decay amplitude for two different scales μ_Q of the running quark mass.

Fig. 2. Real and imaginary parts of the QCD correction to the pseudoscalar two-photon decay amplitude.

in agreement with the explicit expansion of the two-loop contributions.

In Fig.1 the coefficient C_H is shown as a function of the scaling variable $\tau = \tau_Q^{-1}$. For large Higgs masses the large logarithms can be absorbed into the running quark mass by shifting the scale from the quark mass itself to the Higgs mass $m_\Phi/2$.[a] The QCD corrections are small of $\mathcal{O}(\alpha_s)$ so that the processes are theoretically under control. The QCD correction to the pseudoscalar decay develops a Coulomb singularity at threshold [$m_A = 2m_Q$], which is due to the equality of the quantum numbers of the pseudoscalar Higgs bosons and the ground state of heavy quarkonium ($\bar{Q}Q$). This property leads to a step in the imaginary part of C_A and the corresponding logarithmic singularity in the real part [see Fig.2]. Hence the perturbative analysis is not valid within a margin of a few GeV around the threshold, requiring the analysis to be improved in the threshold region [34].

Fig. 3. The relative QCD corrections to the photonic decay width of the $\mathcal{SM}$ Higgs boson.

The relative QCD corrections are shown in Fig.3 for the $\mathcal{SM}$ and Fig.4 for the $\mathcal{MSSM}$. They are large only in those region where strong destructive interferences are present in the lowest order amplitude. This is rather dramatic in the $\mathcal{SM}$ where the top and W loop are nearly cancelling each other at a Higgs mass $m_H \sim 600$ GeV.

5. $\Phi \to gg$ [$\mathcal{SM}$, $\mathcal{MSSM}$]

The gluonic decays $\Phi \to gg$ of the Higgs bosons in the $\mathcal{SM}$ and the $\mathcal{MSSM}$ are mediated in lowest order by loops of colored particles with quarks providing the leading contributions. The lowest order decay width is given by [25,35]

$$\Gamma(\Phi \to gg) = \frac{G_F \alpha_s^2}{36\sqrt{2}\pi^3} m_\Phi^3 \left| \sum_Q g_Q^\Phi A_Q^\Phi(\tau_Q) \right|^2 \tag{33}$$

with the amplitudes

$$A_Q^H(\tau) = \frac{3}{2}\tau[1 + (1-\tau)f(\tau)] \qquad\qquad A_Q^A(\tau) = \frac{3}{2}\tau f(\tau) \tag{34}$$

and g_Q^Φ denoting the corresponding couplings of Table 1. The scaling variable τ_Q and the functions $f(\tau)$ are defined in the previous section. Heavy quarks yield the dominant contribution to the decay width, so that we restrict ourselves to the contributions of the top and bottom quark in the following. In the $\mathcal{MSSM}$ the top quark

[a] The scale $\mu = m_\Phi/2$ has been chosen to define the threshold to be at the correct location $2m_Q(m_Q)$.

Fig. 4. The relative QCD corrections to the photonic decay widths of the $\mathcal{MSSM}$ Higgs bosons for two different values of tgβ.

part is suppressed for large tgβ, whereas the bottom one is enhanced in this case. In the visible mass ranges the branching ratio of the gluonic decay mode amounts to $\lesssim 10^{-1}$.

The two-loop QCD corrections to the decay width can be parametrized by

$$\Gamma(\Phi \to gg(g),\ gq\bar{q}) = \Gamma_{LO}(\Phi \to gg)\left[1 + E_\Phi(\tau_Q)\frac{\alpha_s}{\pi}\right] \tag{35}$$

The evaluation of the coefficients $E_\Phi(\tau_Q)$ requires the computation of five-dimensional Feynman integrals for the virtual corrections, which have been reduced analytically to one-dimensional ones containing trilogarithms in the integrand. Ultraviolet, infrared and collinear singularities are regularized in $n = 4-2\epsilon$ dimensions. As in the photonic decay mode the pseudoscalar γ_5 coupling is defined in the scheme of 't Hooft-Veltman and Breitenlohner-Maison [26]. The counter terms are fixed by defining the quark masses on-shell and the strong coupling α_s in the $\overline{MS}$ scheme with five active light flavors, i.e. the heavy top quark is decoupled. To obtain the full QCD corrections the one-loop real corrections $\Phi \to ggg,\ gq\bar{q}$ have to be added with phase space integration performed in n dimensions. Adding them to the virtual corrections infrared and collinear singularities are cancelled resulting in finite corrections [16,29,36]:

$$\begin{aligned} E_H(\tau) &= \frac{95}{4} - \frac{7}{6}N_F + \frac{33-2N_F}{6}\ \log\frac{\mu^2}{m_H^2} + \Delta E_H \\ E_A(\tau) &= \frac{97}{4} - \frac{7}{6}N_F + \frac{33-2N_F}{6}\ \log\frac{\mu^2}{m_A^2} + \Delta E_A \end{aligned} \tag{36}$$

Fig. 5. The relative QCD corrections to the gluonic decay width of the $\mathcal{SM}$ Higgs boson.

Fig. 6. The relative QCD corrections to the gluonic decay widths of the $\mathcal{MSSM}$ Higgs bosons for two different values of tgβ.

In the limit of heavy quark masses compared to the Higgs masses the contributions ΔE_Φ vanish; this can also be derived from the low-energy theorems for scalar and pseudoscalar Higgs particles. The QCD corrections to the pseudoscalar decay $A \to gg$ develop a Coulomb singularity at threshold [$m_A = 2m_Q$], so that the perturbative analysis is not valid in a small margin around the threshold. The QCD corrections amount to about 50 – 70% and are shown in Fig.5 for the $\mathcal{SM}$ Higgs boson and Fig.6 for the $\mathcal{MSSM}$ Higgs bosons. Hence they provide an important contribution to the theoretical prediction of the gluonic decay rates.

6. References

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