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Particle Currents in a Space-time dependent and CP- Violating Higgs Background: a Field Theory Approach

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Abstract

Motivated by cosmological applications like electroweak baryogenesis, we develop a field theoretic approach to the computation of particle currents on a space-time dependent and CP-violating Higgs background. We consider the Standard Model model with two Higgs doublets and CP violation in the scalar sector, and compute both fermionic and Higgs currents by means of an expansion in the background fields. We discuss the gauge dependence of the results and the renormalization of the current operators, showing that in the limit of local equilibrium, no extra renormalization conditions are needed in order to specify the system completely.

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1. Introduction

In a recent letter, we have presented our results for the computation of the expectation value of particle currents in a two Higgs doublet model, in the case in which the Higgs field background is space time dependent and CP-violating [1]. The original motivation for such a computation comes from the so called spontaneous baryogenesis scenario [2] at the electroweak phase transition. The basic ingredients of this mechanism are CP violation in the Higgs sector, which requires at least two Higgs doublets, and thick bubble walls, so that at least some of the more rapid processes (top Yukawa and gauge flavour diagonal interactions) can be assumed to be in local equilibrium in the background of the electroweak bubble walls and a adiabatic approximation can be applied. Of course, also in this scenario one must require that the baryon number violating processes go quickly out of equilibrium in the broken symmetry phase, which seems by now very hard to achieve, at least in the Standard Model [4] and in the minimal supersymmetric extension of it [5].

Both in the case of explicit and spontaneous [6] CP-violation, a space-time dependent relative phase $\delta(x)$ between the two Higgs fields can be present inside the wall of the expanding bubble. Due to the fact that the background is not space-time translational invariant, the phase $\delta(x)$ can give rise locally to nonvanishing expectation values for the particle currents present in the theory. If baryon number violation is then in equilibrium, these nonvanishing particle currents can rearrange to generate a baryon asymmetry.

In order to reliably compute the final baryon asymmetry, it is then necessary, as a first step, to determine the values of the currents induced by the background. In the approach of refs. [2, 8] a rotation of the fermionic fields is done to make the Yukawa couplings real. As a consequence, a derivative coupling of the form

$$\mathcal{L}_{int} \sim \partial_\mu \delta J^\mu, \tag{1}$$

where J^μ is the current corresponding to the rotation, is induced from the kinetic terms. In the original paper [2] only the time derivative $\dot{\delta}$ was taken into account. $\dot{\delta}$ acts as an effective chemical potential, called "charge potential", and particle densities acquire nonvanishing expectation values $n_i \sim q_i \, \dot{\delta} \, T^2$, where q_i is the charge of the i-th particle under the given rotation. The thermodynamical evolution of the system in presence of baryon number violation then leads to a nonvanishing baryon asymmetry. The gradient part of $\delta(x)$ in eq. (1) was subsequently taken into account in ref. [8] where the role of particle diffusion was also discussed. Nevertheless, as we already pointed our in [1], using the interaction term in eq. (1) as a starting point to compute the perturbations to the thermal averages presents some drawbacks.

Since the phase δ is communicated from the Higgs to the fermion sector through the Yukawa interactions, any perturbation in the fermion densities n_i should vanish in the limit of zero Yukawa couplings h_i. Also, they should vanish in the limit of zero vacuum expectation value for the Higgs fields $H_i(x) = v_i(x)\exp[i\theta_i(x)]$ because no spontaneous CP violation is present in the Higgs sector in this limit. Naively, one could then expect a suppression factor of order $(h_i^2 v_i^2(x)/T^2)$, where h_i is the relevant Yukawa coupling, for the perturbations in the fermionic particle number with respect to the original result. Since one is interested in regions of the bubble wall where sphalerons are still active, *i.e* for values of $v_i(x)/T$ typically smaller than one, then the above mentioned suppression factor might be crucial for the scenario of baryogenesis outlined above [1].

The reason why these suppressions do not appear in the original treatment, is that considering eq. (1) as the only effect of the background is equivalent to perturbing around the Higgs field configuration $\delta(x) = 0$, $v_i(x) \neq 0$, which is not a solution of the field equations. On the other hand, taking the field equations into account, one immediately sees that the expected suppression factors are recovered also in the original treatment, so that the result comes out to be rotation independent, as it should, also in this context. Indeed, by partially integrating eq. (1), we obtain a 'perturbation term' which is given by the hypercharge violating terms in the lagrangian, coming from the fermionic mass terms and from the Higgs potential. Also, from the field equations one can see that $\partial_\mu \delta(x)$ vanishes as $v_i(x)^2$ for vanishing $v_i(x)$. In other words, it is the perturbation (1) itself, if properly considered, to be vanishing in the limit of vanishing top Yukawa and Higgs couplings, or in the limit of vanishing values for the background Higgs fields.

In the limit in which some interaction rates (like for example those for the top Yukawa processes) are so high as to be in thermal equilibrium inside the bubble walls, the local equilibrium values for the corresponding particle numbers are of course independent on the reaction rates.

On the other hand, since their values are fixed by that of the perturbation term (1), they are still suppressed by the above mentioned factor of $(h_t^2 v(x)^2/T^2)$.

In this paper we want to describe in more detail our approach to calculate the expectation values of a composite operator $\hat{O}(z)$ in a CP-violating Higgs space-time dependent background. For definitness, we will work on the background of a bubble wall, as the one described above, however the formalism is quite general and can be easily extended to consider other interesting situations, like for example a non trivial background also for the gauge fields.

The method relies on a functional expansion in power series of $\Phi_i^c(x)$ where the coefficients of the expansion are the n-point 1PI Green's functions with one insertion of the operator $\hat{O}(z)$

computed in the *unbroken* phase.

We shall discuss the dependence of the expectation values from the gauge in which the background $\Phi_i^c(x)$ is expressed, showing explicitely that the expectation values of gauge invariant operators are gauge independent. Moreover, the problem of current renormalization will be addressed. In general, the computation of Green's functions with insertion of a composite operator requires the introduction of new counterterms besides those necessary for the renormalization of the basic lagrangian. This fact leads to the well known phenomenon of the mixing among the different renormalized currents of the theory. We shall discuss the mixing matrix for the renormalized currents present in the Standard Model model with two Higgs doublet and use the non renormalization properties for the conserved currents in order to reduce the number of independent counterterms.

Finally, we will discuss the limit of local equilibrium, showing that, in this case, the only expectation values that can be consistently computed are those for the conserved currents, so that no new renormalization condition is needed to specify the system in this limit.

The paper is organized as follows: in sect. **2** our field theory method is described on general grounds and particular attention is given to the classical equations of motions and to the question of gauge invariance. In sect. **3** the explicit calculations for the fermionic and Higgs currents in the specific model under consideration are given in details. They are relevant for the spontaneous baryogenesis mechanism and their consequence have been discussed in ref. [1]. Sect. **4** deals with the issue of current renormalization, whereas the role of the conserved currents in the thermodynamical limit is described in sect. **5**. Finally, sect. **6** presents our conclusions.

2. The expansion.

In this paragraph we will discuss our approach to the computation of the expectation value of an operator $\hat{O}(z)$ on a non-zero background for the fields of the theory.

Our starting point is the finite temperature generating functional for the 1PI Green's functions with insertion of an operator $\hat{O}(z)$ (in the following $\hat{O}(z)$ will represent a particle current)

$$\Gamma\left[\Phi_i^c(x),\,\Delta(x)\right] = W\left[J_i(x),\,\Delta(x)\right] - \sum_j \int d^4x J_j(x)\Phi_j^c(x), \tag{2}$$

where $\Phi_i^c(x)$ are the classical fields of the theory and $J_i(x)$ the corresponding sources, while $\Delta(x)$ is the source for the operator $\hat{O}(x)$. Note that the Legendre transformation has been performed only on the fields and not on the operator.

The quantity we are interested in is the expectation value of the operator $\hat{O}(z)$ on the background given by the fields $\Phi_i^c(x)$, which we will specify later,

$$\langle \hat{O}(z) \rangle_{\Phi_i^c(x)} = \left. \frac{\delta \Gamma \left[\Phi_i^c,\, \Delta \right]}{\delta \Delta(z)} \right|_{\Delta=0} \equiv \mathcal{O}\left[\Phi_i^c(x) \right](z). \tag{3}$$

We can expand the functional $\mathcal{O}\left[\Phi_i^c(x) \right](z)$ in a power series of Φ_i^c

$$\mathcal{O}\left[\Phi_i^c(x) \right](z) = \sum_{n=0}^{\infty} \sum_{i_1,\ldots,i_n} \frac{1}{n!} \int d^4x_1 \cdots d^4x_n \mathcal{O}^{(n)}_{i_1,\ldots,i_n}(x_1,\cdots,x_n;\; z) \Phi^c_{i_1}(x_1) \cdots \Phi^c_{i_n}(x_n), \tag{4}$$

where the coefficients of the expansion are the n-point 1PI Green's functions with one insertion of the operator $\hat{O}(z)$ computed in the *unbroken* phase

$$\mathcal{O}^{(n)}_{i_1,\ldots,i_n}(x_1,\cdots,x_n;\; z) = \frac{1}{i} \left. \frac{\delta^{n+1} \Gamma \left[\Phi_i^c,\, \Delta \right]}{\delta \Phi^c_{i_1}(x_1) \cdots \delta \Phi^c_{i_n}(x_n) \delta \Delta(z)} \right|_{\Phi_i^c = \Delta = 0}. \tag{5}$$

Now we have to specify the background fields, $\Phi_i^c(x)$, which give the starting point for the expansion in eq. (4). A priori, the relevant background is given by the solution of the full field equations,

$$\left. \frac{\delta \Gamma \left[\Phi_i^c,\, \Delta = 0 \right]}{\delta \Phi_i^c(x)} \right|_{\Phi_i^c = \bar{\Phi}_i^c} = 0 \tag{6}$$

with appropriate boundary conditions. In practice, however, we have to consider some approximation of this complete, and unknown, solution. A possibility could be to consider the solutions of the classical equations of motion as the zero order approximation. However in many interesting cases the radiative corrections to the effective potential, either at zero or at finite temperature, are crucial in order to determine the shape of the potential and consequently of the background. A typical example is that of a first order phase transition induced by finite temperature corrections, in which the existence of bubble solutions is due to the $T \neq 0$ radiative corrections which induce a second minima in the effective potential. Following refs. [9, 10], we will then consider the classical equations of motion in which the tree level effective potential is replaced by the radiatively corrected one.

In the following, we will consider the Standard Model with two Higgs doublets, H_1 and H_2. H_1 has $U(1)_Y$ charge $-1/2$, and couples to the 'down-type' right handed fermions, whereas H_2 (charge $1/2$) couples to the 'up-type' ones. Moreover, since only top and bottom quarks will be relevant in the following we will not consider the lighter quarks and leptons.

Neglecting all the Yukawa couplings but the one for the right handed top, h_t, the classical lagrangian is given by

$$\mathcal{L} \;=\; -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \left(\mathcal{D}_\mu H_1 \right)^\dagger \mathcal{D}^\mu H_1 + \left(\mathcal{D}_\mu H_2 \right)^\dagger \mathcal{D}^\mu H_2 \tag{7}$$

$$+i\bar{t}_L\gamma_\mu\not{\mathcal{D}}^\mu t_L + i\bar{t}_R\gamma_\mu\not{\mathcal{D}}^\mu t_R + (t\to b) + h_t\left(H_2^0\bar{t}_R t_L - H^+\bar{t}_R b_L\right) - V(H_1,H_2), \qquad (8)$$

where the most general tree level scalar potential is given by

$$\begin{aligned} V &= {m_1}^2|H_1|^2 + {m_2}^2|H_2|^2 - ({m_3}^2 H_1 H_2 + \text{h.c.}) + \lambda_1|H_1|^4 + \lambda_2|H_2|^4 \\ &+ \lambda_3|H_1|^2|H_2|^2 + \lambda_4|H_1 H_2|^2 + \left[\lambda_5(H_1H_2)^2 + \lambda_6|H_1|^2 H_1 H_2 + \lambda_7|H_2|^2 H_1 H_2 + \text{h.c.}\right]. \qquad (9) \end{aligned}$$

In eq. (8) we have also omitted tha SU(2) gauge part, which will not be relevant in the following. Note that of the two phases of the Higgs fields, θ_1 and θ_2, defined by $H_i(x) = v_i(x)\exp[1\theta_i(x)]$, only the gauge invariant combination $\delta = \theta_1 + \theta_2$ appears in the scalar potential. The orthogonal combination, $\chi = \theta_1 - \theta_2$, is the gauge phase, corresponding to the would be Goldstone boson after spontaneous symmetry breaking. One can now write down the equations of motion coming from the lagrangian in (8), with the tree level scalar potential $V(H_1, H_2)$ replaced by the (finite temperature) radiatively corrected one, $\overline{V}(H_1, H_2)$. We assume that the parameters of the effective potential (and the temperature) are such that it has only one minimum in the charged directions, given by $H^+ = H^- = 0$, whereas there are two minima in the neutral Higgs directions. Then, a bubble solution exists, with $v_{1,2}(x)$ changing from a non-zero value $v_{1,2}^+$ inside to zero outside, and $H^\pm = 0$ everywhere.

At zero order, we will also take the fermion fields and currents to be zero, so that the background is given by the solution of the following field equations:

$$\partial^2 v_{1,2} + \frac{\partial\overline{V}}{\partial v_{1,2}} - \frac{1}{4}v_{1,2}\left(\partial_\mu\delta \mp \mathcal{D}_\mu\chi\right)^2 = 0 \qquad (10)$$

$$\partial_\mu F^{\mu\nu} = \frac{e}{4}\left[(v_1^2 - v_2^2)\partial^\nu\delta - (v_1^2 + v_2^2)\mathcal{D}^\nu\chi\right] \qquad (11)$$

$$\frac{1}{4}\partial_\nu\left[(v_1^2 + v_2^2)\partial^\nu\delta - (v_1^2 - v_2^2)\mathcal{D}^\nu\chi\right] = -\frac{\partial\overline{V}}{\partial\delta}, \qquad (12)$$

where the gauge invariant quantity $\mathcal{D}_\mu\chi$ is given by

$$\mathcal{D}_\mu\chi = \partial_\mu\chi + eA_\mu. \qquad (13)$$

Assuming also that there are no electric or magnetic fields at zero order, *i.e.* $F_{\mu\nu} = 0$, and chosing the unitary gauge $A_\mu = 0$ (see below), we obtain from eq. (11)

$$v_1^2\partial_\mu\theta_1 = v_2^2\partial_\mu\theta_2, \qquad (A_\mu = 0) \qquad (14)$$

so that eq. (12) now reads

$$\partial^\mu\left(v_1^2\partial_\mu\theta_1\right) = -\frac{\partial\overline{V}}{\partial\delta}. \qquad (15)$$

Inserting the solutions of the above equations into the expansion in eq. (4) we will now be able to compute the various contributions to the expectation values of the currents as a loop expansion. In the following paragraphs we will see that the Higgs currents get a contribution already at the tree level, whereas, consinstently with our assumptions, the contributions to the fermionic currents, and to $\langle \partial_\mu F^{\mu\nu} \rangle$, arise only at one loop.

Before concluding this section, we want to show explicitly that the expectation value for a gauge invariant operator, as obtained by the expansion in eq. (4), is independent on the gauge in which the classical background has been computed. In our discussion above, we have looked for a solution of the field equations with no electromagnetic fields, *i.e.* with $F_{\mu\nu} = 0$, so that it was possible to chose a gauge in which $A_\mu = 0$ everywhere (see eqs. (14,15)). Chosing a different gauge gives rise to a different background, in which A_μ is diferent from zero, and we want to show that the result for the gauge invariant operator $\hat{O}(z)$ is the same as in the original background.

Following the usual procedure for the derivation of the Ward identities it is straightforward to obtain the relation

$$\frac{1}{\alpha}\partial^2 \partial_\mu a^\mu + \partial_\mu \frac{\delta \Gamma}{\delta a_\mu} - i\frac{e}{2}\left(H_1^0 \frac{\delta \Gamma}{\delta H_1^0} - H_1^{0*} \frac{\delta \Gamma}{\delta H_1^{0*}} - H_2^0 \frac{\delta \Gamma}{\delta H_2^0} + H_2^{0*} \frac{\delta \Gamma}{\delta H_2^{0*}} \right) = 0 \tag{16}$$

where α is the gauge parameter, a_μ, $H_{1,2}^0$, and ${H_{1,2}^0}^*$ are the classical gauge and neutral Higgs fields, and we have put the charged Higgses and the fermion classical fields to zero. Differentiating the above expression with respect to the source $\Delta(x)$ keeping the background fields fixed, and then putting $\Delta(x) = 0$ we obtain the relevant identity containing the functional $\mathcal{O}$ of eq. (3),

$$\partial_\mu \frac{\delta \mathcal{O}}{\delta a_\mu} - i\frac{e}{2}\left(H_1^0 \frac{\delta \mathcal{O}}{\delta H_1^0} - H_1^{0*} \frac{\delta \mathcal{O}}{\delta H_1^{0*}} - H_2^0 \frac{\delta \mathcal{O}}{\delta H_2^0} + H_2^{0*} \frac{\delta \mathcal{O}}{\delta H_2^{0*}} \right) = 0. \tag{17}$$

The Green's functions entering the expansion (4) have to be computed in the 'unbroken phase' *i.e.* for vanishing background fields $\Phi_i^c = \{H_{1,2}^0, {H_{1,2}^0}^*, a_\mu\}$. Taking $\Phi_i^c = 0$ in eq. (17), one gets

$$\partial_\mu \left. \frac{\delta \mathcal{O}}{\delta a_\mu} \right|_{\Phi_i^c = 0} = 0, \tag{18}$$

while, differentiating with respect to H_i^0,

$$-i\frac{e}{2} \left. \frac{\delta \mathcal{O}}{\delta H_1^0(y)} \right|_{\Phi_i^c=0} \delta^{(4)}(x-y) + \partial_\mu \left. \frac{\delta^2 \mathcal{O}}{\delta a_\mu(x) \delta H_1^0(y)} \right|_{\Phi_i^c=0} = 0 \tag{19}$$

and similar identities are obtained differentiating with respect to the other fields.

Now, let's indicate with $\overline{\Phi}_i^c = \{\overline{H}^0_{1,2}, \overline{H}^{0\ *}_{1,2}, \overline{a}_\mu = 0\}$ the 'bubble' solution of the field equations discussed above, and with $\overline{\Phi}_i^{c\prime} = \{\overline{H}^{0\ \prime}_{1,2}, \overline{H}^{0\ \prime *}_{1,2}, \overline{a}'_\mu = -1/e\,\partial_\mu\theta\}$ the solution obtained from the former by a gauge transformation. Then, we consider the functional $\mathcal{O}$ on the background $\overline{\Phi}_i^{c\prime}$ and expand it according to eq. (4). Since $\overline{a}'_\mu \neq 0$ in this case also Green functions involving gauge fields in the external legs will contribute to the expansion whereas, if we consider the background $\overline{\Phi}_i^c$, only the Green functions with neutral Higgses in the external legs contribute. However, by means of the above identities, one can easily show that there are cancellations among these new contributions, so that the result is the same as for $\overline{\Phi}_i^c$. Indeed, at the linear order in $\overline{a}'_\mu$ in the expansion we have

$$\int d^4x \left.\frac{\delta\mathcal{O}}{\delta a_\mu(x)}\right|_{\Phi_i^c=0} \overline{a}'_\mu(x) = \frac{1}{e}\int d^4x \partial_\mu \left.\frac{\delta\mathcal{O}}{\delta a_\mu(x)}\right|_{\Phi_i^c=0} \theta(x) = 0 \tag{20}$$

where the last equality is due to eq. (18). At the linear order in $\overline{H}^{0\prime}_1$ we have

$$\int d^4x \left.\frac{\delta\mathcal{O}}{\delta H^0_1(x)}\right|_{\Phi_i^c=0} \overline{H}^{0\prime}_1(x) = \int d^4x \left.\frac{\delta\mathcal{O}}{\delta H^0_1(x)}\right|_{\Phi_i^c=0} \overline{H}^0_1(x)(1 - i\theta(x)/2) + O(\theta^2). \tag{21}$$

The contribution $O(\theta(x))$ is canceled by the $\overline{H}^{0\prime}_1\overline{a}'_\mu$ term in the expansion by virtue of eq. (19),

$$\begin{aligned}
&\int d^4x d^4y \left.\frac{\delta^2\mathcal{O}}{\delta a_\mu(x)\delta H^0_1(y)}\right|_{\Phi_i^c=0} \overline{H}^{0\prime}_1(y)\overline{a}'_\mu(x) = \\
&\frac{1}{e}\int d^4x d^4y\ \partial_\mu \left.\frac{\delta^2\mathcal{O}}{\delta a_\mu(x)\delta H^0_1(y)}\right|_{\Phi_i^c=0} \overline{H}^0_1(y)\theta(y) + O\left(\theta^2\right) = \\
&\frac{i}{2}\int d^4x \left.\frac{\delta\mathcal{O}}{\delta H^0_1(x)}\right|_{\Phi_i^c=0} \overline{H}^0_1(x)\theta(x) + O\left(\theta^2\right).
\end{aligned} \tag{22}$$

The contribution of n-th order in θ in eq. (21), is canceled by the terms coming from the $\overline{H}^{0\prime}_1\left(\overline{a}'_\mu\right)^i$ $(i = 0 \ldots n)$ orders in the expansion, due to the appropriate Ward identities obtained by differentiating eq. (19) with respect to a_μ.

This result generalizes to the other terms of the expansion: all the contributions of the same order in the primed Higgs fields and of any order in a'_μ in the expansion for $\mathcal{O}[\overline{\Phi}_i^{c\prime}]$ sum up to the corresponding term of the same order in the non-primed Higgs fields in the expansion for $\mathcal{O}[\overline{\Phi}_i^c]$, so that

$$\mathcal{O}[\overline{\Phi}_i^{c\prime}] = \mathcal{O}[\overline{\Phi}_i^c]. \tag{23}$$

Since in practical applications one has to truncate the expansion to a finite order in the fields, it is clear from the above discussion that this can be done in a sensible way only if we consider the background $\overline{\Phi}_i^c$, where the classical gauge field is zero, *i.e.* in the unitary gauge.

3. Computation of the currents.

We will now apply the method illustrated in the previous paragraph to the computation of $\langle J_i^\mu(z)\rangle_{\Phi_i^0(x)}$, the expectation value of the current operator $J_i^\mu(x)$ ($i = H_{1,2}^0,\ H^\pm,\ t_{L,R},\ b_{L,R}$) on the background $\bar{\Phi}_{j,k}^c = \{H_1^0,\ H_1^{0*},\ H_2^0,\ H_2^{0*}\}$, which, for definitness, we assume to be the 'bubble' solution to the field equation of motion discussed in the previous paragraph.

In the following, we will assume that there are no chemical potentials. Then, it is easy to realize that a necessary condition for the presence of non-vanishing expectation values for the currents is a non trivial space-time dependence of the background. Indeed, if $H_{1,2}^*(x)$ were constant then the only two vectors available for the construction of $\langle J_i^\mu(z)\rangle_{\Phi_j^c}$ would be z^μ and u^μ (the quadrivector which identifies the motion of the thermal reference). By using translational invariance, we are left with $\langle J_i^\mu(z)\rangle_{\Phi_j^c} \sim \text{const} \times u^\mu$, but this is forbidden by CPT.

Also, by applying a charge congiugation transformation both to the operator and to the background, we see that

$$\langle J_i^\mu(z)\rangle_{\Phi_j^c(x)} = -\langle J_i^\mu(z)\rangle_{\Phi_j^c(x)^*}, \tag{24}$$

i.e. the expectation value for the current would vanish also if the background were real. From these two considerations one can conclude that the expectation values we are looking for, will depend on *space-time derivatives* of a *complex background*, so that the two phases $\theta_1(x)$ and $\theta_2(x)$ discussed previously, (or $\delta(x)$ and $\chi(x)$), constrained by (14), will play a crucial role.

Computing the expectation value as a functional expansion on the background, as in eq. (4), the problem is reduced to the calculation of 1PI Green's functions with one $J_i^\mu(x)$ insertion in the *unbroken* phase, as in eq. (5).

We will work in the imaginary time formalism, so that our expressions for the Green's functions and for the currents are valid in the reference frame of the thermal bath, where the bubble wall is moving.

The zero order of the expansion in eq. (5) vanishes since it is given by the 1PI part of $\langle J_i^\mu(z)\rangle_{\Phi_j^c=0}$ which is zero since translational invariance holds in the unbroken phase.

The linear order is also zero, as can be realized observing that in the unbroken phase the theory has still a *global* $U(1)$ invariance, whereas $\langle J_i^\mu(z)\hat{H}_k^0(y)\rangle_{\Phi_j^c=0}$ is not a singlet under such symmetry.

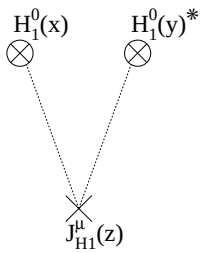

Figure 1: The tree level contribution to the neutral Higgs current.

The first non-vanishing contributions then come from the quadratic terms *i.e.*

$$\langle J_i^\mu(z)\rangle = \frac{1}{2}\sum_{j,k}\int d^4x\, d^4y\, \frac{\delta^3\Gamma\left[\bar{\Phi}_j^c,\, \Delta_\mu^i\right]}{\delta\bar{\Phi}_j^c(x)\delta\bar{\Phi}_k^c(y)\delta\Delta_\mu^i(z)}\bigg|_{\bar{\Phi}_{j,k}^c=\Delta_\mu=0} \bar{\Phi}_j^c(x)\bar{\Phi}_k^c(y). \tag{25}$$

Using again the global $U(1)$, one can see that the only possible non-vanishing contributions to the above expression come from the 1PI part of $\langle J_i^\mu(z)\hat{H}_1^0(x)\hat{H}_2^0(y)\rangle_{\Phi_j^c=0}$, or $\langle J_i^\mu(z)\hat{H}_i^0(x)\hat{H}_i^0(y)^*\rangle_{\Phi_j^c=0}$ $(i=1,\,2)$. These Green's functions will now be computed as usual in perturbation theory, and the result inserted into the expression (25).

We start with the Higgs currents $J_{H_i}^\mu = i\left(H_i^\dagger\mathcal{D}^\mu H_i - \mathcal{D}^\mu H_i^\dagger H_i\right)$. In the case of the neutral Higgses a contribution already appears at the tree level, see Fig.1. Inserting this contribution into the expression (25) we get the classical current

$$\langle J_{H_i^0}^\mu(z)\rangle^{(1)} = -2\, v_i^2(z)\,\partial^\mu\theta_i(z). \tag{26}$$

Note that, as expected, the individual Higgs currents are zero if the phases are constant. Moreover, from the equation of motion (14), we see that the total Higgs hypecharge current vanishes at the tree level.

At one loop there is a contribution to the neutral and charged Higgs currents given by graphs like that in Fig. 2.

Since the computation has to be performed in the unbroken phase, we must use resummed propagators for the Higgs fields in order to deal with the IR divergencies [11]. In the unbroken phase, the Higgs spectrum contains two complex electrically neutral fields and two charged ones. At the tree level, the squared masses of one of the neutral states and of one of the charged ones are negative, since the origin of field space becomes a minimum of the effective potential only after the inclusion of the finite temperature radiative corrections. The resummation can be

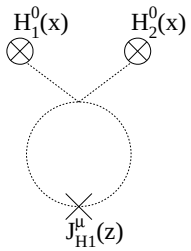

Figure 2: The 1-loop contribution to the neutral and charged Higgs currents.

achieved by considering the propagators for the eigenstates of the thermal mass matrix, which has positive eigenvalues given by

$$M^2_{1,2}(T) = \frac{m_1^2(T) + m_2^2(T) \mp \sqrt{(m_1^2(T) - m_2^2(T))^2 + 4\, m_3^4(T)}}{2}, \tag{27}$$

where the $m_i^2(T)$ are the thermally corrected mass parameters of the potential (9), $m_1^2(T) \simeq m_1^2 + 3g^2T^2/16$, $m_2^2(T) \simeq m_2^2 + h_t^2T^2/4$, while $m_3^2(T)$ receives only logarithmic corrections in T, which were computed in ref. [6]. Correspondingly, the neutral complex eigenstates are given by

$$\begin{cases} h &=& \cos\alpha\, H_1^0 &+& \sin\alpha\, H_2^{0*} \\ H &=& -\sin\alpha\, H_1^0 &+& \cos\alpha\, H_2^{0*} \end{cases}, \tag{28}$$

where

$$\sin 2\alpha = \frac{2m_3^2(T)}{M_1^2(T) - M_2^2(T)}. \tag{29}$$

Completely analogous formulae hold for the charged eigenstates.

The loop in Fig. 2 gives

$$\frac{\delta^3\Gamma}{\delta\Delta^1_\mu(z)\delta H_1^0(x)\delta H_2^0(y)} = -i\,\lambda_5\ \sin 2\alpha\ \ \delta^{(4)}(x-y)\int \frac{d^4q}{(2\pi)^4} e^{iq\cdot(z-x)} \mathrm{H}^\mu(q), \tag{30}$$

where the function $\mathrm{H}^\mu(q)$ is the analytical continuation to Minkowski external momentum q of the finite temperature integral,

$$\mathrm{H}^\mu(\tilde{q}) = \sum\!\!\!\!\!\!\int dp \left\{ \frac{p^\mu}{[p^2 + M_1^2(T)]\ [(\tilde{q}-p)^2 + M_2^2(T)]} - \Big((M_1^2(T) \leftrightarrow M_2^2(T)\Big) \right\}, \tag{31}$$

where $\tilde{q}_4 = iq_0$, $\tilde{q}_i = q_i$, $p_0 = 2\pi n_p T$, $p^2 = p_0^2 + |\vec{p}|^2$, and

$$\sum\!\!\!\!\!\!\int dp \equiv \sum_{n_p} \int \frac{d^3\vec{p}}{(2\pi)^3}.$$

Note that the only quartic coupling contributing to the Higgs currents is λ_5, whereas there are no contributions from λ_6 and λ_7. Moreover, the contribution vanishes if there is no mixing $m_3^2(T)$ between the two Higgs gauge eigenstates (see eq. (29)).

It is easy to check that Green function in (31) is ultraviolet finite, due to the minus sign between the two terms in the integrand. After some algebra, the integral in p can be casted into the form

$$\mathrm{H}^\mu(\tilde{q}) = \frac{i}{2}\tilde{q}^\mu \Big[(\tilde{q}^2 + M_1^2(T)) \int\!\!\!\!\!\!\diagup dp \frac{1}{(p^2 + M_2^2(T))((\tilde{q}-p)^2 + M_1^2(T))((\tilde{q}+p)^2 + M_1^2(T))} - \Big((M_1^2(T) \leftrightarrow M_2^2(T)) \Big) \Big]. \tag{32}$$

Now, the scale of the external momentum q is set by the bubble wall width, which, in the case of interest for us of thick bubble walls, is given by $L_w^{-1} \simeq T/100 - T/10$ [7]. Then, since we have $\tilde{q}^2 \ll M_{1,2}^2(T) \ll (2\pi T)^2$, we approximate the integral by neglecting the $\tilde{q}$ dependence in the integrand and by considering only the zero Matsubara mode for p_0. Inserting the result into the quadratic term of the expansion (4) we obtain

$$\langle J^\mu_{H_1^0}(z)\rangle^{(2)} = \langle J^\mu_{H_2^0}(z)\rangle^{(2)} \simeq \frac{\lambda_5}{8\pi} \frac{T\, m_3^2(T)}{(M_1(T) + M_2(T))^3}\, \partial^\mu \left[v_1(z) v_2(z) \sin\delta(z) \right], \tag{33}$$

Each charged Higgs gets a contribution equal to that of the neutral Higgs belonging to the same doublet. As expected, also the Higgs currents vanish in the limit of vanishing $v_i(z)$.

The first contribution to the fermionic currents arises at one loop and is given by graphs like that in Fig. 3. Once inserted into the expansion (4), they give, in the case of the left handed top,

$$\langle J^\mu_{t_L}(z)\rangle^{(3)} = i h_t^2 \int d^4x\, d^4y\, \mathrm{Im}\left(H_2^0(x) H_2^0(y)^* \right)\, \mathcal{G}^\mu(x,y,z), \tag{34}$$

where h_t is the top Yukawa coupling and

$$\mathcal{G}^\mu(x,y,z) = \int \frac{d^4l}{(2\pi)^4} \frac{d^4m}{(2\pi)^4} e^{i\, l(x-z)} e^{i\, m(y-z)} \mathrm{G}^\mu(m,l) \tag{35}$$

is the Green function corresponding to the diagram in Fig. 3, with

$$\mathrm{G}^\mu(\tilde{m},\tilde{l}) = \int\!\!\!\!\!\!\diagup dk \frac{\mathrm{Tr}\left[(\not{k} + \not{\tilde{m}})\, \not{k} (\not{k} - \not{\tilde{l}}) \gamma^\mu \right]}{(k+\tilde{m})^2 k^2 (k-\tilde{l})^2}, \tag{36}$$

and the zero component of the fermionic loop momentum is $k^0 = (2n+1)\pi T$.

Contrary to the case of the scalar loop, this fermionic loop integral is now infrared finite, since the zero component of the loop momentum never vanishes. As a consequence, we will work with the tree level fermion propagators, without of resummation.

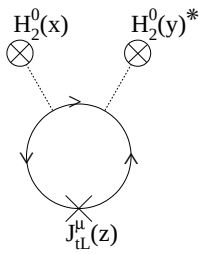

Figure 3: The 1-loop contribution to the left-handed top current.

Again, in presence of a thick wall background we can perform a high temperature expansion. This is achieved by approximating $\mathrm{G}^\mu(\tilde{m},\tilde{l}) \simeq \mathrm{G}^\mu(\tilde{m},0) + \mathrm{G}^\mu(0,\tilde{l})$ (note that $\mathrm{G}^\mu(\tilde{m},\tilde{l}) = -\mathrm{G}^\mu(\tilde{l},\tilde{m})$), and neglecting terms of order $\tilde{l}^2/T^2$ in the computation of $\mathrm{G}^\mu(0,\tilde{l})$.

Now the integral has a logarithmic ultraviolet divergence, so renormalization is needed (see also the next section). In the $\overline{MS}$ scheme we obtain

$$\langle J^\mu_{t_L}(z)\rangle^{(3)} \simeq -\frac{h_t^2}{\pi^2}\, v_2^2(z)\partial^\mu\theta_2(z)\, \left(\log\left(\frac{\mu^2}{A_f\, T^2}\right)+\frac{3}{2}\right), \tag{37}$$

where $A_f = \pi^2\exp(3/2\ -2\gamma_E) \simeq 13.944$.

Eq. (37) shows the expected dependence on h_t^2 and $v_2^2(z)$ which, in comparison to the original result given in ref. [2], gives a suppression factor $\mathcal{O}(h_t v_2/\pi\, T)^2$.

A graph similar to that in Fig. 1 for the right handed top quark leads to a contribution to $\langle J^\mu_{t_R}(z)\rangle$ given by $\langle J^\mu_{t_R}(z)\rangle^{(1)} = -\langle J^\mu_{t_L}(z)\rangle^{(1)}$. For the other fermion species, one finds analogous results, in which h_t is replaced by the appropriate Yukawa coupling, and $v_2(z)$ ($v_1(z)$) appears for the up (down)-type fermions.

A contribution to $\langle J^\mu_{t_L}(z)\rangle$ proportional to $\mathrm{Im}(H_1^0 H_2^0) = v_1 v_2 \sin\delta$, and to h_t^2, appears at two loops, given by the graph in Fig. 4. The corresponding integral is

$$I(k) = \sum\!\!\!\!\!\!\int dq \frac{1}{((q+k)^2+M_1^2(T))(q^2+M_2^2(T))}\mathrm{G}^\mu(-q-k,q) \tag{38}$$

where q is the bosonic loop variable and $\mathrm{G}^\mu(m,l)$ has been defined in (36). Computing the fermionic integral in the approximation described above, we are left with a ultraviolet finite bosonic integration which receives important contributions only from the zero Matsubara mode.

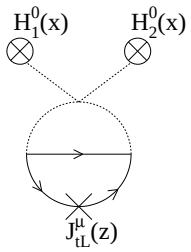

Figure 4: The 2-loop contribution to the left-handed top current. The scalar internal lines can be either neutral or charged Higgs fields.

Inserting the result into the expansion we obtain, again in the high temperature approximation[1],

$$\langle J^\mu_{t_L}(z)\rangle^{(4)} \simeq -\frac{\lambda_5\, h_t^2}{64\pi^3}\frac{T\, m_3^2(T)}{(M_1(T)+M_2(T))^3}\left(\log\left(\frac{\mu^2}{A_f\, T^2}\right)+\frac{3}{2}\right)\partial^\mu\left[v_1(z)v_2(z)\sin\delta(z)\right]. \qquad (39)$$

As for the one-loop result, the two loop contribution with neutral Higgses in the internal lines for $\langle J^\mu_{t_R}(z)\rangle$ is opposite to the one for $\langle J^\mu_{t_L}(z)\rangle$. This is no longer true when the graphs with charged Higgses in the internal lines are taken into account. In such a case, $\langle J^\mu_{t_R}(z)\rangle$ gets a a contribution equal to eq. (39), whereas the result for $\langle J^\mu_{t_L}(z)\rangle$ is analogous but proportional to h_b^2 instead of h_t^2. The charged Higgs loops give also rise to a non-vanishing left handed bottom density opposite to eq. (39).

When applying our calculations to the interesting case of spontaneous baryogenesis, one has to remember that the typical values of $v_i(x)/T$ are smaller than one, as required in regions of the bubble wall where sphalerons are still active. This fact enables us to stop the expansion series at the second order, being the remaining terms suppressed by powers of $v_i(x)/T$ with respect to it.

[1]Note a factor 6 of difference with respect to the result of ref. [1]

4. Current renormalization.

In the previous section we have seen that in order to calculate the finite temperature average of the particle currents, the expansion (4) can be used, whose coefficients are the 1PI Green's functions with the insertion of current operators J_i^μ, computed in the unbroken phase. With J_i^μ's we have indicated the current for the i-th particle, expressed in terms of the renormalized fields.

In general, the computation of Green's functions with the insertion of a composite operator requires the introduction of new counterterms besides those necessary for the renormalization of the basic lagrangian (see ref. [12] for a thorough discussion). For example, the one loop computation of the Green function $\mathcal{G}^\mu(x,y,z) = \langle J^\mu_{t_L}(z) H_2^0(x) H_2^{0,*}(y)\rangle$, defined in eq. (35), gives a divergent part (in the $\overline{MS}$ renormalization scheme, and in the momentum space)

$$\tilde{\mathrm{G}}^{\mu,div}(m,l) = +\frac{1}{8\,\pi^2} h_t^2 \frac{1}{\Gamma(3)} \frac{1}{\varepsilon} (m^\mu - l^\mu)\,. \tag{40}$$

where m, l are the momenta associated to the external Higgs fields and $\varepsilon = 4 - n$. This divergence may be expressed as

$$\mathcal{G}^{\mu,div}(x,y,z) = -\frac{1}{16\,\pi^2} \frac{1}{\varepsilon} \langle J^\mu_{H_2^0}(z) H_2^0(x) H_2^{0*}(y)\rangle \tag{41}$$

it i.e it is proportional to the *tree level* Green function involving the H_2^0 Higgs field current, corresponding to Fig. 1.

This is an example of a quite general phenomenon: a renormalized current $[J_i^\mu]$, can be defined as a linear combination of the currents J_j^μ

$$[J_i^\mu] = \sum_j (\delta_{ij} + \delta Z_{ij}) J_j^\mu \tag{42}$$

in such a way that all the possible Green's functions with $[J_i^\mu]$ insertions are finite. In the example discussed above, we have $\delta Z_{L2} = -\mu^2 h_t^2/(16\pi^2\,\varepsilon)$. The δZ_{ij} are in general new renormalization constants which are not contained in the counterterm lagrangian, so that new renormalization conditions are needed.

An important exception to this is the case in which J^μ is a conserved current for the renormalized lagrangian

$$\mathcal{L} = \mathcal{L}_{basic} + \mathcal{L}_{ct}, \tag{43}$$

where $\mathcal{L}_{basic}$ is expressed in terms of renormalized quantities and $\mathcal{L}_{ct}$ is the counterterm Lagrangian. Let's consider here the simple case of a lagrangian containing only left and right

handed top quarks and the neutral Higgses, which will be enough for discussing the renormalization of the Green's functions relevant for this paper.

The conserved current under the group $U(1)$ is then

$$J^\mu + \delta J^\mu = q_L\,(1+\delta Z_L)\;J_L^\mu + q_R\,(1+\delta Z_R)\;J_R^\mu + q_1\,(1+\delta Z_1)\;J_{H_1^0}^\mu + q_2\,(1+\delta Z_2)\;J_{H_2^0}^\mu, \quad (44)$$

with $q_2 = q_R - q_L = -q_1 = 1/2$ and δJ^μ is the part of the current coming from $\mathcal{L}_{c.t.}$. The δZ_α ($\alpha = L,\ R,\ 1,\ 2$) are now the wave-function renormalization constants for the different fields, coming from $\mathcal{L}_{ct}$. In this case, being the current conserved, no new counterterms besides those already present in (44) (and hence in $\mathcal{L}_{ct}$) are needed [12], *i.e.*

$$J^\mu + \delta J^\mu = [J^\mu] \equiv q_L\,[J_L^\mu] + q_R\,[J_R^\mu] + q_1\left[J_{H_1^0}^\mu\right] + q_2\left[J_{H_2^0}^\mu\right]. \quad (45)$$

Recalling this property of the conserved current it is possible to reduce the number of linearly independent renormalization constants δZ_{ij} (where now $i = L,\ R,\ 1,\ 2$) necessary in order to define the renormalized currents according to eq. (42). Indeed, from (45), recalling (42) and (44), the number of new linearly independent renormalization constants δZ_{ij}'s is reduced from 16 to 8.

The most general renormalization matrix is then given by

$$\begin{pmatrix} 1+\delta Z_{LL} & \delta Z_R - \delta Z_{RR} & \delta Z_{L1} & \delta Z_{L2} \\ \delta Z_L - \delta Z_{LL} & 1+\delta Z_{RR} & -\delta Z_{L1} & -\delta Z_{L2} \\ \delta Z_{1L} & \delta Z_{1R} & 1+\delta Z_{11} & \delta Z_{12} \\ \delta Z_{1L} + \delta Z_{LL} - \delta Z_L & \delta Z_{1R} - \delta Z_{RR} + \delta Z_R & \delta Z_1 + \delta Z_{11} + \delta Z_{L1} & 1+\delta Z_2 + \delta Z_{L2} + \delta Z_{12} \end{pmatrix} \quad (46)$$

We see then that, in general, eight extra renormalization conditions are required in order to define the set of renormalized currents $\{[J_i^\mu]\}$.

The only divergent graph encountered in sect. **3**, is the fermionic one loop (see eq. (37)), so, for the present application, and to our order of computation, it is enough to compute the matrix (46) only to order $(g)^0 h_t^2$. To this order, only one of the eight extra renormalization constants is non zero, δZ_{L2} and moreover we obtain, again from (45), that $\delta Z_{L2} = -\delta Z_2$ (+ higher order terms), so that no new renormalization condition is needed.

The mixing matrix is then given, at this order, by

$$\begin{pmatrix} [J_L^\mu] \\ [J_R^\mu] \\ [J_1^\mu] \\ [J_2^\mu] \end{pmatrix} = \begin{pmatrix} 1 & \delta Z_R & 0 & -\delta Z_2 \\ \delta Z_L & 1 & 0 & \delta Z_2 \\ 0 & 0 & 1 & 0 \\ -\delta Z_L & \delta Z_R & 0 & 1 \end{pmatrix} \begin{pmatrix} J_{t_L}^\mu \\ J_{t_R}^\mu \\ J_1^\mu \\ J_2^\mu \end{pmatrix}, \quad (47)$$

where $\delta Z_{L2} = -\delta Z_2 = (-\mu^2 h_t^2/16\,\pi^2\,\varepsilon)$, $\delta Z_L = \delta Z_R = (-\mu^2 h_t^2/32\,\pi^2\,\varepsilon)$.

Working at higher order, we would need new renormalization conditions, defining the physical currents $[J_i^\mu]$ at a certain scale. Due to the interactions, at a different scale the renormalized currents will be given by different mixings of the unrenormalized ones, so that, in general, the definition of 'pure' (*i.e.* non-mixed) currents is scale dependent. As we have already stressed, this is not the case for the conserved currents, which do not suffer any mixing and do not require any new renormalization condition.

As we will discuss in the following section, the only relevant currents in the limit of local thermodynamical equilibrium are just the conserved ones, so that in this case, the state of the system is completely determined without the need of introducing extra renormalization conditions.

5. The limit of local equilibrium.

Up to this point, we have computed the expectation values of the individual particle numbers starting from a partition function of the form

$$Z[J_i, \Delta_j] = \mathrm{Tr}\ \exp\left[-\beta\left(H + \int \Delta_j N_j\right)\right] \tag{48}$$

where H is the Hamiltonian of the system and we are now considering only the zero components ($N_j \equiv J_j^0$) of the currents considered in the previous paragraphs. We assume that all the chemical potentials are zero.

If we take generic values for the sources J_i and Δ_j the partition function (48) will not correspond to the one describing the limit of thermodynamical equilibrium, in which the individual particle numbers assume values such that the free energy of the system is minimized. In the case under consideration of a space-time dependent Higgs field background, the limit of *local* equilibrium is achieved only if all the interaction rates are much higher that the inverse of the typical space-time scale of the variation of the background, *i.e.* the bubble wall width. This condition is fulfilled in reality only by a few interactions (the gauge flavour conserving ones, the top Yukawa interactions, and possibly some Higgs-Higgs interactions), however it is anyway instructive to consider first this limit before discussing the more realistic situation in which some of the interactions are out of equilibrium inside the bubble wall.

The free energy of the system is obtained by performing the Legendre transformation of the

functional defined in eq. (2) with respect to the sources Δ_j of the density operators,

$$\begin{aligned} \mathrm{F}\left[\Phi_i^c,\ \mathcal{N}_j^c\right] &= W\left[J_i,\ \Delta_j\right] - \textstyle\sum_i \int d^4x J_j(x)\Phi_j^c(x) - \sum_j \int d^4x \Delta_j(x)\mathcal{N}_j^c(x) \\ &= \Gamma\left[\Phi_i^c, \Delta_j\right] - \sum_j \int d^4x \Delta_j(x)\mathcal{N}_j^c(x) \\ &= \langle H\rangle - TS. \end{aligned} \tag{49}$$

The last equality in the above equation is straightforwardly derived recalling that the entropy S is given by $S = -\partial W/\partial T$ and that $W = -T\log Z$ (we are taking the three dimensional volume $\Omega = 1$) .

The classical densities $\mathcal{N}_j^c(x)$ are defined as usual,

$$\mathcal{N}_j^c(x) = \left.\frac{\delta\Gamma[\overline{\Phi}_i^c, \Delta_j]}{\delta\Delta_j(x)}\right|_{\Delta_j=0}, \tag{50}$$

where the background is, as before (eq. (6)), the solution of the field equations with appropriate boundary conditions, *i.e.* we have $J_i(x) = 0$ in eq. (48).

Now, the local equilibrium situation on the background $\overline{\Phi}_i^c(x)$ is obtained once all the processes have driven the free energy to its minimum possible value. This condition corresponds to a system of M equations (where M is the number of processes in equilibrium) of the form

$$\delta^{(l)}\mathrm{F}\left[\Phi_i^c,\ \mathcal{N}_j^c\right] = \sum_j \nu_j^{(l)}\,\frac{\delta\mathrm{F}\left[\Phi_i^c,\ \mathcal{N}_j^c\right]}{\delta\mathcal{N}_j^c(x)} = -\sum_{j=1}^{n}\nu_j^{(l)}\Delta_j(x) = 0 \qquad (l = 1,\ldots,M), \tag{51}$$

where $\nu_j^{(l)}$ is the multiplicity of the j-th particle in the l-th process (*i.e.* in the process $A+2B \leftrightarrow C + D$ we have $\nu_A = -\nu_C = -\nu_D = 1$, and $\nu_B = 2$).

The system (51) is of course strictly analogous to the usual system of equations that one has to solve to determine the equilibrium chemical potentials. In this case, it restricts the set of the linearly independent sources to $\{\Delta'_a\}$ $(a = 1,\ldots,N)$, corresponding to the set of the N conserved charges N_a of the system. This means that the only composite operators that appear in the *equilibrium* partition function

$$Z_{eq}[J_i, \Delta'_a] = \mathrm{Tr}\ \exp\left[-\beta\left(H + \int \Delta'_a N_a\right)\right]\Big|_{\Delta'_a=0} \tag{52}$$

are those corresponding to the conserved charges,

$$\mathcal{N}_a^c(x) = \left.\frac{\delta\Gamma_{eq}[\overline{\Phi}_i^c, \Delta'_a]}{\delta\Delta'_a(x)}\right|_{\Delta'_a=0}, \tag{53}$$

where Γ_{eq} is obtained from Z_{eq} in the standard way.

After solving the system (51), the sources for the individual particle numbers, Δ_i, are expressed in terms of the new sources Δ'_a as

$$\Delta_i = \sum_a q_a^{(i)}\, \Delta'_a, \tag{54}$$

where $q_a^{(i)}$ is the a-charge value of the particle i. Then, the *equilibrium* values for the individual particle numbers are given by

$$\mathcal{N}_i^{eq}(x) = \sum_a \left.\frac{\delta\Gamma_{eq}[\overline{\Phi}^c_i, \Delta'_a]}{\delta\Delta'_a(x)}\right|_{\Delta'_a=0} \left(q_a^{(i)}\right)^{-1} = \sum_a \mathcal{N}_a^c(x) \left(q_a^{(i)}\right)^{-1}. \tag{55}$$

Concerning renormalization we see that since now the only Green's functions that have to be computed are those involving the conserved charges, no new renormalization conditions for the composite operators are necessary when one is dealing with the local equilibrium case.

However in many interesting applications not all the processes have rates fast enough such that a complete local equilibrium situation can be attained. In this case one can consider an adiabatic approximation, in which the 'slow' processes are freezed out inside the bubble wall, while the fast ones are in equilibrium. As a consequence, on the bubble wall only the equations corresponding to such 'fast' processess will contribute to the system (51) and one can repeat the above analysis with new effectively conserved charges now playing the role of the N_a's.

There is in this case a, mainly conceptual, problem, since these charges are now not conserved by the full theory, but only by the effective one obtained by sending to zero all the couplings contributing to the 'slow' processes. Then they do in general mix one another under renormalization, as was discussed in the previous section, so that the interpretation in terms of physical currents will be scale dependent.

However, since the mixing is due only to the small couplings which are neglected in the effective theory, it will be in most of the practical applications a negligible effect.

6. Conclusions

Motivated by the recent interest on the possibility of generating the baryon asymmetry in the early Universe during the electroweak phase transition and, more in particular, by the so called spontaneous baryogenesis mechanism in which the baryon asymmetry is created inside thick CP-violating and expanding bubble walls of the Higgs fields, we have developed a general method to calculate the expectation value of a composite operator in the presence of a generic nonvanishing background for some of the fields of the theory.

The method is based on a functional expansion of the expectation value in powers of the background fields $\Phi_i^c(x)$ which are a solution of the equation of motion with properly chosen boundary conditions. At any order in the functional expansion, the expectation value is determined by a 1P1 Green's function in the unbroken phase, which can be computed in perturbation theory.

Using Ward identities, we have shown that the expectation value for a gauge invariant operator is independent on the gauge in which the background has been computed.

The renormalization of the currents has been also discussed, stressing the crucial role played by the conserved currents in the case of local equilibrium. In this limit, all the individual particle currents are determined in terms of the conserved ones, whose computation do not require the introduction of new renormalization constants besides the ones already contained in the renormalized lagrangian.

In practice, the system is well far away from such an idealized situation, with the possible exception of the processes mediated by the top Yukawa and the gauge flavour diagonal couplings. In this case, a better description of the evolution of particle numbers can be achieved by employing a system of kinetic equations. For each particle number, the values expressed by (55) would then represent the infinite time limit of the corresponding rate equation,

$$\dot{\mathcal{N}}_i \sim \Gamma_{ij}(\mathcal{N}_j - \mathcal{N}_j^{eq}). \tag{56}$$

As we have already stressed in the introduction, the dependence of such equilibrium values on the coupling constants and on the value of the Higgs background should not be confused with the dependence of the rates Γ_{ij} on these quantities.

The computation of the currents is of course only the first step towards a reliable determination of the final value of the baryon asymmetry in this scenario, the major source of uncertainty being represented by the rate of the baryon number violating processes inside and in front of the bubble walls.

In any case, due to its generality, we believe that the method illustrated in this paper can be easily applied to similar situations of interest in Cosmology, in which some of the fields acquire a space-time dependent classical value as for instance in inflationary models [13], or in presence of domain walls, as considered in in ref. [14] in the case of the Next-to-Minimal Supersymmetric Standard Model.

The problem of the computation of the currents on a CP violating background has been addressed recently also by Huet and Nelson [15] for the case of a fermionic axial current. Although they adopt a semiclassical approach quite different from the one discussed in this

paper, their results are in quantitative agreement with ours. In particular the same dependence on h_t and $v(x)$ is found.

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