Theory of Hard Diffraction and Rapidity Gaps

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In this talk we review the models describing the hard diffractive production of jets or more generally high-mass states in presence of rapidity gaps in hadron-hadron and lepton-hadron collisions. By rapidity gaps we mean regions on the lego plot in (pseudo)-rapidity and azimuthal angle where no hadrons are produced, between the jet(s) and an elastically scattered hadron (single hard diffraction) or between two jets (double hard diffraction).

Single hard diffraction has been observed by the UA8 Collaboration (1,2) at the CERN S$p\bar{p}$S Collider ($\sqrt{s} = 630\,\mathrm{GeV}$), and by the H1 and Zeus Collaborations (3–6), in deep inelastic scattering events (DIS) at the DESY HERA $e\,p$ Collider ($\sqrt{s} = 296$ GeV). Double hard diffraction has been observed by the CDF and D0 Collaborations (7,8) at the Fermilab Tevatron $p\,\bar{p}$ Collider ($\sqrt{s} = 1.8$ TeV), and in photoproduction events by the Zeus Collaboration at HERA (9). The distinguishing feature between single and double hard diffraction is the momentum transfer t: while $|t| \simeq 1 - 2\,\mathrm{GeV}^2$ in the UA8 experiment, and $|t| \lesssim \mathrm{a\,few\,GeV}^2$ in the DIS events at HERA, it is very large, $|t| \gtrsim 10^3\,\mathrm{GeV}^2$, in the Tevatron experiments, which suggests that short-distance strong-interaction physics must play a fundamental role in the latter.

I. SINGLE HARD DIFFRACTION

A. The Ingelman-Schlein model

Diffractive production of jets was predicted by Ingelman and Schlein (IS) to occur in hadron-hadron collisions at high energies (10). In order to understand the IS model, let us consider ordinary single diffraction at high c.m. energies $\sqrt{s}$, i.e. a collision between two hadrons, A and B, where hadron B is elastically scattered and hadron A fragments into a high-mass M_X state, $M_X^2 \gtrsim 10\,\mathrm{GeV}^2$ and $(M_X^2/s) \lesssim 0.1$, with a gap in hadron production between hadron B and the fragments of hadron A (Fig. 1a). This process is phenomenologically well described by Regge theory (11) through the exchange

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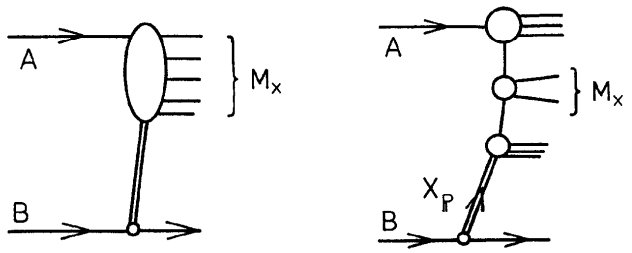

FIG. 1. (*a*) Single diffraction, and (*b*) with jet production, in high-energy hadron-hadron collisions.

of a colorless object, conventionally termed *pomeron*, which accounts for the gap,

$$\frac{d\sigma}{dt\, dx_{I\!P}} = f_{B I\!P}(x_{I\!P}, t)\, \sigma_{tot}(A I\!P)\,, \tag{1}$$

where t is the momentum transfer and $x_{I\!P} = M_X^2/s$. The pomeron flux factor, $f_{B I\!P}$, describes the emission of the pomeron from hadron B, and the cross section, $\sigma_{tot}(A I\!P)$, describes the scattering between the pomeron and hadron A with creation of a high-mass state, and is given in terms of a triple pomeron coupling, which is intuitively apparent when we consider the square of the diagram of Fig. 1a. The gap width must satisfy the kinematic constraint $\Delta\eta_{gap} \gtrsim \ln(1/x_{I\!P})$ (cf. sec.I B).

Regge theory does not say what the pomeron is. There are however perturbative models where the pomeron is pictured as a colorless two-gluon bound state (12,13). Ingelman and Schlein proposed that if the pomeron had a partonic substructure it should manifest itself in the high-mass diffractive scattering through the appearance of jets (or heavy quarks (14)). Inclusive jet production in hadron-hadron collisions is described by the factorization formula,

$$d\sigma(A + B \to jet(s) + X) = \sum_{ab} \int dx_a dx_b f_{a/A}(x_a, \mu) f_{b/B}(x_b, \mu) d\hat{\sigma}_{ab}\,, \tag{2}$$

where $x_{a(b)}$ is the momentum fraction of parton $a(b)$ within hadron $A(B)$, μ is the factorization scale of the order of the jet transverse energy $E_\perp$ and $d\hat{\sigma}_{ab}$ is the jet production rate at the partonic level. Substituting the pomeron-proton cross section $\sigma_{tot}(A I\!P)$ in eq.(1) with the inclusive jet production rate, eq.(2), we obtain the diffractive jet production rate (Fig. 1b),

$$\frac{d\sigma}{dt\, dx_{I\!P}} = f_{B I\!P}(x_{I\!P}, t) \sum_{ab} \int dx_a d\frac{x_b}{x_{I\!P}} f_{a/A}(x_a, \mu) f_{b/I\!P}\left(\frac{x_b}{x_{I\!P}}, \mu\right) d\hat{\sigma}_{ab}\,, \tag{3}$$

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where $x_b/x_{I\!P}$ is the momentum fraction of parton b within the pomeron (10). We take the pomeron flux factor to be used in eq.(3) as given in ref. (15),

$$f_{B/I\!P}(x_{I\!P},t) = \frac{1}{8\pi^2}\,|\beta_{BI\!P}(t)|^2\,x_{I\!P}^{1-2\alpha(t)}\,, \tag{4}$$

normalized[b] in order to agree with the Donnachie-Landshoff flux factor (16) (even though different in its functional form in t). The pomeron-proton coupling, $\beta_{BI\!P}(t)$, and the pomeron trajectory, $\alpha(t)$, may be obtained from fits to the elastic hadron-hadron cross section at small t (15,20),

$$\begin{aligned}\beta_{PI\!P}(t) = \beta_{\bar{P}I\!P}(t) &\simeq 4.6\,mb^{1/2}\,e^{1.9GeV^{-2}t}\,, \\ \alpha(t) &\simeq 1.08 + 0.25\,GeV^{-2}\,t\,.\end{aligned} \tag{5}$$

Then in order to use eq.(3) one must know the parton densities in the pomeron. Ingelman and Schlein assumed a pomeron made of gluons and tested hard, $xf_{g/I\!P}(x) = Ax(1-x)$, and soft, $xf_{g/I\!P}(x) = B(1-x)^5$, gluon densities in the pomeron, with the constants A and B determined from the momentum sum rule, $\int_0^1 dx\,xf_{g/I\!P}(x) = 1$. However, since the pomeron is not a particle there is no reason to expect a momentum sum rule to hold (14,16).

In addition, eq.(3) entails that the factorization picture of eq.(2), well established in inclusive processes (21), carries over to hard diffractive processes, i.e. there is in the parton density in the proton $f_{p/P}(x,\mu)$ a diffractive component which factorizes as the whole function f does. However, we will discuss in sect. IC factorization-breaking effects for which eq.(3) is violated.

B. Diffractive DIS

Let us consider the diffractive deep inelastic scattering (DDIS) $e + p \to e{+}p{+}X$, where a proton of momentum P is elastically scattered to a final-state proton of momentum P' (Fig.2). The relevant kinematic invariants are the squared photon-proton c.m. energy, $W^2 = (q+P)^2$, the momentum transfer $t = (P{-}P')^2$ and the squared mass of the hadronic system $M^2 = (q{+}P{-}P')^2$. The usual variables of DIS are $Q^2 = -q^2$ and $x_{bj} = Q^2/(W^2 + Q^2 - M_P^2)$, in terms of which we can introduce the variables (4,5),

$$\begin{aligned}x_{I\!P} &= \frac{q\cdot(P-P')}{q\cdot P} = \frac{M^2+Q^2-t}{Q^2}x_{bj}\,, \\ \beta &= \frac{x_{bj}}{x_{I\!P}}\,.\end{aligned} \tag{6}$$

[b] Goulianos (17,18) argues that the flux (16) is not appropriate to describe the high-energy $p\bar{p}$ single-diffraction data of the CERN UA4 and the Tevatron E710 and CDF Collaborations (19). Interpreting the flux as a probability density of pomerons in the hadron, he renormalizes it in such a way to never exceed the unity. The ZEUS Collaboration, though, claims (5) that the Regge scaling of F_2^D yielded by the flux of ref. (17,18) does not agree with its data.

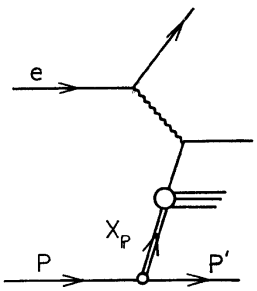

FIG. 2. Diffractive DIS diagram $e + p \rightarrow e + p + X$.

We can parametrize the proton momentum loss in light-cone coordinates as,

$$p_s = P - P' = \left(zP^+, \frac{m_P^2}{P^+} - \frac{m_{P'}^2 + p_{s\perp}^2}{(1-z)P^+}, \mathbf{p_{s\perp}} \right) , \tag{7}$$

with $P^+ = 2P^0$. If $p_s^+ \gg p_s^-, p_{s\perp}$, which for the HERA lab frame ($P^0 = 820$ GeV) holds as long as $z \gtrsim 10^{-6}$, we can rewrite the pomeron momentum as $p_s = zP$, and we readily find that $z = x_{I\!P}$. It is then easy to derive that the invariant mass of the system recoiling against the proton is $M_{eX}^2 \simeq x_{I\!P} s$, and that the gap width has lower bound $\Delta\eta_{gap} \gtrsim \ln(1/x_{I\!P})$.

$e + p \rightarrow e + X$ DIS is fully inclusive over the final-state hadrons, thus it may be parametrized in terms of two structure functions,

$$\frac{d\sigma}{dx_{bj}dQ^2} = \frac{4\pi\alpha^2}{x_{bj}Q^4} \left[(1-y)F_2(x_{bj}, Q^2) + x_{bj}y^2 F_1(x_{bj}, Q^2) \right] , \tag{8}$$

where y is the electron energy loss. If the proton is tagged in the final state, then in order to describe DIS we need two more structure functions, however in the kinematic region of DDIS they are negligible since the transverse momentum of the final-state proton is very small (15), thus we can write,

$$\frac{d\sigma}{dx_{bj}dQ^2dx_{I\!P}dt} = \tag{9}$$
$$\frac{4\pi\alpha^2}{x_{bj}Q^4} \left[(1-y)\frac{dF_2^D(x_{bj}, Q^2, x_{I\!P}, t)}{dx_{I\!P}dt} + x_{bj}y^2 \frac{dF_1^D(x_{bj}, Q^2, x_{I\!P}, t)}{dx_{I\!P}dt} \right] .$$

If the factorization picture of eq.(3) is correct for DDIS we obtain,

$$\frac{d\sigma}{dx_{bj}dQ^2dx_{I\!P}dt} = f_{P/I\!P}(x_{I\!P}, t) \frac{1}{x_{I\!P}} \frac{d\sigma}{d\beta dQ^2} , \tag{10}$$

with the flux factor as given in eq.(4). $d\sigma/d\beta dQ^2$ may be expressed in terms of two pomeron structure functions like in eq.(8), with the parton momentum fraction in the pomeron, β, playing now the role of the Bjorken variable x_{bj}.

Comparing eq.(9) to eq.(10), we obtain for example the diffractive structure function F_2^D in terms of the pomeron structure function $F_2^{I\!P}$,

$$\frac{dF_2^D(x_{bj}, Q^2, x_{I\!P}, t)}{dx_{I\!P} dt} = f_{P/I\!P}(x_{I\!P}, t)\, F_2^{I\!P}(\beta, Q^2)\,. \tag{11}$$

Eq.(11) states that F_2^D exhibits the Regge scaling dictated by single hard diffraction in hadron-hadron scattering, and the Bjorken scaling typical of the usual DIS. The latter entails that F_2^D has a leading twist behavior, i.e. it scales in Q^2 like the ordinary F_2 structure function. These features are presently in agreement with the H1 (4,6) and the ZEUS (5,6) data.

C. Is the factorization picture correct?

The analysis of sec.I A and I B relies upon the factorization picture of eq.(3). However factorization has not been proved for any diffractive process. Following the work of Collins, Frankfurt and Strikman (CFS) (22), we now illustrate the case of diffractive jet production in hadron-hadron scattering where there may be non factorizing contributions which spoil the validity of eq.(3). Let us assume that a pomeron made of two gluons emitted from hadron B goes wholly into the hard scattering with parton a coming from hadron A (Fig.3a). If the two gluon are hard, then this contribution is suppressed by powers of the scale that characterizes the hard process (i.e. the jet transverse energy in jet production) with respect to the non-diffractive contribution due to one-gluon emission, thus it is higher twist and does not spoil eq.(3). However if one of the two gluons is soft, it does not contribute to the power counting in the hard scale and can give a leading-twist contribution. In the usual factorization where no requirement is made on the final states such contributions cancel out after summing over all the final-state soft gluons, however the sum cannot be carried over if one requires a rapidity gap in the final state because some of the diagrams contributing to the sum do not form a gap. This can be seen diagrammatically by squaring the diagram of Fig.3a and drawing the cut line which defines the final state in all the possible positions.

This picture, though, is perturbative and therefore questionable when the momentum transfer $|t|$ is very small. On these grounds one would expect the factorization picture of the IS model still to make sense at very small $|t|$, with non-factorizing contributions growing bigger and bigger as $|t|$ grows, the signature of these being a leading-twist contribution with a $\delta(1-\beta)$ dependence on β.

The UA8 Collaboration (2), has examined diffractive jet production at the S$p\bar{p}$S Collider, with $0.9\,\mathrm{GeV}^2 \leq |\mathrm{t}| \leq 2.3\,\mathrm{GeV}^2$. Assuming the pomeron as made of two gluons the UA8 Collaboration has found that the data could not be explained invoking simply a hard gluon density, of the type $\beta f_{g/I\!P}(\beta) = A\beta(1-\beta)$, and that in order to fit the data it was necessary a 30% contribution from a δ-function-like component, in agreement with CFS prediction (22).

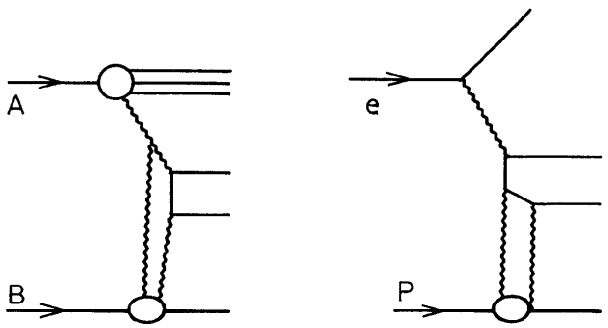

FIG. 3. Two-gluon exchange (a) in hadron-hadron collisions and (b) in DIS.

In diffractive DIS the two gluons forming the pomeron in Fig.3a cannot couple directly to the photon but must couple to the fermion lines (Fig.3b). In the kinematic region where the perturbative picture makes sense, i.e. where both the quark and antiquark transverse momenta are of the order of Q, neither of the two gluons is soft and the contribution of Fig.3b is higher twist (15,22). Thus the lack of initial-state interactions suppresses this contribution.

However, it is still possible that the second gluon is soft and is emitted much later in time as part of the final-state interactions. This factorization-breaking leading-twist mechanism, conceptually analogous to the CFS model, has been considered recently by Buchmüller and Hebecker (BH) (23), who propose that the rapidity gap is due to color fluctuations in the long-range final-state interactions within the proton. BH suppose that the photon-gluon fusion process, which at the perturbative level accounts for the main contribution to F_2 at small x_{bj} in eq.(8), describes the short-range interaction also in DDIS (Fig.3b). They assume then that the $q\bar{q}$ pair formed in the hard-scattering process, while propagating in the color field of the proton, transforms into a color singlet by exchanging a soft gluon with the proton.

The BH model predicts that F_2 and F_2^D have the same Bjorken scaling, in agreement with the IS model (cf. sec.I B). As for the Regge scaling, BH predict that if $F_2 \sim x_{bj}^{-n}$, then $F_2^D \sim x_{I\!P}^{-1-n}$. Thus in the BH model the Regge scaling is determined by the hard scattering, and is directly related to the one of the inclusive process. Conversely, in the IS model the Regge scaling is linked to the pomeron flux factor (cf. sec.I A).

As for the CFS model, though, one would expect that the perturbative picture of the BH model is questionable at very small $|t|$, where the IS model should still anyway be valid.

D. Parton densities in the pomeron

If the picture advocated in sec.I A-I C holds it is possible to fit the data on single hard diffraction to extract the parton densities in the pomeron. The fits should not be global, i.e. should not include data from hadron-hadron scattering because of the factorization-breaking CFS mechanism. However the data from DDIS and from diffractive direct photoproduction of jets should suffice to determine the main parton densities. Besides no assumption should be made on the validity of the momentum sum rule (14,16). We will follow here the program proposed in ref. (15) to measure the parton densities.

First we note that the pomeron, being an object with the quantum numbers of the vacuum, has $C = 1$ and is isoscalar. The former property implies that $f_{q/I\!P}(\beta) = f_{\bar{q}/I\!P}(\beta)$ for any quark q and the latter that $f_{u/I\!P}(\beta) = f_{d/I\!P}(\beta)$. Therefore it is necessary to determine only the up and strange quark densities and the gluon density. In the parton model the pomeron structure function in eq.(11) is,

$$F_2^{I\!P}(\beta, Q^2) = \frac{10}{9}\beta f_{u/I\!P}(\beta, Q^2) + \frac{2}{9}\beta f_{s/I\!P}(\beta, Q^2) + O(\alpha_s)\,, \tag{12}$$

where the gluon density contributes in the $O(\alpha_s)$ term through the DGLAP evolution. The gluon density may be directly measured by using data on jet production from DDIS or diffractive direct photoproduction, whose rate is,

$$\frac{d\sigma}{dt\, dx_{I\!P}}(e + p \to e + p + jet_1 + jet_2 + X) = \tag{13}$$
$$f_{P/I\!P}(x_{I\!P}, t) \sum_b \int d\frac{x_b}{x_{I\!P}}\, f_{b/I\!P}\left(\frac{x_b}{x_{I\!P}}, \mu\right) d\hat{\sigma}(e + b \to e + jet_1 + jet_2 + X)\,.$$

At the lowest order, $O(\alpha\alpha_s)$, the final state of the hard scattering, $jet_1 + jet_2 + X$ consists only of two partons, generated in quark-exchange and Compton-scattering diagrams for quark-initiated hard processes, and in photon-gluon fusion diagrams for the gluon-initiated ones. The parton momentum fraction in the proton, x_b, may be computed from the jet kinematic variables, $x_b = (E/P^0)\exp(2\bar{\eta})$, with E and P^0 the electron and proton energies and $\bar{\eta} = (\eta_{j_1} + \eta_{j_2})/2$ is the rapidity boost of the jet system. The pomeron momentum fraction in the proton, $x_{I\!P}$ may be obtained, as noted in sec.I A and I B, from the invariant mass of the system recoiling against the proton, $x_{I\!P} \simeq M^2_{ej_1j_2}/s$.

If we neglect the strange quark density these two sets of measurements suffice to measure the parton densities in the pomeron. From these one can test if the momentum sum rule, $\int_0^1 dx\, x f_{g/I\!P}(x) = 1$, is correct (24). The strange quark density may then be measured adding to the fit data on charged-current charm production in DDIS (15).

E. Conclusions

Single hard diffraction is a well established phenomenon in hadron-hadron (1,2) and lepton-hadron (3,5) collisions. Several theoretical models have been conceived which predict or explain these events. They range from models which describe the strong-interaction process just in perturbative terms (25,26), to models which rely heavily on soft-interaction modeling and Regge phenomenology (27). We have illustrated the IS model which is a mixture of soft- and hard-interaction physics.

The IS model yields a consistent description of the HERA data (3,5) on DDIS. The predictions for the Regge and Bjorken scaling for F_2^D are in agreement with the data at the present level of accuracy. The IS model relies on the factorization picture, eq.(3), however there may be factorization-breaking leading-twist contributions due to long-range initial-state interactions in diffractive hadron-hadron collisions (2,22), and due to final-state interactions in DDIS (23). These mechanisms should be more relevant as t grows (15,22). Thus, it is very important to measure the t dependence both in DDIS and in diffractive jet production at the Tevatron Collider.

In addition, it is questionable that a momentum sum rule for the parton densities $f_{i/I\!P}$ in the pomeron holds (14,16), since the pomeron is not an on-shell state. If we presume that factorization works only for DDIS and that the momentum sum rule does not apply, we still have enough information from DDIS and diffractive jet production in direct photoproduction to fit the parton densities $f_{i/I\!P}$ and check the momentum sum rule (15,24). In addition, the parton densities should not depend on t if factorization holds (cf. eq.(11)).

The parton densities $f_{i/I\!P}$ measured at HERA should be used to model jet and W-boson diffractive production in hadron-hadron collisions at the Tevatron Collider. If factorization breaks down, the predictions should disagree with the data.

II. DOUBLE HARD DIFFRACTION

A. Gap production in parton scattering

The initial theoretical motivation for examining two-jet production with a rapidity[c] gap in hadron production between the jets was $W-W$ boson scattering via Higgs-boson (or Z-boson) exchange at the SSC Collider (29–31). Unless the Higgs boson is rather heavy (32), $W-W$ boson fusion is not the

[c]Here and in the following we identify the true rapidity used in the theoretical models with the pseudo-rapidity used in the experiments. There is no difference between the two, as long as we deal with particles which are massless or for which $p_\perp \gg m$, but the difference must be kept in mind when dealing with the underlying event (28).

leading Higgs-boson production mode in hadron-hadron collisions, the gluon-gluon fusion channel being more important. However if the final-state W's decay leptonically the $W-W$ boson fusion channel has the unique signature of a rapidity gap in parton production between the quarks initiating the scattering. On the contrary, the gluons producing the Higgs boson would likely radiate more gluons, thus filling the lego plot with color. Still, the rapidity-gap signal could be faked by the scattering between color-singlet two-gluon ladders (31), and finally a rapidity gap at the parton level might not survive the spectator-parton interactions from the underlying event[d].

The simplest case in which one can start addressing these issues is two-jet production in hadron-hadron collisions with a rapidity gap in hadron production between the jets (31). In parton-parton scattering the leading-order process, which is $O(\alpha_s^2)$ and we may picture through one-gluon exchange in the $\hat{t}$ channel, is likely not to produce a gap because the exchanged gluon being a color octet radiates off more gluons (Fig.4a). However a gap may be produced by exchanging two gluons in the $\hat{t}$ channel in a color-singlet config-

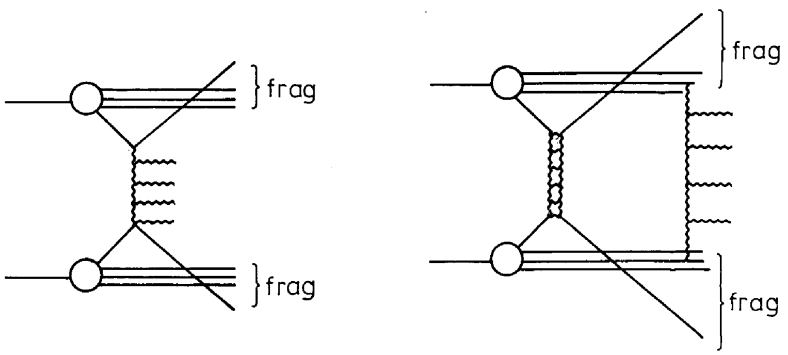

FIG. 4. (*a*) One-gluon and (*b*) two-gluon exchange in the $\hat{t}$ channel. In (*b*) also the soft-gluon emission from the rescattering between the spectator partons has been included.

uration (Fig.4b). This is a $O(\alpha_s^4)$ process, but it is accompanied by infrared logarithms due to the integration over the loop formed by the exchange of the two gluons. Bjorken (31) estimates that

$$\frac{\hat{\sigma}_{sing}}{\hat{\sigma}_{oct}} \sim 0.1\,. \tag{14}$$

The probability that the gap is due to an electroweak exchange is rather

[d] At the LHC Collider, which will operate at high luminosity, the additional problem of overlapping minimum bias events in a single bunch crossing arises. However this could be disposed of by requiring a gap in minijet production rather than in soft-hadron production (33).

small, $\hat{\sigma}_{\gamma,Z,W}/\hat{\sigma}_{sing} \sim 10^{-2}$ (31,34), and can be neglected. The radiation pattern for the emission of gluons has been examined in detail in ref. (35). It has been found that for parton-parton scattering with two-gluon exchange in a color-singlet configuration the gluon radiation is suppressed in the rapidity interval between the scattered partons, analogously to the suppression of gluon radiation due to color coherence in photon exchange in the $\hat{t}$ channel. Conversely, in one-gluon exchange the gluon radiation is found mainly in the central rapidity region.

B. The BFKL pomeron

In the limit of high squared parton c.m. energy $\hat{s}$ and fixed $\hat{t}$, we can describe the gap production at the parton level by using the Balitsky-Fadin-Kuraev-Lipatov (BFKL) model (13,36,37), which resums the leading logarithmic contributions, in $\ln(\hat{s}/\hat{t})$, to the scattering amplitudes to all orders in α_s. Therefore we may consider the exchange of a two-gluon ladder in color-octet or -singlet configurations and compute the leading $\ln(\hat{s}/\hat{t})$ virtual radiative corrections. For a color-octet ladder we obtain (36,38,39),

$$\frac{d\hat{\sigma}_{oct}}{d\hat{t}} \simeq \frac{\pi N_c^2 \alpha_s^2}{2\hat{t}^2} \exp\left(-\frac{N_c \alpha_s}{\pi} \ln\frac{\hat{s}}{\hat{t}} \ln\frac{p_\perp^2}{\mu^2}\right) , \tag{15}$$

where $N_c = 3$ is the number of colors, $p_\perp$ is the transverse momentum of the outgoing gluons, with $\hat{t} \simeq -p_\perp^2$, and μ is a cutoff which regulates the infrared divergence. Eq.(15) defines the scattering as elastic if no soft gluons with $p_\perp \gtrsim \mu$ appear in the final state. The exponential of eq.(15) has the typical form of a Sudakov form factor, and it vanishes as $\mu \to 0$, in agreement with the Bloch-Nordsieck behavior for bremsstrahlung emissions. In addition, the exponential becomes smaller as the rapidity interval between the partons $\eta \simeq \ln(\hat{s}/\hat{t})$ grows, thus as we expected it is very unlikely to produce a large gap between the partons through one-gluon exchange.

In the BFKL model the solution for the exchange of a color-singlet two-gluon ladder is known only at $\hat{t} = 0$ (13), or at $\hat{t} \neq 0$ for the scattering between colorless objects (37). Mueller and Tang (38) have modified the solution of ref. (37) in order to describe the parton-parton elastic scattering at $\hat{t} \neq 0$. Thus the elastic cross section for gluon-gluon scattering is (38,39),

$$\frac{d\hat{\sigma}_{sing}}{d\hat{t}} \simeq \frac{\pi^3 N_c^4 \alpha_s^4}{4\hat{t}^2} \frac{\exp\left(8 \ln 2 \frac{N_c \alpha_s}{\pi} \ln\frac{\hat{s}}{\hat{t}}\right)}{\left(\frac{7}{2}\zeta(3) N_c \alpha_s \ln\frac{\hat{s}}{\hat{t}}\right)^3} . \tag{16}$$

Note that in the high-energy limit the singlet solution, eq.(16), does not depend on the infrared cutoff μ (38). In addition, the probability to produce a gap grows with the gap width, thus even though higher-order the singlet

solution quickly becomes more important than the octet solution as the gap width grows.

Summing eq.(15) and (16), and neglecting the ensuing double counting which is not important at the large rapidities at which the BFKL approximation applies, we obtain the probability of producing a gap in gluon-gluon scattering. If $\mu \gg \lambda_{QCD}$ we obtain the jet production rate with a gap in hadron production between the jets by convoluting the sum of eq.(15) and (16) with the parton densities (38,39). This is legal as long as $\mu \gg \lambda_{QCD}$ because the emission of soft hadrons in the rescattering between spectator partons in the underlying event is allowed, in agreement with the factorization theorems (21).

We compute the jet production rate as function of the rapidity difference, $\Delta\eta = \eta_{j_1} - \eta_{j_2}$, and the rapidity boost $\bar{\eta}$ (cf. sec. I D), since the elastic parton-parton scattering does not depend on $\bar{\eta}$. Thus $\bar{\eta}$ may be fixed or integrated out, thereby introducing a contribution due only to the variation of the parton densities (40). It is then convenient to compute the gap fraction (39), i.e. to normalize the two-jet production with a gap between the jets to the inclusive two-jet production[e] (40) in order to minimize the normalization errors due to using the BFKL approximation. Thus the gap fraction is,

$$\hat{f}(\mu \gg \lambda_{QCD}) = \hat{f}_{sing} + \hat{f}_{oct} \,, \tag{17}$$

with $\hat{f}_{sing(oct)} = \hat{\sigma}_{sing(oct)}/\hat{\sigma}_{incl}$. In ref. (39) the prediction for the gap fraction at Tevatron energies as a function of the gap width shows an abrupt rise of the gap fraction at the largest gap widths kinematically allowed. However much of it is not due to the growth of the singlet contribution in the parton dynamics, eq.(16), but merely to the parton luminosity, which as $x \longrightarrow 1$ falls off faster for the inclusive two-jet production than for the elastic one. This kinematic phenomenon is exactly the reverse of the one noted for the K-factor in inclusive two-jet production in ref. (40,42). Therefore we conclude that at Tevatron energies within the approximation of ref. (39) the gap fraction at large gap widths is basically flat.

In comparing the prediction of ref. (39) with the experimental results (7,8) a caveat is in order. The experiments measure the gap width, $\Delta\eta_{gap}$, between the edges of the jet cones on the lego plot, which differs from the rapidity difference, $\Delta\eta$, between the outgoing partons originating the jets by the cone sizes R, i.e. $\Delta\eta_{gap} = \Delta\eta - 2R$. The cone size used in the experiments (7–9) is $R = 0.7$. Within the BFKL approximation we cannot distinguish between $\Delta\eta_{gap}$ and $\Delta\eta \simeq \ln(\hat{s}/\hat{t})$ since in a leading logarithmic approximation the jets are point-like.

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[e] In ref. (39,40) the jets are ranked by their rapidity, i.e. the two jets with the largest and smallest rapidity are tagged, and the distribution is observed as a function of these two tagging jets. Instead in the experiments (7–9) the jets are ranked by their transverse energies. However, there are preliminary indications, at least in gap production in photoproduction events (41), that the gap fraction does not change substantially ranking the jets by their rapidity or by their transverse energies.

In order to examine the gap fraction as a function of the gap width between the jet-cone edges, while accounting properly for the cone structures, it is necessary to perform a next-to-leading order calculation which includes the basic features of color-singlet exchange. The simplest calculation of this sort would be a full calculation of jet production at $O(\alpha_s^4)$. At the moment this is in general unfeasible because one needs to know the 4-parton 2-loop matrix elements, which have not been computed yet. However the gap fraction, i.e. the ratio of the elastic to the inclusive two-jet production, may be computed subtracting out from the unity the ratio of the inelastic to the inclusive two-jet production, for which at $O(\alpha_s^4)$ the 5-parton 1-loop matrix elements (43), and the 6-parton tree-level matrix elements, suffice (44). The jet production with a gap in rapidity would then be computed by requiring that any extra partons besides the ones we tag on be emitted within the jet cones. Therefore a distinction between octet and singlet contributions would not be done. In addition, the calculation would be infrared stable.

C. Gaps in soft-hadron production

In sec. II B we have required that the threshold μ in soft-hadron production with respect to which we define the jet production as elastic satisfy $\mu \gg \lambda_{QCD}$. Lowering the threshold to $\mu \simeq \lambda_{QCD}$, as suggested by Bjorken (31) and done in the experiments (7–9), the factorization picture of ref. (21) does not apply, and we need a non-perturbative model that lets the gap formed at the parton level survive the rescattering between the spectator partons in the underlying event, which would otherwise fill the gap with soft hadrons. Using an eikonal model, Bjorken (31) estimates the rapidity-gap survival probability, $< |S^2| >$, to be about 5-10%. A study of several phenomenological models has also been done in ref. (45). In a first approximation we can then assume that the fraction of two-jet events with a gap in soft-hadron production is (31),

$$f(\mu \to \lambda_{QCD}) \simeq < |S^2| > \hat{f}(\mu \to \lambda_{QCD}) \,, \tag{18}$$

with $\hat{f}$ as given in eq.(17). The survival probability, $< |S^2| >$, is expected to decrease as the hadron-hadron c.m. energy $\sqrt{s}$ increases (31,45). Indeed the total cross section, σ_{tot}, is related to the area of the soft interactions, πR^2, and to the unitarity bound by the relation, $\sigma_{tot} \simeq \pi R^2 \propto \ln s^2$. Thus as s increases it is less and less likely that the two hadrons do not interact. Then $< |S^2| >$ is expected to be roughly independent of the gap width, $\Delta\eta_{gap}$, since the rapidity interval between the jets $\Delta\eta$ is a kinematic parameter of the hard-interaction process, which according to eq.(18) would factor out of the soft interactions. Finally, $< |S^2| >$ is expected to grow as the momentum fraction x of the incoming partons goes to 1, because there is less and less energy available for the underlying event, i.e. for the spectator partons, in analogy with the suppression of the underlying event observed in photoproduction events as $x \to 1$ (46).

In gap production in photoproduction events at HERA, the smaller c.m. energy $\sqrt{s}$, the smaller radius of the resolved photon as compared to the proton, and the greater stiffness of the parton densities in the photon as compared to the ones in the proton, all conspire to make the value of $< |S^2| >$ larger. This might explain the higher value of the gap fraction in the HERA data (9), as compared to the one in the Tevatron data (7,8).

Since the theoretical calculation of $< |S^2| >$ is not very firm, it would be better to measure it. In eq.(18), $< |S^2| >$ appears tangled to the gap fraction at the parton level, thus a single measurement cannot give any information on $< |S^2| >$. However, if we raise the threshold μ in such a way as to saturate the soft-gluon emission from the underlying event as done in eq.(17) then,

$$f(\mu \gg \lambda_{QCD}) \simeq \hat{f}(\mu \gg \lambda_{QCD}) \,. \tag{19}$$

Subtracting the octet contribution (8), the gap fraction of eq.(18) becomes,

$$(f - f_{oct})(\mu \rightarrow \lambda_{QCD}) \simeq < |S^2| > (\hat{f} - \hat{f}_{oct})(\mu \rightarrow \lambda_{QCD}) \,, \tag{20}$$

Subtracting out the octet contribution from eq.(19) and using the insensitivity of the singlet contribution to the threshold μ we obtain,

$$(f - f_{oct})(\mu \gg \lambda_{QCD}) \simeq (\hat{f} - \hat{f}_{oct})(\mu \gg \lambda_{QCD}) = (\hat{f} - \hat{f}_{oct})(\mu \rightarrow \lambda_{QCD}) \,. \tag{21}$$

Thus taking the ratio of measurements of the gap fraction according to the prescription of eq.(20) and (21) would allow us to determine $< |S^2| >$.

D. Conclusions

The existence of rapidity gaps between jets in hadron-hadron (7,8) and photon-hadron (9) collisions seems well established. The high value of $|t|$ in these events suggests that short-distance strong-interaction physics must play a vital role in them. Therefore these events may be interpreted as evidence for a perturbative color-singlet exchange.

Bjorken's predictions, based on the simplest one-gluon and two-gluon exchange and the survival of the rapidity gap at the soft-hadron level, have been essentially confirmed by the data. Also the BFKL-pomeron picture of ref. (38,39), which considers the radiative corrections to the lowest-order one-gluon and two-gluon exchange, is in agreement with the data, however the distinctive feature of the BFKL pomeron, i.e. the growth of the gap fraction at very large gap widths, is not to be seen at the Tevatron.

There is large room for improvement of the model; on the theoretical side we should make a more detailed model of color-singlet exchange at the parton level, to keep into account the structure of the jets (cf. sec. II B); on the experimental side the non-perturbative features of the gap production, like the rapidity-gap survival probability (cf. sec. II C), should be measured.

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